Chapter

# Transfer of Heat through a Thin Liquid Film

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## Abstract

The present work discusses heat transfer in thin liquid films past a stretching surface in the presence of Joule heating, radiation, viscous dissipation, and magnetic effects. Using a suitable similarity transformation, the nonlinear coupled partial differential equations are reduced to nonlinear ordinary differential equations. The coupled nonlinear ordinary differential equations are then solved by using the shooting method. The physical quantities, such as the unsteadiness parameter, the Prandtl number, the magnetic field parameter, the radiation parameter, and the viscous dissipation parameter on temperature distributions, are depicted graphically. It is observed that the effects of Prandtl and Eckert numbers are same over the temperature distribution. Furthermore, it is also evident that the increasing values of the unsteadiness parameter enhance the thermal conductivity of the fluid.


**Keywords:** thin film, viscous dissipation, joule heating, radiation effect, Rosseland number

## 1. Introduction

Due to industrial uses, researchers have developed a strong interest in boundary-layer flow and heat transmission in thin liquid films on stretching surfaces. Understanding heat transfer inside a thin liquid film is critical in engineering for operations, such as wire and fiber coating, food processing, cooling metallic plates, reactor fluidization, drawing polymer sheets, and aerodynamic extrusion of plastic sheets. This knowledge is critical during the extrusion process to ensure the extrudate's surface quality remains consistent. By minimizing friction, transparency, and strength, a smooth surface enhances the aesthetic and performance of the product [1]. The flow and heat transfer of laminar thin films across an unstable stretched sheet with a changing transverse magnetic field is investigated. The graph shows how various parameters affect the temperature distribution. Listed below are some of the key findings from our graphic research. Sakiadas' research began with an investigation of MHD flow's boundary-layer behavior on a stretching surface [2]. Wang [3] made significant contributions to the study of thin-film flow across a stretched sheet. Additionally, Andersson et al. [4, 5] improved Wang's work by including a power-law fluid with changeable physical characteristics. Wang and Pop [6] later devised a homotopy analytic approach for analyzing the heat transfer features of a power-law fluid's liquid film flow.

Khan et al. [7] showed that when the thin film parameter is increased, the temperature profiles get lower and the concentration profiles get higher. They did this while also looking at the thermal conductivity and viscosity of fluids moving through a stretching sheet. Pop et al. [8] used computational methods to explore the effect of injection and suction on the flow of a continuous laminar gravity-driven film down a vertical wall, emphasizing the importance of mass transfer and the Prandtl number. According to Khan et al. [9], a second-grade fluid flowing through a porous medium through a stretched sheet exhibits thin-film flow and heat transfer.

Sun et al. [10] investigated the thin-film flow over a rotating disc subjected to heat source and nonlinear radiation and found that increase in the rotation parameter result in a decrease in the azimuthal component of velocity, whereas increase in the temperature function of the radiation parameter. Ma et al. [11] studied how liquids flow and evaporate in a rocket combustion chamber with high temperatures and high shear forces. He investigated the flow and evaporation of a liquid film in a rocket combustion chamber with high temperatures and high shear forces. Khan et al. [12] explored Darcy–Forchheimer laminar thin-film flow with MHD and the investigation of heat transfer on an unstable horizontal stretched surface. Thermal radiation and viscous dissipation effects on thin-film flow are also examined. Using boundary-layer flow, you can look at a source of heat that emits thermal radiation. This could be very useful in industrial engineering operations like electric power generation and solar energy modernization and in astrophysical flow. Shahzad et al. [13] explored thin-film flow for radially expanding surfaces using magnetic effects and viscous dissipation for the first time. They analyzed that as the magnetic and unsteadiness parameters are raised, the film thickness decreases.

To the best of the author's knowledge, the provided facts on thermal analysis have not been investigated previously for the radially stretching surface by taking in account the effects of Joule heating and radiation effects. Section 2 detailed the governing coupled PDEs, which were later changed into coupled nonlinear ODEs by suitable transformations; Section 3 explained the approach used to solve the problem; Section 4 summarised the findings and discussed them, while Section 5 expanded on the final remarks.

## 2. Physical problem and its mathematical form

If an incompressible fluid flows through a flat plate with a uniform temperature $T_w$ that is different from the ambient temperature $T_\infty$, then we have magnetohydrodynamic boundary-layer flow. **Figure 1** depicts fluid flow in a simplified manner.

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = \upsilon\frac{\partial^2 u}{\partial z^2} + B(t)^2 u, \tag{2}$$

$$\begin{aligned} &\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + +w\frac{\partial T}{\partial z} = \\ &\alpha\,\frac{\partial^2 T}{\partial z^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial z}\right)^2 + \frac{1}{\sigma\rho C_P}B(t)^2u^2 - \frac{1}{\sigma\rho C_p}\frac{\partial}{\partial z}q_r. \end{aligned} \tag{3}$$

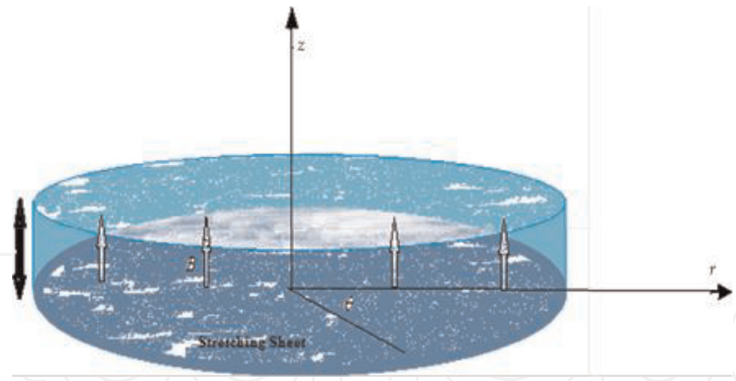


**Figure 1.**
*Schematic diagram of fluid flow*

We obtain $q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$, using the Rosseland approximation for radiation, where $k^*$ and $\sigma^*$ denote the absorption coefficient and Stefan Boltzmann constant, respectively. By expanding $T^4$ in a Taylor's series in the vicinity of $T_\infty$ and neglecting higher-order terms, we get

$$T^4 = 4{T_\infty}^3 T - 3{T_\infty}^4.$$

So Eq. (3) can be rewritten as

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + +w\frac{\partial T}{\partial z} = \left(\alpha + \frac{1}{\rho C_p}\frac{16\sigma^*}{3k^*}T_\infty^3\right)\frac{\partial^2 T}{\partial z^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial z}\right)^2 + \frac{\sigma}{\rho C_P}B(t)^2 u^2, \quad (4)$$

where $u(x,y)$ and $v(x,y)$ are velocities; $T$ represents the temperature. Also, $\nu$ and $\alpha$, respectively, are the thermal diffusivity and kinematics viscosity of the fluid. The boundary conditions may be expressed as

$$u = U, w = 0, T = T_s, \text{at } z = 0, \frac{\partial u}{\partial z} = \frac{\partial T}{\partial z} = 0, w = \frac{dh}{dt}, \text{at } z = h, \quad (5)$$

An acceptable stream function $\psi = \psi(r,z)$ satisfying continuity Eq. (1) identically, with similarity transformations, so that the flow equations are translated to nonlinear ODEs.

$$\eta = \frac{z}{r}Re^{\frac{1}{2}}, \psi = -r^2 U Re^{-\frac{1}{2}} f(\eta), \text{and } \theta(\eta) = \frac{T_0 - T(r,z)}{T_s - T_0}, \quad (6)$$

where $\eta$ is the independent variable, $\mathrm{Re} = \frac{rU}{\upsilon}$ is the local Reynold number, and $\psi(r,z)$ is Stokes stream function such that $u = \left[-\frac{1}{r}\left(\frac{\partial\psi}{\partial z}\right)\right]$ and $w = \left[\frac{1}{r}\left(\frac{\partial\psi}{\partial r}\right)\right]$. Eqs. (2)–(4) becomes after using the aforementioned modification.

$$f'''(\eta) + 2f(\eta)f''(\eta) - S\left(f'(\eta) + \frac{\eta}{2}f''(\eta)\right) - (f'(\eta))^2 - Mf'(\eta) = 0, \quad (7)$$

$$(1 + Rd)\theta''(\eta) = Pr\left(\frac{S}{2}(3\theta(\eta) + \eta\theta'(\eta)) + 2f'(\eta)\theta(\eta) - 2\theta'(\eta)f(\eta) + 2Ec(f''(\eta))^2 + EcMf'^2\right), \quad (8)$$

$$f'(0) = 1, f(0) = 0, \theta(0) = 1, f'(\beta) = 0, \theta'(\beta) = 0, f(\beta) = \frac{S\beta}{2}, \tag{9}$$

where $Pr = \frac{\nu}{\alpha}$ and $Ec = \frac{U^2}{C_{p(T_w - T_\infty)}}$ are dimensionless Prandtl and Eckert numbers. $S = \alpha/b$ is the measure of unsteadiness parameter and $\beta$ (unknown constant) is the dimensionless thickness of the film and is given as $\beta = (b/\upsilon(1 - \alpha t))(1/2)\, h$. The rate at which film thickness varies can be obtained as $dh/dt = (-\alpha\beta/2)(\upsilon/b(1 - \alpha t))(1/2)$. The important physical quantity of interest in this problem is Nusselt number $Nu_r = (-r/Tref)\left(\frac{\partial T}{\partial Z}\right)_{Z=0}$. In non-dimensional form, it can be expressed as

$$2(1 - \alpha t)^{1/2} Re^{-3/2} Nur = -(1 + Rd)\theta'(0).$$

The rate of heat transfer is represented by the quantity of physical interest $-(1 + Rd)\theta'(0)$. As a result, our goal is to figure out how the controlling factors $Pr, Ec, M,$ and $n$ affect these values.

## 3. Solution methodology

The system of nonlinear PDEs is given in Eqs. (2) and (4) subject to given boundary conditions given by Eq. (5) are transformed to ODEs via similarity transformation and solved via the shooting method. The following are the third-order differential equations for the second-order nonlinear ODE and the second-order nonlinear ordinary differential for first order.

$$f = y(1), f' = y(2), f'' = y(3), f''' = yy1, \tag{10}$$

$$yy1 = S\left(y(2) + \frac{\eta}{2}y(3)\right) + y(2)^2 - 2y(1)y(3) + My(2) \tag{11}$$

$$\theta = y(4), \theta' = y(5), \theta'' = yy2, \tag{12}$$

$$yy2 = \frac{Pr}{(1 + Rd)}\left(\frac{S}{2}(3y(4) + \eta y(5)) + 2y(4)y(2) - 2y(1)y(5) + 2Ecy(3)^2 + EcMy(2)^2\right) \tag{13}$$

The iterative procedure will end with the appropriate level of precision.

## 4. Result and discussion

This work fundamentally alters our understanding of thin films. Unsteadiness, magnetic parameters, Eckert number, Prandtl number, and radiation parameters are the regulating parameters. Numerical simulation is utilized in conjunction with analytical solutions of nonlinear differential equations. Additionally, dimensionless parameters were investigated in axisymmetric MHD thin liquid film only for heat transfer past the stretching surface.

### 4.1 Temperature profile influenced by unsteadiness parameters

According to **Figure 2**, a higher unsteadiness parameter (S) results in a decrease in temperature, which in turn enhances heat transmission in thin liquid films.

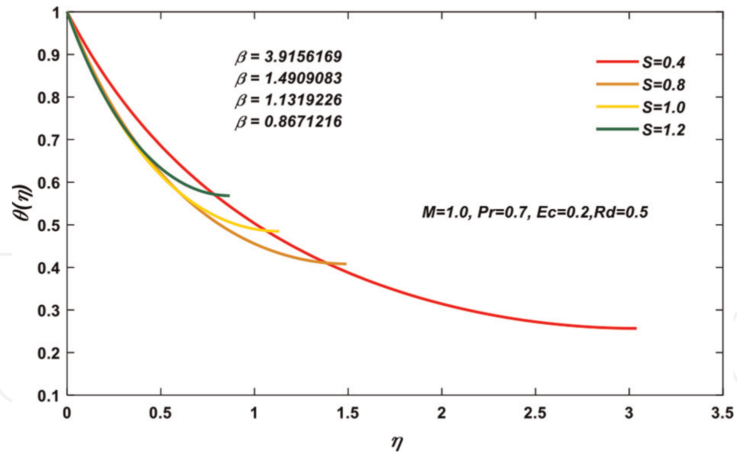


**Figure 2.**
*Illustration of S on the θ(η).*

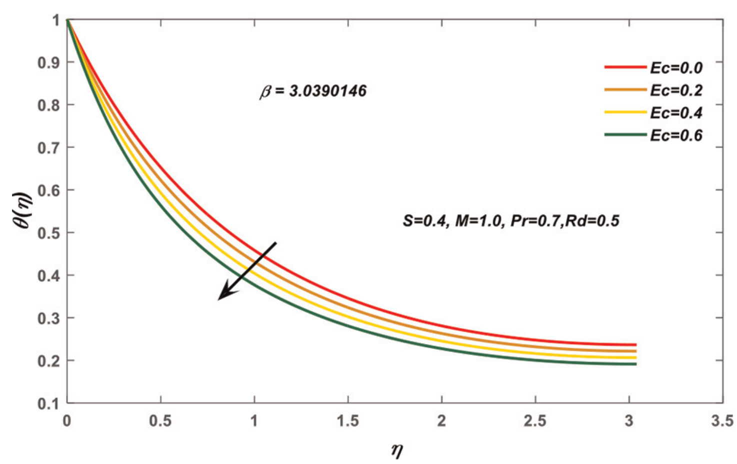


**Figure 3.**
*Illustration of Ec on θ(η).*

### 4.2 Temperature profile influenced by Eckert number

The temperature distribution for various Eckert numbers is depicted in **Figure 3**. This physical characteristic characterizes the relationship between kinetic and internal energy.

### 4.3 Temperature profile influenced by Prandtl number

The Prandtl number presents the relationship between momentum and thermal diffusivity. When the Prandtl number is decreased, the thickness of the boundary-layer drops, further restricting heat transfer. The Prandtl number is plotted against the temperature distribution in **Figure 4**. The thermal boundary layer was decreased as a result of the enhanced dissipation.

### 4.4 Temperature profile influenced by radiation parameter

The influence of the radiation parameter $Rd$ on the temperature profile for the unsteadiness parameter $S$ is seen in **Figure 5**. It is clearly demonstrated that radiation

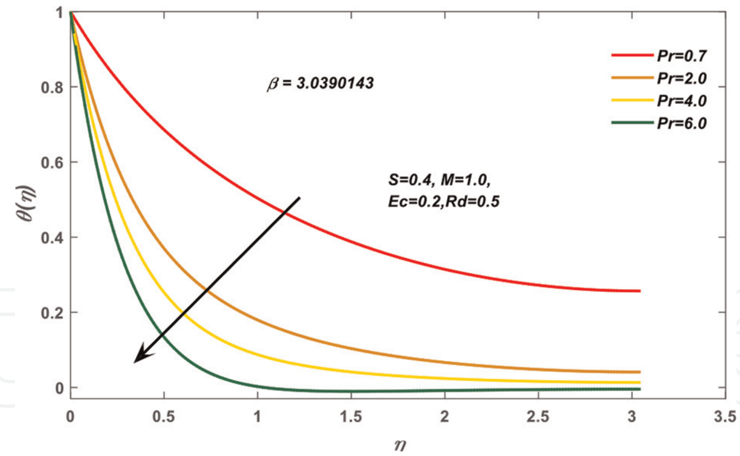


**Figure 4.**
*Illustration of Pr on* $\theta(\eta)$.

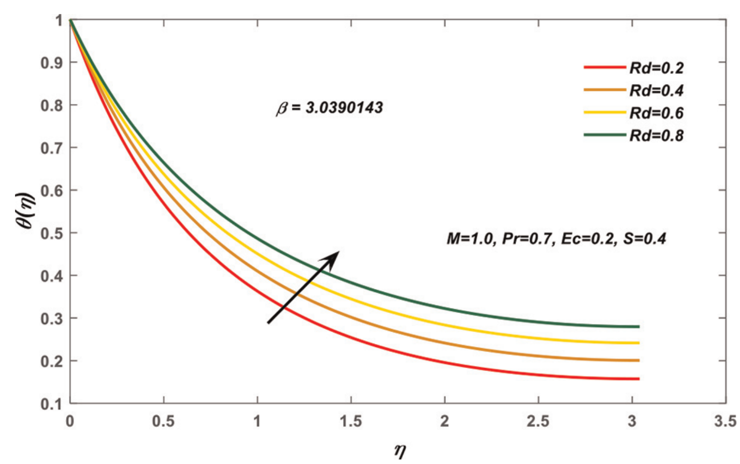


**Figure 5.**
*Illustration of Rd on* $\theta(\eta)$.

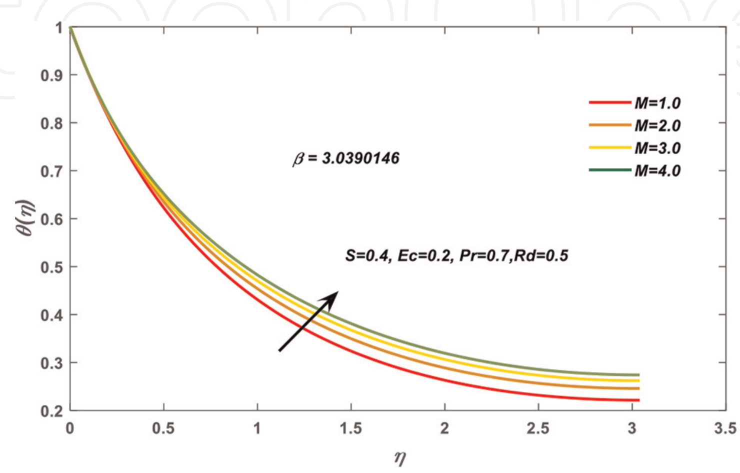


**Figure 6.**
*Illustration of M on* $\theta(\eta)$.

| $M$ | $S$ | Pr | Ec | $-\theta'(0)$ | |
|---|---|---|---|---|---|
| | | | | **[13]** | **Present** |
| 1 | 0.8 | 0.7 | 0.2 | 1.4105271 | 1.528545 |
| | 1.0 | | | 1.4200617 | 1.5319487 |
| | 1.2 | | | 1.3936895 | 1.4958518 |
| 1 | 0.4 | 0.7 | 0.2 | 1.3214547 | 1.4443904 |
| | | 4 | | 3.81002363 | 4.3083451 |
| | | 6 | | 4.7918785 | 5.4647752 |
| 1 | 0.4 | 0.7 | 0.0 | 1.2314224 | 1.2314194 |
| 1 | | | 0.2 | 1.3214547 | 1.3214547 |
| 1 | | | 0.4 | 1.4114869 | 1.6573614 |

**Table 1**
*Effect of pertinent parameters on heat transfer rate.*

raises the temperature in the fluid film's boundary- layer region, lowering the cooling rate for thin-film flow.

### 4.5 Temperature profile influenced by magnetic parameter

For a fixed unsteadiness parameter S, the temperature distribution is shown in **Figure 6** in relation to the magnetic field value M. As M rises, so does the thickness of the thermal boundary layer. As a result of the Lorentz force, the thermal boundary layer becomes thicker because frictional drag increases. In turn, this causes an increase in the thermal boundary-layer thickness. The purpose of **Table 1** is to examine the influence of several important factors on the dimensionless heat transfer rate at the surface. The Prandtl number and the unsteadiness parameter have an effect on the rate of heat transfer at the surface.

## 5. Conclusion

The flow and heat transfer through a stretching sheet with a variable transverse magnetic field is studied in this work. The shooting strategy is used to solve the governing equations numerically. The temperature distribution is shown in the graph below as a function of several parameters. In summary, the following are some of the most significant discoveries from our graphical investigation:

i. Enhance the thermal boundary layer, decrease fluid particle movement, and increase the temperature field when a magnetic field is applied to an electrically conducting fluid.

ii. The temperature decreases for the increasing values of Prandtl and Eckert numbers, i.e., both have the same effect.

iii. The influence of radiation parameter Rd. rises in thermal boundary- layer thickness, which enhances heat transmission.

## Nomenclature

| | |
|---|---|
| $(r, \theta, z)$ | polar coordinate |
| $\alpha = \left(\frac{k}{\rho c_p}\right)$ | thermal diffusivity |
| $(u, v, w)$ | velocity component |
| $\beta(t)$ | applied magnetic field |
| $T$ | fluid's Temperature |
| $T_{ref}$ | reference temperature |
| $c_p$ | specific heat |
| $P$ | pressure |
| $\psi(r, z)$ | Stokes stream function |
| PDF | Partial differential equations |
| $\sigma$ | electrical conductivity |
| $\rho$ | density of fluid |
| $Re$ | Local Reynolds number |
| $Cf$ | Local skin-friction coefficient |
| $Nu$ | Nusselt number |
| $Pr$ | Prandtl number |
| $Ec$ | Eckert-number |
| $\beta$ | thickness of the film |
| $\upsilon$ | kinematic viscosity |
| ODE | Ordinary differential equations |

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