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TRANSFORMATION FORMULAS RELATING GEODETIC COORDINATES TO A TANGENT TO EARTH, PLANE COORDINATE SYSTEM

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BY

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SUMMARY

In order to simulate an air traffic terminal area with dynamic deterministic traffic, the aircraft's body centered axes must be related to the geodetic site of navigation aides, runways, and other fixes for on-board navigation. In addition, the aircraft locations are shown as perceived by an airport radar. The approach used is to map geodetic positions to an earth tangent plane with an airport centered rectangular coordinate system. This report develops both the transformation equations and their approximations. The TCV program uses a different set of approximations to relate MLS derived aircraft positions to geodetic values for navigation. A short error analysis in appendix B shows the TCV approximations to be sufficiently close to the exact analytical expression for precision MLS navigation use.

INTRODUCTION

A Terminal Area Air Traffic Model (TAATM) was developed in conjunction with NASA's Terminal Configured Vehicle (TCV) Program. The principal objective of the TCV program is to increase the productivity of the aircraft/airport/air traffic control (ATC) system with primary emphasis on the airborne elements of the system. The TAATM provides a representative real-time, multiple aircraft environment for both the TCV simulator cockpit and the TCV B-737 aircraft to interact with in the conduct of a wide range of experiments. In addition a fast time version of TAATM exists for computer batch processing.

When simulating aircraft traffic within the confines of a typical terminal area airspace, the requirement of a simple coordinate system quickly manifests itself. One must relate the geodetic location of runways, navigation sites, fixes, etc. to the aircraft body-centered axis for navigation purposes and in addition show aircraft positions as would be perceived by an airport radar. This problem is simplified by chosing an airport centered rectangular coordinate system with an assumed flat earth surface. Though reducing the complexity of the navigation problem, this approach still requires the mapping of geodetic positions to an earth tangent plane. The algorithms necessary for the mapping are the subject of the following discussion.

SYMBOLS AND ABBREVIATIONS

a	semimajor or equatorial radius of the earth's ellipsoid
A/C	aircraft
b	semiminor or polar radius of the earth's ellipsoid
e	eccentricity of the earth's ellipsoid
f	flattening of the earth's spheroid
h	height above mean sea level
h_0	height above mean sea level of the rectangular coordinate **origin** at the airport
R_L	local or Gaussian radius of curvature
R_L'	$R_L + h_0$
R_M	radius of the circle defining north-south curvature of the earth's ellipsoid at a specified latitude
R_M'	$R_M' + h_0$
R_p	radius of the circle use to define east-west curvature of the earth's ellipsoid at a specified latitude
R_p'	$R_p' + h_0$
MLS	Microwave Landing System
MSL	mean sea level
TAATM	Terminal Area Air Traffic Model
TCV	Terminal Configured Vehicle
ϕ	geodetic or geographic latitude

ϕ'	geocentric latitude
ϕ_0	reference latitude of TAATM coordinate orgin
$\Delta\phi$	change in latitude about the reference latitude
λ_0	reference longitude of TAATM coordinate origin
$\Delta\lambda$	change in latitude from the reference longitude

DEVELOPMENT OF TRANSFORMATIONS

Description of Earth's Ellipsoid Surface

For navigation purposes the earth's sea level surface can be approximated by an ellipsoid of revolution around its spin or minor axis which is the diameter joining the north and south pole. The geometry of the ellipsoid is defined by an ellipse whose semimajor axis is the equatorial radius a and whose semiminor axis is the polar radius b, as shown in figure 1. The eccentricity of the elliptic section is defined by

$$e^2 = \frac{a^2 - b^2}{a^2} \tag{1}$$

and the oblateness or flattening of the spheroid as

$$f = \frac{a - b}{a} \tag{2}$$

The relation between e and f is

$$e^2 = f(2 - f) \tag{3}$$

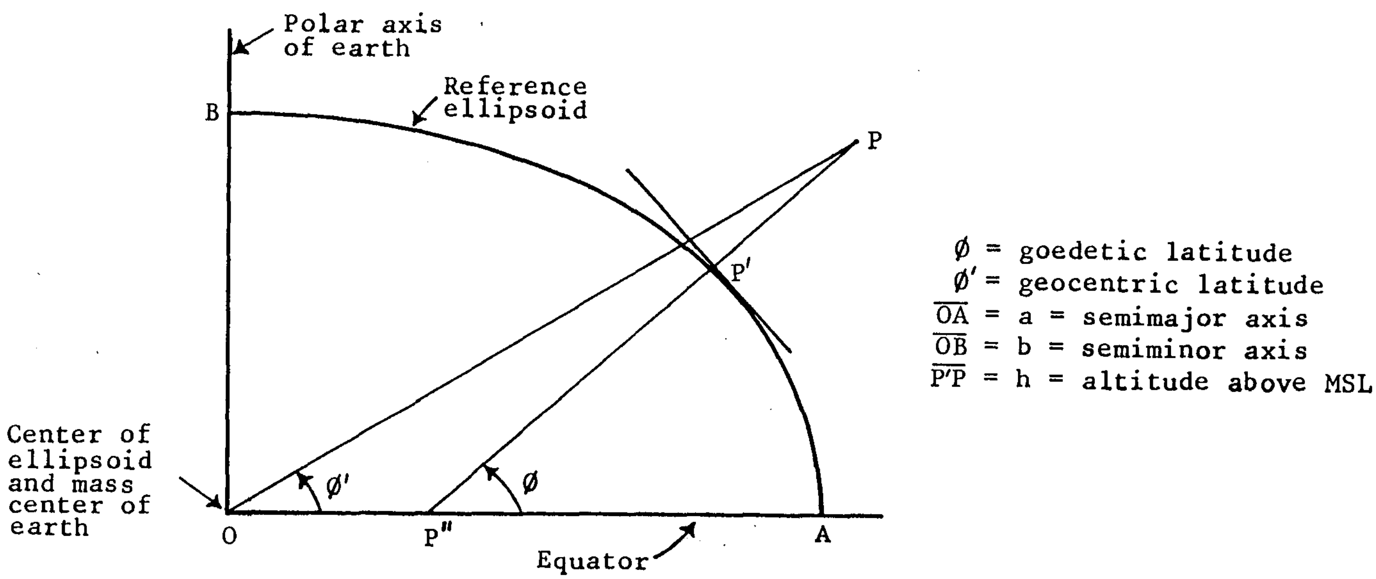


Figure 1. - Geocentric and geodetic latitude.

There have been several reference or standard ellipsoids used. Table I list some of these:

Description	a (meters)	f
Adopted by TCV for MLS Work	6,378,169.79	3.3901×10^{-3}
FAA Route Development - US Army Engineers	6,378,206.40	3.3901×10^{-3}
IAU - 1952	6,378,388	$\frac{1}{297}$
NASA Values for Trajectory Calculations (NASA SP-7)	6,378,166$\pm$25	$\frac{1}{298.3}$
IASU - 1965	6,378,160	$\frac{1}{298.25}$

TABLE 1. Reference Ellipsoid Approximations of Geoid

From Figure 1 we see the geodetic (also called geographic or map) latitude ϕ of a point P is the angle between the normal to the ellipsoid and the equatorial plane. For navigation purposes the radii of curvature of the ellipsoid are of primary importance. The meridian radius R_M is the radius of the circle of best north-south curvature fit to the meridian section of the ellipsoid at geodetic latitude ϕ. Its utility comes from the fact that a given relatively small north-south distance subtends approximately the same increment of latitude on a circle of radius R_M as that north-south distance does on the geodetic ellipsoid at the reference latitude ϕ. The value R_M and its approximation at some latitude ϕ is determined in appendix A to be

$$R_M = \frac{a(1 - e^2)}{(1 - e^2 \sin^2\phi)^{3/2}} \tag{4}$$

and

$$R_M \approx a[1 + e^2 (3/2 \sin^2\phi - 1)] \tag{5}$$

The prime radius R_p, used to define the east-west curvature, is the radius of a circle model of the meridian section such that at latitude ϕ its transverse circle of revolution around the polar axis is equal to that of the geodetic ellipsoid at latitude ϕ. From Appendix A we can write

$$R_p = \frac{a}{(1-e^2 \sin^2\phi)^{1/2}} \tag{6}$$

and

$$R_p \approx a[1 + \frac{e^2}{2} \sin^2\phi] \tag{7}$$

The local or Gaussian radius of curvature R_L, is the radius of the best fitting sphere to the ellipsoid at a local point. From Appendix A we get

$$R_L = \frac{a\ (1 - e^2)^{1/2}}{1 - e^2 \sin^2\phi} \tag{8}$$

and

$$R_L \approx a[1 - \frac{e^2}{2} \cos 2\phi] \tag{9}$$

<u>Transformation from an Earth Tangent Plane, Actual Aircraft Projection to Geodetic Coordinates</u>

Let the TAATM origin at the airport be at reference latitude ϕ_0, longitude λ_0 and MSL altitude h_0. The TAATM axis system is X-North, Y-East and Z down from the XY plane. From Figure 2, the point (x' , y' , z' = 0) on the flat plane through which a line passes from the x, y, z position to the local earth vertical is determined as follows:

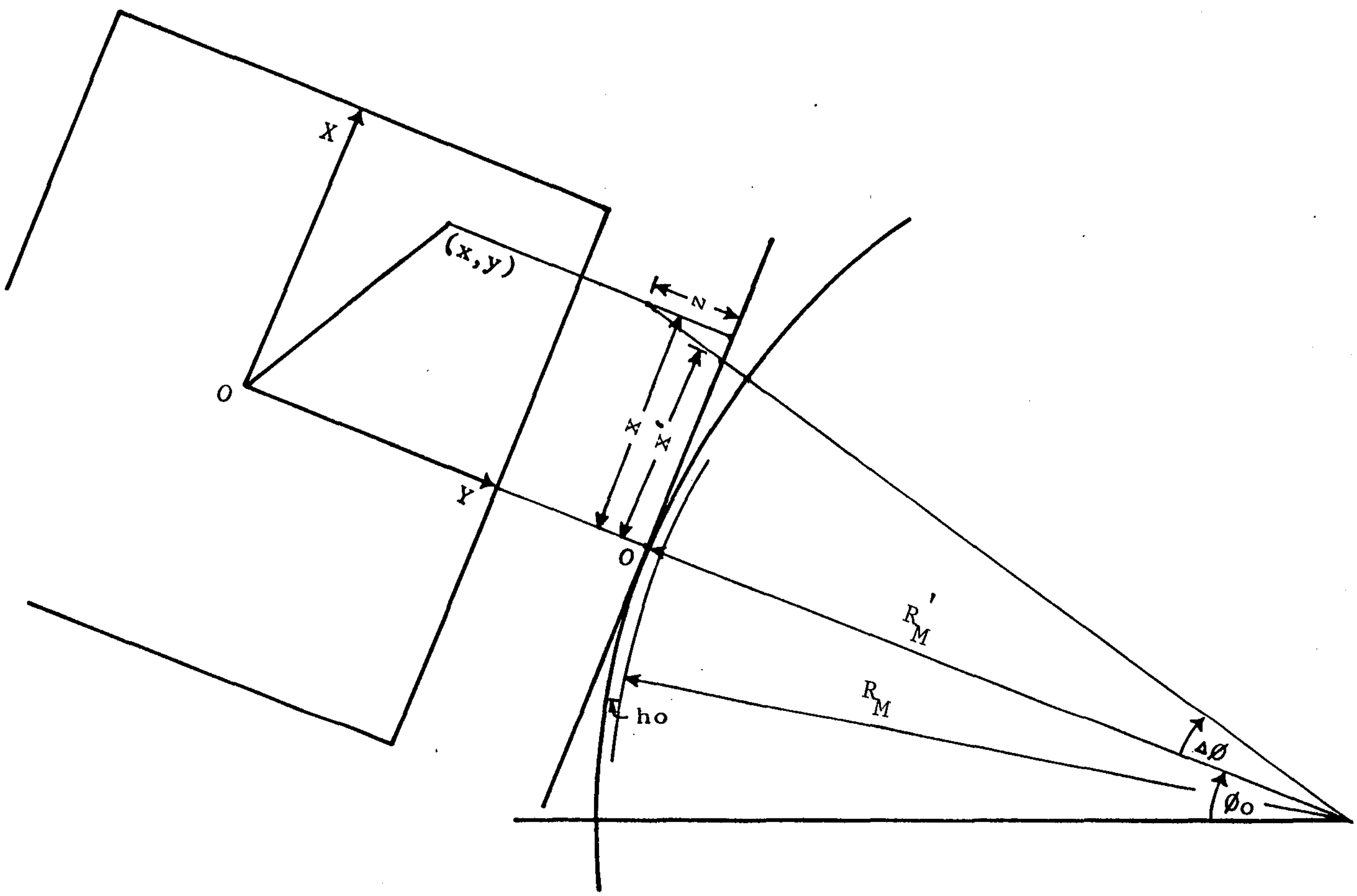


Figure 2. - Relation of TAATM tangent plane to earth coordinates.

Define $R'_M \triangleq R_M + h_0$

then

$$\frac{z}{R'_M} = \frac{x - x'}{x'}$$

or

$$x' = x\left[\frac{1}{1+z/R'_M}\right] \tag{10}$$

$$x' = x[1-z/R'_M + (z/R'_M)^2 - (z/R'_M)^3 + \ldots]$$

and

$$x' \approx x[1 - z/R'_M] \quad \text{for } z/R'_M \text{ small} \tag{11}$$

likewise for $R'_p \triangleq R_p + h_0$

$$y' = y\left[\frac{1}{1 + z/R'_p}\right] \tag{12}$$

and

$$y' \approx y[1 - z/R'_p] \quad \text{for } z/R'_p \text{ small} \tag{13}$$

With x' and y' known exactly from (10) and (12) respectively and approximated in (11) and (13), the expression for $\Delta\phi$ and $\Delta\lambda$ can be developed. Since the TAATM axes are X-North and Y-East then from figure 2

$$\Delta\phi = \arctan\left(\frac{\Delta N}{R'_m}\right) = \frac{x'}{R'_m}$$

and

$$\Delta\lambda = \arctan\left(\frac{-\Delta E}{R'_p \cos\phi}\right) = \frac{-y'}{R'_p \cos\phi_o}$$

for

$+ \Delta\phi \triangleq$ radians north of TAATM origin
$+ \Delta\phi \triangleq$ radians west of TAATM origin

The corresponding geodetic coordinates in latitude and longitude are

$$\phi = \phi \;\; + \Delta\phi \tag{16}$$

and

$$\lambda = \lambda_o + \Delta\lambda \tag{17}$$

Let us now consider the relation of altitude above sea level and the tangent plane height z at some arbitrary location (x, y). For this we use the local or Gaussian radius of curvature. Figure 3 illustrates the relation of the quantities used. If h_0 is the height of the tangent plane's origin above MSL then the altitude h is

$$h = h_0 + h_1 + h_2 \tag{18}$$

to determine h_1 and h_2 we define

$$R_L' = R_L + h_0$$

$$r^2 = x^2 + y^2$$

so that

$$r' = \frac{r R_L'}{z + R_L'} \tag{19}$$

We can then say

$$(R_L' + h_2)^2 = (R_L')^2 + (r')^2$$

$$h_2 = - R_L' + R_L' [1 + (\frac{r'}{R_L'})^2]^{\frac{1}{2}}$$

$$h_2 = - R_L' + R_L'[1 + (\frac{r}{z + R_L'})^2]^{\frac{1}{2}} \tag{20}$$

Determine h_1 from

$$\frac{z}{h_1} = \frac{R_L'}{R_L' + h_2}$$

$$h_1 = z(1 + \frac{h_2}{R_L'})$$

$$h_1 = z [1 + (\frac{r}{z + R_L'})^2]^{\frac{1}{2}} \tag{21}$$

thus the MSL altitude h is

$$h = h_0 + z[1 + (\frac{r}{z + R_L'})^2]^{\frac{1}{2}} - R_L' + R_L'[1 + (\frac{r}{z + R_L'})^2]^{\frac{1}{2}} \tag{22}$$

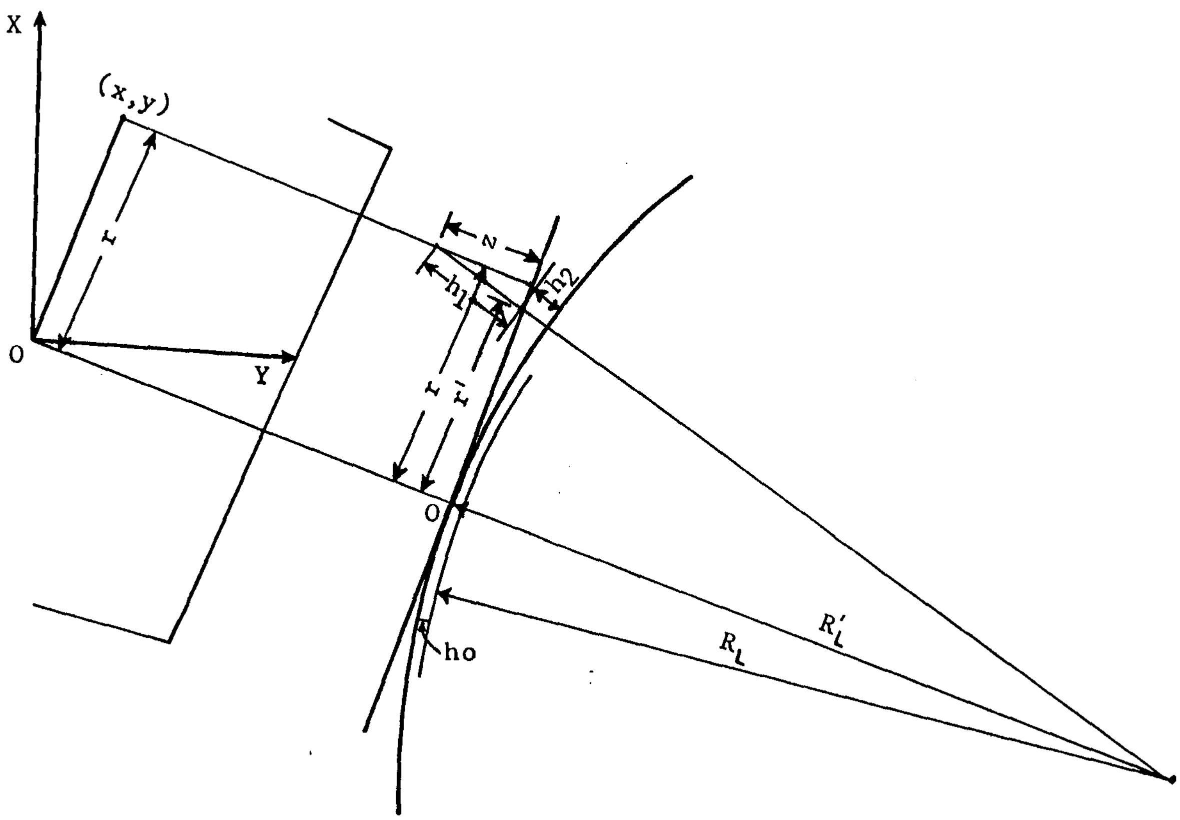


Figure 3. - Relation of tangent plane Z to MSL altitude.

this can be approximated by

$$h \approx h_0 + z + \frac{r^2}{2R_L'} \tag{23}$$

Transformation from TAATM Projection to Geodetic Coordinates

We started with a generalized tangent plane actual aircraft projection. The exact transformation to geodetic coordinates are given by equations 16, 17, and 22. It should be noted, as a result of either the exact equations for x' and y' (eqs. 10 and 12) or their approximations (eqs. 11 and 13), an aircraft with flat plane coordinates x, y, $z \neq 0$ will have a different latitude and longitude from that of the flat plane coordinates x, y, $z = 0$. That is, an aircraft over a TAATM plane, ground navigation fix would have a different latitude and longitude from that of the fix. To avoid this problem for the TAATM simulation we will say $x = x'$ and $y = y'$. As a result, an aircraft at some altitude over a TAATM ground fix will have, after transformation, the same geodetic latitude and longitude as the transformed fix location. This is a reasonable simplification because of the distances considered ($x, y \leq 50$ n. mi), and for operational terminal area altitudes the ratios z/R_M' and z/P_p' are small.

When we set $x = x'$ and $y = y'$ for ground fix and aircraft latitude and longitude compatability the problem reduces to what is called gnomonic projection. Gnomonic projection is obtained by placing a plane tangent to the earth and projecting geoid points geometrically from the center of the earth. Figure 4 a illustrates this process.

Since TAATM axis ar X-North and Y-East the distances are

$$\Delta N = x$$

$$\Delta E = y \tag{24}$$

For the gnomonic projection the relation between the TAATM distances ΔN and ΔE and that of latitude and longitude are

$$\Delta\phi = \arctan \frac{\Delta N}{R_M'} \tag{25}$$

$$\Delta\lambda = \arctan\left(\frac{-\Delta E}{R_p' \cos \phi_o}\right) \quad (26)$$

Azimuthal Equidistant Projection

Besides gnomonic projection, another approach employed for converting the curved earth to a flat plane is azimuthal equidistant projection. This is obtained by converting north-south and east-west radial or great circle distances to plane x, y. Figure 4b illustrates this projection. In this system the relation between the flat plane x, y distances and that of latitude and longitude (in radians) is defined by

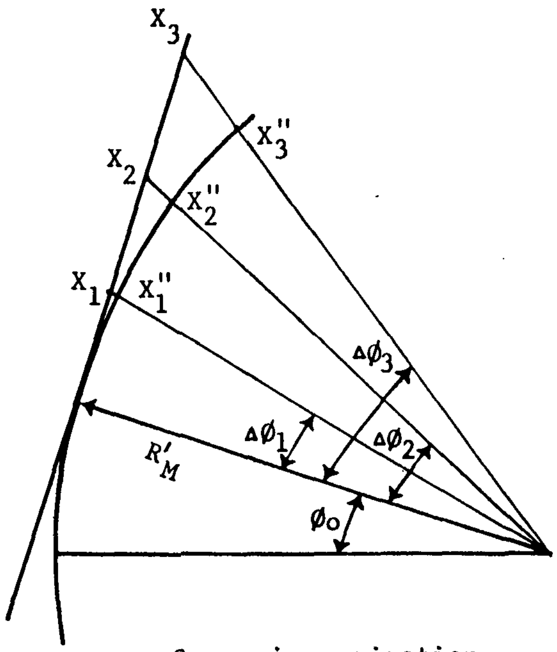
$$\Delta\phi = \frac{x}{R_M'} \quad (27)$$

$$\Delta\lambda = \frac{y}{R_p' \cos \phi_o} \quad (28)$$

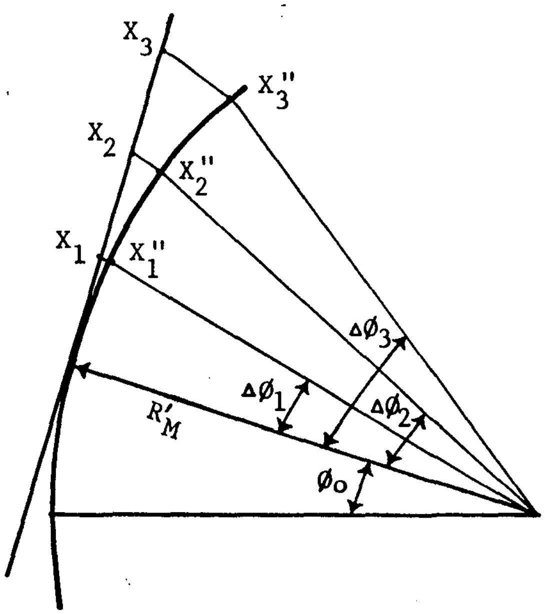
Looking back to Equations 25 and 26 we see that for small angles of $\Delta\phi$ and $\Delta\lambda$ the gnomonic and azimuthal equidistant projection are approximately equal.

TCV's Transformation Approximations

Other approximations that those developed in this report have been used for mapping tangent earth plane positions to geodetic location. An example is the Terminal Configured Vehicle Program's approximation equations used to relate MLS derived position to geodetic latitude and longitude for on-board navigation. Appendix B contains a listing of their approximation equations and compares the results using these approximations with the output from the exact analytical expression derived earlier in this report.



a. Gnomonic projection



b. Azimuthal equidistant projection

Figure 4. - Geoid to flat plane mapping.

CONCLUDING REMARKS

The tranformations developed for mapping geodetic positions to an earth tangent rectangular plane via gnomonic projection is a relatively straight forward process which simplifies the coordinate system requirements for simulating a terminal area air traffic environment. These requirements are to relate the geodetic positions of runways navigation sites, fixes etc. to the aircraft body centered axis to enable simulated aircraft to navigate, and also to display aircraft positions as would be detected by an airborne radar. Inverse transformation can be explicitly determined from the approximation equations developed.

This report's development proceeded from a generalized tangent plane actual aircraft projection to that of a flat earth gnomonic projection. The generalized position projection represents the ideal position as perceived by a high frequency, line of sight system such as radar or the Microwave Landing System (MLS). Standard formulas may be used to transfer the spherical values of elevation, azimuth and range to the rectangular X, Y, Z. The generalized relation between geodetic position in latitude, longitude, and mean sea level altitude and the corresponding location in a rectangular flat plane X, Y, Z as developed in this report, provide a procedure to convert MLS values to geodetic position. The Terminal

Configured Vehicle Program used approximation equations to relate MLS position to geodetic values used by their on-board computer. Appendix B contains an error analysis comparison of the values obtained by the TCV approximation and the values obtained using our derived analytic expressions. Values of distance and altitude used in the error analysis and a representative range of values an aircraft would experience when landing at some air traffic terminal area. The results indicate the TCV approximations are adequate for precision MLS navigation use.

APPENDIX A

DERIVATION OF ELLIPSOID RADII OF CURVATURE

If a meridian section of the earth's ellipsoid of resolution is taken, the result is the ellipse shown in Figure A-1. For a conventional X, Y coordinate system, the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{A-1}$$

where a is the semimajor (earth equatorial) radius and b is the semi-minor (polor) radius. When the eccentricity e is defined by

$$e^2 = \frac{a^2 - b^2}{a^2} \tag{A-2}$$

then we can rewrite Equation A-1 as

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1 \tag{A-3}$$

Solving for y in Equation A-3 and differentiating with respect to x yields the slope y' at the point (x, y) on the ellipse.

$$y' = (1 - e^2)(- \frac{x}{y}) \tag{A-4}$$

Since the geodetic latitude ϕ at a point (x, y) is the angle between the normal to the ellipsoid and the equatorial plane

$$y' \tan \phi = -1 \tag{A-5}$$

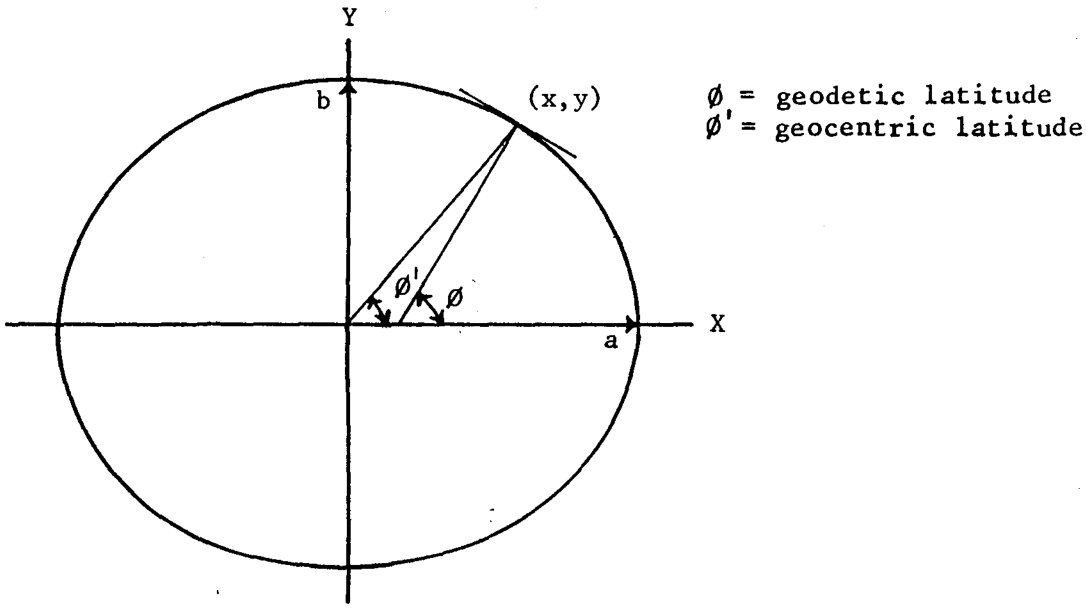


Figure A-1. - Meridian section of geodetic ellipsoid.

Substituting Equation A-4 into A-5 yields the relation

$$y = x(1 - e^2)\tan\phi \tag{A-6}$$

Using A-6 in Equation A-3 results in

$$x = \frac{a\cos\phi}{(1 - e^2\sin^2\phi)^{\frac{1}{2}}} \tag{A-7}$$

Taking Equations A-6 and A-7 together yields

$$y = \frac{a(1-e^2)\sin\phi}{(1 - e^2\sin^2\phi)^{\frac{1}{2}}} \tag{A-8}$$

For the meridian radius R_M, what is desired is a circle of radius R_M such that its north-south curvature or rate of change (second derivative of y with respect to x) at its latitude ϕ is the same as that of the geodetic ellipsoid at latitude ϕ.

The rate of change y'' of the ellipse at point (x, y) is determined by differentiating Equation A-4 with respect to x and using Equation A-5 to obtain

$$y'' = -\frac{1}{y}\left(\frac{1-e^2\sin^2\phi}{\sin^2\phi}\right) \tag{A-9}$$

Substituting from Equation A-8 yields:

$$y'' = \frac{-(1-e^2\sin^2\phi)^{3/2}}{a(1-e^2)\sin^3\phi} \tag{A-10}$$

A circle with its center at the origin of coordinates (X_c Y_c) with radius R_M has the equation

$$x_c^{\,2} + y_c^{\,2} = R_M^{\,2} \tag{A-11}$$

For this circle the following relations hold

$$y_c' \tan \phi_c = -1$$

$$\tan \phi_c = \frac{y_c}{x_c}$$

$$y_c = R_M \sin \phi_c \qquad \text{(A-12)}$$

Using the relations of Equation A-12, the rate of change of the circle y_c'' is determined to be

$$y_c'' = -\frac{R_M^{\,2}}{y_c^3} = \frac{-1}{R_M \sin^3\phi_c} \qquad \text{(A-13)}$$

If the circle is placed such that the point (x, y) of the ellipse in Figure A-1 and (x_c, y_c) of the circle are the same for $\phi_c = \phi$, then we can rewrite Equation A-13 as

$$y_c'' = \frac{-1}{R_M \sin^3\phi} \qquad \text{(A-14)}$$

Equating the rates of change in Equations A-10 and A-14 gives

$$R_M = \frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}} \qquad \text{(A-15)}$$

which is the radius of the best fitting circle for north-south curvature to the meridian section of the ellipsoid at latitude ϕ. Its utility comes from the fact that a given relatively small north-south distance subtends the same increment of latitude on a circle of radius R_M as that south-north distance does on the geodetic ellipsoid at the reference latitude ϕ.

If Equation A-15 is expanded in the form

$$R_M = a(1-e^2)\ [1 + (e^2\sin^2\phi)]^{-3/2} \qquad \text{(A-16)}$$

and dropping higher order terms of e we get the following approximation

$$R_M \approx a[1 + e^2(\tfrac{3}{2}\sin^2\phi - 1)] \qquad \text{(A-17)}$$

A similiar radius can be determined which relates the east-west curvature of a circle to that of the earth's ellipsoid. This radius, called the prime radius R_p, is defined as the radius of a circle model of the meridian section such that at latitude ϕ, its transverse circle of revolution around the polar axis is equal to that of the geodetic ellipsoid at latitude ϕ.

From Figure A-2 we see this radius of east-west curvature is equal to the earth's ellipsoid value x at latitude ϕ. For an circle model of the meridian section the radius of rotation about the solar axis at latitude ϕ must be equal to the value x.

$$R_p \cos\phi = x \tag{A-18}$$

When Equation A-7 is substituted into Equation A-18, the value R_p is determined in terms of the geodetic latitude ϕ.

$$R_p = \frac{a}{(1-e^2\sin^2\phi)^{\frac{1}{2}}} \tag{A-19}$$

If Equation A-19 is expanded as it was in Equation A-16, the following approximation for R_p results

$$R_p \approx a[1 + \frac{e^2}{2} \sin^2\phi] \tag{A-20}$$

The local or Gaussian radius of curvature R_L, is the radius of the best fitting sphere to the ellipsoid at a local point

$$R_L = \sqrt{R_M R_p} \tag{A-21}$$

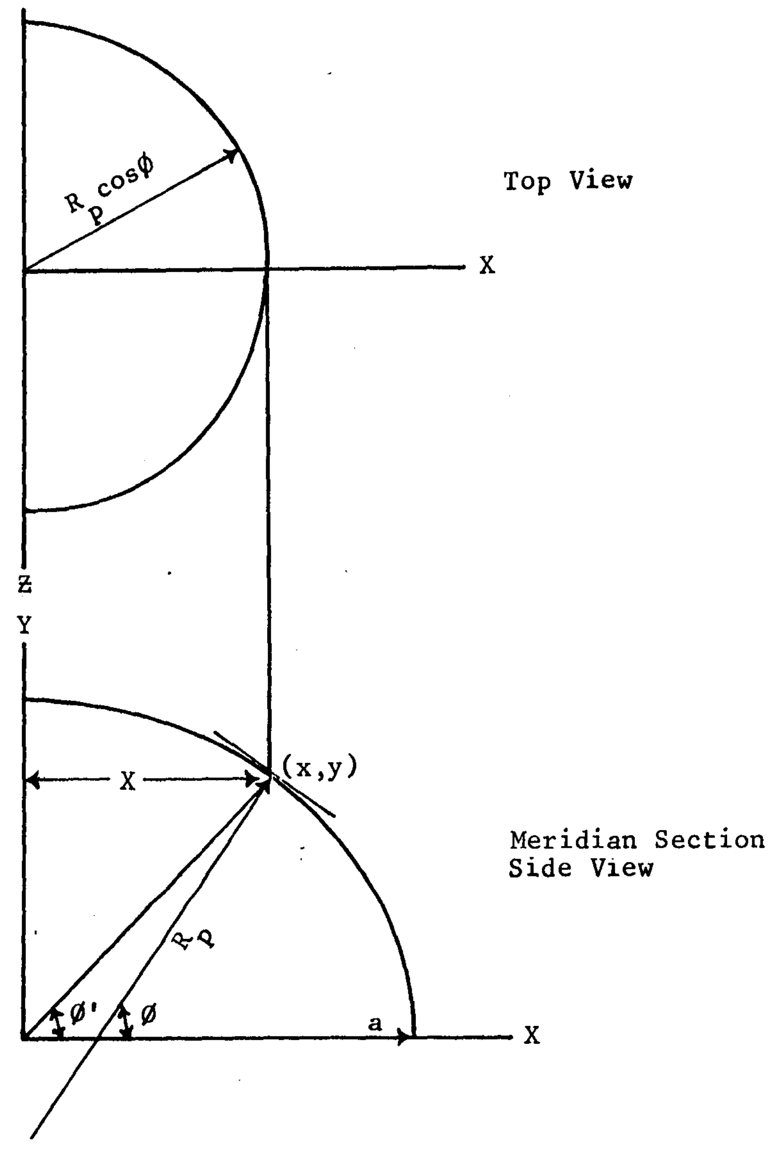


Figure A-2. - Relation of east-west curvature to a meridian section of geodetic ellipsoid.

If Equation A-15 and Equation A-19 are substituted one gets

$$R_L = \frac{a(1-e^2)^{\frac{1}{2}}}{(1-e^2\sin^2\phi)} \qquad \text{(A-22)}$$

If Equation A-22 is binomially expanded and higher order terms of e are dropped then we get for an approximation

$$R_L \approx a[1- \frac{e^2}{2} \cos 2\phi] \qquad \text{(A-23)}$$

APPENDIX B

COMPARISON OF ANALYTICAL EXPRESSIONS TO TCV's APPROXIMATION EQUATIONS

The Terminal Configured Vehicle (TCV) Program has used a number of approximation equations relating the earth's ellipsoid to a rectangular coordinate system in its Microwave Landing System (MLS) work (ref. 3). Following is a list of these equation:

PARAMETERS OF TCV ELLIPSOID

$a = 6{,}378{,}169.79$ meters

$f = 3.3901 \times 10^{-3}$

APPROXIMATION EQUATIONS USED BY TCV

$$R_M = a\left[\frac{1}{1 + f\,(2-3\,\sin^2\,\phi_0)}\right] \tag{B-1}$$

$$R_M' = R_M + h_0 \tag{B-2}$$

$$R_p = a\left[\frac{1}{1-f\,\sin^2\,\phi_0}\right] \tag{B-3}$$

$$R_p' = R_p + h_0 \tag{B-4}$$

$$R_L = a\left[\frac{1}{1+f(1-2\,\sin^2\phi_0)}\right] \tag{B-5}$$

$$R_L' = R_L + h_0 \tag{B-6}$$

$$x' = x\left(1- \frac{z}{R_L'}\right) \tag{B-7}$$

$$y' = y\left(1- \frac{z}{R_L'}\right) \tag{B-8}$$

$$r^2 = x^2 + y^2 \tag{B-9}$$

$$h = h_0 + z + \frac{r^2}{2R_L'} \qquad \text{(B-10)}$$

$$\Delta\phi = \frac{x'}{R_M'} \qquad \text{(B-11)}$$

$$\Delta\lambda = \frac{y'}{R_p' \cos \phi_0} \qquad \text{(B-12)}$$

Table B-1 gives a comparison of results using TCV approximations and that obtained by using the exact expressions. For a chosen latitude of +40°, we used the IASU - 1965 reference ellipsoid and the exact analytical expressions of a generalized tangent plane actual aircraft projection derived in the report to obtain the following three values:

1. the delta latitude from reference point
2. the delta longitude from reference point
3. height above mean sea level

These were obtained for a rectangular tangent plane specific (x, y, z) point. A set of the above three values was obtained using the IASU - 1965 reference ellipsoid together with the TCV approximation equations. Finally for the specific (x, y, z) point, another set of the previously mentioned three values was obtained using both TCV's ellipsoid and TCV's approximation equations. A comparison of the values obtained was made using the report's analytical expressions and the IASU - 1965 reference ellipsoid as the standard. The N/S and E/W errors in distance using the IASU - 1965 reference ellipsoid for calculated errors in $\Delta\phi$ and $\Delta\lambda$ was then determined using

ERROR = Analytical Expression Value - Approximation Value

This whole procedure was repeated for a range of x, y, z values. The resulting numbers in Table B-1 indicate that the TCV approximations are sufficiently close to the exact analytical expressions for precision MLS navigation use.

TABLE B-1. COMPARISON OF TCV APPROXIMATION EQUATIONS AND ELLIPSOID VALUES TO EXACT EXPRESSIONS WITH IASU - 1965 ELLIPSOID MODEL FOR A LATITUDE OF 40°

	Analytical Expression IASU - 1965 Ellipsoid	TCV Approximation IASU - 1965 Ellipsoid	TCV Approximation TCV Ellipsoid
Radius of Curvature Values (meters)			
R_M	6,361,838.371	6,361,938.450	6,361,768.661
R_p	6,386,999.409	6,387,008.141	6,387,116.270
R_L	6,374,406.476	6,374,448.643	6,374,417.266
Comparison for A/C at (x, y) = (5,5) n.mi.; z = 1,600 ft; h_0 = 0			
h (ft)	1644.130	1644.133	1644.133
$\Delta\phi$ (rad.)	0.001455441	0.001455420	0.001455459
$\Delta\lambda$ (rad.)	0.001892459	0.001892458	0.001892426
h error (ft)	---	-0.003	-0.003
N/S error (ft)	---	0.452	-0.359
E/W error (ft)	---	0.010	0.524

TABLE B-1 Continued

Comparison for A/C at (x, y) = (10,10) n.mi.; z = 3,200 ft; h_0 = 0

h (ft)	3376.506	3376.532	3376.533
$\Delta\phi$ (rad.)	0.002910654	0.002910617	0.002910695
$\Delta\lambda$ (rad.)	0.003784615	0.003784627	0.003784563
h error (ft)	---	-0.027	-0.027
N/S error (ft)	---	0.767	-0.854
E/W error (ft)	---	-0.187	0.841

Comparison for A/C at (x, y) = (15,15)n.mi.; z = 4,800 ft; h_0 = 0

h (ft)	5197.106	51797.198	5197.200
$\Delta\phi$ (rad.)	0.004365630	0.004365591	0.004365708
$\Delta\lambda$ (rad.)	0.005676456	0.005676506	0.005676410
h error (ft)	---	-0.092	-0.094
N/S error (ft)	---	0.818	-1.614
E/W error (ft)	---	-0.808	0.734

Comparison for A/C at (x, y) = (20,20)n.mi.; z = 6,350 ft; h_0 = 0

h (ft)	7055.908	7056.129	7056.133
$\Delta\phi$ (rad.)	0.005820380	0.005820357	0.005820512
$\Delta\lambda$ (rad.)	0.007567985	0.007568114	0.007567985
h error (ft)	---	-0.222	-0.225
N/S error (ft)	---	0.477	-2.765
E/W error (ft)	---	-2.070	-0.013

TABLE B-1 Continued

Comparison for A/C at (x, y) - (30,30)n.mi.; z = 9,500 ft; h_0 = 0

h (ft)	11138.016	11138.791	11138.799
$\Delta\phi$ (rad.)	0.008729109	0.008729199	0.008729432
$\Delta\lambda$ (rad.)	0.011349974	0.011350433	0.011350241
h error (ft)	---	-0.775	-0.783
N/S error (ft)	---	-1.888	-6.751
E/W error (ft)	---	-7.373	-4.288

Comparison for A/C at (x, y) - (40,40)n.mi.; z = 12,750 ft; h_0 = 0

h (ft)	15572.625	15574.518	15574.532
$\Delta\phi$ (rad.)	0.011636798	0.011637150	0.011637461
$\Delta\lambda$ (rad.)	0.015130483	0.015131594	0.015131338
h error (ft)	---	-1.893	-1.904
N/S error (ft)	---	-7.346	-13.828
E/W error (ft)	---	-17.823	-13.710

Comparison for A/C at (x, y) - (50,50)n.mi.; z = 16,000 ft; h_0 = 0

h (ft)	20409.500	20413.309	20413.331
$\Delta\phi$ (rad.)	0.014543366	0.014544176	0.014544564
$\Delta\lambda$ (rad.)	0.018909362	0.018911551	0.018911231
h error (ft)	---	-3.809	-3.831
N/S error (ft)	---	-16.910	-25.011
E/W error (ft)	---	-35.134	-29.994

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15. Supplementary Notes

16. Abstract

Formulas and their approximation were developed to map geodetic position to an earth tangent plane with an airport centered rectangular coordinate system. The transformations were developed for use in a Terminal Area Air Traffic Model (TAATM) with deterministic aircraft traffic. The exact formulas were used in an appendix to perform an error analysis of the Terminal Configured Vehicle's (TCV) approximation equations used in their precision Microwave Landing System (MLS) navigation experiments.

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