

Transformation of 4-dimensional Rindler spacetime

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ABSTRACT

In special relativity theory, we discover 4-dimensional transformation of general Rindler space-time from 4-dimensional Lorentz transformation in inertial frames.

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I. Introduction

In special relativity theory, we discover 4-dimensional transformation of general Rindler space-time from 4-dimensional Lorentz transformation in inertial frames.

At first, 2-dimensional-Lorentz transformation is in inertial frame,

$$ct = \gamma(ct' + \frac{V_0}{c} \cdot x'),$$

$$x = \gamma(x' + \vec{v}_0 t'), y = y', z = z' \quad \gamma = 1 / \sqrt{1 - \frac{V_0^2}{c^2}} \quad (1)$$

2-dimensional transformation is in Rindler spacetime,

$$ct = \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right), x = \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}$$

$$y = \xi^2, z = \xi^3 \quad (2)$$

2. 4-dimensional transformation in Rindler spacetime

4-dimensional-Lorentz transformation is in inertial frame,

$$ct = \gamma\left(ct' + \frac{\vec{V}_0}{c} \cdot \vec{x}'\right)$$

$$\vec{x} = \vec{x}' + \gamma \vec{v}_0 t' - (1 - \gamma) \frac{\vec{V}_0 \cdot \vec{x}'}{V_0^2} \vec{V}_0, \quad \gamma = 1 / \sqrt{1 - \frac{V_0^2}{c^2}} \quad (3)$$

4-dimensional-differential Lorentz transformation is in inertial frame,

$$cdt = \gamma\left(cdt' + \frac{\vec{V}_0}{c} \cdot d\vec{x}'\right)$$

$$d\vec{x} = d\vec{x}' + \gamma \vec{v}_0 dt' - (1 - \gamma) \frac{\vec{V}_0 \cdot d\vec{x}'}{V_0^2} \vec{V}_0, \quad \gamma = 1 / \sqrt{1 - \frac{V_0^2}{c^2}} \quad (4)$$

Hence, the proper time is

$$d\tau^2 = dt^2 - \frac{1}{c^2} d\vec{x} \cdot d\vec{x} = dt^2 - \frac{1}{c^2} (dx^2 + dy^2 + dz^2)$$

$$= dt^2 - \frac{1}{c^2} (dx'^2 + dy'^2 + dz'^2) \quad (5)$$

If we suggest 4-dimensional transformation in Rindler spacetime,

$$ct = \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{c^2}{a_0} \left(1 + \frac{\vec{a}_0}{c^2} \cdot \vec{\xi}\right)$$

$$\vec{x} = \vec{\xi} + \frac{c^2}{a_0^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) \vec{a}_0 - (1 - \cosh\left(\frac{a_0 \xi^0}{c}\right)) \frac{\vec{a}_0 \cdot \vec{\xi}}{a_0^2} \vec{a}_0 - \frac{c^2}{a_0^2} \vec{a}_0 \quad (6)$$

Therefore, 4-dimensional-differential transformation is in Rindler spacetime

$$cdt = \cosh\left(\frac{a_0 \xi^0}{c}\right) c d\xi^0 \left(1 + \frac{\vec{a}_0}{c^2} \cdot \vec{\xi}\right) + \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\vec{a}_0}{a_0} d\vec{\xi}$$

$$d\vec{x} = d\vec{\xi} + \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{c}{a_0} \vec{a}_0 d\xi^0 + \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{d\xi^0}{c} \frac{\vec{a}_0 \cdot \vec{\xi}}{a_0} \vec{a}_0$$

$$-(1 - \cosh\left(\frac{a_0 \xi^0}{c}\right)) \frac{\vec{a}_0 \cdot d\vec{\xi}}{a_0^2} \vec{a}_0 \quad (7)$$

Hence, the proper time is [8]

$$d\tau^2 = dt^2 - \frac{1}{c^2} d\vec{x} \cdot d\vec{x} = dt^2 - \frac{1}{c^2} (dx^2 + dy^2 + dz^2)$$

$$= \left(1 + \frac{\vec{a}_0 \cdot \vec{\xi}}{c^2}\right)^2 (d\xi^0)^2 - \frac{1}{c^2} ((d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2) \quad (8)$$

3. Conclusion

We know general Rindler coordinate transformation from 4-dimensional Lorentz transformation.

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