

## Transforming the optical landscape

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**Abstract:** Electromagnetism provides us with some of the most powerful tools in science, encompassing lasers, optical microscopes, MRI scanners, radar and a host of other techniques. To understand and develop the technology requires more than a set of formal equations. Scientists and engineers have to form a vivid picture that fires their imaginations and enables intuition to play a full role in the process of invention. It is to this end that transformation optics has been developed exploiting Faraday's picture of electric and magnetic fields as lines of force which can be manipulated by the electrical permittivity and magnetic permeability of surrounding materials. Transformation optics says what has to be done to place the lines of force where we want them to be.

The nature of light was revealed by Maxwell a century and a half ago. His equations stand today as an description of light, and all electromagnetic phenomena, exact at the classical level. It has long been known that Maxwell's equations are invariant under a coordinate transformation (1-3). If  $\epsilon(\mathbf{x}), \mu(\mathbf{x})$  are the permittivity and permeability tensors expressed in some coordinate frame, then a transformation to a new coordinate frame  $\mathbf{x}'$  does not alter the form of Maxwell's equations but only changes the values of permittivity and permeability to be used. Formally we have,

$$\epsilon'^{i'j'} = [\det(\Lambda)]^{-1} \Lambda^{i'} \Lambda^{j'} \epsilon^{ij}$$

$$\mu'^{i'j'} = [\det(\Lambda)]^{-1} \Lambda^{i'} \Lambda^{j'} \mu^{ij}$$

where,

$$\Lambda_j^{j'} = \frac{\partial x^{j'}}{\partial x^j}$$

This simple mathematical statement has been elaborated over the past couple of decades into an intuitive working tool of electromagnetic theory (4-8). Here, we shall give a physical interpretation of the equations and re-derive the formulae in an intuitive fashion, followed by examples of where transformation optics has been effectively applied.

Bald mathematical statements hide the real power of the transformation method which is only revealed when we appeal to Michael Faraday's representation of electric and magnetic fields in terms of lines of force. In this scheme the displacement field,  $\mathbf{D}(\mathbf{x})$ , and the magnetic field  $\mathbf{B}(\mathbf{x})$  are represented by field lines that begin and end on charges in the case of the displacement field, and are continuous in the case of the magnetic field. Their density represents the field strength. A coordinate transformation can be thought of as a distortion of space which moves the field lines around as if they were embedded in a block of rubber. As we distort the coordinates,

we carry the field lines along too. This yields an intuitive picture of how to manipulate fields: just as Snell's law tells how rays of light can be redirected and focused using the refractive index, transformation optics shows how field lines can be manipulated and gives an exact prescription for the values of  $\epsilon(\mathbf{x}), \mu(\mathbf{x})$  ensuring that the distorted field lines obey Maxwell's equations. Replacing the rays of Snell's law with Faraday's fields extends our powers of visualization into regimes untouched by Snell, such as the sub wavelength electric fields found in plasmonic systems and even to the regime of static electric and magnetic fields (9).

Not only does this manipulation apply to  $\mathbf{D}$  and  $\mathbf{B}$ , but also any electromagnetic quantity that is conserved and therefore represented by field lines: for example the Poynting vector representing the flow of energy serves as a generalization of the ray picture. Another example is the flow of charge either as an electrical current or as the trajectory of an individual particle.

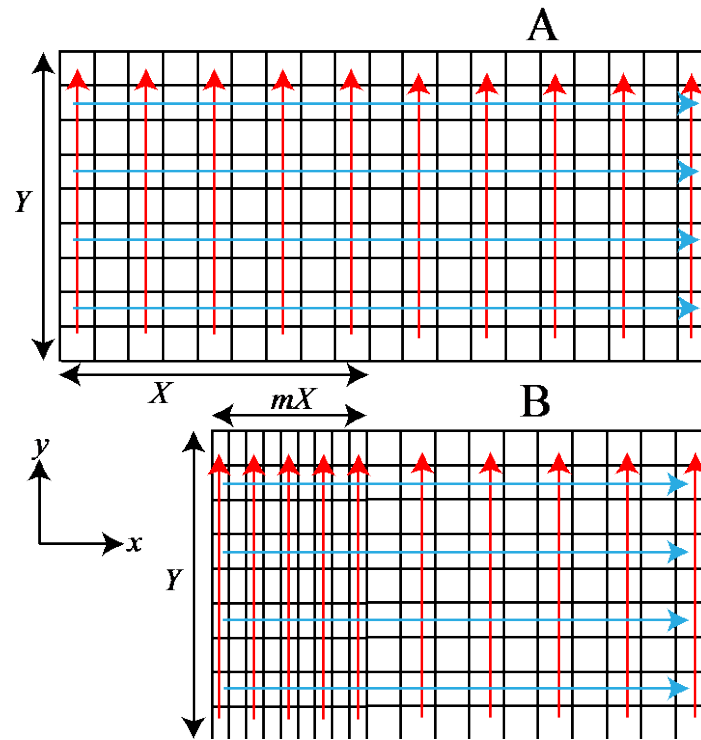
**The power of transformation.** We shall give a few examples that demonstrate the power of the technique, but first let us make an alternate, intuitive, derivation of the transformation equations.

Consider an electric displacement field,  $D_y$ , parallel to the  $y$  axis in a dielectric medium (as shown red in Fig. 1A). Next make a coordinate transformation in which one half of space is uniformly compressed by a factor  $m$  (Fig. 1B). We then ask how the permittivity in the compressed medium must change. To calculate  $\epsilon'_y$  we require that  $E_y$  is continuous. Noting that owing to the compression the lines of force have been pushed closer together and hence

$D'_y$  has been increased by a factor  $m^{-1}$ ,

$$D_y/\epsilon_y = D'_y/\epsilon'_y = m^{-1} D_y/\epsilon'_y$$

and hence  $\epsilon'_y = m^{-1}\epsilon_y$ . The same argument for magnetic fields shows that  $\mu'_y = m^{-1}\mu_y$ .



**Fig. 1. Visualising a transformation in the electric displacement fields.** The field lines (A) before compression of half the coordinates, and (B) after compression. The flux lines perpendicular to the axis of compression (cyan) are pushed closer together, whereas those parallel to the axis (blue) have their spacing and therefore the  $D$  field intensity unaltered.

To find  $\epsilon'_x$  consider an electric displacement field,  $D_x$ , parallel to the  $x$  axis (as shown cyan in Fig 1A). After compression of the coordinate system this field is unaltered,  $D'_x = D_x$  (Fig. 1B).

To find  $\epsilon'_x$  we require that the work done by a test charge passing from one side of the compressed region to the other is unaltered by the compression. Because the charge traverses a region shortened by a factor  $m$ , the electric field,  $E'_x$  must increase by a factor  $m^{-1}$ , and as  $D_x$  is unchanged this requires that,

$$\epsilon'_x = m\epsilon_x.$$

The final result is simple: compression along an axis by a factor of  $m$  decreases  $\epsilon$  along that axis by a factor of  $m$ , and along the two perpendicular axes  $\epsilon$  is increased by a factor of  $m^{-1}$ . Similarly for the permeability. In fact this is the most general statement of the transformation, since the most general distortion of a local coordinate system can be represented as sequential compressions about three axes followed by a rotation. So not only do we have an intuitive feel as to how field lines behave as we distort the coordinates, but we also get an intuitive feel for the changes in the electric and magnetic response of the transformed medium.

**Example applications: Cloaking.** Perhaps the most famous application of transformation optics has been the design of a cloak of invisibility (10-15). The technique is ideally suited to this task as fields are only disturbed where the coordinates are distorted, so confining the distortion to the interior of the cloak itself leaves external fields undisturbed and therefore the act of cloaking undetected. However early experimental realizations of cloaks were all much larger than the wavelength of radiation from which they were hiding, and could in principle, though with difficulty, have been designed using ray optics. More recently cloaking principles have been applied to the near field, and in extremis to static magnetic fields, an instance where ray optics obviously has no application. The same transformation used for an optical cloak (10) was used to design a cloak for static magnetic fields (15),

$$r' = R_1 + r(R_2 - R_1)/R_2, \quad \theta' = \theta, \quad \phi' = \phi$$

resulting in the following cloaking parameters for the permeability,

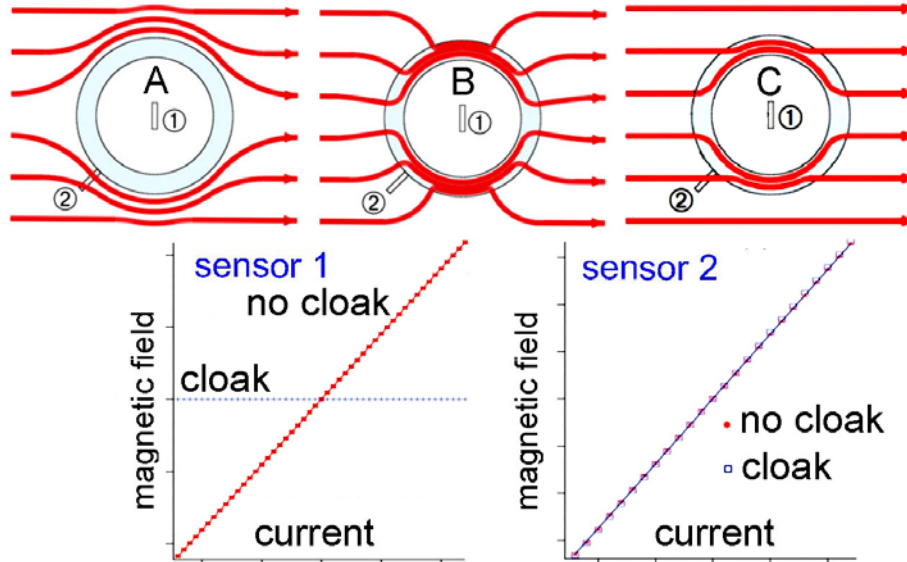
$$\mu'_{r'} = \frac{R_2}{R_2 - R_1} \frac{(r' - R_1)^2}{r'^2}, \quad \mu'_{\theta'} = \frac{R_2}{R_2 - R_1}, \quad \mu'_{\phi'} = \frac{R_2}{R_2 - R_1}$$

Note how these parameters obey the compression rules derived above: radial values of  $\mu$  are decreased and angular ones increased.

The magnetic field cloak (Fig. 2C) is designed using these formulae with the simplification that because magnetic fields alone are assumed to be present we need only adjust the permeability.

The design strategy is to make a transformation that takes all the 'space' inside the cloak and compress it in the radial direction into an annulus, leaving an empty inner space in which we can hide objects. As we have noted, compressing space increases the permeability perpendicular to the direction of compression, and decreases the permeability along the direction of compression, in this instance along the radius.

The latter specification was the key challenge that Narayana and Sata met (15). They built a cylindrical cloak for static magnetic fields using a novel design for the diamagnetic metamaterials demanded by the formula (16). They placed the cloak in a magnetic field created by two electromagnets and measured the fields inside and outside the cloak as a function of the current in the electromagnets (Fig. 2). They also measured the fields for 3 different devices each of which excludes the magnetic field: (A) a superconducting screen that exploits the Meissner effect to exclude magnetic fields but, by repelling the fields into the surrounding space, distorts the external fields; (B) a mumetal shield that attracts the field lines into the highly permeable metal, but also distorts the external fields, and (C) a cloak designed according the transformation optical principles which excludes the magnetic fields in a fashion that does not distort the external fields. The measured data for the cloak confirm that the cloak not only excludes fields but is also undetectable.



**Fig. 2. Three schemes for excluding magnetic flux from an enclosed region of space.** (A) A superconducting shell exploits the Meissner effect to exclude flux, but results in a strong disturbance in the external field which has to accommodate the excluded flux lines. (B) Mumetal is used to attract flux lines away from the central space, but also results in a distortion of the external field. (C) A true magnetic cloak removes flux lines from the protected space but confines the displaced flux lines within the body of the cloak leaving external fields unaltered. Below are the results of an experiment on the cloak shown in (C). A magnetic field generated by current in an electromagnet is applied to the cloak. Sensor 1 inside the cloak demonstrates zero flux inside the cloak; sensor 2 outside the cloak shows an undisturbed external field.

There are several situations in which it may be desirable to hide a sensitive object from a magnetic field, but not to disturb the field outside the cloak. In this way external functions of the field, such as resolving objects in an MRI scan, are preserved. The solutions shown in Figs. 2A and 2B would not achieve this objective and would also result in mechanical forces on the cloak itself due to interaction with the induced dipole moment. In contrast the true cloak has no dipole moment and therefore no net force is exerted.

**Nanophotonics.** Plasmonics has also been a fruitful area of application as reported in (7,17,18). The surfaces of metals such as gold and silver support density oscillations of the electrons, much like waves on a sea. These can couple to external radiation, but have a much shorter wavelength.

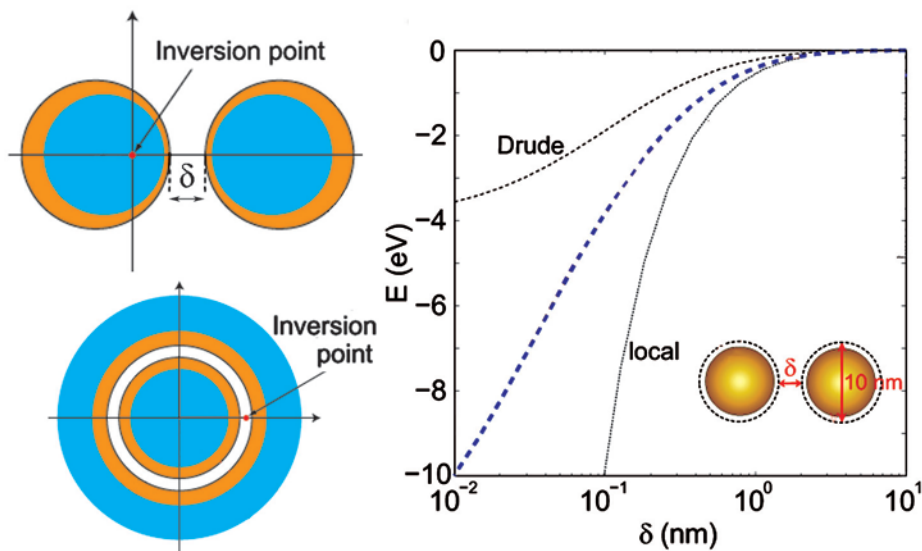


The plasmonic excitations are greatly influenced by the shape of the surface and in particular by any singularities such as sharp corners, touching surfaces, or other rough features, which tend to attract very high field intensities: they act as harvesting points for any incident radiation. This is the origin of surface enhanced Raman spectroscopy (SERS) in which the intense field concentrations greatly increase the Raman signals from adsorbed molecules, sometimes by as much as  $10^6$ . Without this effect many molecules would be Raman invisible at the surface.

By applying transformations to simple structures, such as plasmonic waveguides consisting of two parallel sheets of silver, many of the singular structures can be generated through a singular transformation and their spectra understood through the spectrum of the original simple waveguide. Thus apparently diverse structures such as sharp edges, points, nearly touching spheres, can be shown to have a common origin and can in many cases be treated analytically. This deep understanding enables further properties of these structures to be elucidated such as the dispersion forces acting at short range between surfaces that are otherwise out of physical contact (19,20). The origin of these forces lies in the zero point energy,  $\hbar\omega/2$ , of the electromagnetic modes whose frequencies shift as surfaces approach. By transforming apparently complex surface configurations to simple waveguides spectral shifts (invariant under a transformation) can be calculated analytically and the forces determined (21).

Two gold spheres in close proximity (Fig. 3) are shown to transform into two concentric spheres. The transformation consists of an inversion about a point carefully chosen so that the inverted structure comprises two concentric spheres. The highly symmetric geometry of the inverted structure leads to greatly simplified calculations – another instance of ‘hidden symmetries’.

This theory has been applied to the interaction between two gold nanospheres. First the experimental permittivity of gold (22,23) was fitted using the Drude-like term for the lower frequencies and Lorentzian terms for higher frequencies where core excitations in the gold are important. Data extending to 100eV were fitted. Included in the calculation are non local effects which smear out surface charges and hence introduce a natural cut off in wave vector without which the forces would not saturate as the touching point is approached. On the right are the van der Waals energies calculated in the coordinate system of the concentric spheres. The calculations demonstrate the importance of going beyond the Drude approximation to the permittivity and also of including non local corrections.



**Fig. 3. van der Waals energy between two 10-nm-diameter gold nanoparticles as a function of the separation.** For the blue dashed line, nonlocal effects are considered. The black dotted line shows the van der Waals energy calculated neglecting nonlocal effects. The energy considering nonlocal effects but neglecting the Lorentzian terms in the permittivity is shown by the short dashed line.

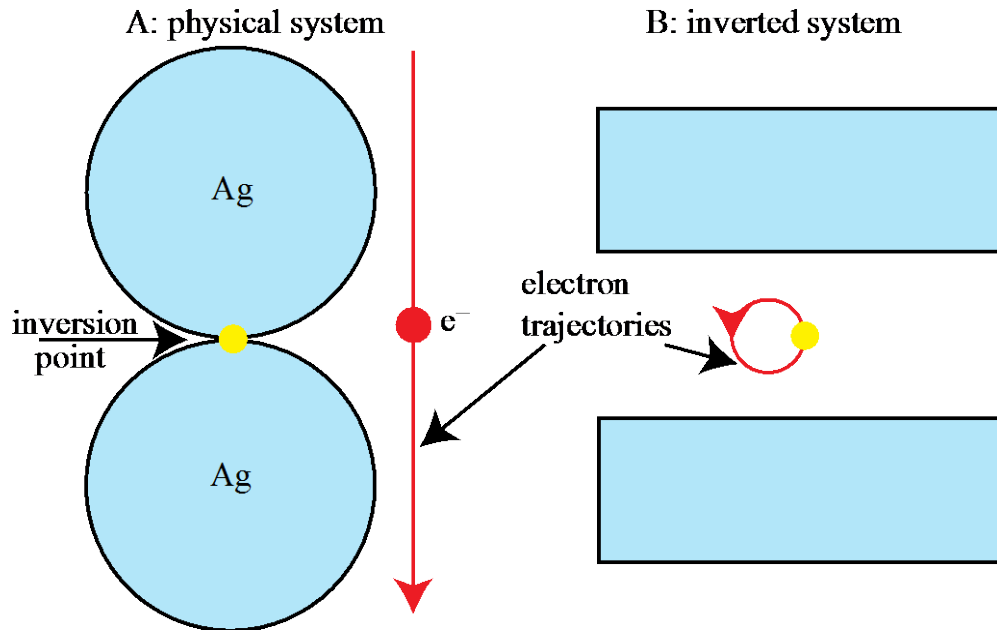
When investigating a system's properties, whether electromagnetic, electronic, or acoustic, to mention only three aspects, considerations of symmetry are often vital. In spectroscopy symmetry classifies transitions, and when calculating the electronic properties of matter

symmetry is vital to simplifying computations because states are classified by wave vector, courtesy of Bloch's theorem. In contrast disordered solids are still not well understood because of their lack of symmetry. However it is possible to start with a highly symmetric system, such as a planar plasmonic waveguide, whose properties are classified by a wave vector and therefore well understood thanks to the symmetry, and apply a transformation that destroys the geometric symmetry, but leaves the spectroscopic properties intact. This we call a 'hidden symmetry' (24). For example the planar cross section of the waveguide can be transformed variously into a knife edge, two cylinders in close proximity, or a crescent. All these structures inherit the spectrum of the mother system: they have a common spectroscopic 'DNA'.

**Electron microscopy.** Yet another example of both insight and computational facility brought by transformation optics is found in electron energy loss calculations. Electron microscope technology is highly developed and can image materials at the sub nanometer level. In addition by applying energy analysis to the emerging beam it is possible to detect the electromagnetic spectrum of an object resolved at the nanoscale (25). This is a very powerful technique.

Figure 4 shows on the left an electron beam, typically 50keV, passing close to a metallic nanostructure. Electrons lose energy and momentum to the surface plasmons of the metal giving access to the electromagnetic spectrum of the structure. Electrons also have the advantage that unlike photons, the excitations are not limited by a dipole selection rule. The electromagnetic field surrounding an electron in uniform motion is easily calculated, but in the past it has been generally supposed that the modes with which it interacts require a complex harmonic expansion involving many terms. However a simple inversion about the touching point takes us to a highly symmetric planar waveguide whose modes are easily calculated and which can immediately be transformed to the modes of the touching cylinders. The electron trajectory in the transformed

frame maps to a circle traversed at a non uniform rate, but that need not concern us because the field has already been simply calculated in the other frame. Hence the loss problem is easily solved in this way.



**Fig. 4. Electron energy loss can be calculated simply using transformation optics.** (A) shows the physical system with a high energy electron passing close to a pair of touching nanowires represented as cylinders. (B) an inversion about the touching point takes points at the origin to infinity and points at infinity to the origin. The cylinders morph into a waveguide contained between two parallel metal plates, and the electron now makes a circular trajectory starting slowly at the origin, moving more rapidly at the furthest point from the origin, and slowing again before returning to the origin. The electromagnetic field of the electron is most simply calculated in (A), and that of the metallic system in (B).

**Perfect lensing.** Another theoretical challenge to which TO has been applied is the perfect lens.

Veselago (26) made a study of the then hypothetical materials which show negative refraction.

He identified the necessary conditions for this effect which comprise  $\epsilon < 0$ ,  $\mu < 0$ . Many years

later metamaterials have enabled his ideas to be realized (27-30). Amongst many suggestions

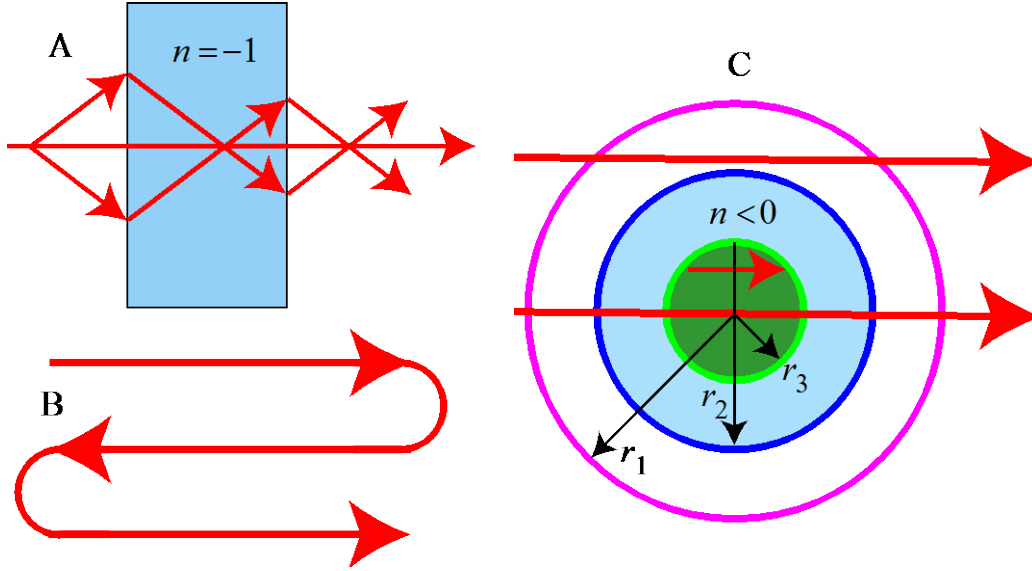
from that earlier work was the possibility of focusing light using negative refraction: Veselago

used the ray picture to show that light is naturally focused by a negative index slab of material

(Fig. 5A). Of itself this result is unremarkable: there are many devices that focus light, but an

alternative derivation using transformation optics casts new light on the Veselago lens. We have discussed how compressing a region of space by a factor  $m$  results in  $\epsilon$  &  $\mu$  parallel to the compression being reduced by a factor of  $m$ , whereas in the perpendicular direction that  $\epsilon$  &  $\mu$  are increased by  $m^{-1}$ . Now let us push matters to extreme: infinite compression leads to infinities in one direction and zeros in the other, and going beyond that to negative values of  $m$  implies negative values of  $\epsilon$  &  $\mu$  if we take the mathematics seriously. If  $m = -1$  we regain an isotropic system but with  $\epsilon = -1$ ,  $\mu = -1$  and hence according to Veselago the refractive index is  $n = -1$ . In geometrical terms we have folded space over onto itself (Fig. 5B). This means that according to the transformation light passes through the same region of space three times coming to the same focus each time. Furthermore, because transformation optics is exact at the level of Maxwell's equations, this is an exact statement and the focus has the same level of perfection on each of the three manifolds. This conclusion was reached some time ago (31) by a more elaborate multiple scattering treatment of the problem with the conclusion that the Veselago lens is perfect in the sense that its resolution is limited only by the perfection of manufacture, not by fundamental constraints. This gives yet another instance that transformation optics is not merely a tool for computation, but provides insight and understanding of complex problems.

The apparent paradox of the light being in three places at once is resolved by the fact that the condition  $n = -1$  can be satisfied exactly only at a single frequency and therefore a causal interpretation in the time domain is not available.



**Fig. 5. Geometry and negative refraction.** (A) The Veselago lens. (B) If negative refraction is interpreted as ‘negative space’ the light appears to move in a folded space. (C) Transformation to a system of spherical coordinates makes a perfect magnifying glass which to an external observer displays everything within the green sphere appear within the magenta sphere in magnified form.

Transformation optics has extended the concept of the perfect lens to the case of a magnifying lens. If we apply to the perfect lens a transformation,

$$\begin{aligned} x &= r_0 \exp(r'/R) \sin \theta' \cos \phi', \\ y &= r_0 \exp(r'/R) \sin \theta' \sin \phi', \\ z &= r_0 \exp(r'/R) \cos \theta' \end{aligned}$$

then in the primed frame we have a spherically symmetric system shown in figure 5C with the parameters,

$$\begin{aligned} \epsilon_x = \epsilon_y = \epsilon_z &= +r_2^2/r_3^2, & 0 < r < r_3 \\ \epsilon_x = \epsilon_y = \epsilon_z &= -r_2^2/r^2, & r_3 < r < r_2 \\ \epsilon_x = \epsilon_y = \epsilon_z &= +1, & r_2 < r < \infty \end{aligned}$$

where  $r_1, r_2, r_3$  are the radii of consecutive spheres, outermost to innermost. This new lens, colored in cyan, is described in more detail in (5,32,33) and has the property that everything inside the green inner sphere is perfectly imaged as a magnified object into the region within the

outer magenta sphere, and conversely any object between the magenta sphere and the lens, appears demagnified within the inner sphere. This leads to a paradox: an observer to the right of the lens would observe the red upper ray as having passed through the central region of the lens because the theory says that it has to. In particular if the region inside the green sphere were filled with a black material, to an external observer the whole of the large sphere outlined in magenta would appear black. The ray picture would say that this is not possible because rays that do not strike the blue sphere cannot be refracted into the central sphere. This superficially logical statement is wrong: one of the many curious consequences of negative refraction. The paradox is resolved if we allow for optical ‘tunneling’ between rays which is allowed in the wave picture.

**Concluding remarks.** Transformation optics is a tool that can be applied across the entire electromagnetic spectrum. Here we have selected a few examples of its power. Already the concept has spread to acoustics (34), and other wave like phenomena (35). In the future we can expect many more applications as its utility comes to be recognized.

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