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TRANSIENT DISTURBANCES IN A RELAXING THERMOELASTIC HALF SPACE DUE TO MOVING INTERNAL HEAT SOURCE

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ABSTRACT. This paper is concerned with the transient waves created by a line heat source that suddenly starts moving with a uniform velocity inside a thermoelastic semi-infinite medium with thermal relaxation of the type of Lord and Shulman The source moves parallel to the boundary surface which is traction-free. The problem is reduced to the solution of three differential equations, one involving the elastic vector potential, and the other two coupled, involving the thermoelastic scalar potential and the temperature. Using Fourier and Laplace transforms, the solution for the displacements have been obtained in the transform domain. The displacements have been calculated on the boundary surface for small time

KEY WORDS AND PHRASES: Thermally relaxing medium, moving source, thermal wave
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1. INTRODUCTION

The problem of heat sources acting in an elastic body is one of mathematical interest as well as of physical importance The dynamic heat source problem was first investigated by Danilovskaya [1] using the uncoupled theory of thermoelasticity. The problem of instantaneous and moving heat sources in infinite and semi-infinite space, and static line heat sources in semi-infinite space were considered by Eason and Sneedon [2], Nowacki [3] and others, under the coupled theory of thermoelasticity Dhaliwal and Singh [4] gave the short time approximation due to a suddenly applied point source inside an infinite space. Nariboli and Nyayadhish [5] gave exact solutions of the one-dimensional coupled problem of impulse and thermal shock at the end of a semi-infinite rod for small time.

However, the coupled theory of thermoelasticity suffers from a serious drawback, namely, the heat conduction equation is parabolic and consequently predicts an infinite velocity for heat propagation To remedy this defect the theory of generalized thermoelasticity with one relaxation time was formulated by Lord and Shulman [6]. The heat conduction equation here is a hyperbolic one so that there is a finite speed of propagation for thermal waves.

Nayfeh and Nemat Nasser [7, 8] studied the problem of transient waves in thermally relaxing solids Wang and Dhaliwal [9] have studied the fundamental solutions for generalized thermoelasticity, including problems of body force and heat source.

In this paper, we make a study of the thermoelastic disturbances created by an internal line heat source that suddenly starts moving uniformly inside a semi-infinite space with thermal relaxation. The problem is solved by using joint Fourier and Laplace transforms. The expressions for displacements in the transform domain indicate the existence of dilatational, transverse and thermal waves inside the medium. The displacement components have been evaluated on the boundary for small time only, the general inversion being too complicated. Presence of high thermal damping makes the short time solution meaningful.

2. FORMULATION OF THE PROBLEM

We consider a homogeneous isotropic thermoelastic solid occupying the region $x_2 \geq -h$ which is initially at rest, and the free surface $x_2 = -h$ which is stress free. A line source starts moving suddenly inside the medium at a depth h below the free surface uniformly in the x_1 direction. The line source is parallel to the x_3 axis so that all quantities are independent of x_3, and the third component u_3 of the displacement vector vanishes. The governing equations are:

1. Strain displacement relation

$$2e_{ij} = u_{i,j} + u_{j,i}, \quad i,j = 1,2. \tag{2.1}$$

2. Stress-strain relation

$$\tau_{ij} = 2\mu e_{ij} + \lambda e_{kk}\delta_{ij} - \gamma\theta\delta_{ij}, \quad i,j = 1,2. \tag{2.2}$$

3. Heat conduction equation

$$\rho c_v(\dot{\theta} + \tau_0\ddot{\theta}) + \gamma\theta_0(\dot{u}_{i,i} + \tau_0\ddot{u}_{i,i}) - Q - \tau_0\dot{Q} = k\theta_{,ii}, \quad i = 1,2. \tag{2.3}$$

4. Equations of motion

$$(\lambda+\mu)u_{j,ij} + \mu u_{i,jj} - \gamma\theta_{,i} = \rho\ddot{u}_i, \quad i,j = 1,2. \tag{2.4}$$

5. Initial conditions and boundary conditions

The initial conditions are

$$\left.\begin{array}{llll} u_i = 0, & \theta = 0, & \text{at } t \leq 0 & \text{in } x_2 \geq -h \\ \dot{u}_i = 0, & \dot{\theta} = 0, & \text{at } t \leq 0 & \text{in } x_2 \geq -h, \end{array}\right\} \quad i = 1,2. \tag{2.5}$$

The stress-free boundary conditions are

$$\tau_{12} = \tau_{22} = 0 \quad \text{on} \quad x_2 = -h \quad \text{for} \quad t \geq 0. \tag{2.6}$$

The regularity conditions are

$$\theta, u_i \to 0 \quad \text{as} \quad x_2 \to \infty,\ x_1 \to \pm\infty. \tag{2.7}$$

The thermal boundary condition at $x_2 = -h$ is

$$H_1\theta + H_2\theta_{,2} = 0. \tag{2.8}$$

Scalar and vector potentials ϕ, $(0,0,\psi)$ are introduced as follows:

$$\left.\begin{array}{l} u_1 = \phi_{,1} + \psi_{,2} \\ u_2 = \phi_{,2} - \psi_{,1} \end{array}\right\}, \tag{2.9}$$

where

$$\phi = \phi(x_1,x_2,t) \quad \text{and} \quad \psi = \psi(x_1,x_2,t).$$

Taking divergence and curl of equations (2.4) gives

$$\nabla^2\phi - m\theta = \frac{1}{c_L^2}\ddot{\phi}, \tag{2.10}$$

$$\nabla^2\psi = \frac{1}{c_T^2}\ddot{\psi}, \tag{2.11}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}.$$

From equation (2.3), we have

$$k\nabla^2\theta = \rho c_v\left(\dot{\theta} + \tau_0\ddot{\theta}\right) + r\theta_0\left(\nabla_2\dot{\phi} + \tau_0\nabla^2\ddot{\phi}\right) - Q - \tau_0\dot{Q}. \tag{2 12}$$

Introducing the following dimensionless quantities

$$\begin{aligned}
&\overline{x}_i = x_i/(C_T\omega),\ \ \overline{u}_{0i}/(C_T\omega),\ \ \overline{\theta} = m\theta,\\
&\overline{\theta} = \phi/(C_T\omega)^2,\ \ \overline{\psi}/(C_T\omega)^2,\ \ \overline{t} = t/\omega,\\
&\overline{\tau} = \tau_0/\omega,\ \ \overline{Q} = m\omega Q/\rho c_v,\ \ \overline{\tau}_{ij} = \tau_{ij}/\mu,
\end{aligned} \tag{2.13}$$

$$\text{where } \omega = k/\rho c_v C_T^2 \text{ is of the dimension of time,} \tag{2 14}$$

the equations (2.10)-(2.12) become

$$\overline{\nabla}^2\overline{\phi} - \overline{\theta} = \frac{1}{\beta^2}\ddot{\overline{\phi}}, \tag{2.15}$$

$$\overline{\nabla}^2\overline{\psi} = \ddot{\overline{\psi}}, \tag{2 16}$$

$$\overline{\nabla}^2\overline{\theta} = \left(\dot{\overline{\theta}} + \overline{\tau}\ddot{\overline{\theta}}\right) + \epsilon\left(\overline{\nabla}^2\dot{\overline{\phi}} + \overline{\tau}\overline{\nabla}^2\ddot{\overline{\phi}}\right) - \overline{Q} - \overline{\tau}\dot{\overline{Q}}, \tag{2 17}$$

where

$$\overline{\nabla}^2 \equiv \frac{\partial^2}{\partial \overline{x}_1^2} + \frac{\partial^2}{\partial \overline{x}_2^2} \tag{2.18}$$

and $\epsilon = \frac{\gamma^2\theta_0}{\rho^2 c_v C_L^2}$ is the coupling parameter.

Henceforth we shall omit the bars in equation (2.15)-(2.17). We shall also write x, y for x_1, x_2 and u, v for u_1, u_2 respectively.

The Laplace transform of a function $A(x, y, t)$ with respect to t is defined as

$$\mathfrak{L}(A(x,y,t)) = \int_0^\infty e^{-pt}A(x,y,t)dt = \tilde{A}(x,y,p)$$

and the Fourier transform of $\tilde{A}$ as

$$\mathfrak{F}\left(\tilde{A}(x,y,p)\right) = \int_{-\infty}^\infty e^{-i\xi x}\tilde{A}(x,y,p)dx = A^*(\xi,y,p).$$

Inverting the transforms gives

$$A(x,y,t) = \mathfrak{L}^{-1}\left\{\frac{1}{2\pi}\int_{-\infty}^\infty A^*(\xi,y,p)e^{i\xi x}d\xi\right\}$$

where $\mathfrak{L}^{-1}$ denote the inverse Laplace transform.

Taking Laplace and Fourier transform of both sides of equations (2.15)-(2.17), we have

$$\left[\frac{d^2}{dy^2} - \xi^2 - \frac{p^2}{\beta^2}\right]\phi^* = \theta^*, \tag{2 19}$$

$$\left[\frac{d^2}{dy^2} - \xi^2 - p^2\right]\psi^* = 0, \tag{2.20}$$

$$\left(\frac{d^2}{dy^2} - \xi^2 - p(1+p\tau)\right)\theta^* - \epsilon p(1+p\tau)\left(\frac{d^2}{dy^2} - \xi^2\right)\phi^2 = -(1+p\tau)Q^*. \tag{2.21}$$

The stresses are transformed as

$$\tau_{xx}^* = 2i\xi\frac{d\psi^*}{dy} + (2\xi^2 + p^2)\phi^*, \tag{2.22}$$

$$\tau_{yy}^* = (p^2 + 2\xi^2)\phi^* - 2i\xi\frac{d\psi^*}{dy}, \tag{2.23}$$

$$\tau_{xy}^* = 2i\xi\frac{d\phi^*}{dy} + \frac{d^2\psi^*}{dy^2} + p^2\psi^*. \tag{2.24}$$

The moving source is located at the origin, at time $t = 0^+$ and starts moving along the positive x-axis, with uniform velocity V. The source is assumed in the form

$$Q = Q_0\delta(x - Vt)\delta(y)H(t), \quad Q^* = Q_0\delta(y)/(p + i\xi V). \tag{2.25}$$

In solving the above equations (2.19) (2.21), we impose that ϕ^*, ψ^*, θ^*, satisfy the regularity conditions at infinity. Moreover,

$$\tau_{xy}^* = \tau_{yy}^* = 0 \quad \text{on} \quad y = -h. \tag{2.26}$$

Further, since the stress components are continuous across $y = 0$, it follows that

$$\phi^*, \ \frac{d\phi^*}{dy}, \ \frac{d^2\phi^*}{dy^2}, \ \psi^*, \ \frac{d\psi^*}{dy}, \ \frac{d^2\psi^*}{dy^2}, \tag{2.27a}$$

are all continuous across $y = 0$.

To obtain the jump discontinuity due to the presence of $\delta(y)$ in Q^*, equation (2.21) is integrated from $y = -\eta$ to $y = \eta$, $(\eta > 0)$, and finally η made to tend to 0^+

$$\left.\frac{d^3\phi^*}{dy^3}\right|_{y\to 0^\pm} = \left.\frac{d\theta^*}{dy}\right|_{y\to 0\pm} = -(1+p\tau)\frac{Q_0}{p + i\xi V}. \tag{2.27b}$$

3. SOLUTION OF THE PROBLEM

Substituting from equation (2.19) in equation (2.21), we get a fourth order equation in ϕ^*

$$\left[d^2 - \left(\frac{p^2}{\beta^2} + p(1+\epsilon)(1+p\tau)\right)D + \frac{p^2}{\beta^2}(1+p\tau)\right]\phi^* = -Q_0\frac{(1+p\tau)}{(p \rightarrow i\xi V)}\delta(y), \tag{3.1}$$

where

$$D \equiv \frac{d^2}{dy^2} - \xi^2. \tag{3.2}$$

The solution of (3.1) satisfying regularity conditions is

$$\phi^* = \begin{cases} A_1e^{-\alpha_1 y} + A_2e^{-\alpha_2 y} + A_3e^{\alpha_1 y} + A_4e^{\alpha_2 y}, & -h < y < 0 \\ B_1e^{-\alpha_1 y} + B_2e^{-\alpha_2 y}, & y > 0 \end{cases} \tag{3.3}$$

$$\psi^* = C\exp\{-(\xi^2 + p^2)^{1/2}y\}, \quad -h < y < \infty \tag{3.4}$$

where

$$a_{1,2}^2 = \xi^2 + \frac{1}{2}\left[p(1+\epsilon)(1+p\tau) + \frac{p^2}{\beta^2}\right] \pm \frac{1}{2}\left[\left(\frac{p^2}{\beta^2} + p(1+p\tau)(1+\epsilon)\right)^2 - \frac{4p^3}{\beta^2}(1+p\tau)\right]^{1/2}, \quad (3.5)$$

$a_{1,2}^2$ both are assumed to be real and positive.

The displacement fields u^*, v^* are

$$u^* = \begin{cases} i\xi(A_1e^{-a_1y} + A_2e^{-a_2y} + A_3e^{a_1y} + A_4e^{a_2y}) - Cb_1e^{-b_1y}, & -h < y < 0 \\ i\xi B_1e^{-a_1y} + i\xi B_2e^{-a_2y} - Cb_1e^{-b_1y}, & y > 0 \end{cases} \quad (3.6)$$

$$v = \begin{cases} -a_1A_1e^{-a_1y} - a_2A_2e^{-a_2y} + a_1A_3e^{a_1y} + a_2A_4e^{a_2y} - i\xi Ce^{-b_1y}, & -h < y < 0 \\ -a_1B_1e^{-a_1y} - a_2B_2e^{-a_2y} - i\xi Ce^{-b_1y}, & 0 < y < \infty \end{cases} \quad (3.7)$$

where
$$b_1 = \left(\xi^2 + p^2\right)^{1/2}. \quad (3.8)$$

Using conditions (2.27a) and (2.27b), we obtain a set of four equations involving the six constants A_1, A_2, A_3, A_4, B_1, B_2:

$$A_1 + A_2 + A_3 + A_4 - B_1 - B_2 = 0 \quad (3.9)$$

$$a_1A_1 + a_2A_2 + a_1A_3 + a_2A_4 - a_1B_1 - a_2B_2 = 0 \quad (3.10)$$

$$a_1^2A_1 + a_2^2A_2 + a_1^2A_3 + a_2^2A_4 - a_1^2B_1 - a_2^2B_2 = 0 \quad (3.11)$$

$$a_1^3A_1 + a_2^3A_2 + a_1^3A_3 + a_2^3A_4 - a_1^3B_1 - a_2^3B_2 = Q_0\,\frac{(1+p\tau)}{p+i\xi V}. \quad (3.12)$$

The conditions (2.26) imply

$$d\left(A_1e^{a_1h} + A_2e^{a_2h} + A_3e^{-a_1h} + A_4e^{-a_2h}\right) + 2i\xi b_1Ce^{b_1h} = 0, \quad (3.13)$$

$$2i\xi\left(a_1A_1e^{a_1h} + a_2A_2e^{a_2h} - a_1A_3e^{-a_1h} - a_2A_4e^{-a_2h}\right) - dCe^{b_1h} = 0, \quad (3.14)$$

where
$$d = p^2 + 2\xi^2. \quad (3.15)$$

The thermal boundary condition (2.8) gives

(A) $\quad H_1 = 1,\ H_2 = 0$ i.e., $\theta = 0$ on $y = -h$, (3.16)

(the zero temperature boundary condition)

(B) $\quad H_1 = 0,\ H_2 = 1$ i.e., $\theta_{,y} = 0$ on $y = -h$, (3.17)

(the zero heat-flux condition on the boundary).

For case (A), on using equation (3.3) and (2.19) condition (3.16) reduces to

$$A_1\left(a_1^2 - b_2^2\right)e^{a_1h} + A_2\left(a_2^2 - b_2^2\right)e^{a_2h} + A_3\left(a_1^2 - b_2^2\right)e^{-a_1h} + A_4\left(a_2^2 - b_2^2\right)e^{-a_2h} = 0 \quad (3.18)$$

where
$$b_2 = \xi^2 + \left(p^2/\beta^2\right). \quad (3.19)$$

From equations (3.9)-(3.14) and (3.18), we obtain

$$A_1 = -K[e^{(a_2-a_1)h}(a_1+a_2)\{d^2(a_1-a_2) - 4\xi^2b_1a_1a_2 + 4\xi^2b_1b_2^2\} + 8a_1\xi^2b_1\left(a_2^2 - b_2^2\right)]/a_1\triangle e^{(a_1+a_2)h} \quad (3.20)$$

$$A_2 = -K\left[e^{(a_2-a_1)h}(a_1+a_2)\left\{d^2(a_2-a_1)-4\xi^2b_1a_1a_2+4\xi^2b_1b_2^2\right\} + 8a_2\xi^2b_1\left(a_1^2-b_2^2\right)\right]/a_2\triangle e^{(a_1+a_2)h}. \tag{3.21}$$

$$A_3 = K/a_1, \quad B_1 = A_1 + A_3, \quad B_2 = A_2 + A_4,$$

and

$$C = ide^{-b_1h}\left(A_1e^{a_1h} + A_2e^{a_2h} + A_3e^{-a_1h} + A_4e^{-a_2h}\right)/2\xi b_1,$$

where

$$K = \frac{Q_0(1+p\tau)}{2(a_1^2-a_2^2)(p+i\xi V)}, \tag{3.21}$$

$$\triangle = (a_1-a_2)\left\{d^2(a_1+a_2) - 4\xi^2b_1a_1a_2 - 4\xi^2b_1b_2^2\right\}. \tag{3.22}$$

The surface displacements are

$$u^*\Big|_{y=-h} = \frac{2_iQ_0p^2\xi b_1(1+p\tau)}{(p+i\xi V)\triangle}\left(e^{-a_1h}-e^{-a_2h}\right), \tag{3.23}$$

$$v^*\Big|_{y=-h} = \frac{Q_0p^2d(1+p\tau)}{(p+i\xi V)\triangle}\left(e^{-a_1h}-e^{-a_2h}\right). \tag{3.24}$$

For the case B, the condition (3.17) can be written as

$$a_1A_1\left(a_1^2-b_2^2\right)e^{a_1h} + a_2A_2\left(a_2^2-b_2^2\right)e^{a_2h} - a_1A_3\left(a_1^2-b_2^2\right)e^{-a_1h} - a_2A_4\left(a_2^2-b_2^2\right)e^{-a_2h} = 0, \tag{3.25}$$

which when combined with (3.9)-(3.14) yield the corresponding values of the constants. The surface displacements are given by

$$u^*\Big|_{y=-h} = \frac{2_iQ_0p^2\xi b_1(1+p\tau)}{(p+i\xi V)\triangle_1}\left(a_2e^{-a_1h}-a_1e^{-a_2h}\right), \tag{3.26}$$

$$v^*\Big|_{y=-h} = \frac{Q_0p^2d(1+p\tau)}{(p+i\xi V)\triangle_1}\left(a_2e^{-a_1h}-a_1e^{-a_2h}\right), \tag{3.27}$$

where

$$\triangle_1 = (a_1-a_2)\left[d^2\left(a_1^2+a_1a_2+a_2^2-b_2^2\right) = 4\xi^2b_1a_1a_2(a_1+a_2)\right]. \tag{3.28}$$

4. SMALL TIME APPROXIMATION

To the first order of approximation in ϵ_1, a_1^2 and a_2^2 may be written as

$$a_1^2 = \xi^2 + \frac{p^2}{\beta^2} - \frac{p^2(1+p\tau)}{\beta^2(1+p\tau')}\epsilon, \tag{4.1}$$

$$a_2^2 = \xi^2 + \tau p^2 + p + \frac{p(1+p\tau)^2}{1+p\tau'}\epsilon, \tag{4.2}$$

where

$$\tau' = \tau - \frac{1}{\beta^2}. \tag{4.3}$$

For a short time approximation to the displacement components, we expand a_1, a_2, b_1 in terms of powers of p, and consider relevant terms as $p \to \infty$.

Then

$$a_1 \sim \frac{\epsilon}{2\beta^3\tau'^2} + \frac{pm_1}{\beta} \tag{4.4}$$

$$a_2 \sim \frac{1}{2\sqrt{\tau}} - \frac{\epsilon\sqrt{\tau}}{2\beta^2\tau'^2} + p\sqrt{\tau}\, m_2 \tag{4.5}$$

$$b \sim p + \frac{\xi^2}{2p} \tag{4.6}$$

where

$$m_1 = 1 - \frac{\epsilon\tau}{2\tau'}, \quad m_2 = 1 + \frac{\epsilon\tau}{2\tau'}. \tag{4.7}$$

It is clear from (4.4)-(4.6) that there are three waves with velocities β/m_1, $1/m_2\sqrt{\tau}$ and 1 respectively representing the dilatational, the thermal and the transverse elastic waves.

However, the inversion of u^*, v^* inside the medium is too complicated, we evaluate the surface displacements only from (3.29) and (3.30) for the zero temperature on the boundary.

For a small time, using (4.4), (4.5) in (3.23), (3.24),

$$u^*\Big|_{y=-h} \sim \frac{2iQ_0\xi(1+p\tau)(e^{-a_1 h} - e^{-a_2 h})}{(p+i\xi V)p^3 L} \tag{4.7}$$

$$v^*\Big|_{y=-h} \sim \frac{Q_0(1+p\tau)(e^{-a_1 h} - e^{-a_2 h})}{(p+i\xi V)p^2 L}, \tag{4.8}$$

where

$$L = \left[\left(\frac{1}{\beta^2} + \tau + \epsilon\tau\right)^2 - \frac{4\tau}{\beta^2}\right]^{1/2}. \tag{4.9}$$

Finally, taking inverse Laplace and Fourier transform from (4.7) and (4.8) gives

$$u|_{y=-h} \approx \frac{2Q_0}{LV}\left[\exp\left\{-\left(\epsilon h/2\beta^3\tau'^2\right)\right\}(f_1 - f_2) - e^{\frac{-h}{2\sqrt{\tau}}\left(1-\frac{\epsilon\tau}{\beta^2\tau'^2}\right)}(f_3 - f_4)\right], \tag{4.10}$$

$$v|_{y=-h} \approx \frac{Q_0}{L}\left[\exp\left\{-\left(\epsilon h/2\beta^3\tau'^2\right)\right\}f_2 - e^{-\frac{h}{2\sqrt{\tau}}\left(1-\frac{\epsilon\tau}{\beta^2\tau'^2}\right)}f_4\right], \tag{4.11}$$

where

$$f_1 = \left(t - \frac{m_1 h}{\beta}\right)\left(\tau + \frac{1}{2}\left(t - \frac{m_1 h}{\beta}\right)\right)\delta(x) H\left(t - \frac{m_1 h}{\beta}\right), \tag{4.12}$$

$$f_2 = \frac{1}{V}\left(t + \tau + \frac{x}{V} - \frac{m_1 h}{\beta}\right) H\left(t - \frac{m_1 h}{\beta} - \frac{x}{V}\right), \tag{4.13}$$

$$f_3 = \left(t - m_2 h\sqrt{\tau}\right)\left(\tau + \frac{1}{2}\left(t - m_2 h\sqrt{\tau}\right)\right)\delta(x) H\left(t - m_2 h\sqrt{\tau}\right), \tag{4.14}$$

$$f_4 = \frac{1}{V}\left(t + \tau + \frac{x}{V} - m_2 h\sqrt{\tau}\right) H\left(t - m_2 h\sqrt{\tau} - \frac{x}{V}\right). \tag{4.15}$$

It is observed that the surface displacements for small time consist of dilatational waves propagating with velocity (β/m_1) and a thermal wave moving with velocity $\left(1/m_2\sqrt{\tau}\right)$. Also the waves are attenuated by exponential factors depending on ϵ and τ. The terms containing f_1, f_3 in u represent the displacement at the point $x = 0$, while terms containing f_2, f_4 represent the surface disturbance up to the point above the position of source at the time. The non-relaxing thermoelastic case may be obtained simply by putting $\tau = 0$ in the above results.

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