Transient Hyperbolic Heat Conduction in a Functionally Graded Hollow Cylinder

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Hyperbolic heat conduction in a functionally graded hollow cylinder is investigated in this paper. Except for uniform thermal relaxation time, all other material properties of the cylinder are assumed to vary along the radial direction following a power-law form with arbitrary exponents known as the nonhomogeneity indices. When the cylinder is infinitely long, end effects can be ignored, and the one-dimensional heat conduction problem in the radial direction is solved analytically in the Laplace domain. The final transient solution of the problem in the time domain is obtained by numerical inversion of the Laplace transformed temperature and heat flux. The exact speed of the thermal wave in the nonhomogeneous cylinder is also obtained. Moreover, the effects of the nonhomogeneity indices and thermal relaxation time on the results are shown graphically by some illustrative examples. The current results are corroborated by the steady-state results for the homogeneous cylinder in the literature.

Nomenclature

A_1, A_2	=	integration constants given by Eqs. (18)
a	=	arbitrary real number that is greater than all the real parts of singularities of a Laplace transformed function
C	=	nondimensional velocity of the thermal waves
c_p	=	specific heat at constant pressure, J/(kg K)
c_{po}	=	specific heat of the outer surface of the cylinder, J/(kg K)
F	=	nondimensional function defined by Eq. (11)
$f, \tilde{f}$	=	general function and its Laplace transform
G, H and I	=	nondimensional parameters defined by Eqs. (15)
i	=	imaginary unit
J_G	=	Gth-order Bessel function of the first kind
K	=	thermal conductivity, W/(m K)
K_o	=	thermal conductivity of the outer surface of the cylinder, W/(m K)
L	=	number of terms needed in the Laplace inversion process
M	=	parameter defined by Eqs. (17)
N	=	number of time points to which the total time is divided in the Laplace inversion process
n_i	=	nonhomogeneity index; $i = 1, 2$ and 3
P	=	parameter defined by Eqs. (17)
$Q, \tilde{Q}$	=	nondimensional heat flux and its Laplace transform
q	=	heat flux vector, W/m²
q_r	=	radial component of heat flux, W/m²
R	=	internal heat generation, W/m³
r	=	radial coordinate, m
r_i, r_o	=	inner and outer radii of the cylinder, respectively; m
r_γ	=	relative thickness of the hollow cylinder, r_i/r_o
s	=	Laplace variable
T	=	absolute temperature, K
T_{total}	=	total time over which the Laplace inversion is performed
T_{wi}	=	absolute temperature of the inner surface of the cylinder, K
T_{wo}	=	absolute temperature of the outer surface of the cylinder, K
T_γ	=	relative temperature change, $(T_{wi} - T_\infty)/(T_{wo} - T_\infty)$
T_∞	=	ambient temperature, K
t	=	time, s
$v_{thermal}$	=	velocity of thermal waves in hyperbolic heat conduction theory, $\sqrt{K/\rho c_p \tau}$; m/s
W, X	=	parameters defined by Eqs. (19)
Y_G	=	Gth-order Bessel function of the second kind
Z	=	parameter defined by Eqs. (19)
Δt	=	time increment, s
ε_o	=	nondimensional thermal relaxation time, $\kappa_o \tau/r_o^2$
η	=	nondimensional radial coordinate, r/r_o
$\theta, \tilde{\theta}$	=	nondimensional temperature change, $(T - T_\infty)/(T_{wo} - T_\infty)$ and its Laplace transform, respectively
κ_o	=	thermal diffusivity of the outer surface of the cylinder, $K_o/(\rho_o c_{po})$; m²/s
ξ	=	nondimensional time, $\kappa_o t/r_o^2$
ρ	=	density, kg/m³
ρ_o	=	density of the outer surface of the cylinder, kg/m³
τ	=	thermal relaxation time, s
$()_{,s}$	=	the partial differentiation of ( ) with respect to s

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Introduction

FUNCTIONALLY graded materials (FGMs) are nonhomogeneous materials within which physical properties vary continuously. The smooth variation of properties results from continuous transition of the volume fraction of constituents. First introduced in 1984 in the aerospace industry as a thermal shock barrier [1], FGMs have found many other engineering applications. FGMs are currently used in many applications, such as heat engine components, wear resistant linings, and even prostheses [2].

Heat conduction analysis of FGMs is of great importance, as they are used as thermal shields. Temperature gradients cause thermal stresses that may result in crack growth and eventually fracture of the structure. Hence, to design a reliable FGM structure working under severe thermal loadings, it is crucial to know the temperature distribution within it.

There are different theories about heat conduction in solids. The famous Fourier heat conduction theory relates heat flux directly to the temperature gradient using a proportionality coefficient known as thermal conductivity. The accuracy of Fourier's heat conduction law is sufficient for many practical engineering applications. However,

this theory cannot accurately explain conduction of heat caused by highly varying thermal loading such as pulsed laser heating. For example, the surface temperature of a slab measured immediately after a sudden thermal shock is 300°C higher than that predicted by Fourier's law [3]. The Fourier heat conduction theory also breaks down at very low temperatures and when the applied heat flux is extremely large [4]. In addition, Fourier's law results in an infinite speed of thermal wave propagation, which is physically unrealistic. To better explain heat conduction in solids, non-Fourier heat conduction theories have been developed. One of the non-Fourier theories is hyperbolic heat conduction theory. This theory, separately proposed by Vernotte [5] and Cattaneo [6], accounts for the finite speed of thermal energy propagation by introducing a new material property called thermal relaxation time. Thermal relaxation time is the time that the temperature field needs to adjust itself to thermal disturbances. This theory is called hyperbolic heat conduction because it results in a hyperbolic differential equation for temperature rather than the parabolic one obtained using Fourier's law.

There are many papers in the literature dealing with hyperbolic heat conduction in homogeneous solids. A few of these papers are described in the following discussion. Sadd and Cha [7] analytically solved the conduction problem for regions interior and exterior to long cylinders. However, their solution is limited to small time durations. Lin and Chen [8] employed the Laplace transform technique in the time domain and the numerical finite volume method in the spatial domain to give the solution for hyperbolic heat conduction in both cylindrical and spherical objects. Periodic, steady heat conduction in long, hollow cylinders was investigated by Zanchini and Pulvirenti [9]. Jiang and Sousa [10] provided a completely analytical solution for hyperbolic heat conduction problems in homogeneous spheres. Tsai and Hung [11] investigated thermal wave propagation in a bilayered composite sphere due to a sudden temperature change on the outer surface. The finite difference method has been employed by Darabseh et al. [12] to find thermal stresses induced by hyperbolic heat conduction in orthotropic cylinders. A review of available analytical solutions for hyperbolic heat conduction in homogeneous solids is given by Antaki [13].

The literature on Fourier heat conduction in FGMs is not widely developed as compared with that of hyperbolic heat conduction in homogeneous solids. Some papers dealt with power-law FGMs. Hosseini et al. [14] solved the Fourier heat conduction problem in an FGM cylindrical shell using direct separation of variables, viz., the radial coordinate and time. Eslami et al. [15] used Fourier's law to find temperature-induced stress in a hollow FGM sphere. The best transition of material properties is also given in [15] to optimize tangential stresses in a nonhomogeneous spherical vessel. Tarn and Wang [16] used the state-space approach to analyze transient and steady Fourier heat conduction in a power-law FGM cylinder.

To the authors' best knowledge, there are a few articles related to hyperbolic heat conduction in FGM structures. Fang and Hu [17] investigated the propagation of hyperbolic thermal waves caused by a spherical substrate in a semi-infinite FGM medium. To simplify the solution procedure, they assumed that density and thermal relaxation are constant, while other properties vary exponentially in the direction normal to the boundary of the medium. Babaei and Chen [18] solved a transient hyperbolic heat conduction problem in an FGM hollow sphere.

In the present paper, similar to [18], the transient hyperbolic heat conduction problem is considered for an FG, long, hollow cylinder. Except for thermal relaxation, which is taken to be uniform to simplify the analytical solution in the Laplace domain, all other material properties are assumed to vary along the radial direction following a power law with an arbitrary exponent. As a first step in analyzing the non-Fourier heat conduction in FGM structures, dependence of material properties on temperature was not taken into account for simplicity. To find temperature and heat flux in this one-dimensional problem, the Laplace transform technique is used to eliminate time. The problem is solved analytically in the Laplace domain. Time dependency is then restored by numerically inverting the Laplace transform. Effects of the thermal relaxation and nonhomogeneity index of the cylinder on the results are further

investigated by a numerical example. In the steady-state case, the results are also verified for homogeneous cylinders. It is worth noting that, unlike a hollow sphere [18], the thermal wave speeds for the hollow cylinder can be expressed explicitly.

Problem Definition and Governing Equations

As shown in Fig. 1, we consider heat conduction in a radially FG, long, hollow cylinder. The initial temperature of the cylinder is the ambient temperature, T_∞. To represent a thermal shock (e.g., a sudden change of the surrounding temperature), the temperatures of the inner and outer surfaces of the cylinder are suddenly elevated to new values:

$$T(r,t)|_{r=r_i} = T_{wi} \qquad T(r,t)|_{r=r_o} = T_{wo} \qquad (1)$$

The hyperbolic heat conduction equation for isotropic solids is written as follows [19]:

$$\boldsymbol{q} + \tau \partial \boldsymbol{q}/\partial t = -K\nabla T \qquad (2)$$

On the other hand, for a conduction problem, the first law of thermodynamics accounting for energy conservation reads [13]:

$$\rho c_p \partial T/\partial t = R - \nabla \cdot \boldsymbol{q} \qquad (3)$$

The uniformly imposed boundary conditions and geometry make the current problem axisymmetric. Moreover, end effects in the axial direction are also eliminated as the cylinder is infinitely long. Thus, temperature and heat flux just depend on the radial coordinate and time. Then, Eqs. (2) and (3) are reduced to

$$-(1 + \tau\partial/\partial t)q_r = K\partial T/\partial r \qquad (4a)$$

$$-(1/r)\partial(rq_r)/\partial r = \rho c_p \partial T/\partial t \qquad (4b)$$

It is also assumed that the continuous transition of material properties in the radial direction follows a power law, except for thermal relaxation, which is taken to be constant:

$$K(\eta) = K_o \eta^{n_1}, \qquad \rho(\eta) = \rho_o \eta^{n_2}, \qquad c_p(\eta) = c_{po}\eta^{n_3} \qquad (5)$$

where $\eta = r/r_o$.

For convenience, we normalize Eqs. (4) by introducing the following parameters:

$$\xi = \kappa_o t/r_o^2, \qquad \varepsilon_o = \kappa_o\tau/r_o^2, \qquad r_\gamma = r_i/r_o$$
$$\theta = (T - T_\infty)/(T_{wo} - T_\infty), \qquad T_\gamma = (T_{wi} - T_\infty)/(T_{wo} - T_\infty)$$
$$Q = r_o q_r/(K_o T_\infty) \qquad (6)$$

Using these parameters and Eqs. (5), we can rewrite Eqs. (4) as

$$(1 + \varepsilon_o\partial/\partial\xi)Q = -\eta^{n_1}(T_{wo} - T_\infty)/T_\infty \partial\theta/\partial\eta \qquad (7a)$$

$$\partial(\eta Q)/\partial\eta = -\eta^{n_2+n_3+1}(T_{wo} - T_\infty)/T_\infty \partial\theta/\partial\xi \qquad (7b)$$
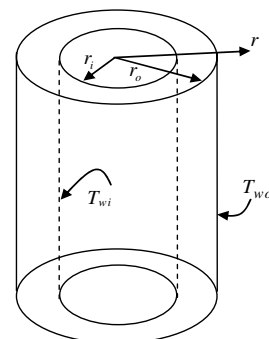

Fig. 1   Geometry and loading conditions of the FG hollow cylinder.

The boundary and initial conditions of the problem mentioned earlier, when normalized, read

$$\theta(\eta, \xi)|_{\eta=r_\gamma} = T_\gamma, \qquad \theta(\eta, \xi)|_{\eta=1} = 1$$

$$\theta(\eta, \xi)|_{\xi=0} = \theta_{,\xi}(\eta, \xi)|_{\xi=0} = 0 \qquad (8)$$

Solution Procedure

Eliminating the heat flux term, Q, between Eqs. (7a) and (7b), we can obtain a second-order differential equation for the normalized temperature change, θ, as follows:

$$\eta\theta_{,\eta\eta} + (n_1 + 1)\theta_{,\eta} - \eta^{n_2+n_3-n_1+1}(\theta_{,\xi} + \varepsilon_0\theta_{,\xi\xi}) = 0 \qquad (9)$$

The existence of second derivatives of the temperature change with respect to both the radial coordinate and time implies a wave nature of the temperature in the hyperbolic heat conduction theory.

We rewrite Eq. (9) in the following form:

$$\theta_{,\xi\xi} - (C(\eta))^2\theta_{,\eta\eta} = F(\eta, \partial\theta/\partial\xi, \partial\theta/\partial\eta) \qquad (10)$$

where

$$F = \frac{1}{\varepsilon_o}\left(\frac{n_1 + 1}{\eta^{n_2+n_3-n_1+1}}\theta_{,\eta} - \theta_{,\xi}\right) \qquad (11)$$

$C(\eta)$ is the velocity of the thermal waves within the FG cylinder:

$$C(\eta) = \eta^{1/2(n_1-n_2-n_3)}/\sqrt{\varepsilon_o} \qquad (12)$$

It is seen that the nondimensional velocity of the thermal waves depends both on the material nonhomogeneity indices, n_i, and the position, η. Discussion on the speed of the thermal waves is given in the next section.

To find the temperature change and heat flux, we now apply the Laplace transform to Eq. (9). The resulting equation is

$$\eta\tilde{\theta}_{,\eta\eta} + (n_1 + 1)\tilde{\theta}_{,\eta} - \eta^{n_2+n_3-n_1+1}s(1 + \varepsilon_o s)\tilde{\theta} = 0 \qquad (13)$$

where $\sim$ indicates the Laplace transform of a function.

The solution of the aforementioned homogeneous ordinary differential equation is

$$\tilde{\theta}(\eta, s) = (A_1 J_G(I\eta^H) + A_2 Y_G(I\eta^H))\eta^{-1/2n_1} \qquad (14)$$

where

$$G = n_1/(2 + n_2 + n_3 - n_1), \qquad H = 1 - 1/2(n_1 - n_2 - n_3)$$

$$I = 2\sqrt{-(1 + \varepsilon_o s)s}/(2 + n_2 + n_3 - n_1) \qquad (15)$$

Substituting Eq. (14) into the Laplace transform of Eq. (7a), we can obtain the heat flux in the transformed domain, $\tilde{Q}$, as follows:

$$\tilde{Q} = P/2\eta^{1/2n_1-1}(A_1(MJ_G(I\eta^H) - 2IH\eta^H J_{G+1}(I\eta^H))$$
$$+ A_2(MY_G(I\eta^H) - 2IH\eta^H Y_{G+1}(I\eta^H))) \qquad (16)$$

where

$$M = -n_1 + 2HG, \qquad P = -(T_{wo} - T_\infty)/((1 + \varepsilon_o s)T_\infty) \qquad (17)$$

We are now able to find A_1 and A_2 by satisfying the boundary conditions [Eqs. (8)] of the problem in the Laplace domain, as follows:

$$A_1 = Z/X, \qquad A_2 = W/X \qquad (18)$$

where

$$Z = -Y_G(Ir_\gamma^H) + r_\gamma^{1/2n_1}Y_G(I)T_\gamma$$

$$W = J_G(Ir_\gamma^H) - r_\gamma^{1/2n_1}J_G(I)T_\gamma \qquad (19)$$

$$X = s(-J_G(I)Y_G(Ir_\gamma^H) + J_G(Ir_\gamma^H)Y_G(I))$$

To restore the time effect, the solutions in the Laplace domain, $\tilde{\theta}$ and $\tilde{Q}$, must be inverted. As this inversion may not be simply done analytically, a numerical treatment is employed. We use the fast Laplace inversion technique method first proposed by Durbin [20].

In this method, the Laplace inverse of a function $(\tilde{f}(\eta, s))$ at time ξ_j is found as follows:

$$f(\eta, \xi_j) = D(j)\left[-1/2\,\mathrm{Re}\{\tilde{f}(\eta, a)\} + \mathrm{Re}\left\{\sum_{k=0}^{N-1}(A(\eta, k)\right.\right.$$
$$\left.\left. + iB(\eta, k))U^{jk}\right\}\right] \qquad j = 0, 1, 2, \ldots, N - 1 \qquad (20)$$

where

$$A(\eta, k) = \sum_{t=0}^{L}\mathrm{Re}\{\tilde{f}(\eta, a + i(k + lN)2\pi/T_{\text{total}})\}$$

$$D(j) = 2/T_{\text{total}}e^{aj\Delta t}, \qquad \Delta t = T_{\text{total}}/N$$

$$B(\eta, k) = \sum_{l=0}^{L}\mathrm{Im}\{\tilde{f}(\eta, a + i(k + lN)2\pi/T_{\text{total}})\}, \qquad U = e^{i2\pi/N} \qquad (21)$$

It should also be noted that a, an arbitrary real number, should be greater than any of the real parts of the function's singularities; parameters L and N influence the accuracy of the solution; Δt is the time increment; and T_{total} is the total time over which the numerical inversion is performed.

To minimize both discretization and truncation errors, it is recommended that the following constraints be observed :

$$5 \le aT_{\text{total}} \le 10 \qquad 50 \le NL \le 5000 \qquad (22)$$

For the present work, these parameters are taken as

$$aT_{\text{total}} = 7.5 \qquad L = 10 \qquad N = 500 \qquad (23)$$

Numerical Example

In the numerical calculations, we use the following values for the initial and boundary conditions: 1) the initial temperature of the cylinder is 300 K (ambient); 2) the temperature of the inner surface of the cylinder is kept at ambient temperature, that is, $T_\gamma = 0$; and 3) the temperature of the outer surface of the cylinder is $T_{wo} = 500$ K. The sudden increase in the temperature of the outer surface results in the generation of thermal waves from the outer surface toward the inner surface of the cylinder. The outer radius of the cylinder is taken to be $r_o = 1$ m, and the inner radius is such that $r_\gamma = 0.6$.

Figure 2 illustrates the effect of nonhomogeneity indices, n_i, on the position of thermal waves at three different times. In these plots, nonhomogeneity indices are taken to be the same for all properties varying along the radial coordinate, that is, $n_1 = n_2 = n_3 = n$. This allows us to focus exclusively on the effect of the nonhomogeneity.

When hyperbolic heat conduction theory is employed, the temperature field requires a finite amount of time to adjust to thermal disturbances. This can be observed in Fig. 2a at nondimensional time 0.1260. The inner portion of the cylinder has not yet responded to the temperature increase of the outer surface, that is, the temperature change and heat flux are still zero.

As seen in Figs. 2a and 2b, the wave fronts of the larger nonhomogeneity indices are ahead of those of the smaller nonhomogeneity indices. This means that a higher n results in a higher speed of the thermal wave propagation, just as Eq. (12) indicates. It is known that a higher thermal conductivity results in a higher speed of the thermal wave propagation. When n is increased, the average value of the thermal conductivity in the cylinder is increased, leading to a higher velocity of the thermal waves. It is also worth noting that the thermal waves dissipate with time and eventually vanish. Figure 2c shows the steady-state distribution of the temperature and heat flux for different nonhomogeneity indices. A higher n leads to larger temperature and heat flux values.
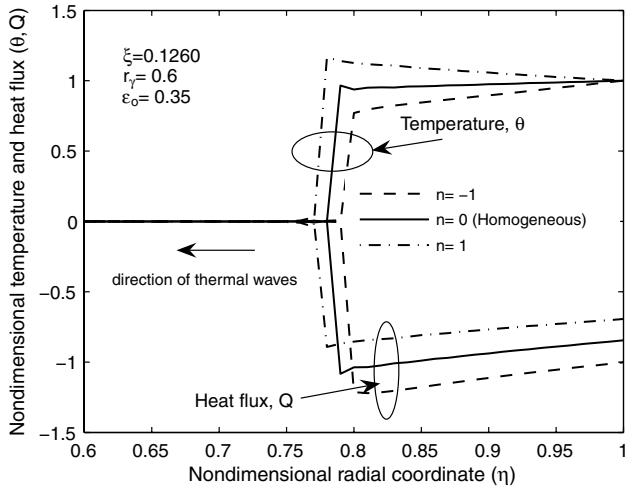
a)
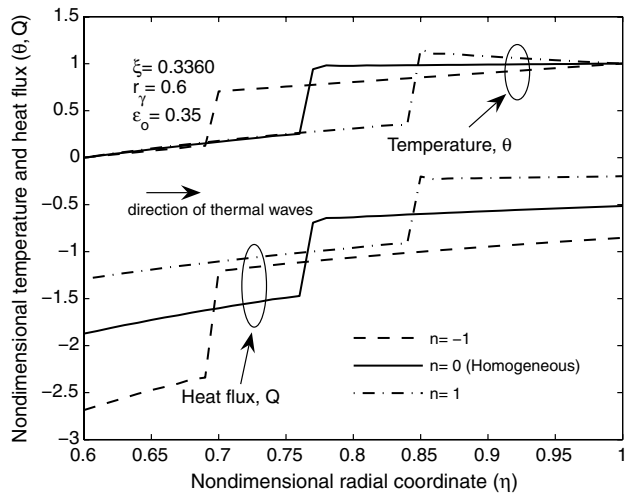
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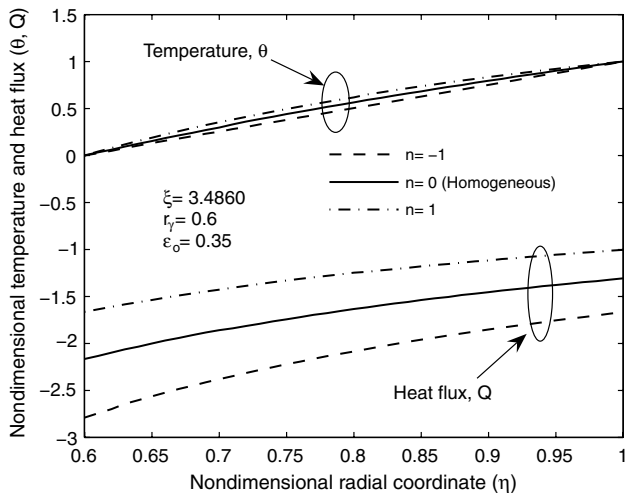
c)

Fig. 2 Effect of nonhomogeneity indices on the distribution of temperature and heat flux at different times: a) $\xi = 0.1260$, b) $\xi = 0.3360$, and c) steady-state solution.

Although not depicted graphically, the curves in Fig. 2c related to the homogeneous case, $n = 0$, are exactly the same as those of the completely analytical solution given in [21]. This corroborates the correctness of the current results.

Although it may not be possible to manufacture such an FGM cylinder, by further investigating Eq. (12), it is revealed that when the
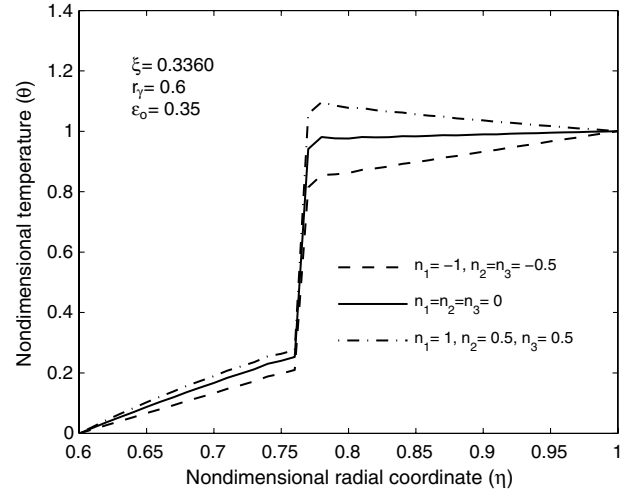

Fig. 3 Independence of the position of the thermal waves from the nonhomogeneity indices when $n_1 = n_2 + n_3$.

nonhomogeneity indices have the specific relationship $n_1 = n_2 + n_3$, the speed of wave propagation is neither a function of the nonhomogeneity index nor a function of the radial coordinate. In this case, all wave fronts are in the same position regardless of the n value, as shown in Fig. 3.

Figure 4 depicts the effect of the thermal relaxation time on the temperature history at the middle of the annulus. To focus on the effect of thermal relaxation, a homogeneous cylinder, $n_i = n = 0$, is considered in the calculations. Figure 4 shows that higher thermal relaxation times lead to higher transient amplitudes and longer durations of the thermal waves, whereas smaller relaxation times result in diffusive behavior of the temperature. When $\varepsilon_o = 0$, the hyperbolic heat conduction theory is reduced to Fourier's theory.

Figure 5 shows the effect of nonhomogeneity of different properties on the temperature history when $\varepsilon_o = 0.35$. Figure 5a shows the effect of nonhomogeneity of thermal conductivity, with $n_1 \neq 0$, while density and specific heat are uniform, $n_2 = n_3 = 0$. The final value of the temperature in the steady-state case depends on the nonhomogeneity index of thermal conductivity. Higher nonhomogeneity of thermal conductivity results in higher amplitudes of the thermal wave and the steady-state temperature. Figure 5b shows the effect of nonhomogeneity of density, $n_2 \neq 0$, while thermal conductivity and specific heat are uniform, $n_1 = n_3 = 0$. Similar to the nonhomogeneity of thermal conductivity, a higher nonhomogeneity of density results in higher amplitudes of the thermal wave; however, the steady-state temperature distribution is not affected by the nonhomogeneity index of density. Although not shown here for
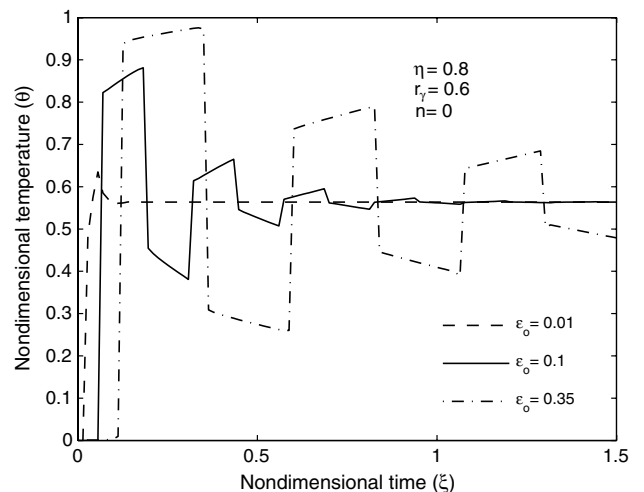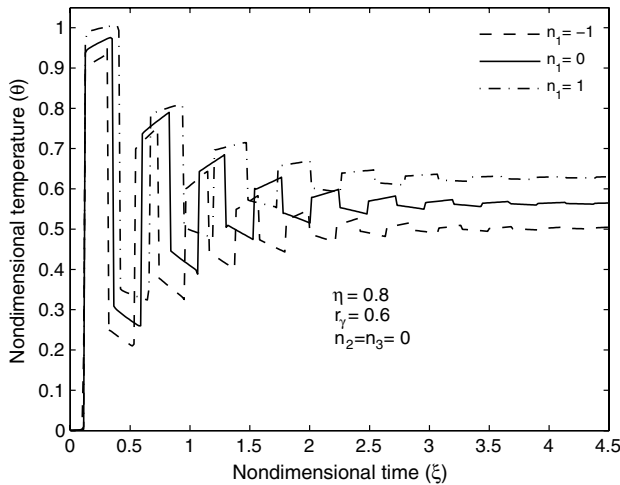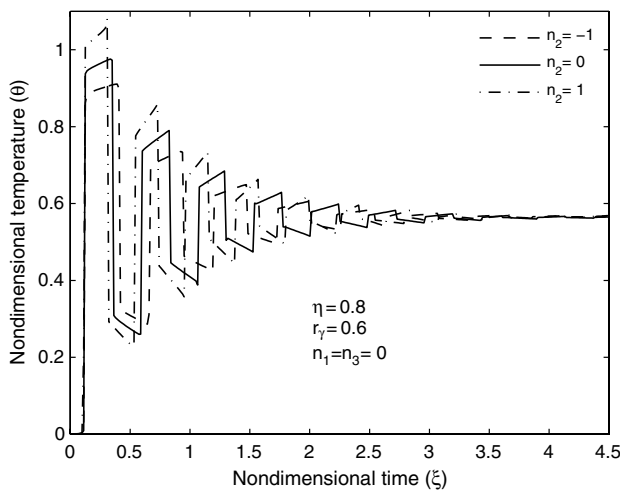

Fig. 4 Effect of thermal relaxation time on the temperature at the middle of the annulus ($\eta = 0.8$).

a)

b)

Fig. 5 Effect of nonuniformity on the temperature of the middle surface of the annulus ($\eta = 0.8$): a) thermal conductivity, and b) density.

brevity, the steady-state thermal field is also not affected by the nonhomogeneity of specific heat. The key effect of thermal conductivity on the steady-state temperature values can be mathematically deduced from Eq. (4b). As seen in the equation, in the steady-state case, in which $\partial T/\partial t = 0$, density and specific heat are eliminated in the governing equations.

Conclusions

Transient hyperbolic heat conduction in an FG hollow cylinder is investigated herein. The cylinder is power-law graded with arbitrary nonhomogeneity indices. The one-dimensional heat conduction problem in a long cylinder is solved analytically in the Laplace domain. The speed of thermal wave propagation is also given in the paper. Time dependency is restored by the numerical inversion of the Laplace-transformed temperature and heat flux. With some illustrative examples, effects of the nonhomogeneity index and thermal relaxation time on the thermal field are investigated. The summary of the results are as follows:

1) The speed of thermal wave propagation depends strongly on the nonhomogeneity indices. Higher nonhomogeneity indices lead to higher wave speeds, as shown in Figs. 2a and 2b.

2) The higher the nonhomogeneity index is, the higher the steady-state values of both the temperature and heat flux are, as shown in Fig. 2c.

3) In the case in which nonhomogeneity indices satisfy the relationship $n_1 = n_2 + n_3$, the thermal wave propagates at a constant speed regardless of the values of the nonhomogeneity indices, as shown in Fig. 3.

4) Higher thermal relaxation results in larger amplitudes and longer durations of the thermal waves before the steady-state is reached, as shown in Fig. 4.

5) The steady-state values of the temperature and heat flux are affected only by nonhomogeneity in thermal conductivity. The nonhomogeneity of density or specific heat has no influence on the final (steady-state) value of the temperature, as shown in Fig. 5.

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