TRANSIENT RESPONSE TECHNIQUE APPLIED TO ACTIVE MAGNETIC BEARING MACHINERY DURING ROTOR DROP

by

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(ABSTRACT)

The active magnetic bearing (AMB) is a relatively new technology which has many advantages compared with conventional bearing design. In an AMB system, the rolling-element back-up bearings are indispensable to protect the magnetic bearing rotor and stator, and other stationary seals along the rotor shaft. In this paper, a theoretical formulation is proposed and solved numerically to examine the transient response of the flexible rotor, from the time just previous to the AMB shuts down and including the rotor drop onto the back-up bearing. The backward whirl of the rotor, which may lead to the destructive damage of the machinery, has been analytically predicted at very light support damping and very high support damping. Also, the vibration due to the non-linearity of the contact point geometry has been included in the analysis. The influence of the support damping on the displacement of the disk and also the contact force between the journal and the inner-race of the back-up bearing have been computed for various rotor system parameters. By comparing these results with the optimum support damping for the simple flexible rotor model, it is shown that this support damping optimization can be applicable for specifying the required optimum range of support damping for the back-up bearings of AMB systems.

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Nomenclature

C	damping (N-sec/m).
C_ψ	torsional damping (kg-m^2/sec).
$F_{fric.}$	frictional force between inner-race and journal.
J	moment of inertia (kg-m^2).
K	stiffness (N/m) .
K_m	stiffness matrix of the rotor.
K_ψ	torsional stiffness (N-m).
M	mass (kg).
M_m	mass matrix of the rotor.
N	contact normal force between the journal and the inner-race of the back-up bearing (N).
N_o	pre-load of the back-up bearing (N).
R	radius (m).
R_m	pitch radius of the back-up bearing (m).
T_d	torque required for the disk (N-m).
$c_{sup.}$	support damping (N-sec/m).
e	eccentricity of unbalance (m).
gr	acceleration of gravity (m/sec^2).
k	stiffness ratio ($k_{sup.} / k_i$) (dim).
k_i	modal stiffness for the i-th mode of the rotor (N/m).
$k_{sup.}$	support stiffness (N/m).
m_i	modal mass for the i-th mode of the rotor (kg).
t	time (sec).

Δt time increment in a Runge-Kutta algorithm (sec).

x x-direction (vertical) displacement (m).

y y-direction (horizontal) displacement (m).

α angular location of the journal in the bearing (rad).

δ deformation at the contact point (m).

ϕ_i normilized i-th mode shape of the rotor.

ξ support damping ratio ($c_{sup.} / (2\sqrt{m_i\, k_{sup.}})$) (dim).

μ_d the coefficient of dynamic sliding friction between the journal and inner-race of the bearing (dim).

μ_s the coefficient of static sliding friction between the journal and inner-race of the bearing (dim).

μ_ψ the coefficient of rolling friction of the back-up bearing (dim).

ψ angular displacement (rad).

ω angular velocity (rad/sec).

$\omega_{cr.\,i}$ i-th critical speed of the rotor (rad/sec).

Subscripts

b refers to back-up bearing.

c refers to contact point.

cg refers to center of gravity of the disk.

d refers to disk.

h refers to housing.

mb refers to active magnetic bearing.

s refers to shaft or journal.

Chapter 1

Introduction

1.1 Introduction

Active magnetic bearings (AMBs) are being used in industrial rotating machinery such as high performance centrifugal compressors and pumps [1], [2]. This non-contact support system has the following advantages compared with conventional bearings :

<u>No contact with rotor</u> ..... no wear, no friction, very little energy loss, quiet operation.

<u>No lubricant</u> ..... clean (no contamination of the process with it), simple configuration, lower operating cost, wide-range temperature operation.

<u>Controllable stiffness and damping</u> ..... accurate rotor positioning.

These advantages enable very high peripheral speed operation. In addition, AMBs offer almost unlimited control over the rotor they support.

The magnetic bearing system needs a back-up rotor support system (back-up bearings) in case the active magnetic bearing system fails or the machine does not operate. The typical section of an AMB compressor is shown in Fig. 1a, and the simple model of AMB machinery is shown in Fig. 1b. The back-up bearing, which is also called an "auxiliary bearing" or a "catcher bearing," is a rolling-element bearing located at the outer-bound of the magnetic bearing. Being assembled on the stator, the back-up bearing is not active during the normal operation of the magnetic bearing. The clearance of the back-up bearing is smaller than that of the magnet bearing to protect it. Typically, fifty percent of the magnetic bearing clearance is used for the back-up bearing clearance.

The back-up bearings are indispensable for the magnet bearing system not only to protect the magnet bearings but also to protect the rotor and machinery itself. However, some troubles about the back-up system [3] have been reported such as the serious damage of the journal, the back-up bearings and the seals. These results imply that the back-up bearings did not fill their roles or they could not support the transient rotor drop loading.

1.2 Literature Review

There has been little research on the rotor drop on the back-up bearings because major concerns about AMB systems have been the control and the stability of the system. However, some applicable research has been conducted in rotor dynamics.

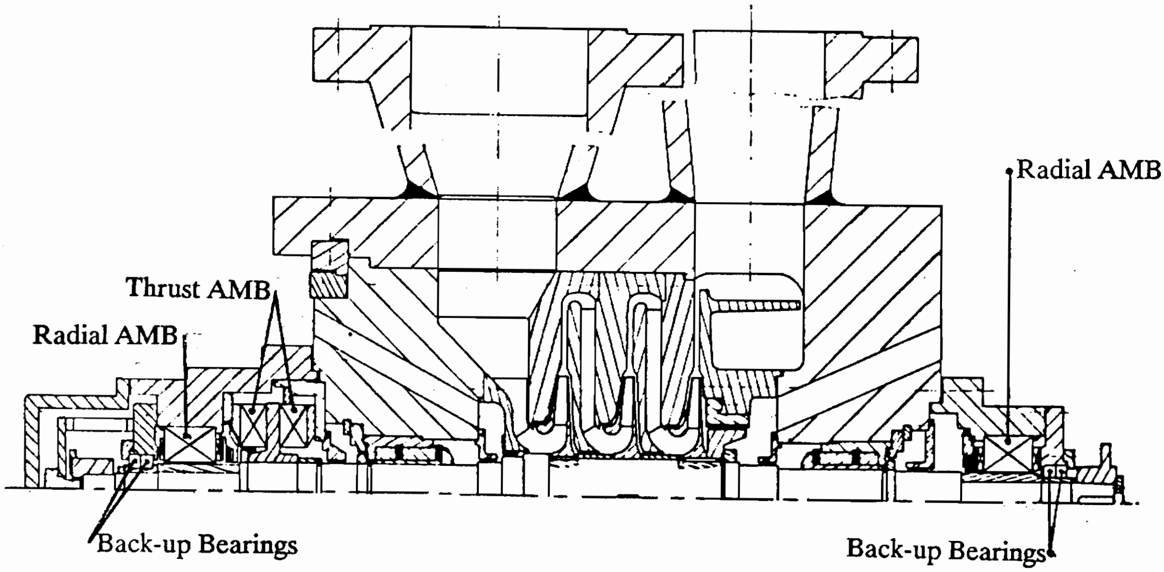

Figure 1a. Typical Section of an AMB Compressor (Ref. [1])

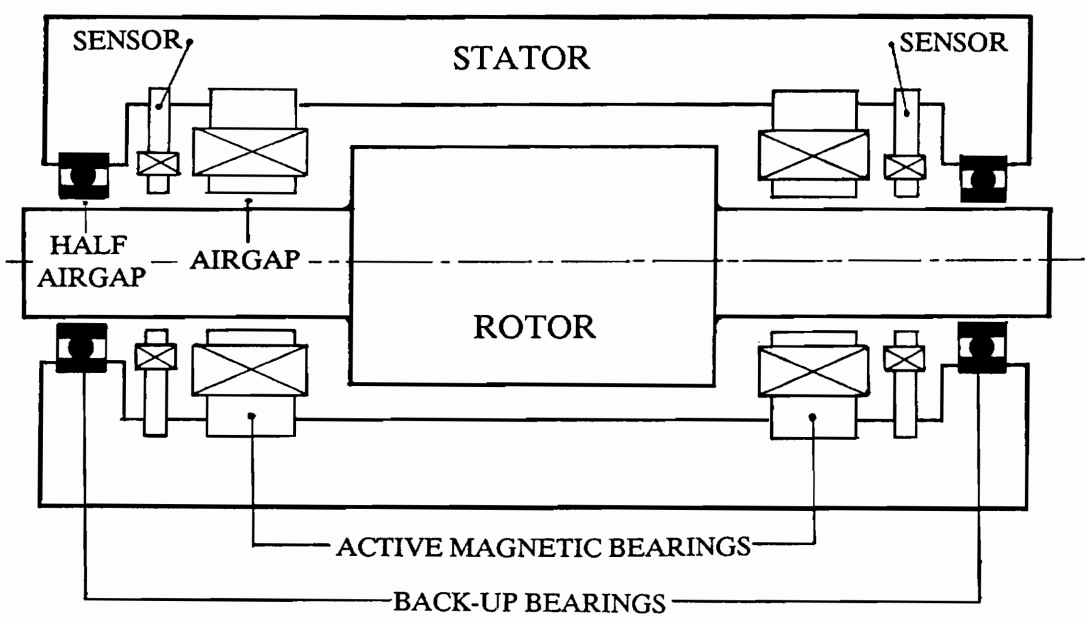

Figure 1b. Simple Model of AMB Machinery

Ehrich and O'Connor [4] presented the rotor model which was supported by the flexible stator with some dead-band clearance. They took the interaction between the rotor and the stator including friction into consideration, and identified the vibrational behavior of the rotor and the stator in the bearing clearance. They also pointed out that the stator whirl, wherein large stator response occurs, may be experienced at super critical speeds.

Muszynska [5] focused on the interaction between a rotor and a stator. She modified the stiffness, which becomes larger due to the contact, and gave a simple explanation of the mechanisms causing the steady-state rotor response due to the partial rub between the stator and the rotor.

Day [6] and Zalik [7] examined the nonlinear Jeffcott model with a dead-band in the supporting system. This dead-band was considered to be located between the bearing and the bearing support, and the rotational speed of the rotor was considered to be constant.

Gelin, et al.[8] investigated the dynamic response of the rotor on the back-up bearings. They considered a constant coefficient of friction and calculated the transient response of the rotor during the coast down.

Kirk and Gunter [9] studied the effect of the support stiffness and damping by evaluation of forced response for numerous rotor-support system parameters, they found the optimum support characteristics of the flexible rotor by examination of cross-plots of rotor optimum response for a tuned rotor support system.

Barrett, et al. [10] proposed the approximate method of calculating the optimum damping of the support system using the modal mass and modal stiffness of the flexible rotor.

1.3 Research Objectives

The major objective in this project is to get the orbit of the rotor in the back-up bearing after the rotor drop and to know what kind of forces act on the rotor and the back-up bearing. Then, the optimum support conditions for the back-up bearing can be determined.

First an approximate model of the AMB system, which includes the rotation of the inner-race of the back-up bearing and Coulomb friction, is presented in Chapter 2. Also a theoretical formulation is proposed and the transient response of the rotor drop after AMB shut down is calculated using the fourth-order Runge-Kutta method.

Chapter 3 presents the results of the transient response for various support system damping values. The influence of the bearing rotational constraint and frictional coefficient between the journal and the inner-race of the back-up bearing is discussed. The effect of the support damping is examined and the optimum damping is introduced. Then, in Chapter 4, the conclusions and recommendations for further research are presented.

Chapter 2

Transient Response Analysis

2.1 Equations of Motion of the Rotor Supported by the Magnetic Bearings

Figure 2a represents a rotor and the active magnetic bearings model considered in this project. The rotor is composed of a disk mass which has an eccentricity e_d, journal masses, and a massless elastic shaft. When the magnet bearings are active, the symmetric rotor is supported only by them with the supporting stiffness K_{mb} and the supporting damping C_{mb}. Gyroscopics need not to be considered for the symmetric model considered in this project. The resulting equations are valid for only symmetric rotor motion. These assumptions are consistent with previous research (ref. [9], [10]). The governing equations of motion for the disk are written as :

$$M_d \ddot{X}_{cg} + C_{ss} (\dot{X}_d - \dot{X}_s) + K_{ss} (X_d - X_s) = M_d gr - C_d \dot{X}_d \qquad [1]$$

$$M_d \ddot{Y}_{cg} + C_{ss} (\dot{Y}_d - \dot{Y}_s) + K_{ss} (Y_d - Y_s) = - C_d \dot{Y}_d \qquad [2]$$

where

$$X_{cg} = X_d + e_d \cos\psi_d \qquad [3]$$

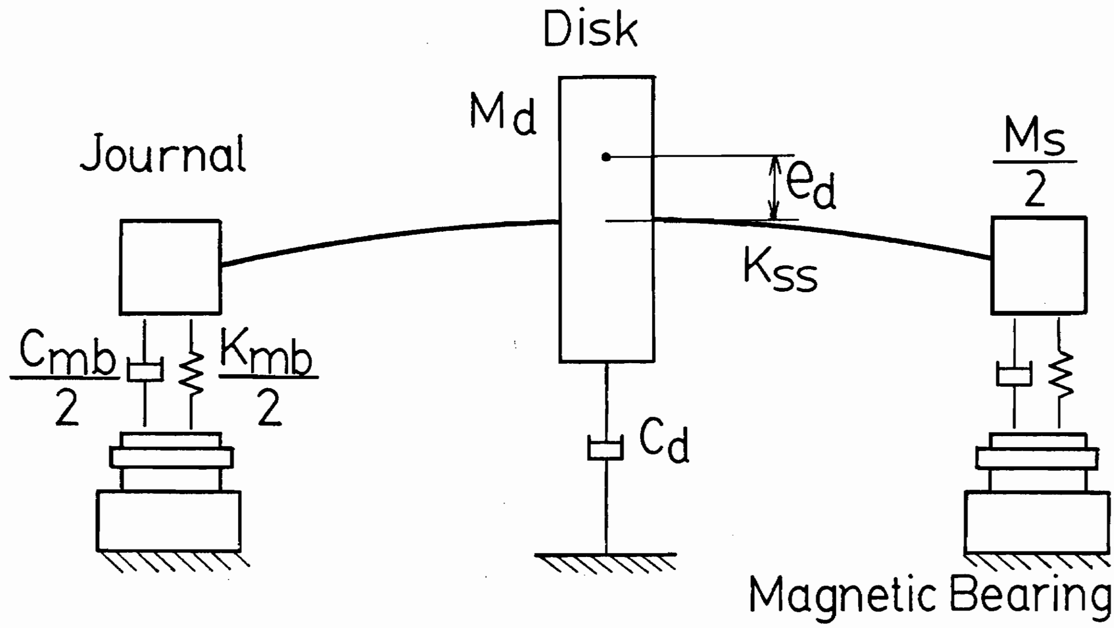

Figure 2a. AMB Rotor Model

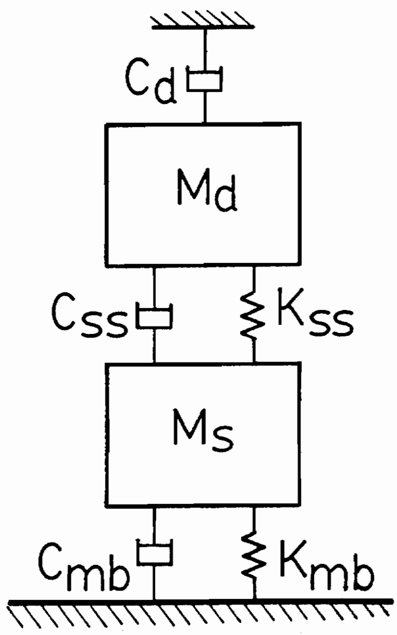

Figure 2b. Reduced 2DOF Representation of AMB Rotor Model

$$Y_{cg} = Y_d + e_d \sin\psi_d \qquad [4]$$

and

$\psi_d =$ angular location of mass center of the disk

By differentiating equations 3 and 4 with respect to time,

$$\ddot{X}_{cg} = \ddot{X}_d - e_d\,(\dot{\psi}_d^{\,2} \cos\psi_d + \ddot{\psi}_d \sin\psi_d) \qquad [5]$$

$$\ddot{Y}_{cg} = \ddot{Y}_d - e_d\,(\dot{\psi}_d^{\,2} \sin\psi_d - \ddot{\psi}_d \cos\psi_d) \qquad [6]$$

During the normal operation, $\ddot{\psi}_d = 0$, $\dot{\psi}_d = \omega_d$, $\psi_d = \omega_d t$, therefore equations 5 and 6 are reduced to equations 7 and 8 :

$$\ddot{X}_{cg} = \ddot{X}_d - e_d\,\omega_d^{\,2} \cos(\omega_d t) \qquad [7]$$

$$\ddot{Y}_{cg} = \ddot{Y}_d - e_d\,\omega_d^{\,2} \sin(\omega_d t) \qquad [8]$$

So, the equations of motion of the disk during the normal operation are written as :

$$M_d \ddot{X}_d + C_{ss}(\dot{X}_d - \dot{X}_s) + K_{ss}(X_d - X_s) - M_d\,gr + C_d \dot{X}_d$$

$$= M_d\, e_d\, \omega_d^{\,2} \cos(\omega_d t) \qquad [9]$$

$$M_d \ddot{Y}_d + C_{ss}(\dot{Y}_d - \dot{Y}_s) + K_{ss}(Y_d - Y_s) + C_d \dot{Y}_d$$

$$= M_d\, e_d\, \omega_d^{\,2} \sin(\omega_d t) \qquad [10]$$

And the governing equations of the journal masses can be written as :

$$M_s \ddot{X}_s + C_{ss}(\dot{X}_s - \dot{X}_d) + K_{ss}(X_s - X_d) + C_{mb}\dot{X}_s + K_{mb}X_s$$

$$= M_s gr \qquad [11]$$

$$M_s \ddot{Y}_s + C_{ss}(\dot{Y}_s - \dot{Y}_d) + K_{ss}(Y_s - Y_d) + C_{mb}\dot{Y}_s + K_{mb}Y_s = 0 \qquad [12]$$

2.2 Equations of Motion after the Rotor Drop

After the magnetic bearings become inactive, the rotor drops on the back-up bearings. Figure 3 represents the AMB rotor model after the rotor drop. Figure 4a represents the geometry of the model. The back-up bearing model used in this project is also shown in Fig. 4b.

When the journal comes into contact with the back-up bearing , the contact force N is induced due to the contact stiffness K_c and the contact damping C_c. Coulomb frictional forces are acting between the journal and the inner-race of the back-up bearing, and between the inner-race and outer-race of the bearing. Once the magnetic bearings become inactive, the torque is cut off. Therefore, ω_d is no longer constant. Neglecting the cross coupling terms and the gyroscopics, the equations of motion of the disk can be written using equations 5 and 6 :

$$M_d \ddot{X}_d + C_s(\dot{X}_d - \dot{X}_s) + K_s(X_d - X_s) - M_d gr + C_d\dot{X}_d$$

$$= M_d e_d(\dot{\psi}_d^2 \cos\psi_d + \ddot{\psi}_d \sin\psi_d) \qquad [13]$$

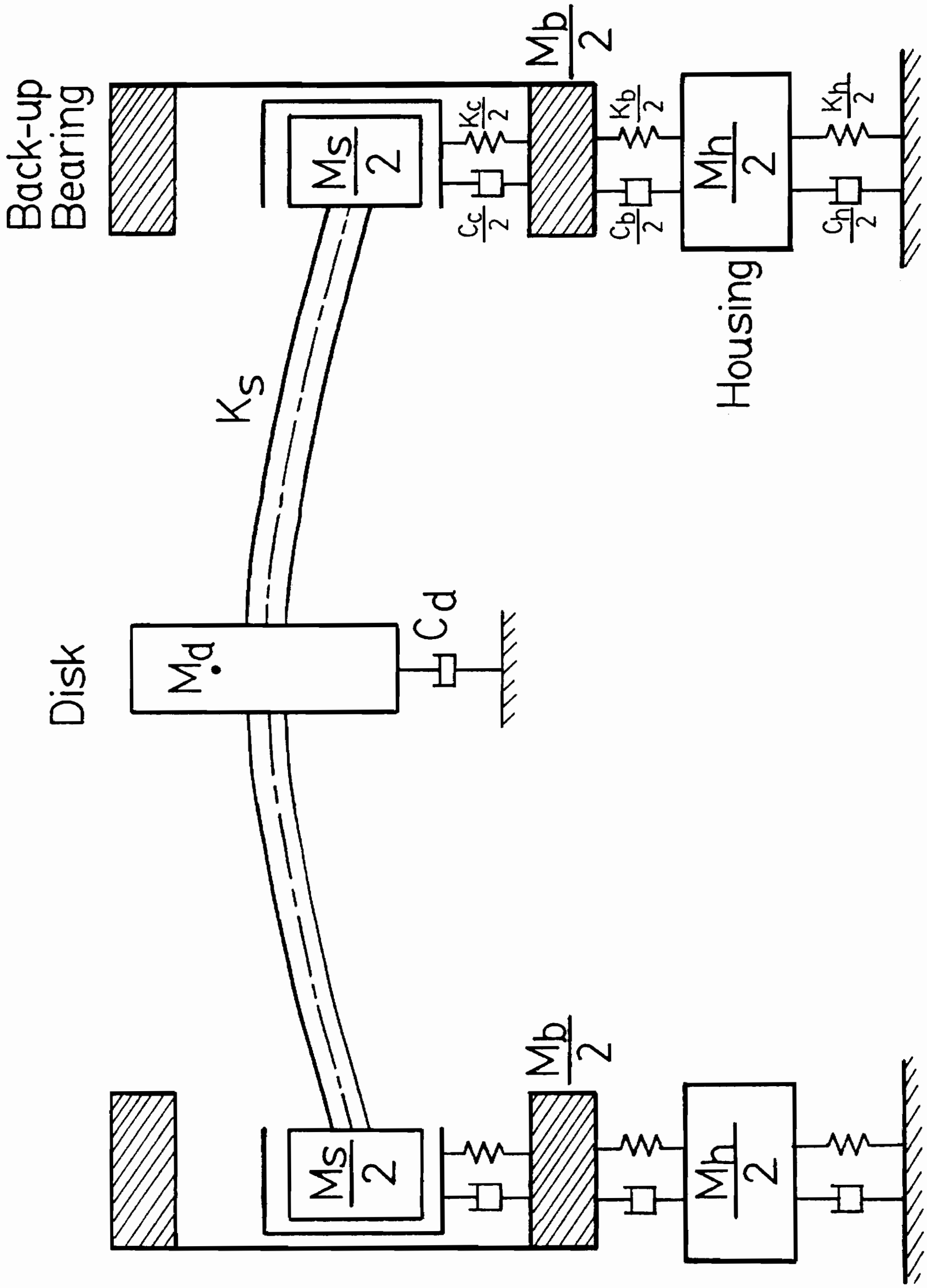

Figure 3. AMB Rotor Model during Rotor Drop

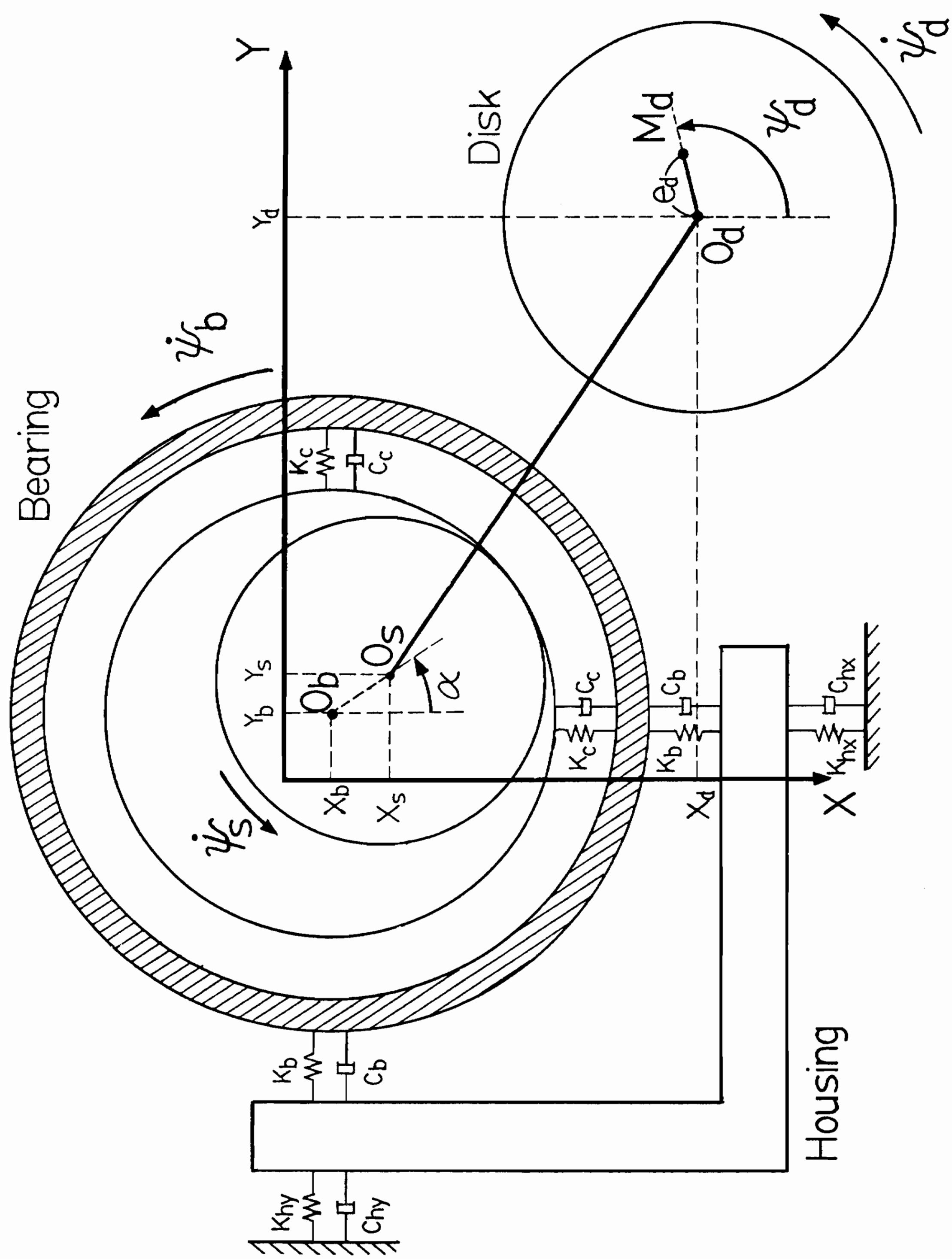

Figure 4a. Geometry of AMB Rotor Model after Rotor Drop

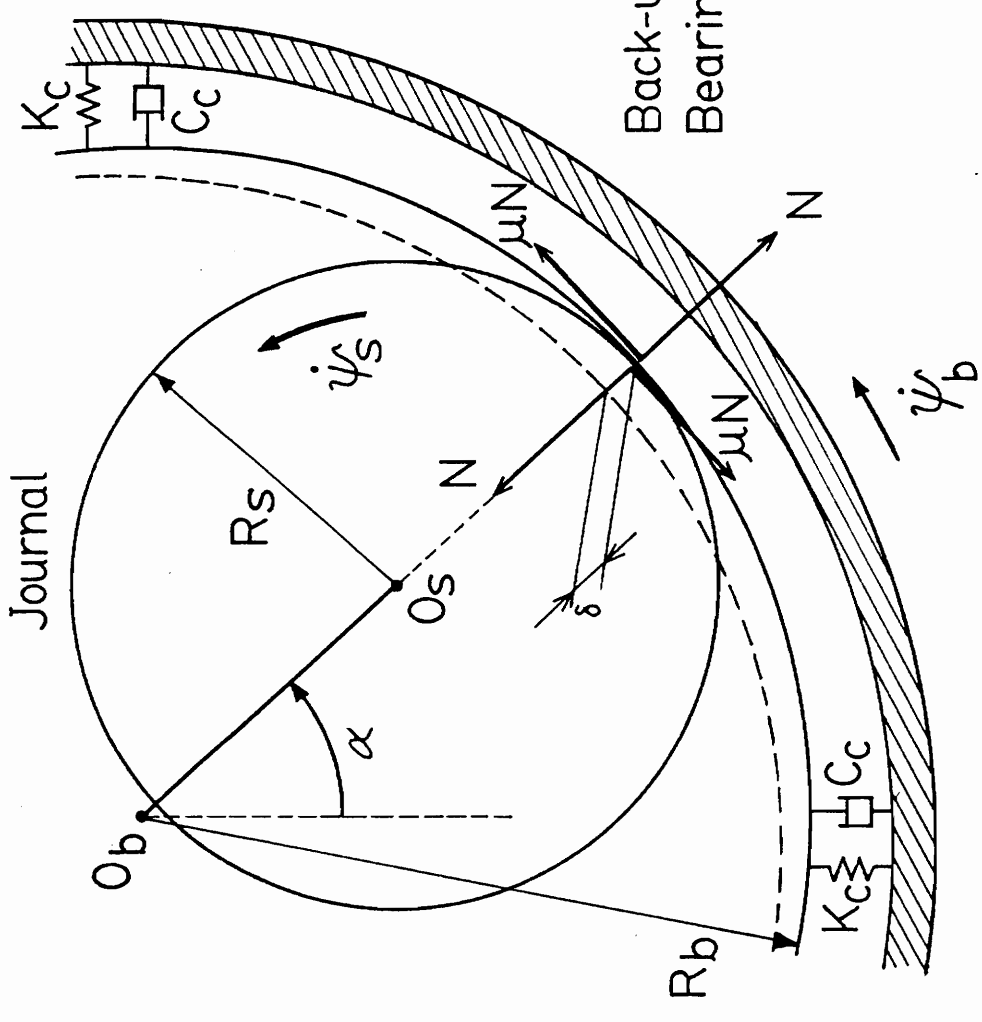

Figure 4b. Back-up Bearing Model

$$M_d \ddot{Y}_d + C_s (\dot{Y}_d - \dot{Y}_s) + K_s (Y_d - Y_s) + C_d \dot{Y}_d$$

$$= M_d e_d (\dot{\psi}_d^2 \sin\psi_d - \ddot{\psi}_d \cos\psi_d) \qquad [14]$$

Concerning to the angular movement,

$$J_d \ddot{\psi}_d + C_{s\psi} (\dot{\psi}_d - \dot{\psi}_s) + K_{s\psi} (\psi_d - \psi_s) + C_{d\psi} \dot{\psi}_d$$

$$= - M_d \, gr \, e_d \sin\psi_d - T_d \qquad [15]$$

As for the journal, the equations of motion are written as follows :

$$M_s \ddot{X}_s + C_s (\dot{X}_s - \dot{X}_d) + K_s (X_s - X_d) - M_s gr = - N (\cos\alpha - \mu \sin\alpha) \qquad [16]$$

$$M_s \ddot{Y}_s + C_s (\dot{Y}_s - \dot{Y}_d) + K_s (Y_s - Y_d) = - N (\sin\alpha + \mu \cos\alpha) \qquad [17]$$

$$J_s \ddot{\psi}_s + C_{s\psi} (\dot{\psi}_s - \dot{\psi}_d) + K_{s\psi} (\psi_s - \psi_d) = - \mu N R_s \qquad [18]$$

where

$$\alpha = \tan^{-1} \frac{Y_s - Y_b}{X_s - X_b} \qquad [19]$$

$$\delta = (X_s - X_b) \cos\alpha + (Y_s - Y_b) \sin\alpha - R_b + R_s \qquad [20]$$

if $\delta > 0$

then

$$N = K_c \delta + C_c \dot{\delta} \qquad [21]$$

and $N \geq 0$

if $\delta \leq 0$

then

$$N = 0$$

The frictional torque of the back-up bearing is assumed to be proportional to the bearing load, which is expressed as a summation of the contact force (N) and the pre-load of the back-up bearing (N_o). The rolling coefficient of friction (μ_ψ) is constant as long as the inner-race of the back-up bearing rotates.

The equations of motion of the back-up bearing are shown in Eqs. 22 -24, and those of the housing are shown in Eqs. 25 and 26.

Back-up bearings :

$$M_b \ddot{X}_b + C_b (\dot{X}_b - \dot{X}_h) + K_b (X_b - X_h) = N (\cos\alpha - \mu \sin\alpha) \qquad [22]$$

$$M_b \ddot{Y}_b + C_b (\dot{Y}_b - \dot{Y}_h) + K_b (Y_b - Y_h) = N (\sin\alpha + \mu \cos\alpha) \qquad [23]$$

$$J_b \ddot{\psi}_b + C_{b\psi} \dot{\psi}_b = \mu N R_b - \mu_\psi (N + N_o) R_m \qquad [24]$$

Housing :

$$M_h \ddot{X}_h + C_{hx} \dot{X}_h + K_{hx} X_h = C_b (\dot{X}_b - \dot{X}_h) + K_b (X_b - X_h) \qquad [25]$$

$$M_h \ddot{Y}_h + C_{hy} \dot{Y}_h + K_{hy} Y_h = C_b (\dot{Y}_b - \dot{Y}_h) + K_b (Y_b - Y_h) \qquad [26]$$

2.3 Coulomb Friction at the Contact Point

The frictional force is proportional to the contact normal force (N) and the dynamic sliding coefficient of friction (μ_d) is applied as long as there is a slip between the journal and the inner-race of the back-up bearing. However, the frictional force changes once the circumferential speed at the contact point becomes equal; that is, there is no slip at the contact point.

The journal and the inner-race keep contacting without a slip as long as the frictional force does not exceed the maximum static frictional force (μ_s N). From the geometry, the circumferential velocity at the contact point relative to the center of the back-up bearing can be written as follows :

The relative velocity of the journal-side contact point :

$$V_{s\,cont.} = R_s (\dot{\psi}_s - \dot{\alpha}) + R_b \dot{\alpha} \qquad [27]$$

The relative velocity of the bearing-side contact point :

$$V_{b\,cont.} = R_b \dot{\psi}_b \qquad [28]$$

When $V_{s\,cont.} = V_{b\,cont.}$ then,

$$| F_{fric.} | \leq \mu_s N$$

$V_{s\,cont.} \geq V_{b\,cont.}$ then,

$$F_{fric.} = \mu_d N$$

$V_{s\,cont.} \leq V_{b\,cont.}$ then,

$$F_{fric.} = - \mu_d N$$

2.4 Transient Response Calculation

The previous equations of motion are solved for the rotor model shown in Table 1. First, the equations for the normal operation (Eqs. 9- 12) are solved. After the initial transient motion has damped out, the system steady-state motion is the circular synchronous precession. When the center of the disk crosses the X-axis at the lower point, the active magnetic bearings' power is assumed to be cut off. Then, by using the condition of this point as the initial condition, Eqs. 13 - 18, and 22 - 26 are solved for the rotor drop.

Because the orbit of the rotor when AMB is active is very small compared with the bearing clearance in this model, the difference of the point which the AMB becomes inactive is not considered to be a significant contributor of the results. If the initial orbit is much larger, the effect of a different release point should be considered.

The numerical solution is obtained by using a fourth-order Runge-Kutta algorithm. The time increment (Δt) is selected to give at least twenty increments in a cycle of the highest undamped natural frequency of the system. The undamped natural frequencies are shown in Table 2.

The program was written in Fortran and can be seen in Appendix. It was compiled by Microsoft Fortran v. 5.0. The typical calculation time using Swan 486 25 MHz, IBM compatible computer, is about twenty minutes for 1.5 second of the transient response, which has 300,000 data points.

Table 1. Summary of Geometric Dimensions of the AMB Rotor Model

Disk Mass	4.0	(kg)
Journal Mass	1.6	(kg)
Bearing Mass	0.4	(kg)
Disk Eccentricity	30.0 E-6	(m)
Journal Diameter	29.6 E-3	(m)
Back-up Bearing Bore	30.0 E-3	(m)
Back-up Bearing Pitch Diameter	43.5 E-3	(m)
Disk Moment of Inertia	2.3 E-3	($kg-m^2$)
Journal Moment of Inertia	1.2 E-3	($kg-m^2$)
Back-up Bearing Moment of Inertia	0.1 E-3	($kg-m^2$)
Shaft Stiffness	2.0 E+6	(N/m)
Contact Stiffness	500.0 E+6	(N/m)
Back-up Bearing Support Stiffness	3.0 E+6	(N/m)
Magnetic Bearing Stiffness (x)	2.0 E+6	(N/m)
(y)	1.8 E+6	(N/m)
Shaft Torsional Stiffness	17.7 E+3	(N-m)
Disk Damping	1.0	(kg/s)
Shaft Damping	150.0	(kg/s)
Back-up Bearing Support Damping	10. - 150.E+3	(kg/s)
Contact Damping	2000.0	(kg/s)
Magnetic Bearing Damping (x)	1000.0	(kg/s)
(y)	900.0	(kg/s)
Disk Torsional Damping	0.2 E-3	($kg-m^2/s$)
Pre-load of the Back-up Bearing	2.0	(N)
Disk Required Torque	0.2	(N-m)

Table 2. Undamped Liner Natural Frequencies of the Model after the Drop
(neglecting the dead-band between the journal and the back-up bearing)
(assuming the housing is rigidly supported)

1st Undamped Natural Frequency	83.38	(Hz)
2nd	262.55	(Hz)
3rd	6303.73	(Hz)

Chapter 3

Results of Transient Response Analysis

3.1 Constraint in Rotational Movement of the Back-up Bearings

Figures 5a -5e show orbits of the journal and the disk of the rotor model in a relatively light back-up bearing support damping case ($C_b = 90.$N-s/m). In this calculation, it is assumed that the rotational movement of the inner-race of the back-up bearings is frozen; in other words, the back-up bearing does not act as a bearing but as a simple bushing. The inner circle ($r = 20 \times 10^{-2}$ mm) represents the clearance circle of the back-up bearing. The journal often exceeds the clearance circle because of the deformation of the back-up bearing support as well as the contact point. The small circles near the origin are the steady-state orbits while the rotor is supported by an AMB system. After the AMB system becomes inactive, the rotor drops on the back-up bearing and bounces several times on the inner-race. Because of the friction between the journal and the inner-race, the rotor begins to rock back and forth in the bottom half of the inner-race at the same time. The rocking motion grows larger and the rotor finally begins to rotate all around and gets into the backward whirl motion. Once the backward whirl occurs, the displacement of the rotor grows larger and larger, which may leads to the

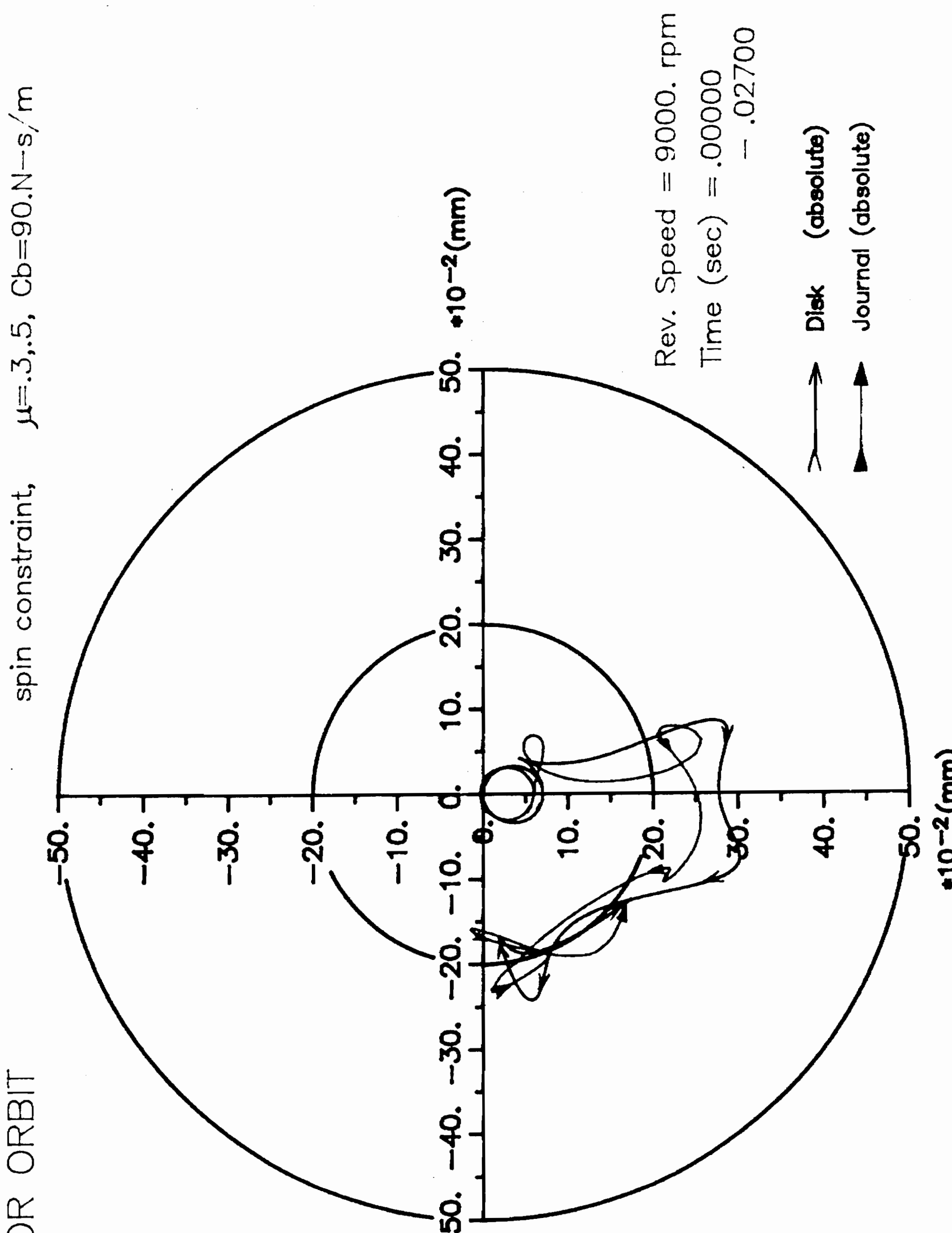

Figure 5a. The Orbits of the Disk and the Journal after the Rotor Drop
$T = 0.000$ - 0.027 s, spin constraint, $C_b = 90.$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

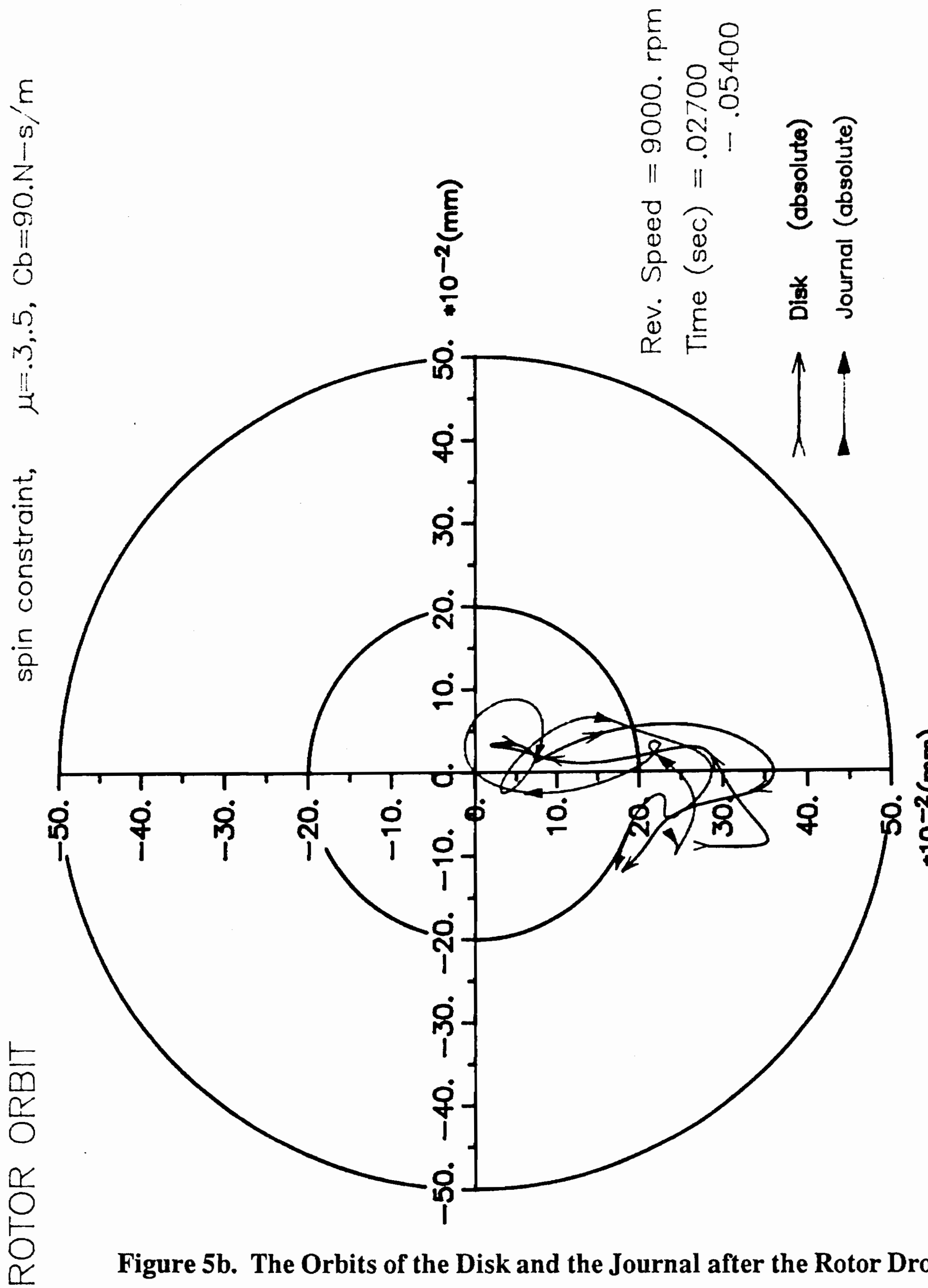

Figure 5b. The Orbits of the Disk and the Journal after the Rotor Drop
T = 0.027 - 0.054 s, spin constraint, C_b = 90. N-s/m, μ_d = 0.3, μ_s = 0.5

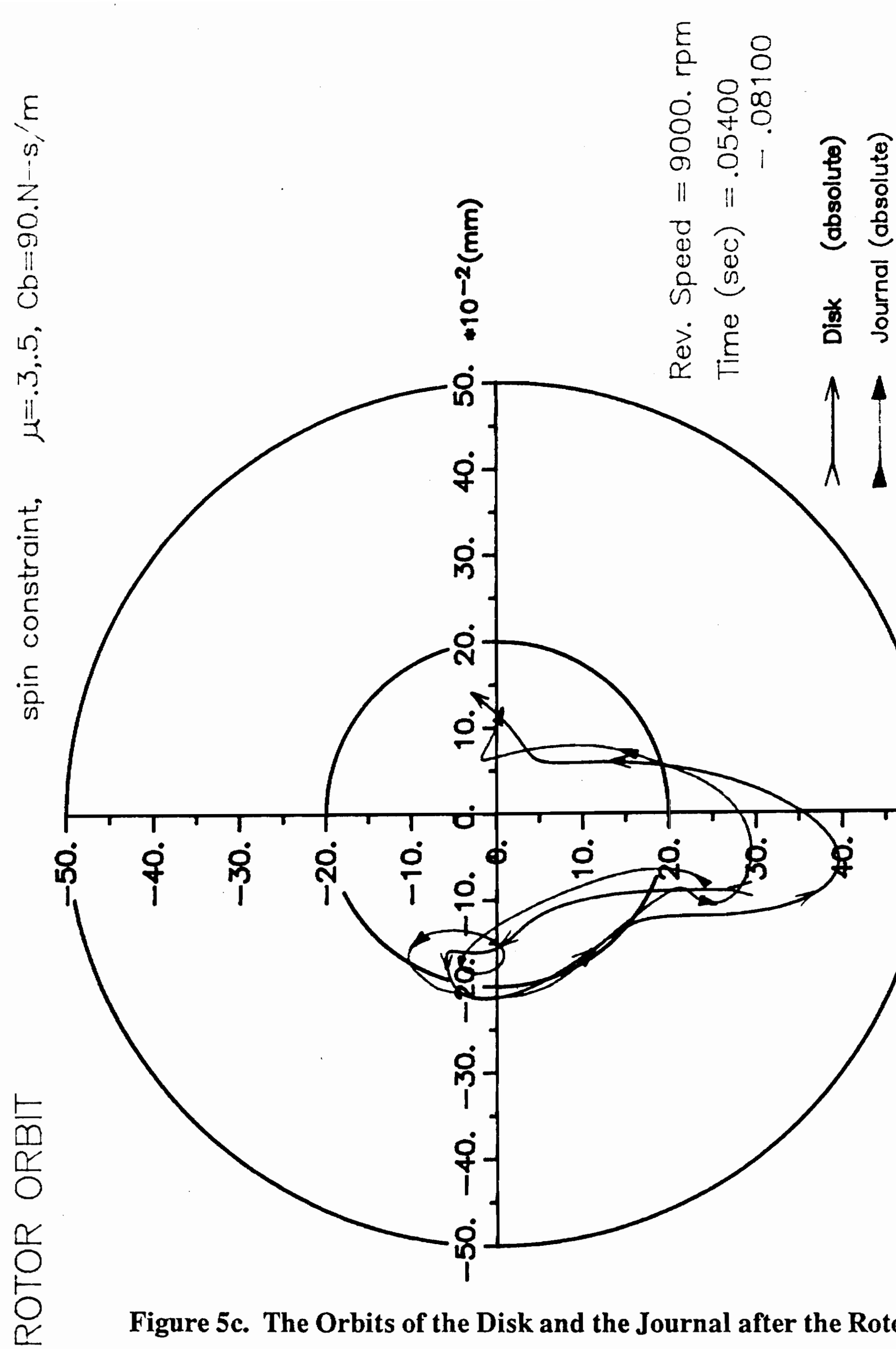

Figure 5c. The Orbits of the Disk and the Journal after the Rotor Drop

T = 0.054 - 0.081 s, spin constraint, C_b = 90. N-s/m, μ_d = 0.3, μ_s = 0.5

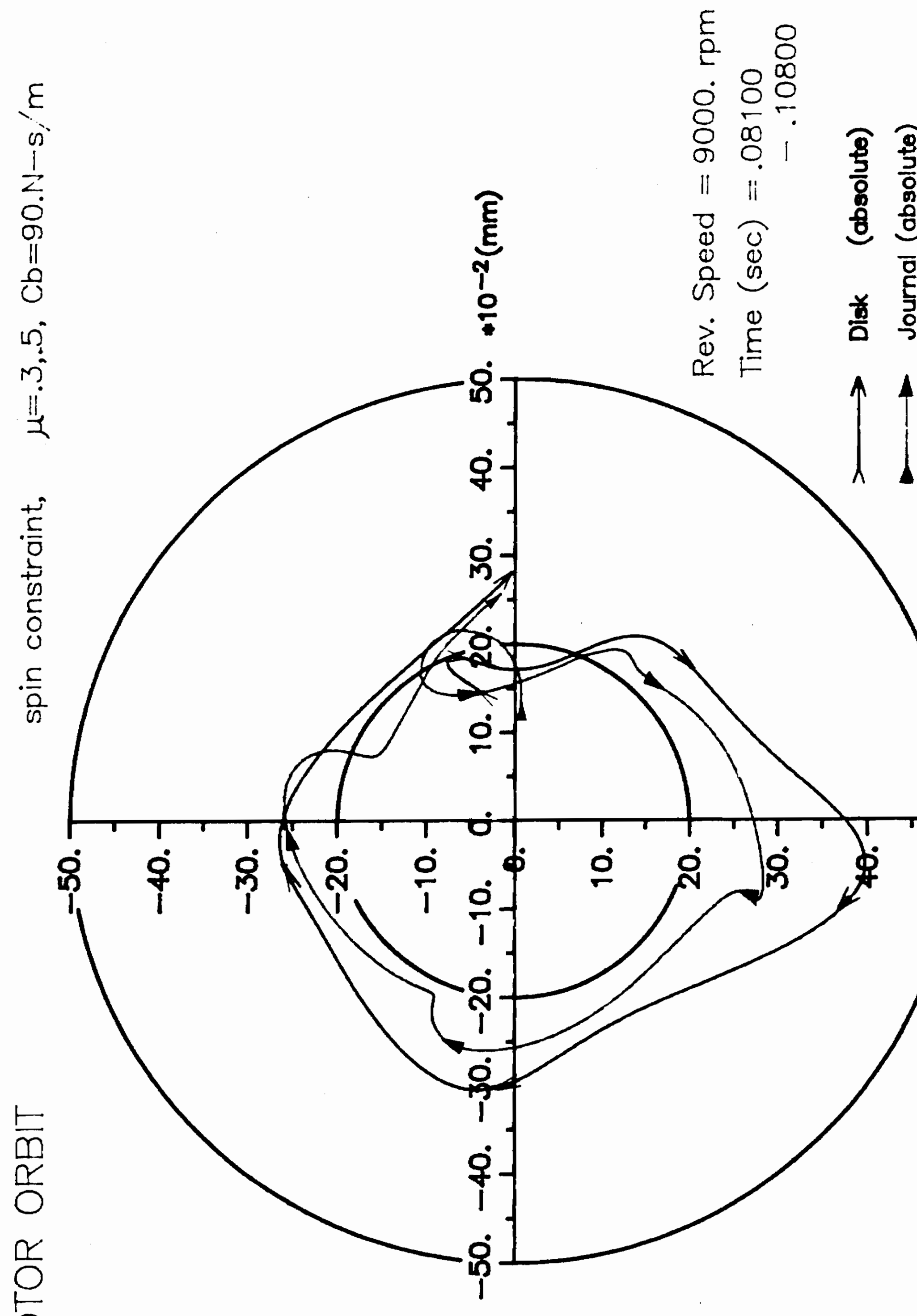

Figure 5d. The Orbits of the Disk and the Journal after the Rotor Drop
T = 0.081 - 0.108 s, spin constraint, C_b = 90. N-s/m, μ_d = 0.3, μ_s = 0.5

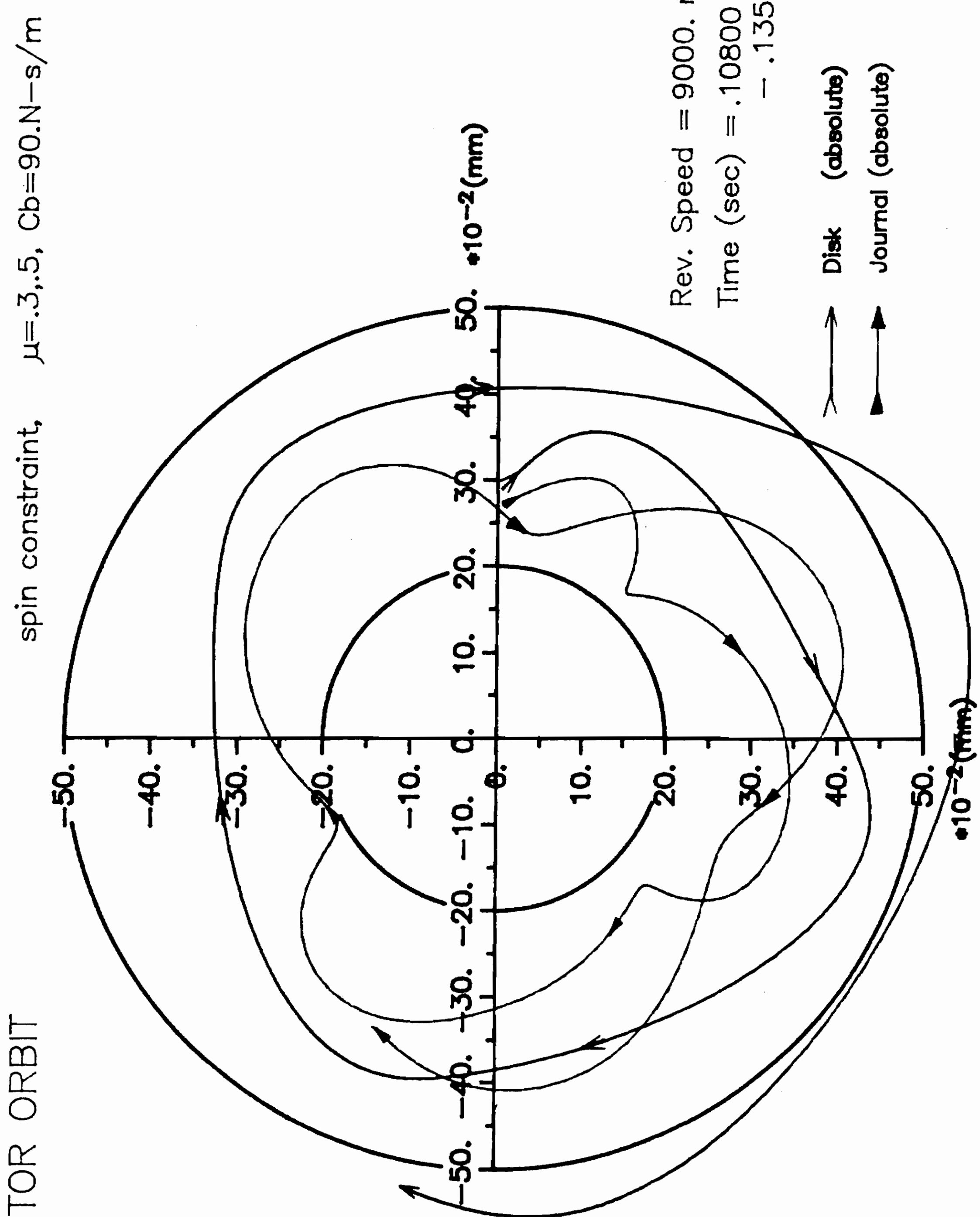

Figure 5e. The Orbits of the Disk and the Journal after the Rotor Drop

$T = 0.108$ - 0.135 s, spin constraint, $C_b = 90.$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

destructive damage of the machinery.

Figures 6a -6d show the orbits for the various back-up bearing support damping. As the support damping becomes larger, the rocking motion becomes smaller. The rotor stays at the bottom half of the inner-race and will not take off into backward whirl motion. But, for very high support damping, it takes off again into backward whirl motion. In this case, the response of the back-up bearings does not grow large because very high support damping freezes the movement of the bearing support structure.

Figures 7a and 7b show the contact force and the angle location of the journal in the back-up bearing in a very light support damping case ($C_b = 90.$N-s/m). After 0.1 second, the rotor starts to whirl backward and the contact force keeps growing. The backward whirl rate stays constant and it is equal to the first natural frequency of the system (83 Hz) which is calculated by neglecting the dead-band between the journal and the back-up bearing, even though the rotational speed of the rotor (150 Hz) is higher than that.

The radial displacement of the disk (Fig. 7c) as well as that of the back-up bearing (Fig. 7d) becomes larger as the rotor keeps whirling backward. In addition to the backward whirl, the contact force begins to change violently at 0.2 seconds (Fig. 7a). Figure 7e shows the power spectrum density of the back-up bearing displacement from 0.2 to 0.4 seconds. The first two peaks (80 and 250 Hz) represent the linear natural frequencies of the system. There are two other peaks between 800 Hz and 1100 Hz, which have nothing to do with the linear natural frequencies. This vibration is considered to be caused by the non-linearity of the contact point, that is, the dead-band and the friction between the journal and the

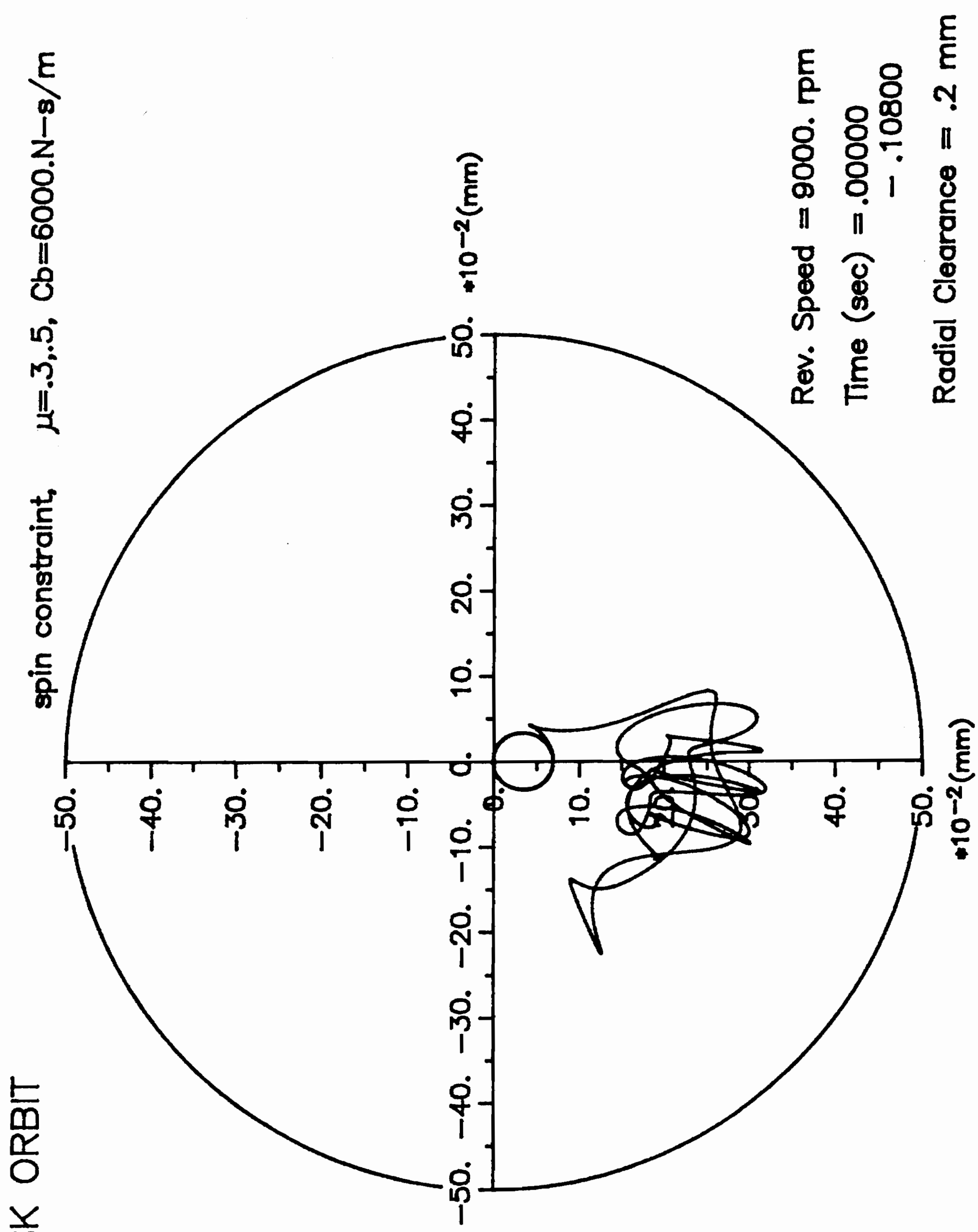

Figure 6a. The Orbit of the Disk after the Rotor Drop
T = .000 - .0108 s, spin constraint, $C_b = 6000.$ N-s/m, $\mu_d = .3$, $\mu_s = .5$

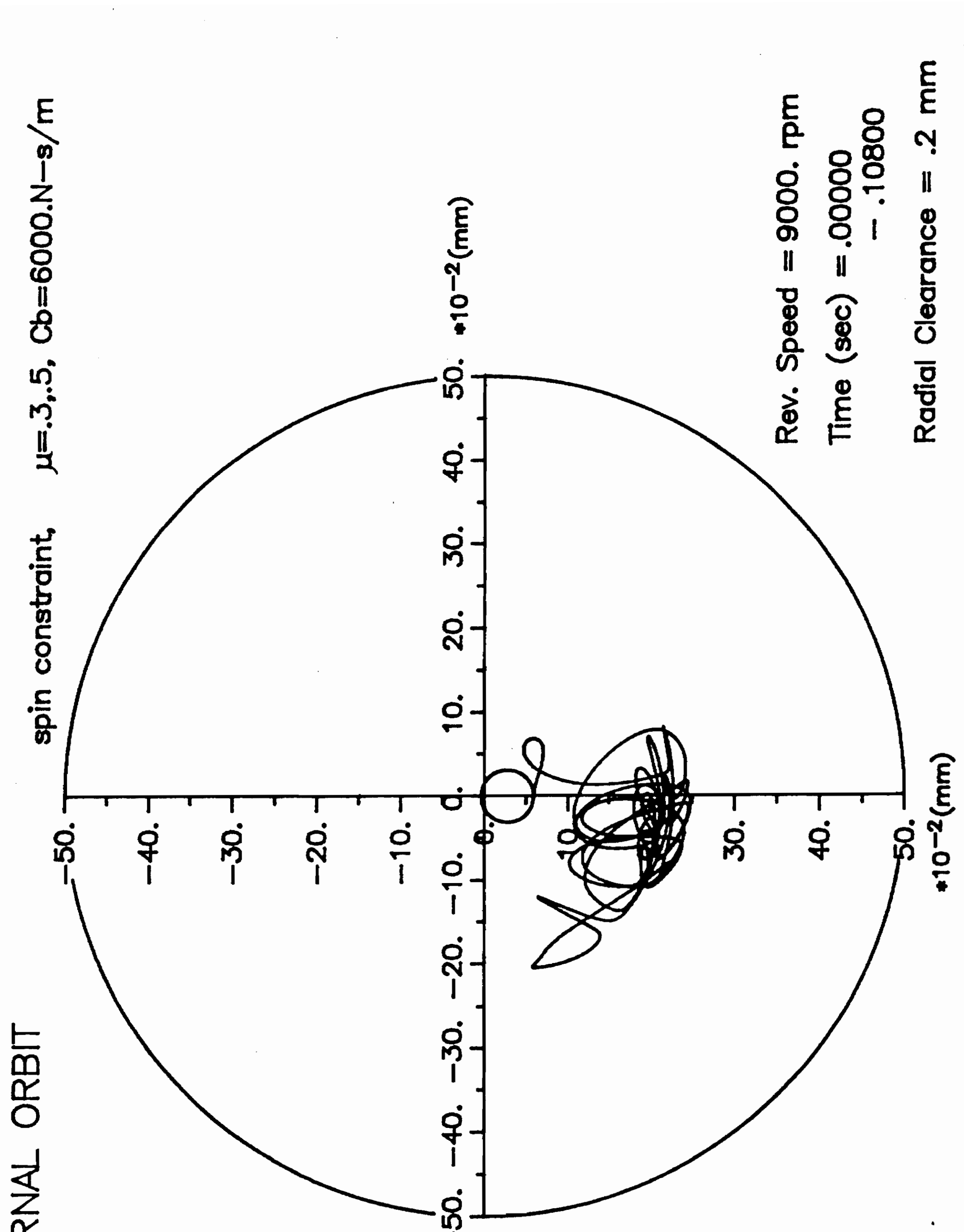

Figure 6b. The Orbit of the Journal after the Rotor Drop

T = .000 - .0108 s, spin constraint, $C_b = 6000.$ N-s/m, $\mu_d = .3$, $\mu_s = .5$

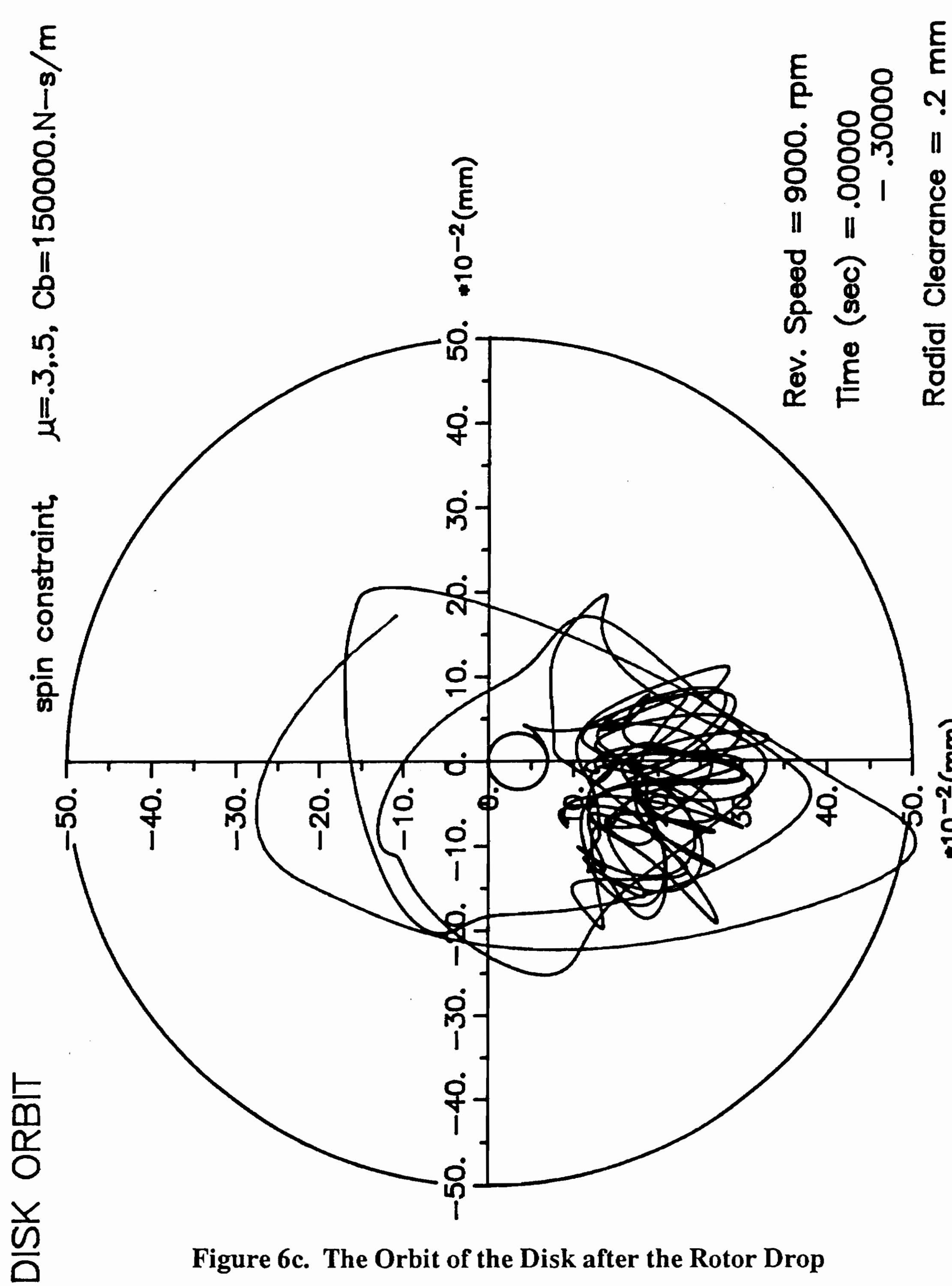

Figure 6c. The Orbit of the Disk after the Rotor Drop

T = .00 - .30 s, spin constraint, C_b = 150000. N-s/m, μ_d = .3, μ_s = .5

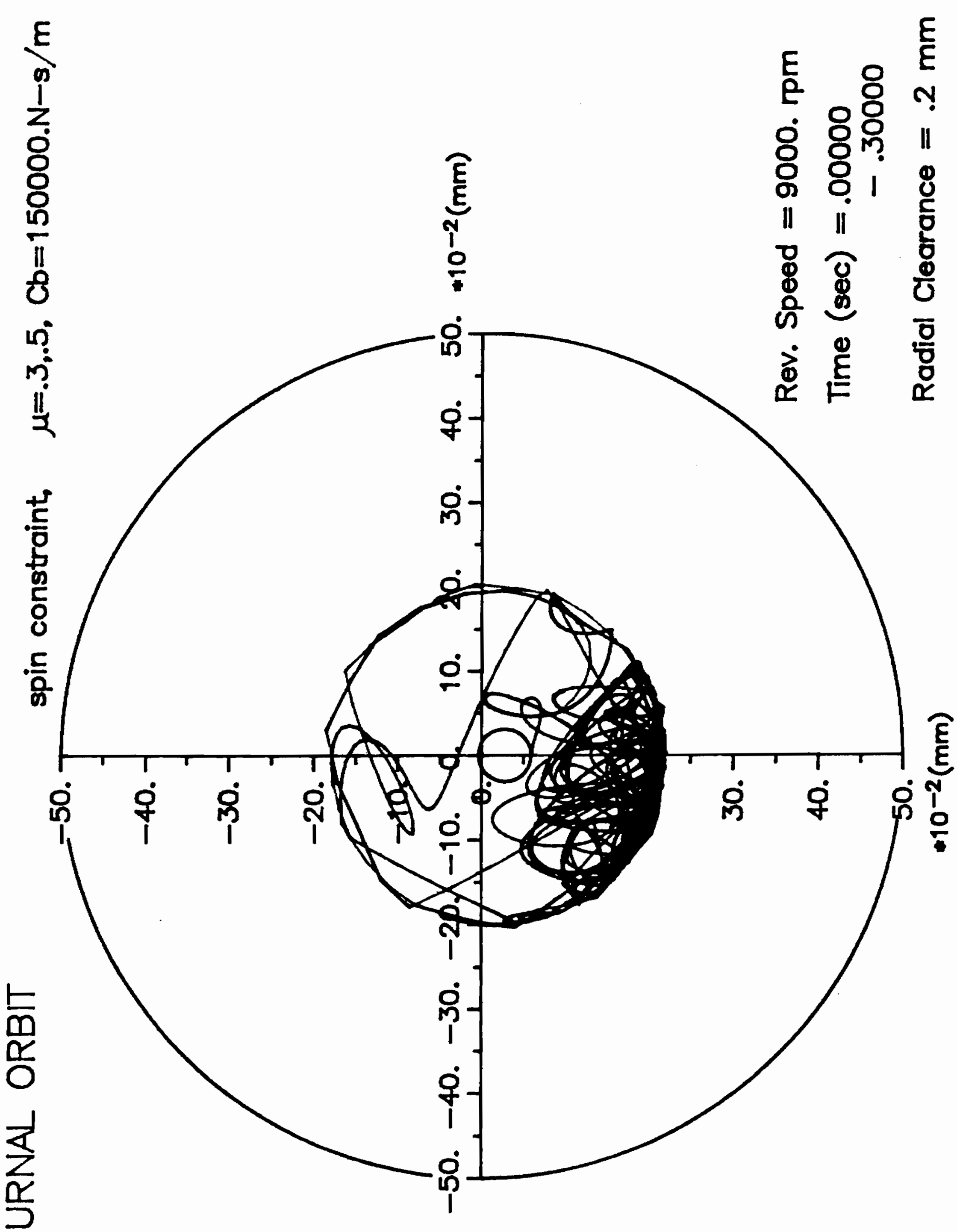

Figure 6d. The Orbit of the Journal after the Rotor Drop

T = .00 - .30 s, spin constraint, C_b = 150000. N-s/m, μ_d = .3, μ_s = .5

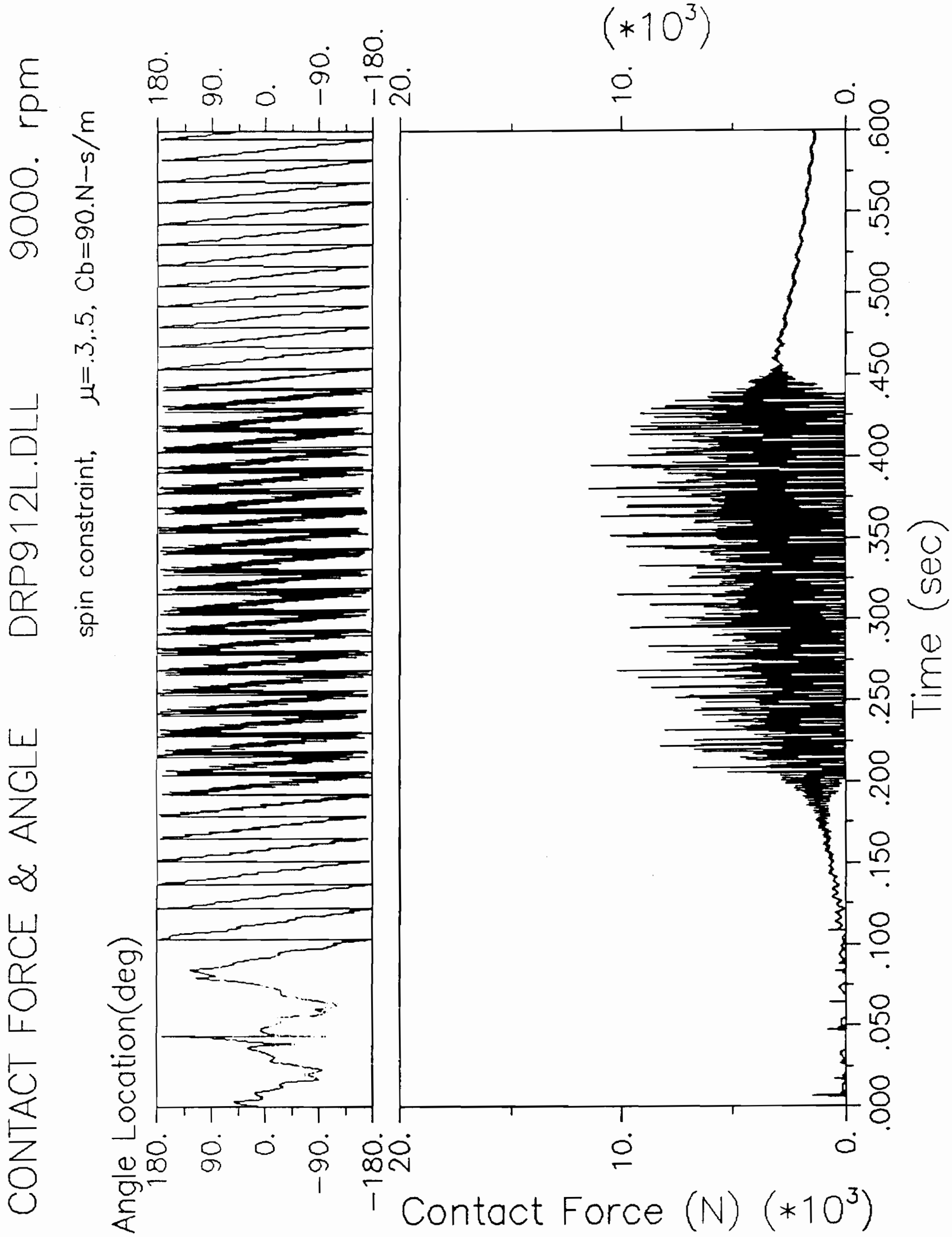

Figure 7a. The Contact Force and the Angular Location of the Journal
$T = 0.0 - 0.6$ s, spin constraint, $C_b = 90.$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

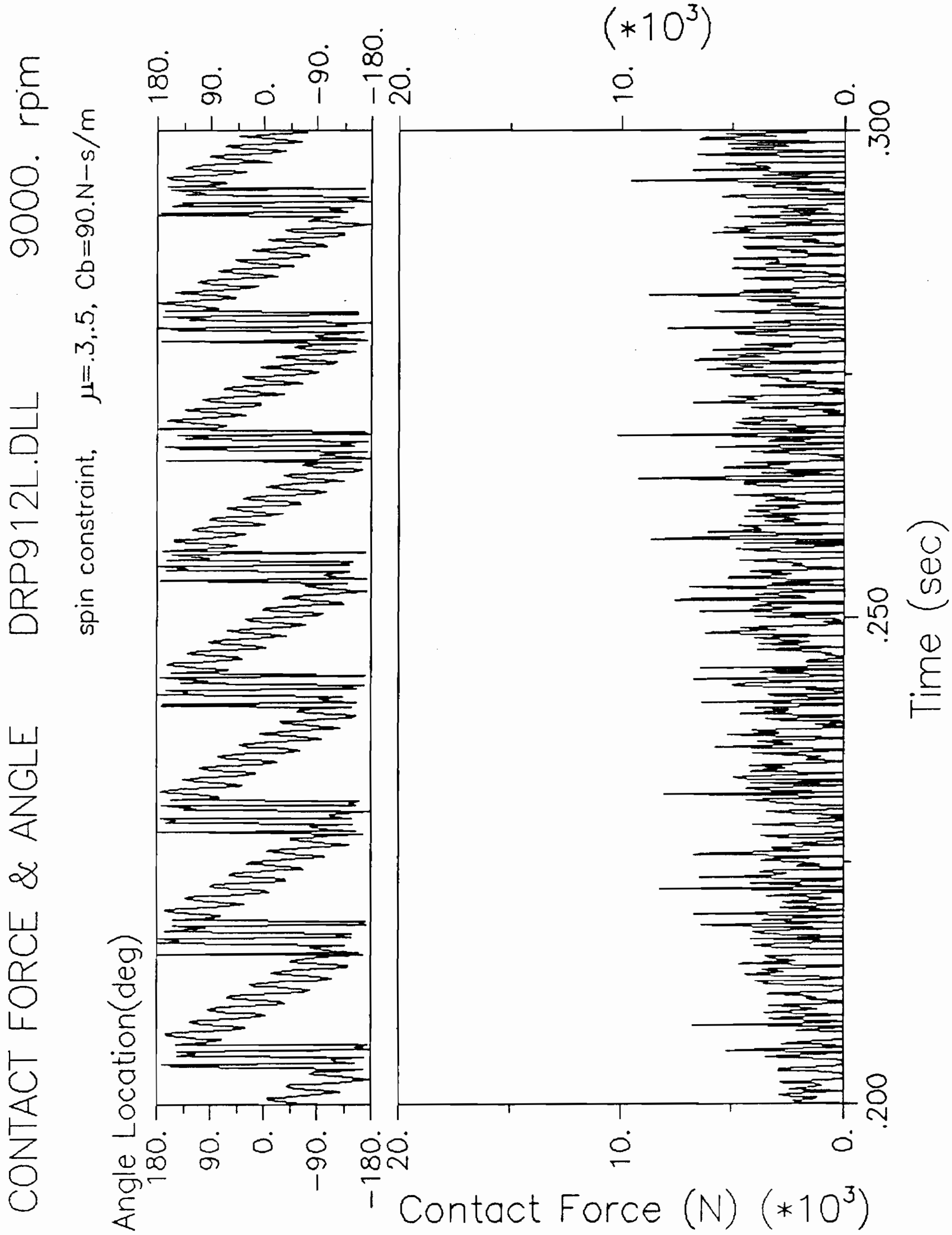

Figure 7b. The Contact Force and the Angular Location of the Journal
T = 0.2 - 0.3 s, spin constraint, $C_b = 90.$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

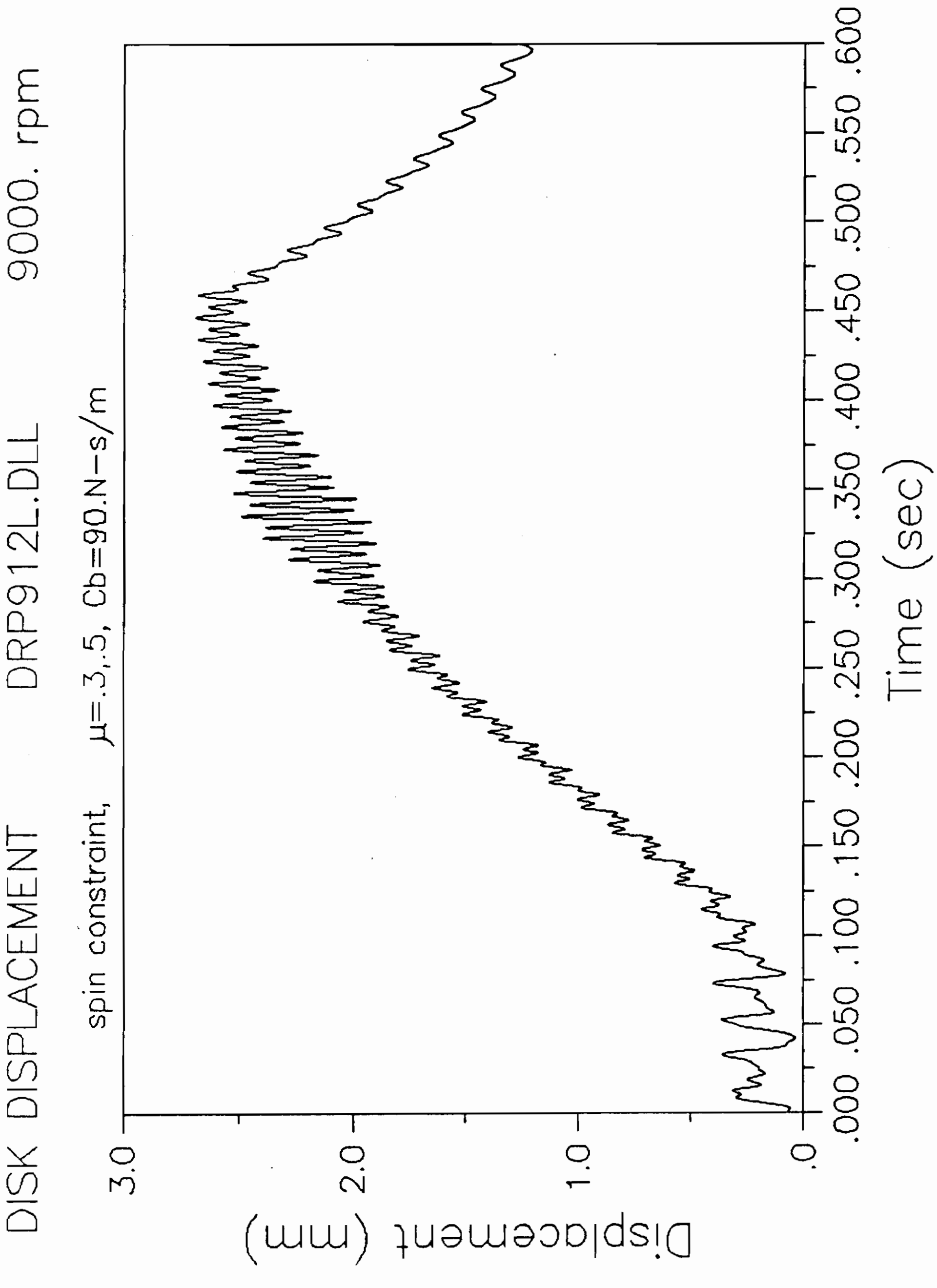

Figure 7c. The Radial Displacement of the Disk

T = 0.0 - 0.6 s, spin constraint, $C_b = 90$. N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

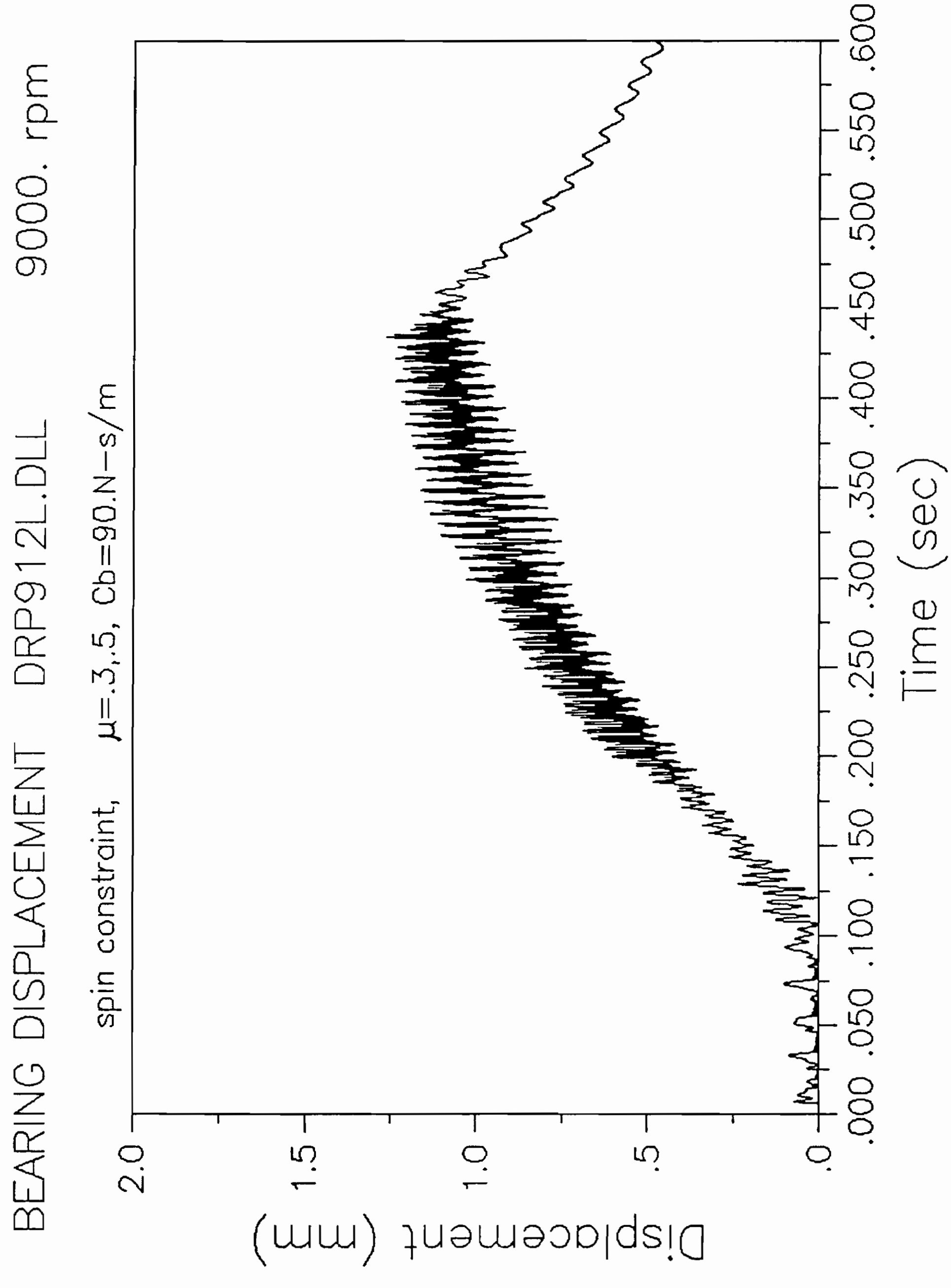

Figure 7d. The Radial Displacement of the Back-up Bearing
T = 0.0 - 0.6 s, spin constraint, $C_b = 90.$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

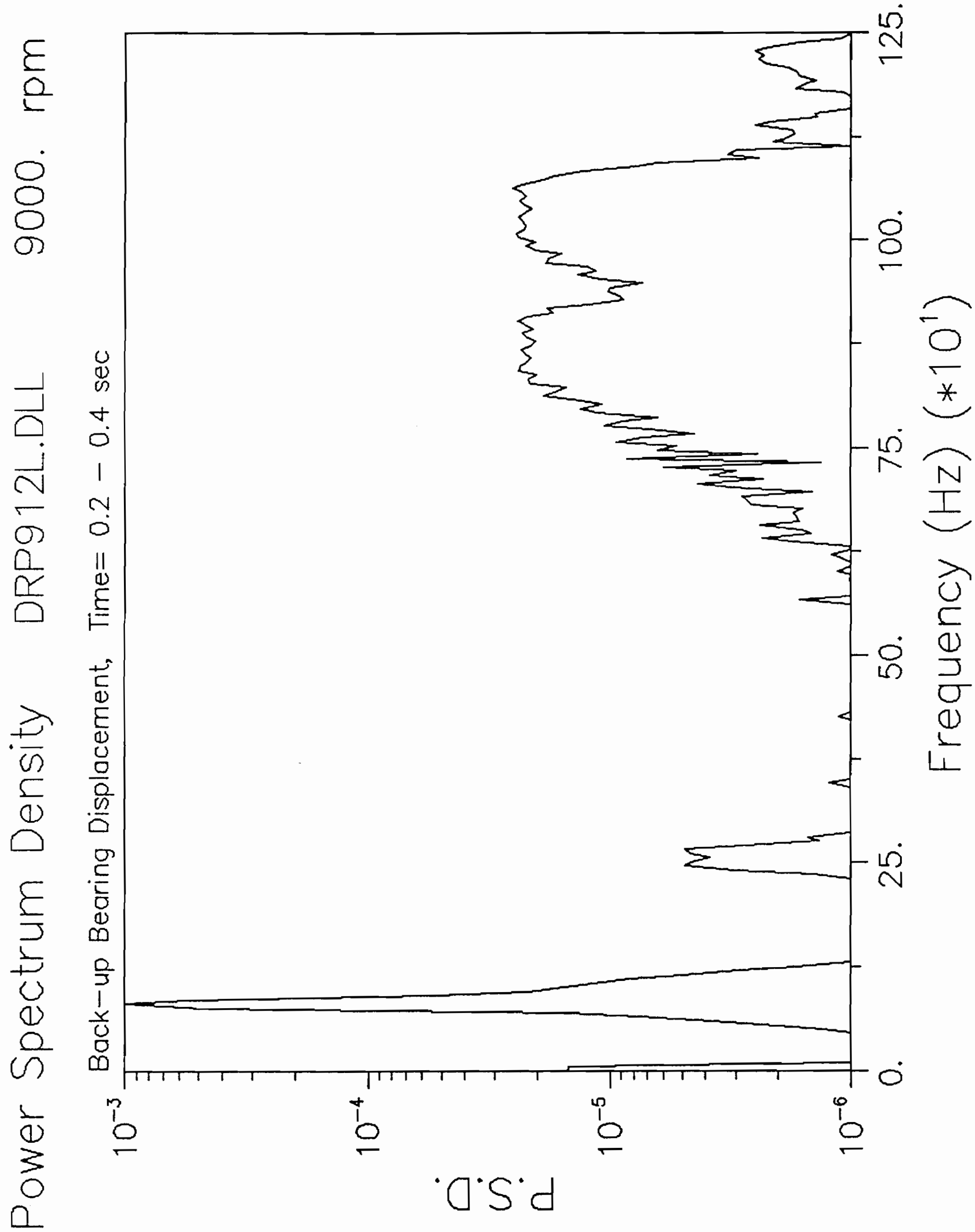

Figure 7e. The Power Spectrum Density of the Back-up Bearing Displacement

T = 0.2 - 0.4 s, spin constraint, $C_b = 90.$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

inner-race of the back-up bearing. The vibration does not affect the displacement of the disk so much because of its high frequency, but it is very harmful for the back-up bearings or the journals because it causes the large contact force between them. The influence of the nonlinear vibration can be observed in the change of the radial displacement of the back-up bearing (Fig. 7d).

Chattopadhyay and Saxena [11] presented the experimental results of the vibration of the beam which is supported with a clearance gap. Their experimental set-up is shown in Fig. 8a and the results are shown in Fig. 8b. The upper sinusoidal curve represents the input displacement by the shaker. The nonlinear vibration appears in the output response shown below. This nonlinear vibration is very similar to that was calculated in the previous case between the journal and the inner-race of the model though the beam does not rotate in their experiment.

When the bearing support damping is between 2000. - 18000. N-s/m, the backward whirl does not occur. The rotor rocks in the bottom half of the back-up bearing. The maximum displacement of the disk as well as the maximum contact force is much smaller than that of the previous case. Figures 9a and 9b show the contact force and the radial displacement for $C_b =$ 6000.N-s/m.

Figures 10a - 10c show the results in a very high support damping case ($C_b =$ 50000.N-s/m). The backward whirl rate is about 100 Hz, which is a little bit larger than the undamped first natural frequency because high support damping makes the system natural frequency higher. The displacement of the disk becomes large when the rotor starts whirling backward. Because of the very high damping, the movement of the back-up bearing is limited, which makes the disk response smaller than that for a light support damping.

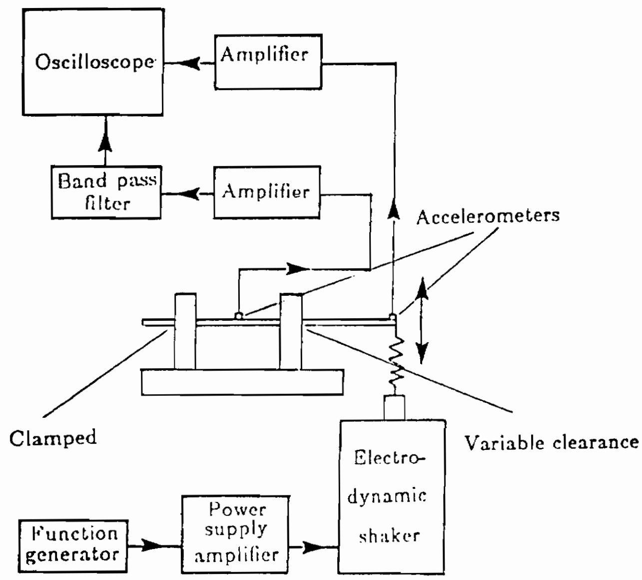

Figure 8a. Experimental Set-up. (Ref. [11])

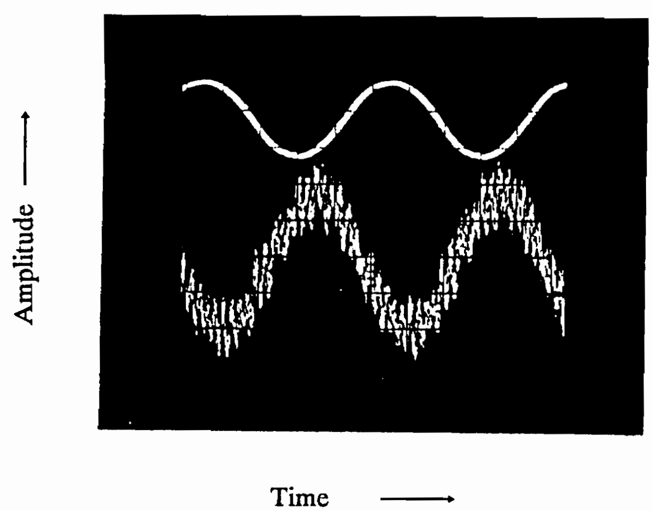

Figure 8b. Experimental Result of the Input and Output Displacement of the Beam which is Supported with a Gap. (Ref. [11])

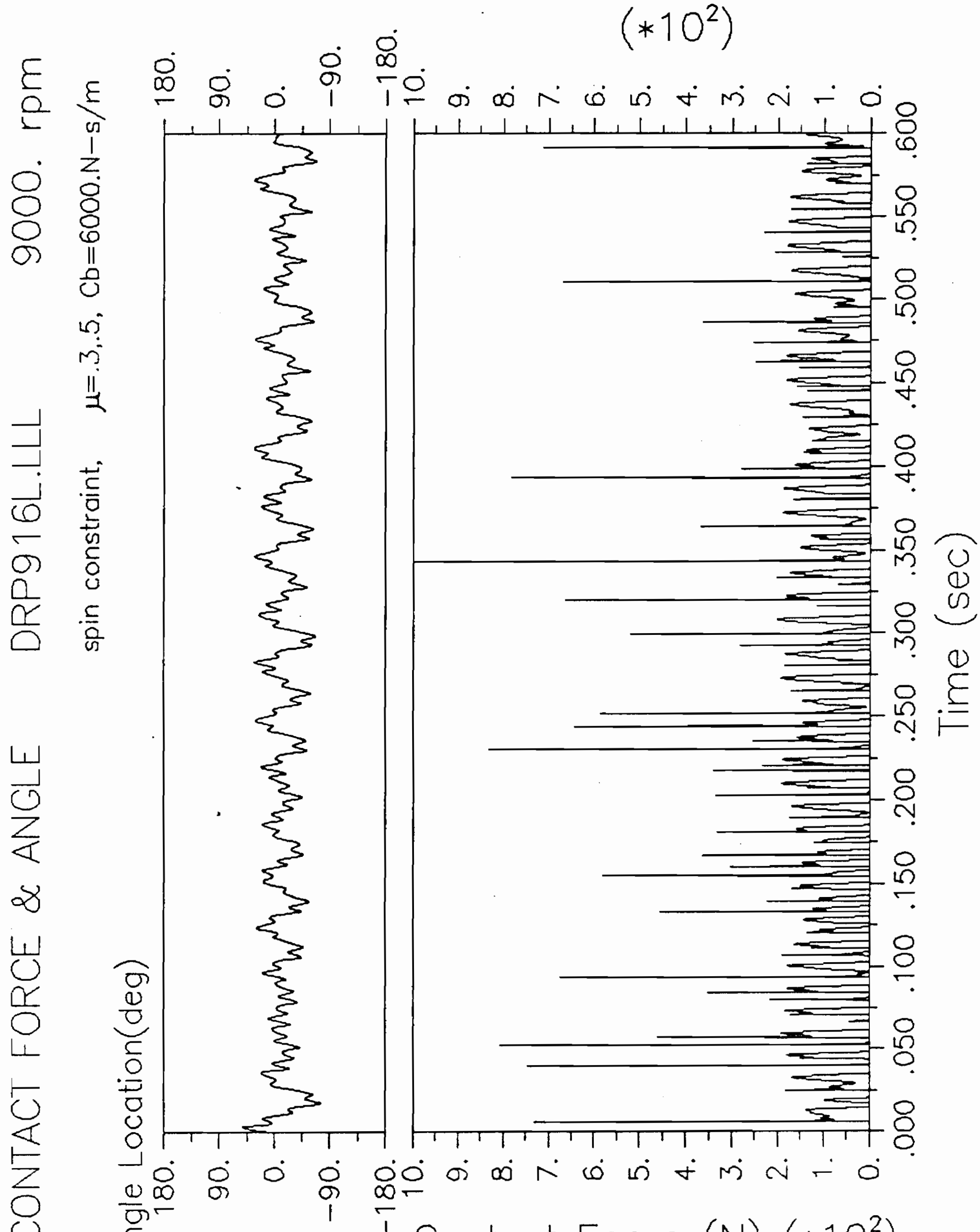

Figure 9a. The Contact Force and the Angular Location of the Journal
T = 0.0 - 0.6 s, spin constraint, $C_b = 6000.$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

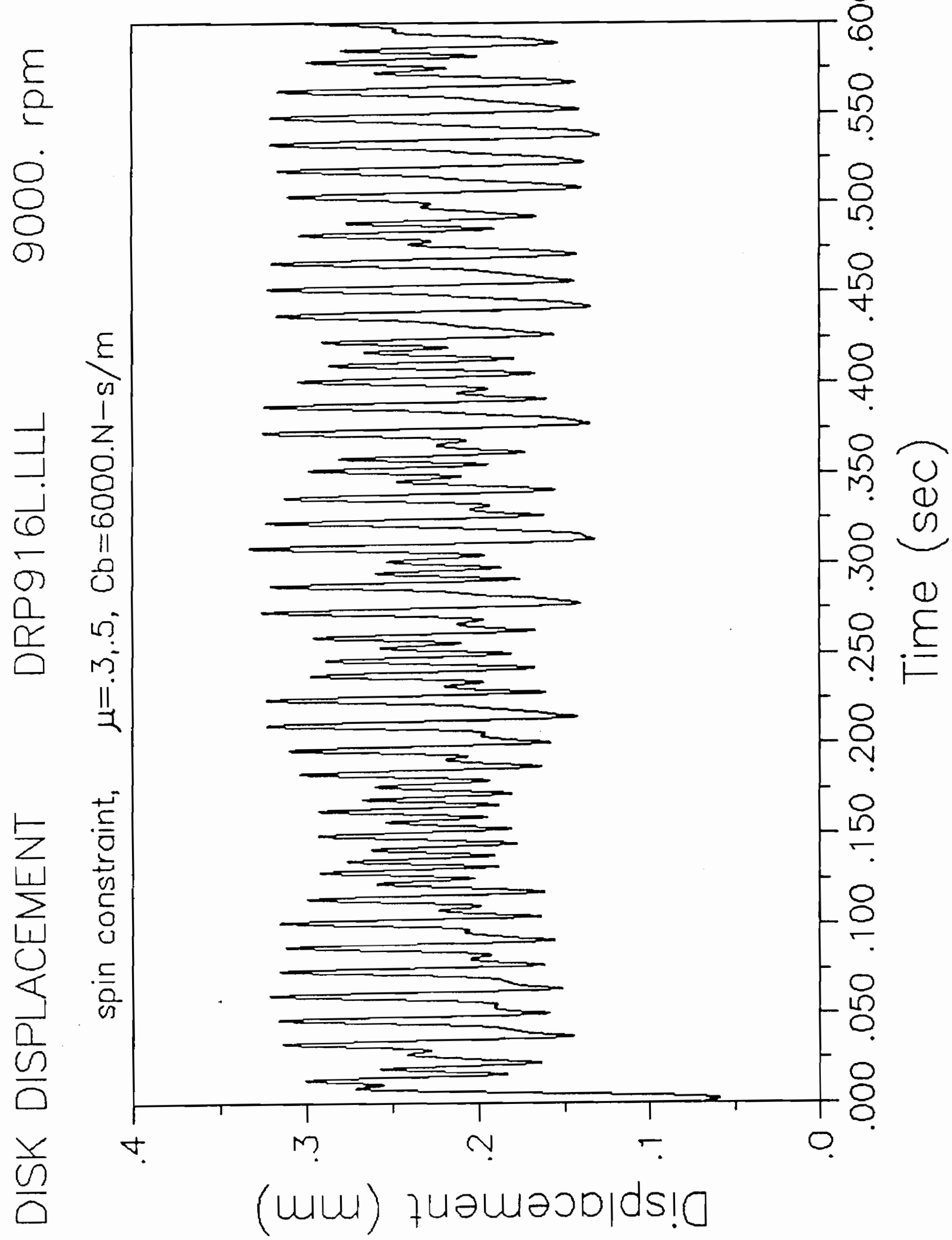

Figure 9b. The Radial Displacement of the Disk
$T = 0.0$ - 0.6 s, spin constraint, $C_b = 6000.$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

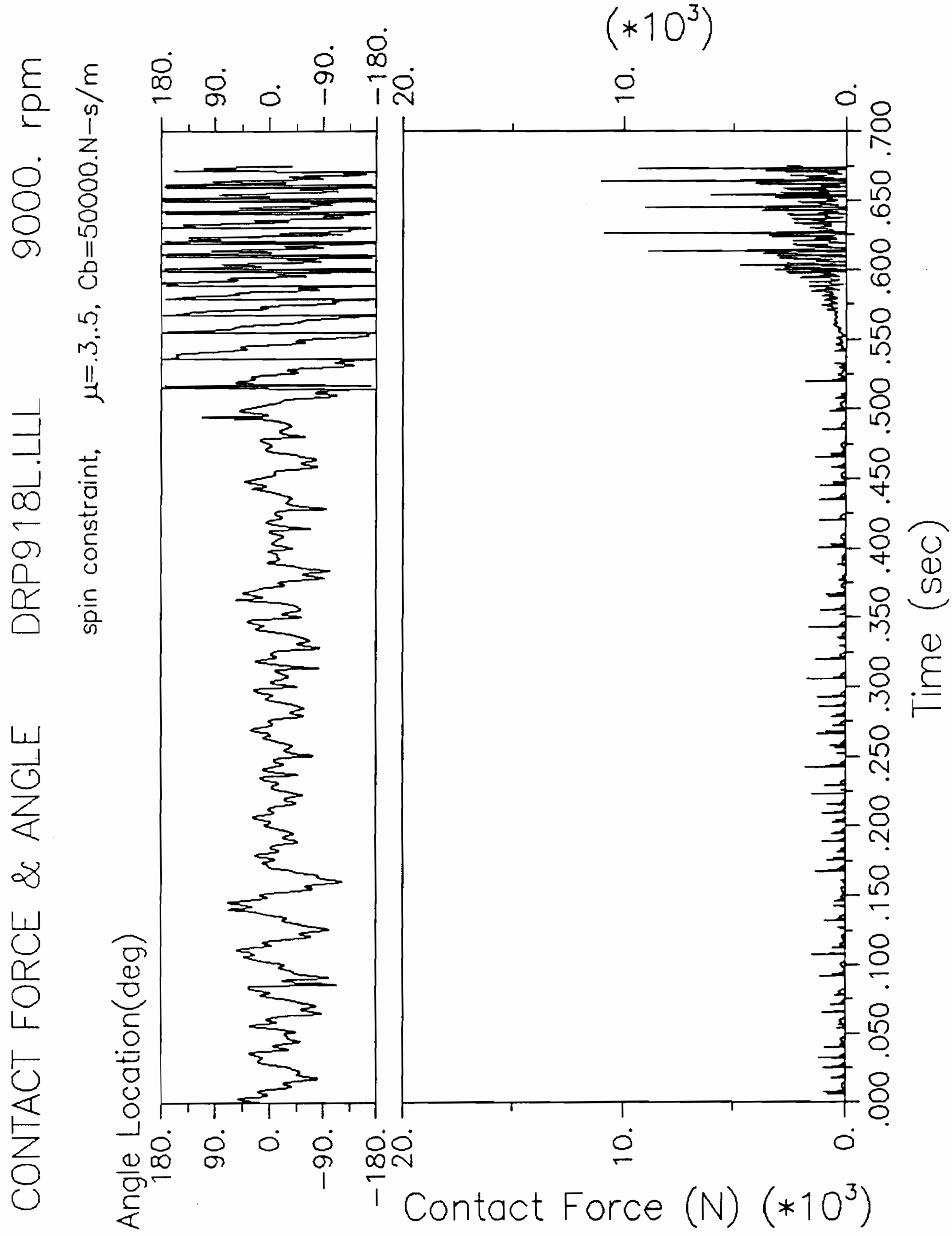

Figure 10a. The Contact Force and the Angular Location of the Journal

T = 0.0 - 0.675 s, spin constraint, $C_b = 5.E4$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

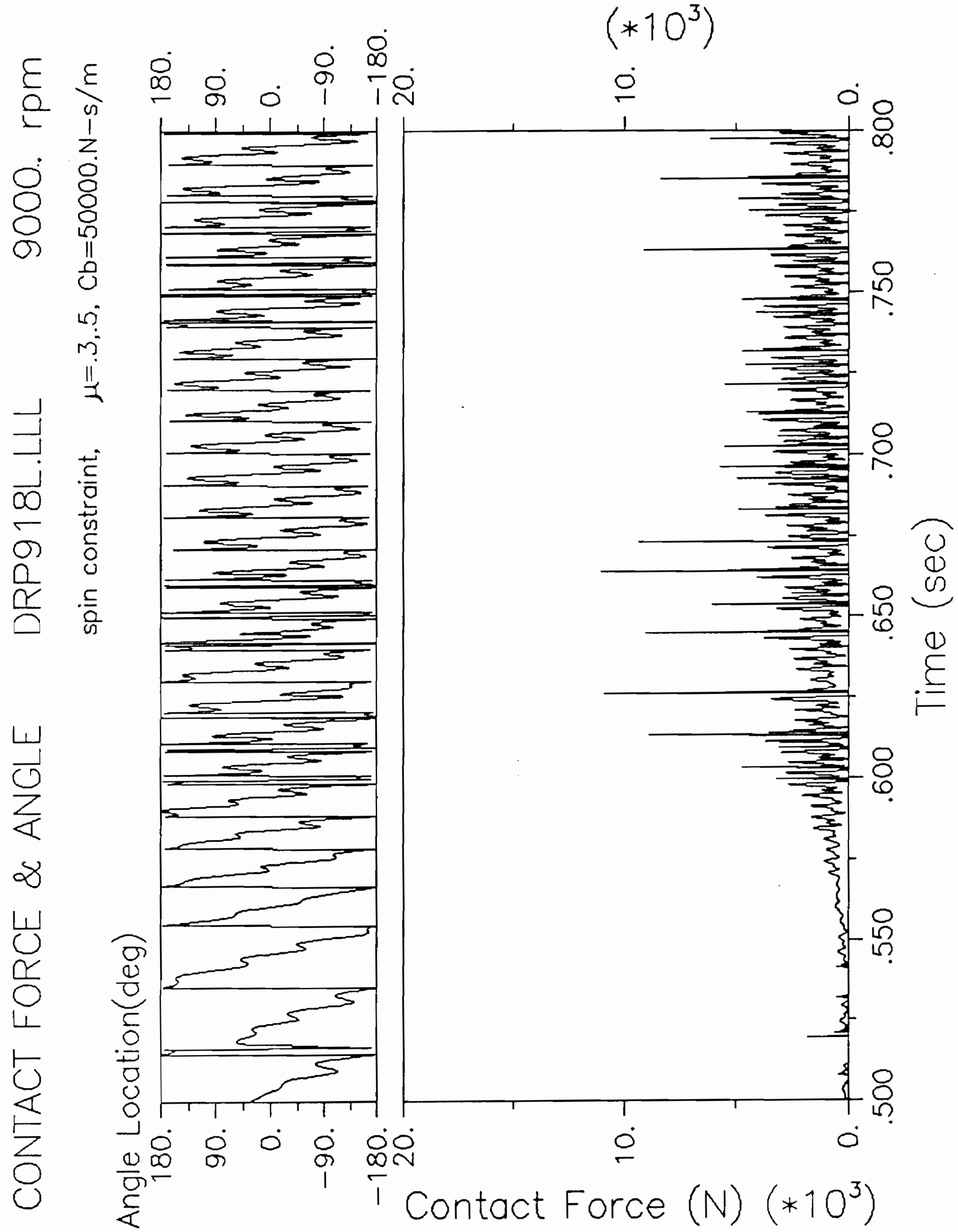

Figure 10b. The Contact Force and the Angular Location of the Journal
T = 0.5 - 0.8 s, spin constraint, $C_b = 5.E4$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

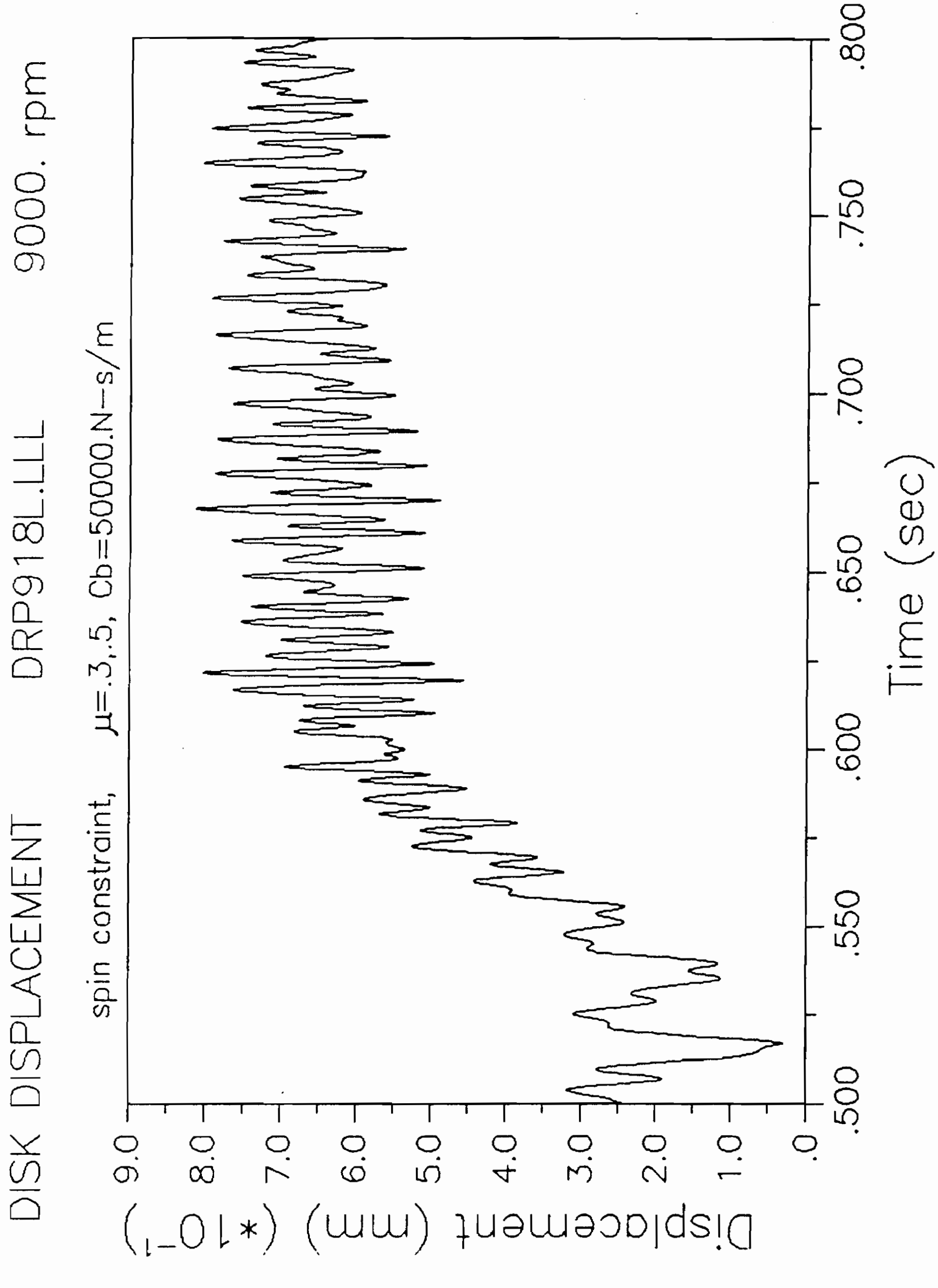

Figure 10c. The Radial Displacement of the Disk

T = 0.5 - 0.8 s, spin constraint, $C_b = 5.E4$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

The higher damping support causes higher backward whirl rate as shown in Figs. 11a -11c. In the previous cases, the backward whirl rate is nearly equal to the first natural frequency of the system, and there is a slip between the journal and the inner-race owing to the different velocity of the contact point. Since very high support damping freezes the movement of the back-up bearing, the contact force becomes larger than those of the previous cases when the backward whirl occurs. The larger contact force, which keep the journal in contact with the inner-race without slip, makes the backward whirl rate very high (nearly 2800 Hz), and contributes not to excite the non-linear vibration. In spite of the large contact force, the maximum disk displacement is relatively small because the high support damping freezes the movement of the back-up bearing support structure.

3.2 Freely Spinning Back-up Bearings

In case there is no rotational constraint in the back-up bearing but the moment of inertia and the rotational friction due to the load, the backward whirl is also observed in light support damping cases as shown Figs. 12a and 12b. The backward whirl rate is nearly equal to the first natural frequency as observed in case that the spin constraint exists. The rotational speed of the inner-race of the back-up bearing catches up with that of the journal. This is the reason for the decrease of the contact force and the disk displacement after a few backward whirls. Owing to the freely spinning inner-race, the rotor keeps contacting with the inner-race without a slip, and the contact force remains small compared with

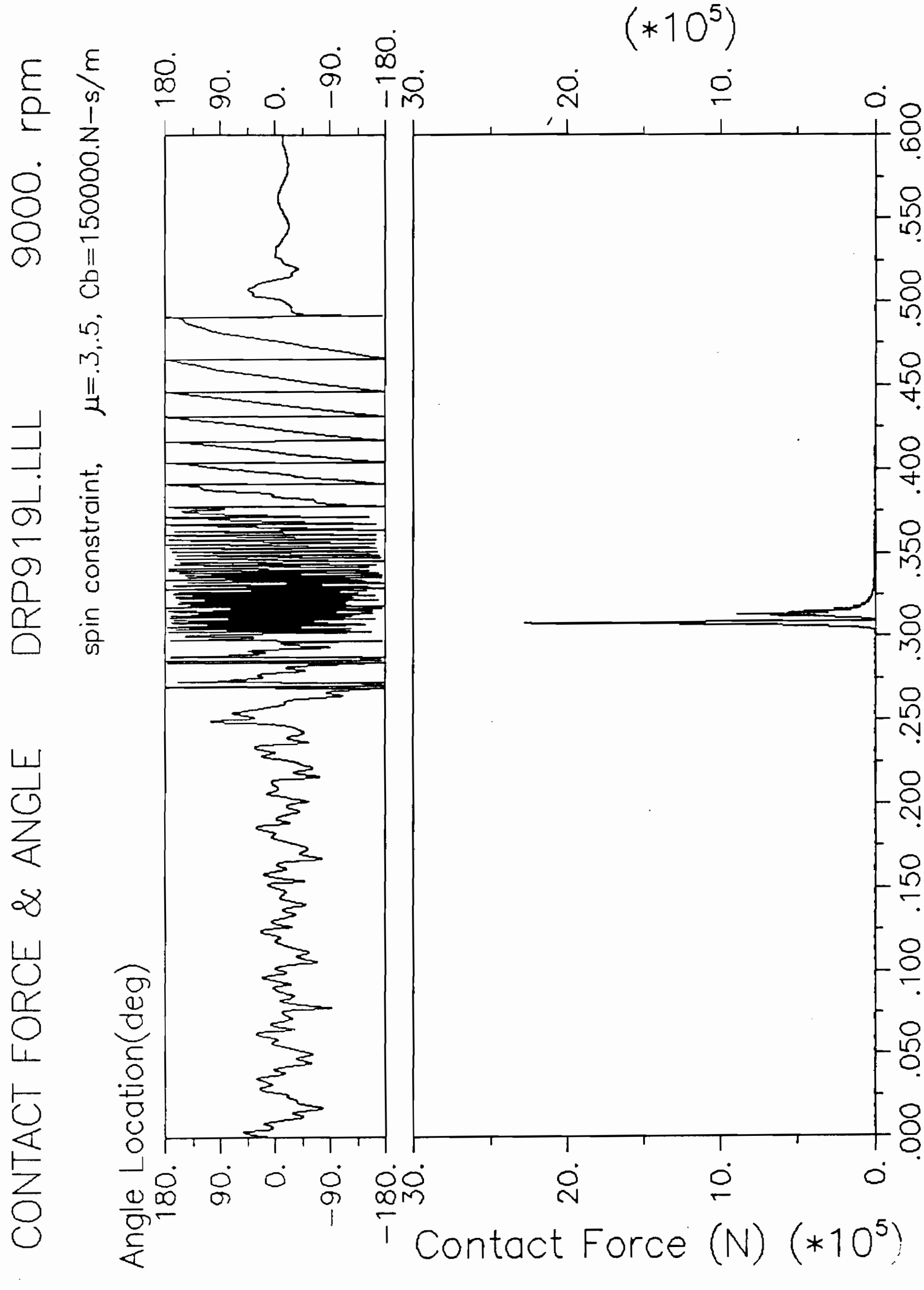

Figure 11a. The Contact Force and the Angular Location of the Journal
T = 0.0 - 0.6 s, spin constraint, $C_b = 15.E4$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

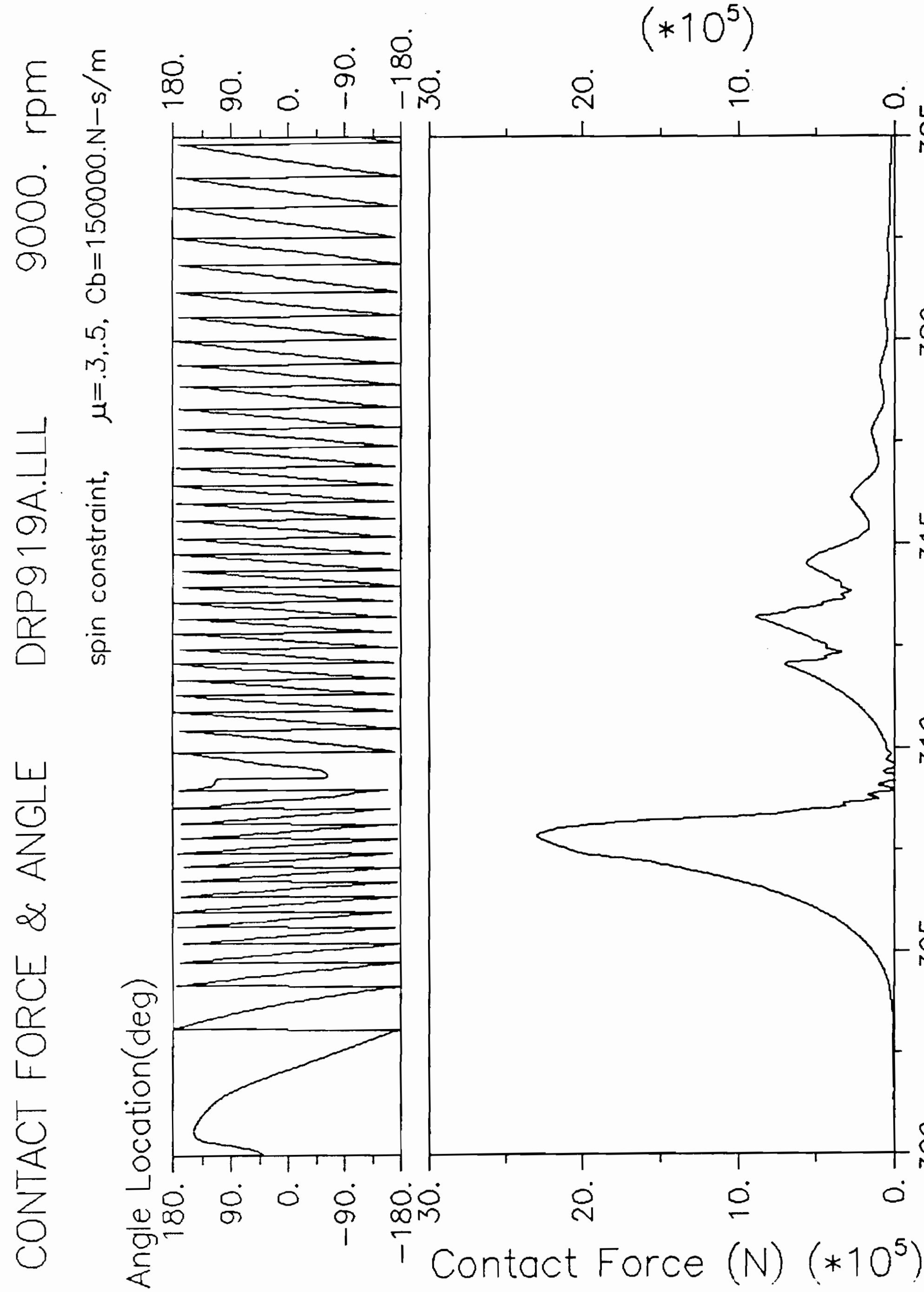

Figure 11b. The Contact Force and the Angular Location of the Journal

T = 0.300 - 0.325 s, spin constraint, $C_b = 15.E4$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

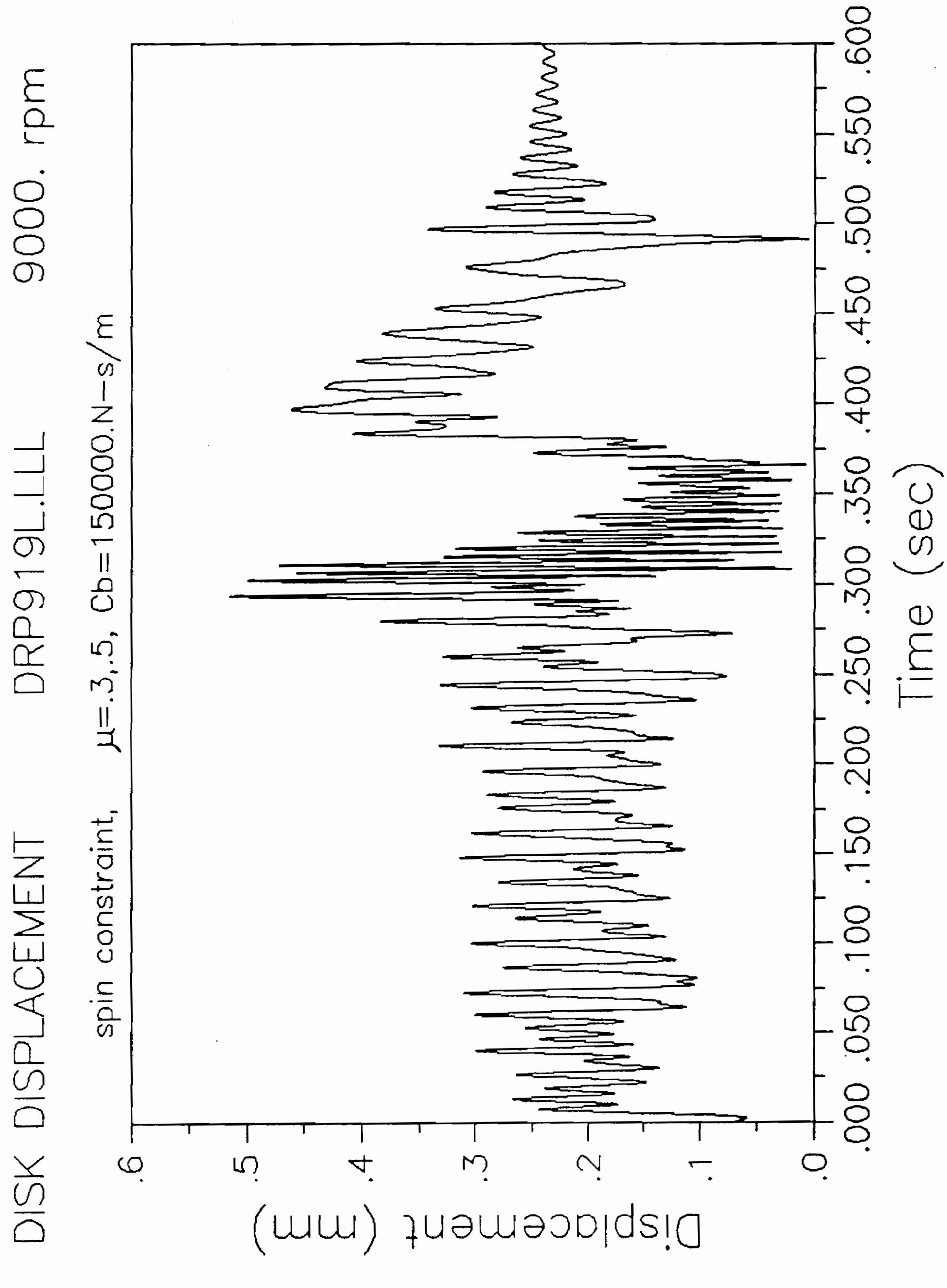

Figure 11c. The Radial Displacement of the Disk

$T = 0.0$ - 0.6 s, spin constraint, $C_b = 15.E4$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

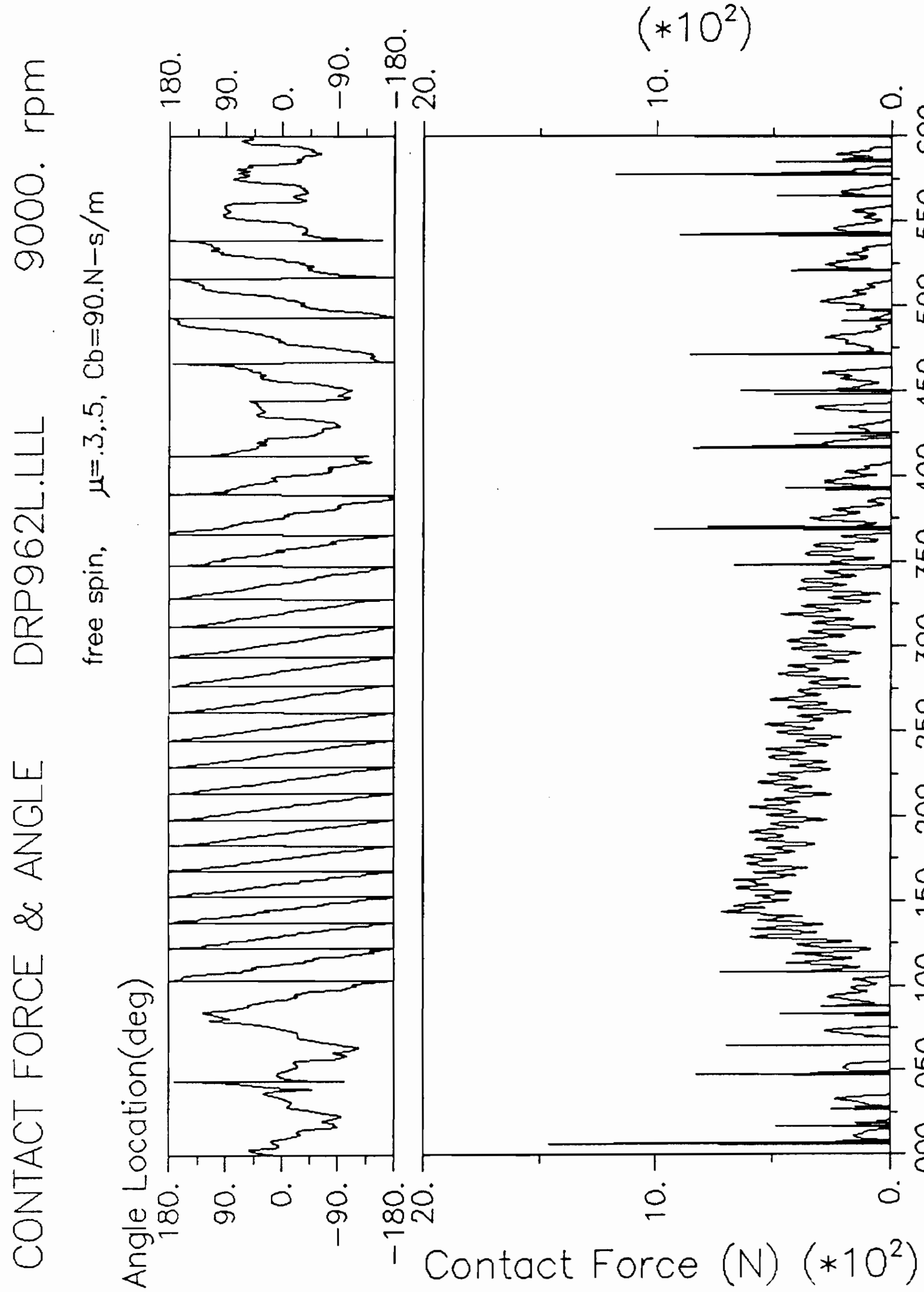

Figure 12a. The Contact Force and the Angular Location of the Journal
T = 0.0 - 0.6 s, freely spinning, C_b = 90. N-s/m, μ_d = 0.3, μ_s = 0.5

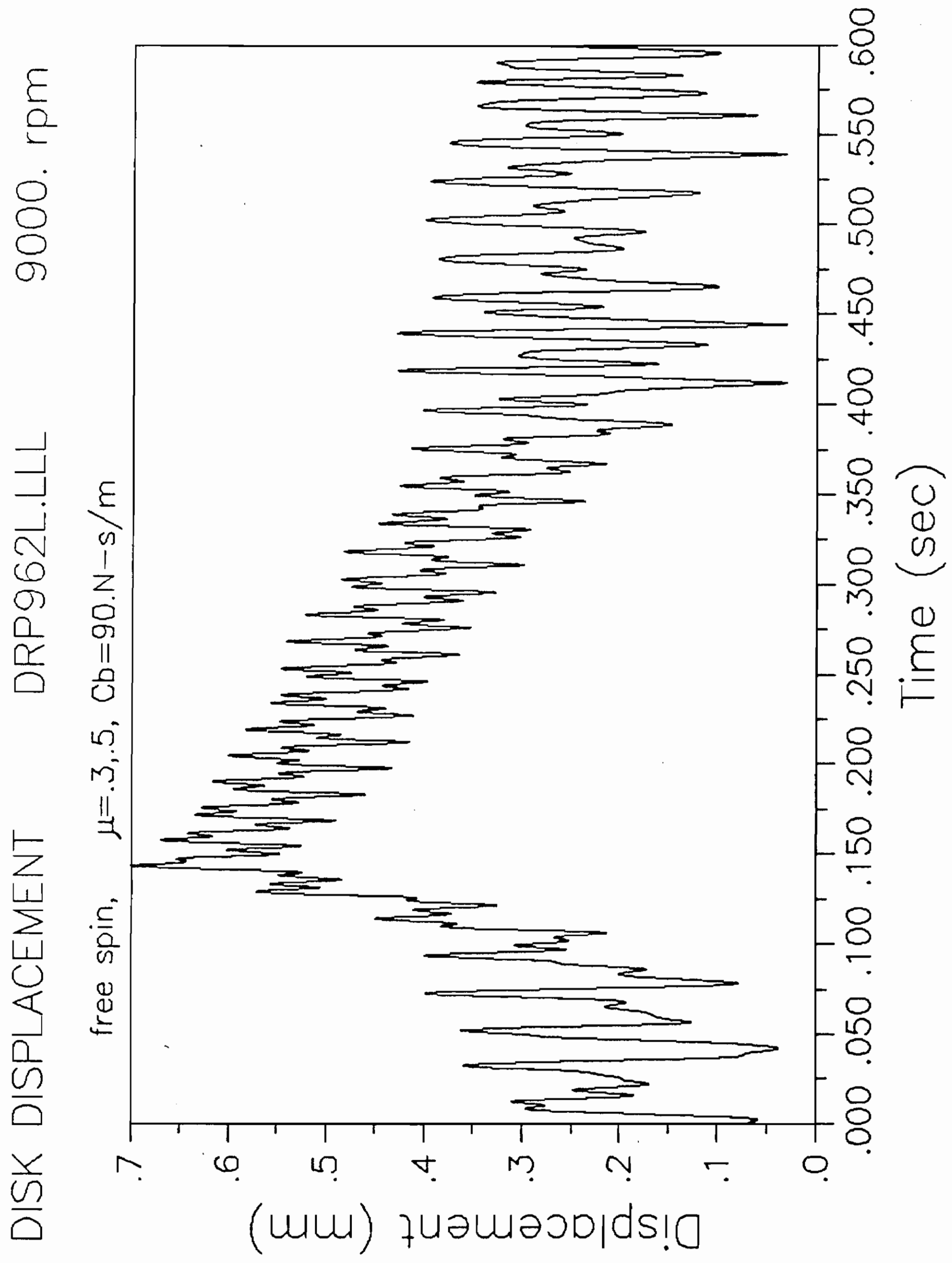

Figure 12b. The Radial Displacement of the Disk

T = 0.0 - 0.6 s, freely spinning, $C_b = 90.$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

the case in which the spin constraint exists. The non-linear vibration is not observed because the journal keeps contact with the inner-race without slip. The maximum disk displacement is observed in the first three backward whirls (Fig. 12b) ; on the other hand, the maximum contact force is observed at the first contact just after the rotor drops.

Due to the large braking torque of the rotor, its rotational speed becomes smaller than that of the inner-race. Therefore, after the backward whirl stops, the forward whirl is observed.

When the bearing support damping is between 6000. - 50000. N-s/m, the whirl does not occur. The results of the disk response and contact force are very similar to those observed when the spin constraint exists. The only difference is that the angle location of the rotor is shifted to the bottom of the bearing because of the freely spinning inner-race (Figs. 13a, 13b).

Figure 14a shows that the backward whirl is also observed in a very light support damping case. In this case, the whirl begins at 0.275 sec., but it does not last long because the rotational speed of the inner-race catches up with that of the journal. The rotor begins to rock at the bottom of the inner-race immediately. The contact force and the radial displacement of the disk take the maximum values during the backward whirl at about 0.3 sec. as shown in Figs. 14a and 14b, but these values are not as large as those occurring in case the constraint in rotational motion of the inner-race exists, which is discussed the previous section.

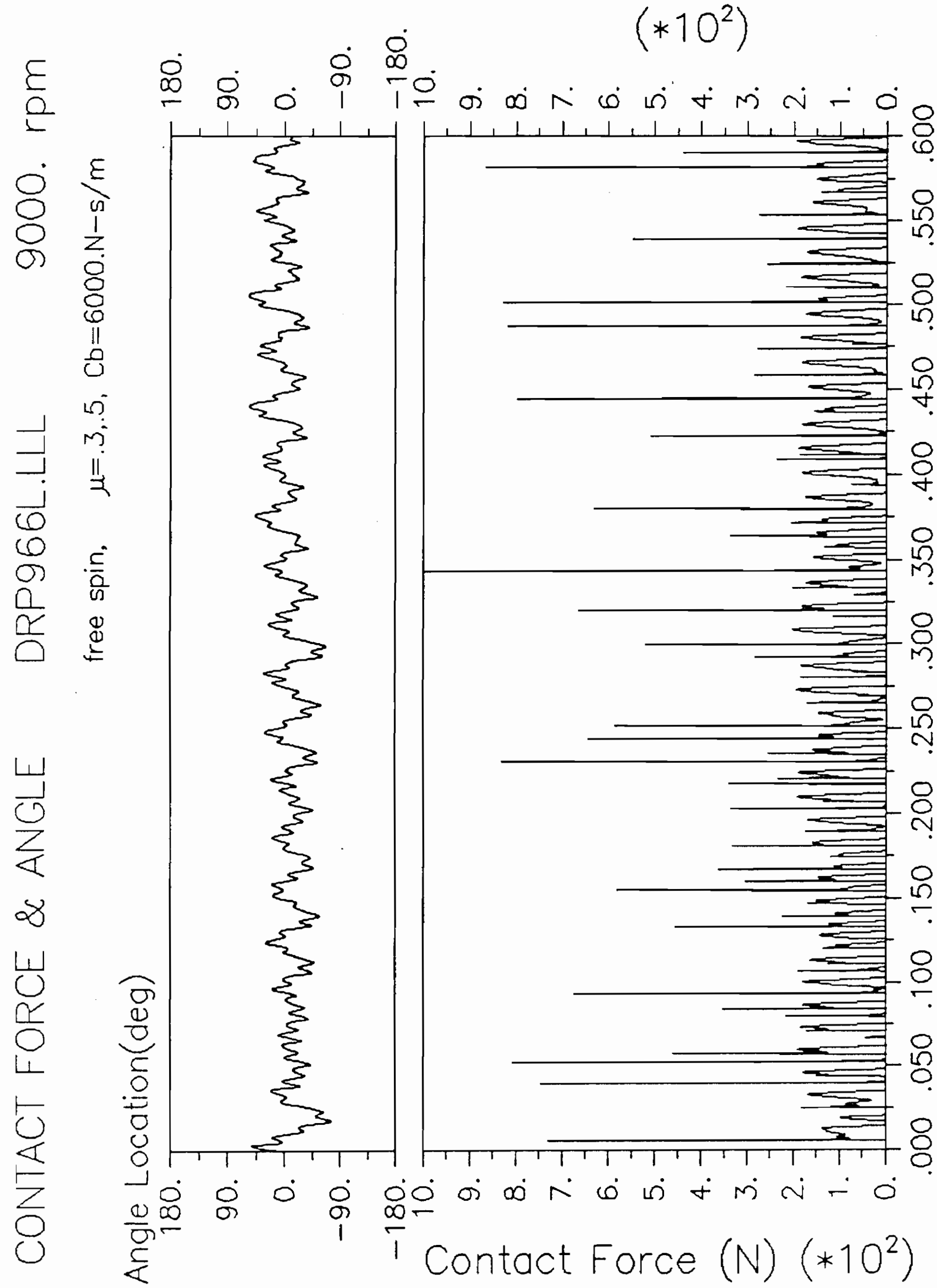

Figure 13a. The Contact Force and the Angular Location of the Journal
T = 0.0 - 0.6 s, freely spinning, $C_b = 6000.$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

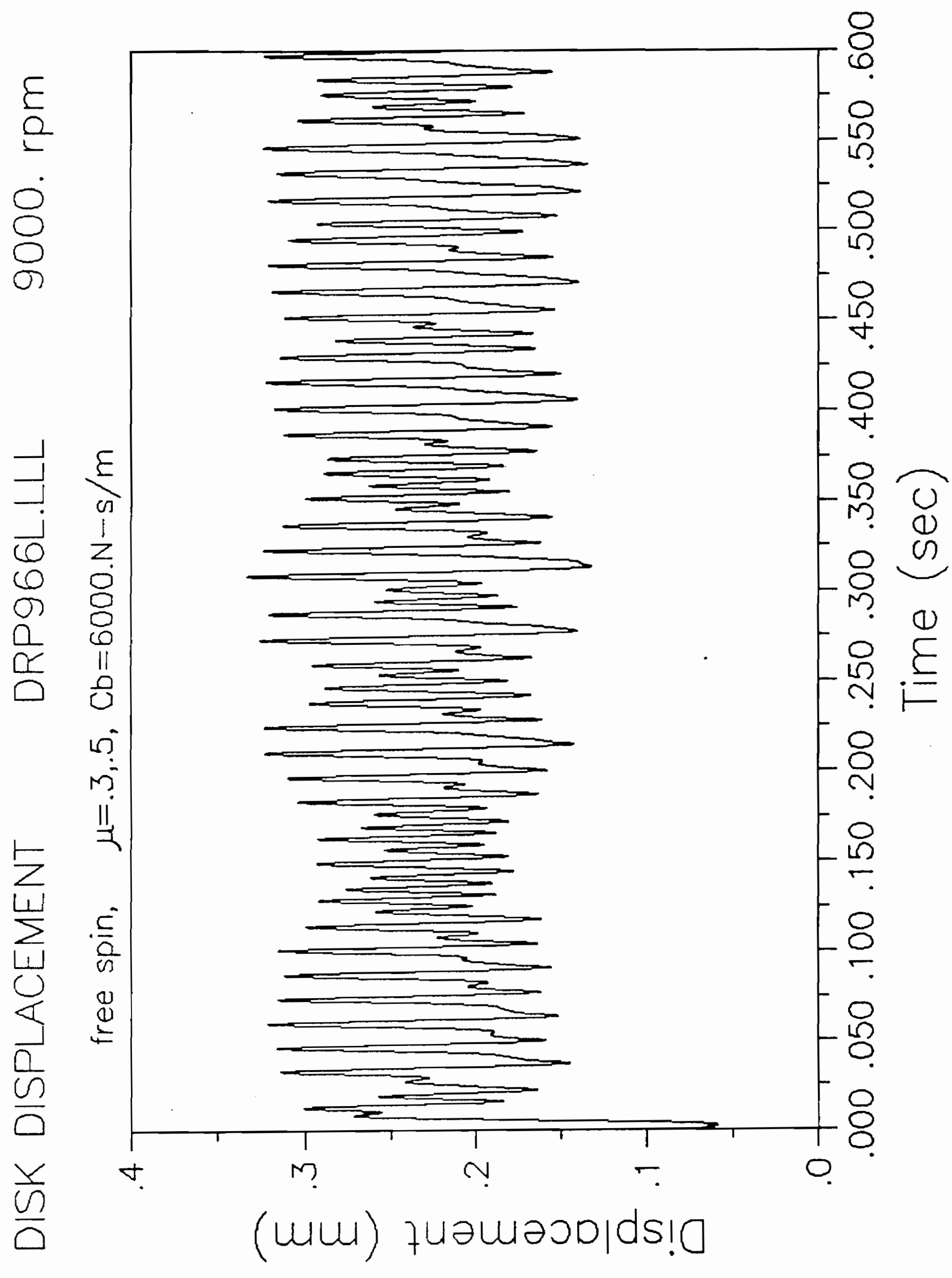

Figure 13b. The Radial Displacement of the Disk

T = 0.0 - 0.6 s, freely spinning, $C_b = 6000.$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

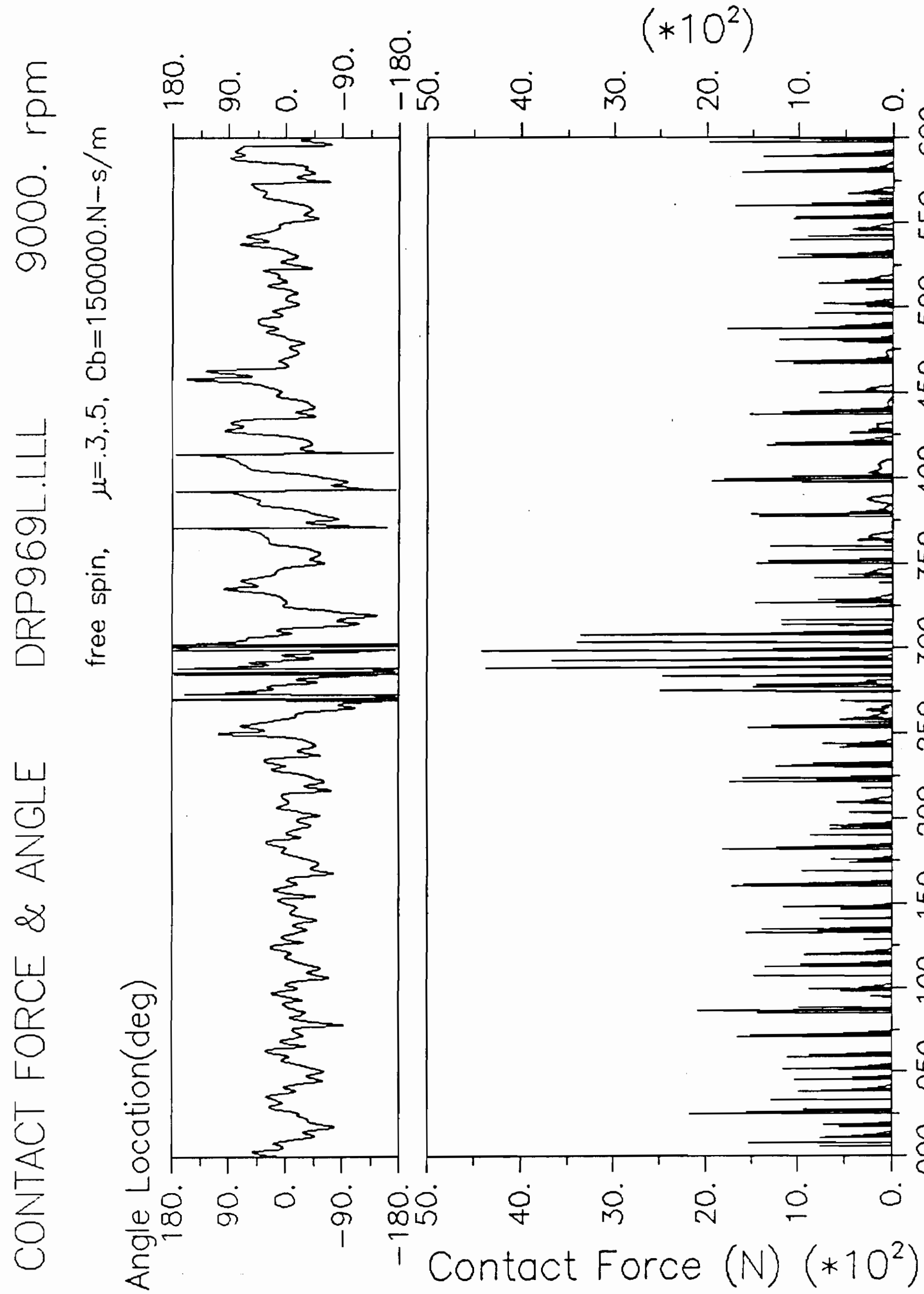

Figure 14a. The Contact Force and the Angular Location of the Journal

T = 0.0 - 0.6 s, freely spinning, $C_b = 15.E4$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

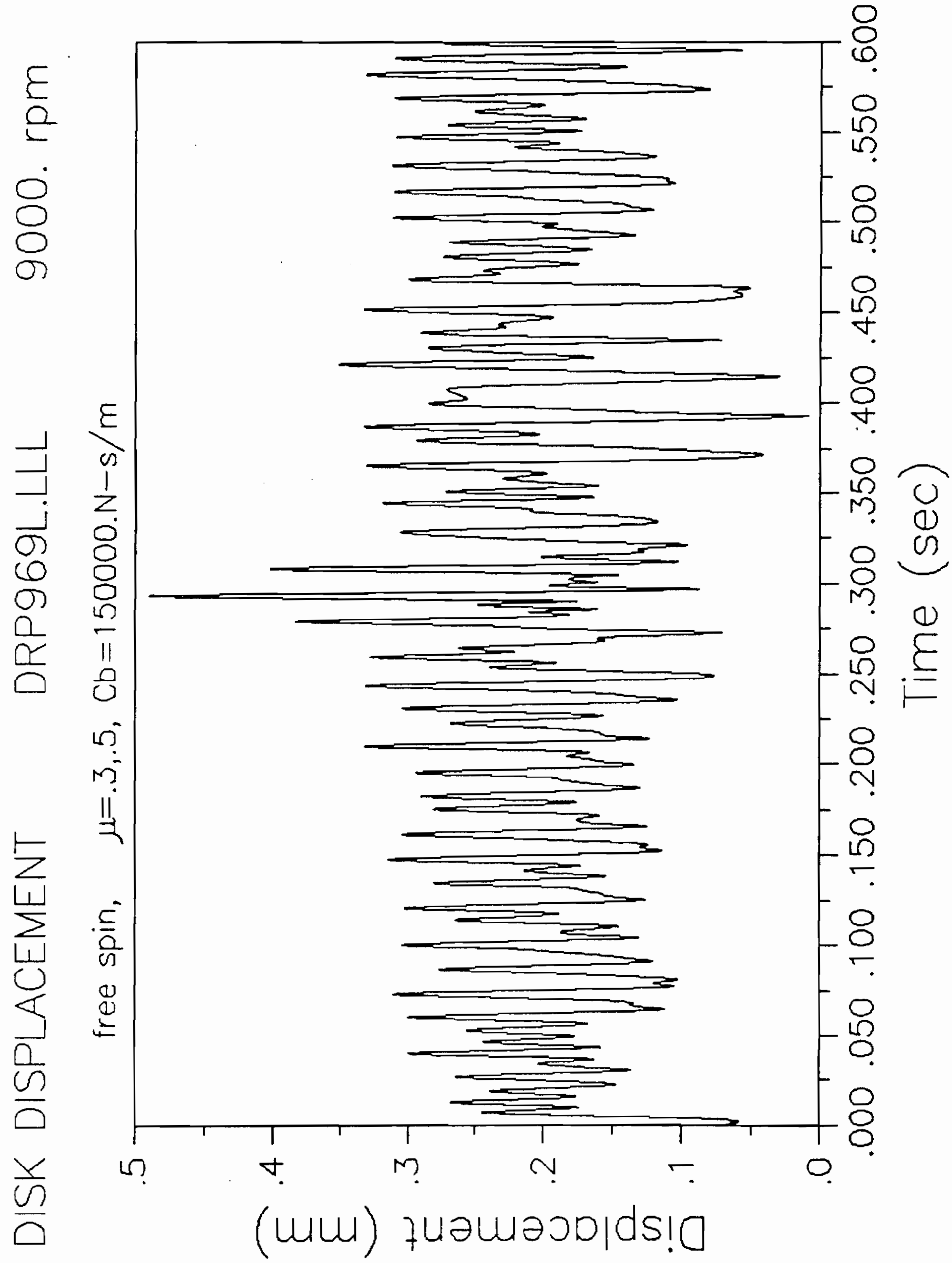

Figure 14b. The Displacement of the Disk
T = 0.0 - 0.6 s, freely spinning, $C_b = 15.E4$ N-s/m, $\mu_d = 0.3$, $\mu_s = 0.5$

3.3 The Influence of the Coefficient of Friction and Rotational Speed

Figure 15a and 15b show the maximum contact force and the maximum radial displacement of the disk versus the damping ratio (C_b / C_s) at the operational speed (9000 rpm) for three different coefficients of friction and two different cases of rotational constraint for the back-up bearing. When the damping ratio is between 30 - 100, there is little difference in the maximum contact force and the maximum displacement for all cases.

Concerning to the maximum contact force, in lighter damping cases, the constraint of the bearing has a dominant effect of making the contact force larger, compared with the effect of the coefficient of friction. Still the larger coefficient of friction leads to the larger contact force. The disk displacement is very sensitive to the coefficient of friction and the constraint in rotational movement when the damping ratio is less than 2.

Figures 16a -16d show the maximum contact forces and the maximum radial displacement of the disk at 7000 rpm and 5000 rpm, which is the lowest undamped linear natural frequency of the system. When the damping ratio is between 30 - 100, both the maximum contact force and the maximum radial displacement of the disk have the minimum values, which is similar to the previous results at the operational speed.

With respect to the contact force, as the rotational speed becomes faster, it becomes larger because it is largely affected by the force due to the unbalance of the rotor, that is, the rotational speed and the eccentricity.

As for the radial displacement of the disk, in case the damping ratio is

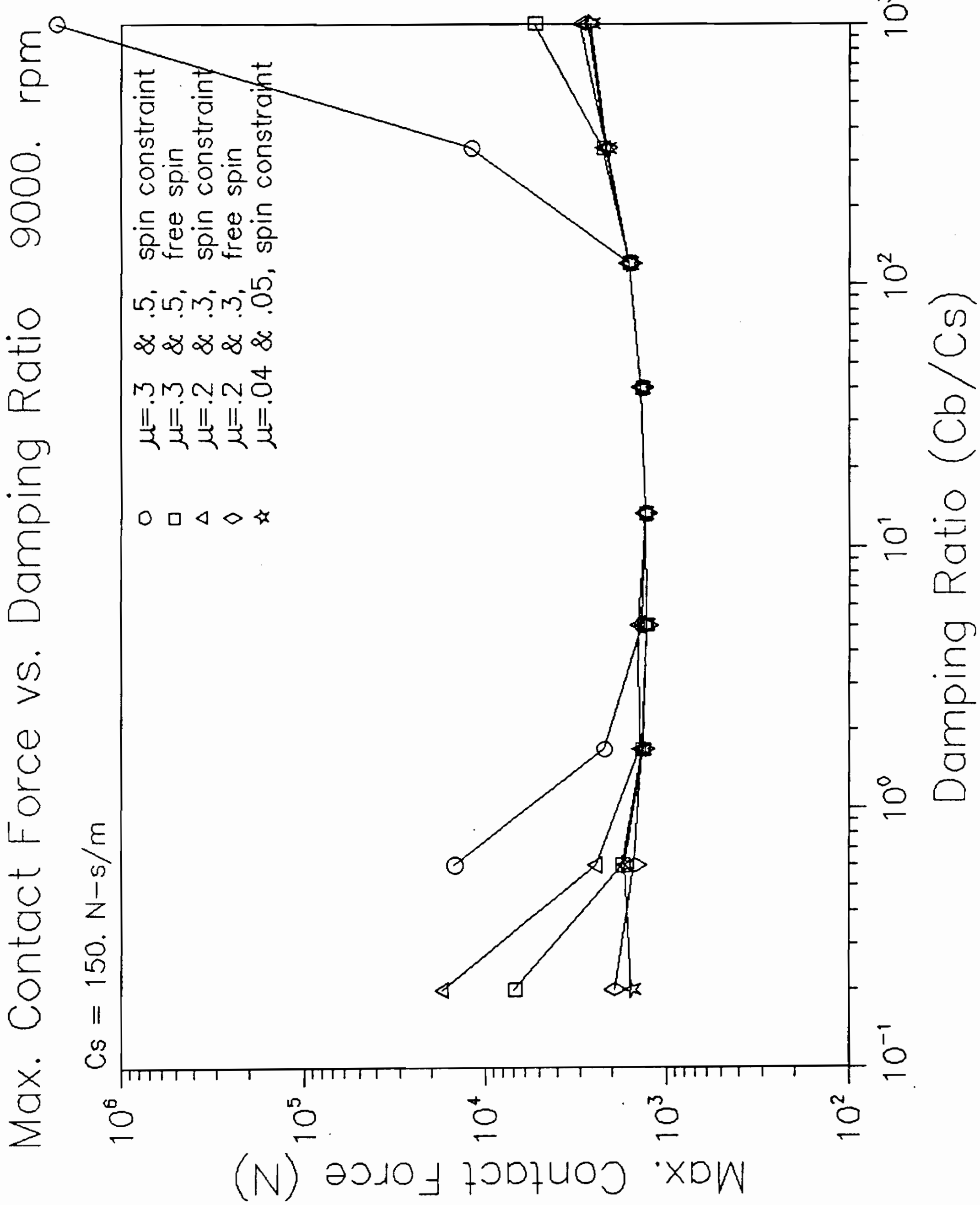

Figure 15a. The Maximum Contact Force and Damping Support Ratio
9000. rpm, $C_s = 150.$ N-s/m.

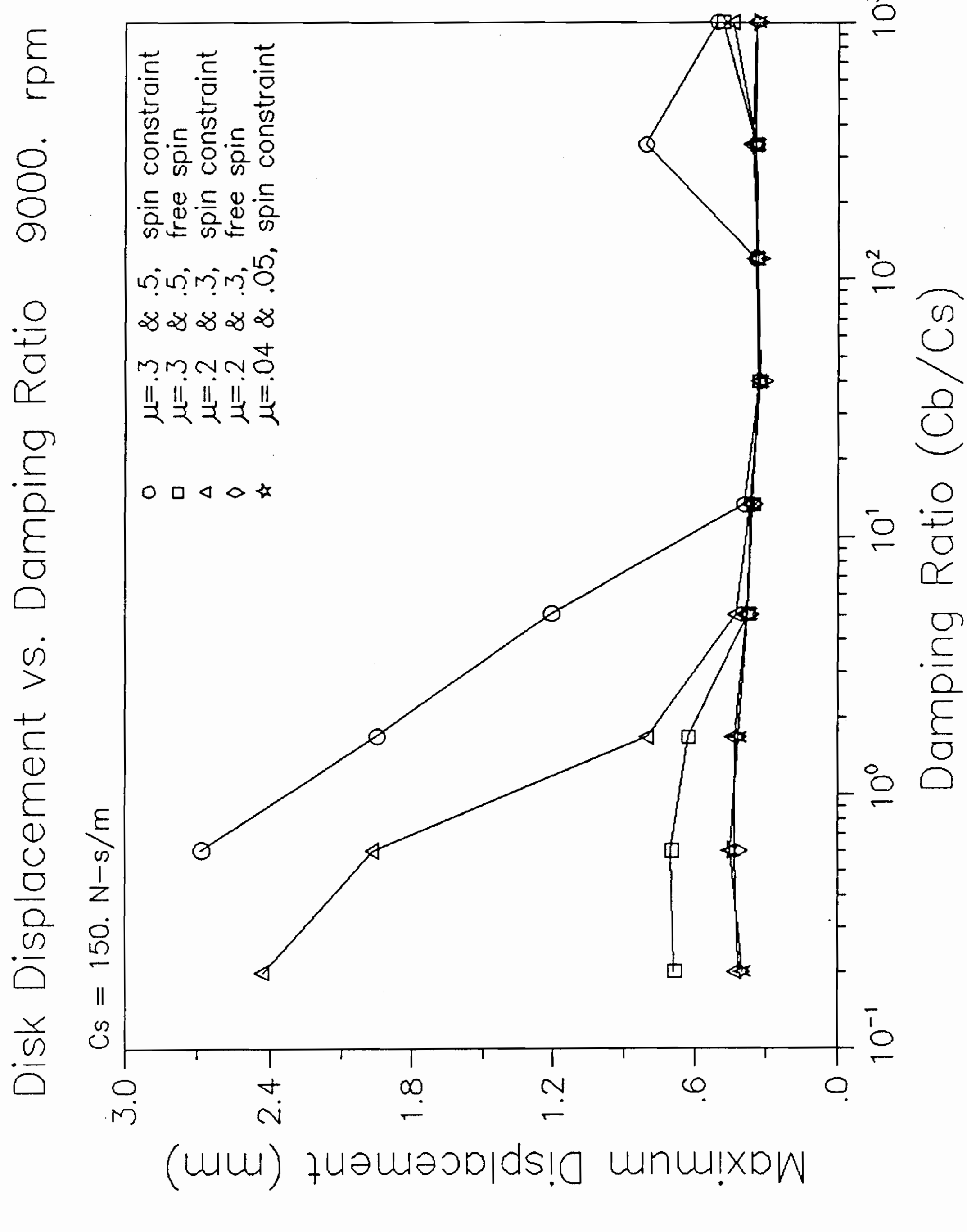

Figure 15b. The Maximum Disk Displacement and Damping Support Ratio
9000. rpm, $C_s = 150.$ N-s/m.

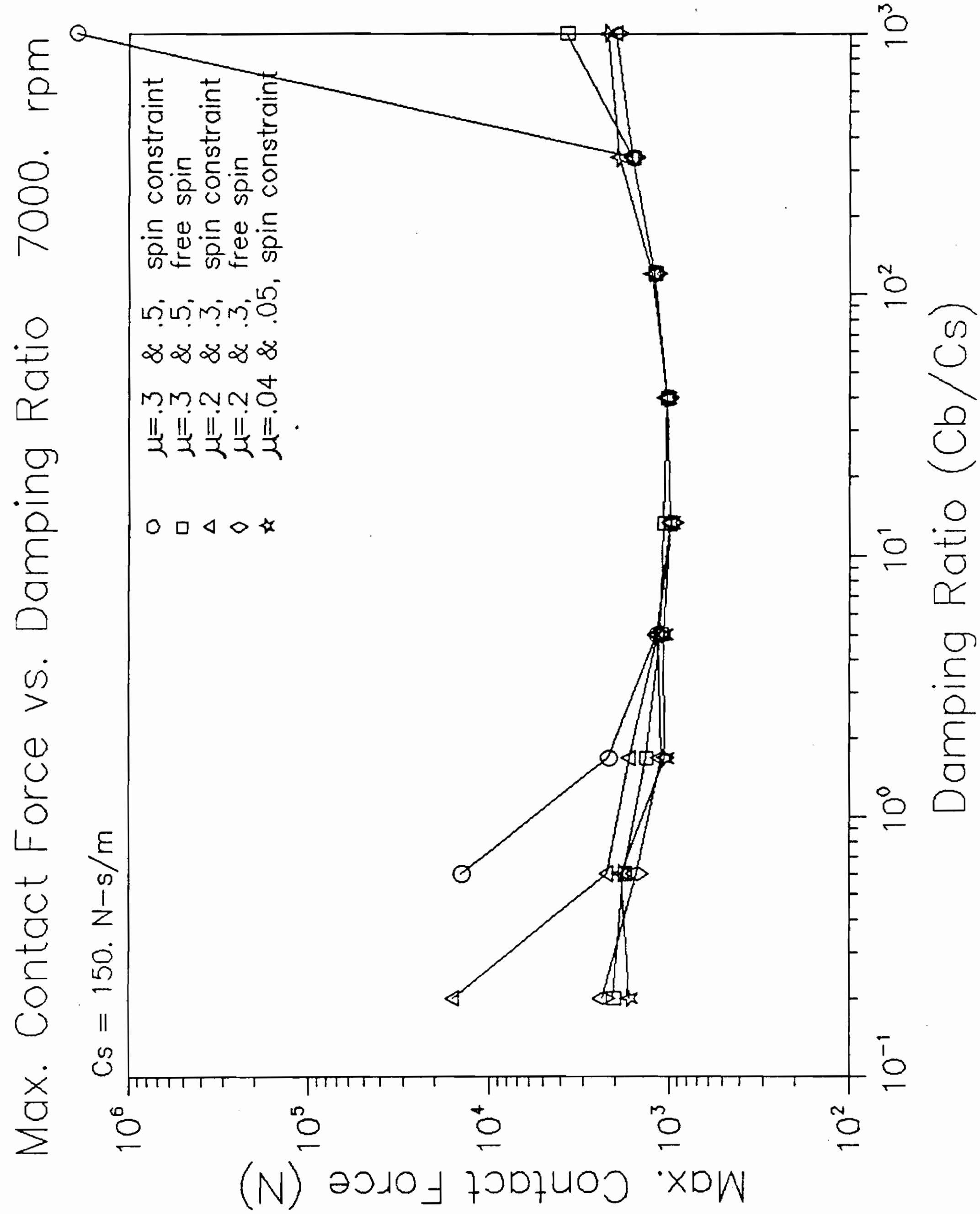

Figure 16a. The Maximum Contact Force and Damping Support Ratio
7000. rpm, $C_s = 150.$ N-s/m.

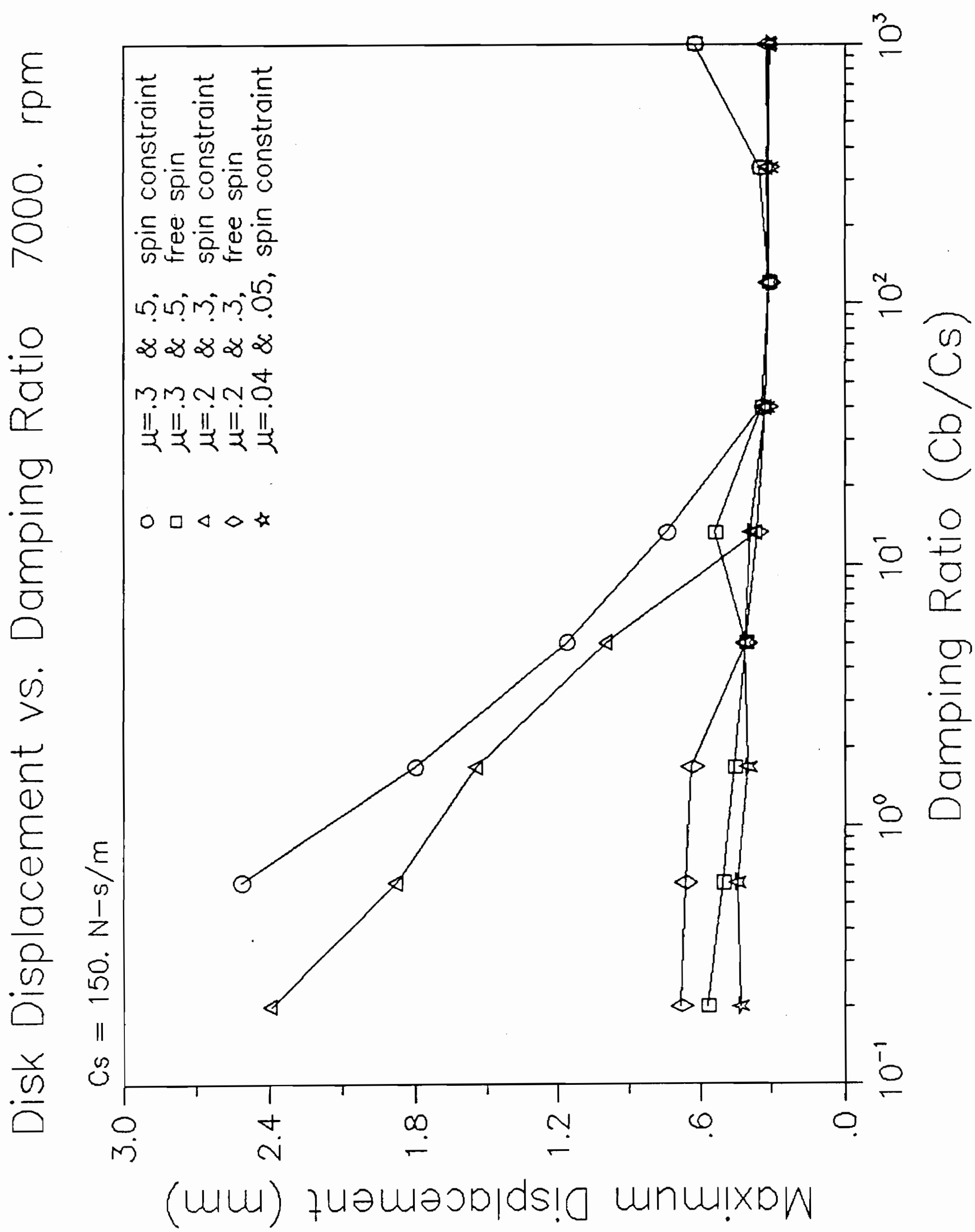

Figure 16b. The Maximum Disk Displacement and Damping Support Ratio
7000. rpm, $C_s = 150.$ N-s/m.

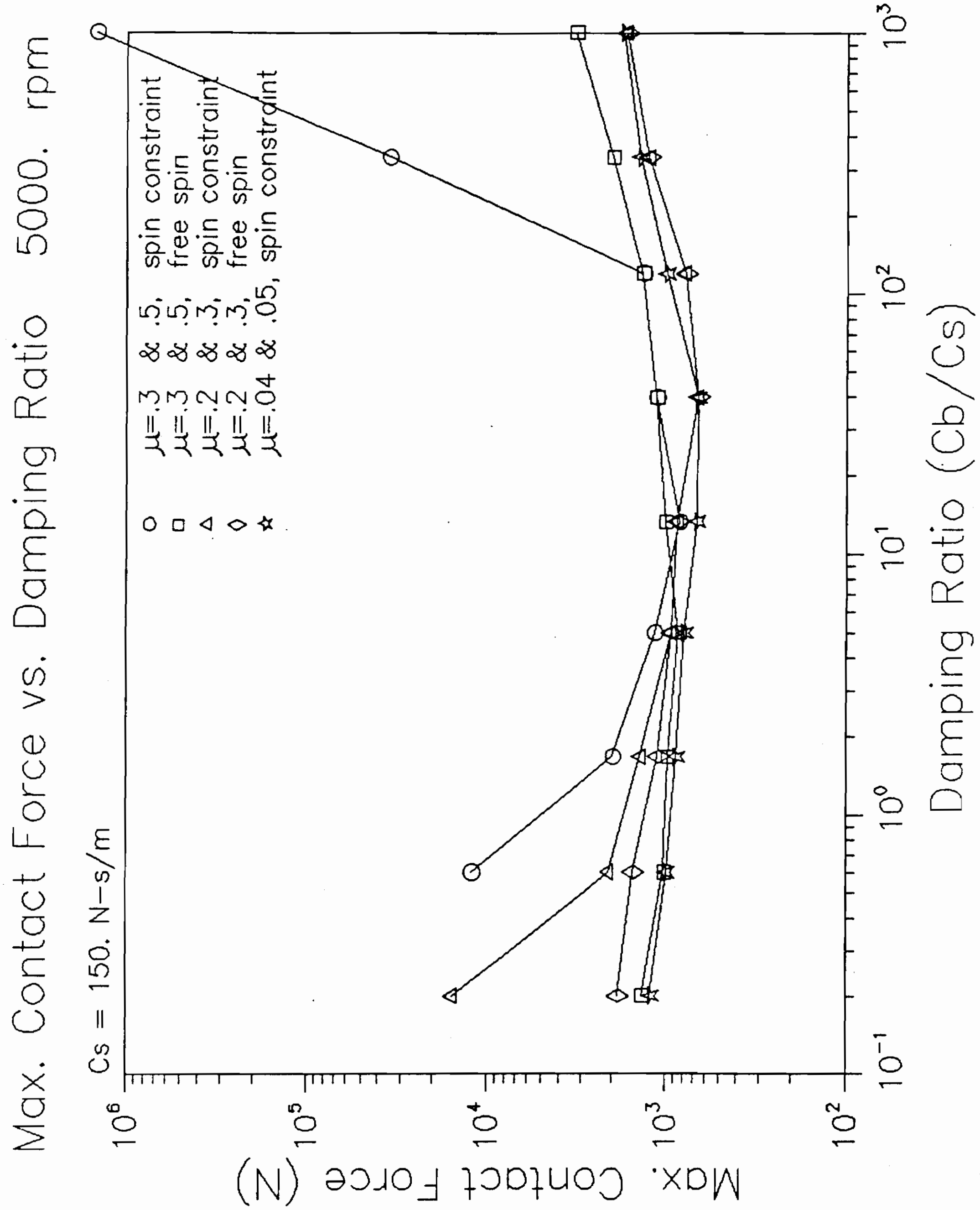

Figure 16c. The Maximum Contact Force and Damping Support Ratio
5000. rpm, $C_s = 150.$ N-s/m.

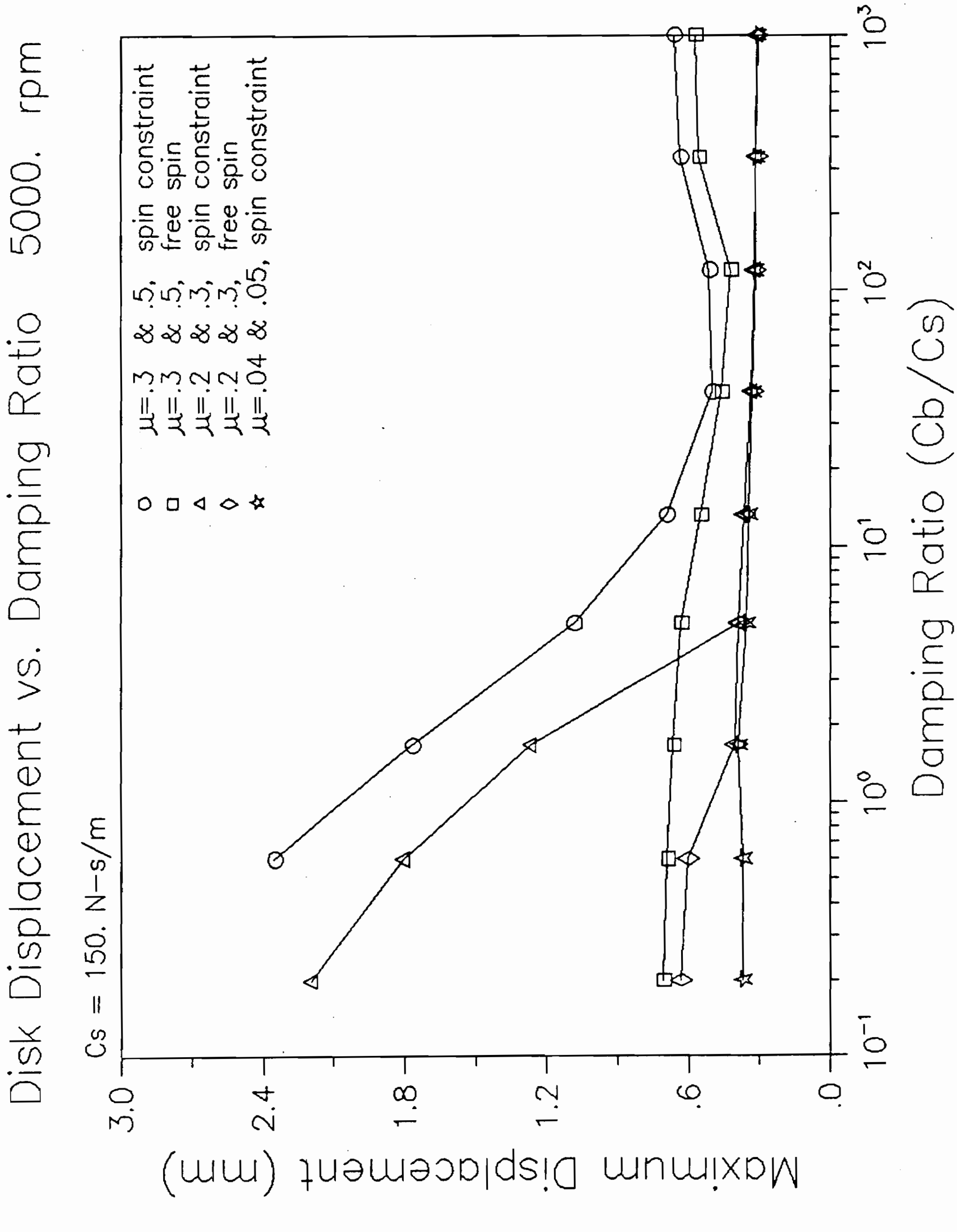

Figure 16d. The Maximum Disk Displacement and Damping Support Ratio
5000. rpm, $C_s = 150.$ N-s/m.

between 30 - 100, the larger coefficient of friction results in the lager disk response at 5000 rpm; however, the disk response is almost the same at 7000 - 9000 rpm.

3.4 The Optimum Support Damping

From the results of the transient response analysis, the optimum back-up bearing support damping ratio (C_b / C_s) to minimize the disk response and the contact force between the journal and the back-up bearing was found to be between 30 - 100, or an optimum bearing support damping of 4500 - 15000 N-s/m.

Barrett, et al.[10] introduced a rapid approximate method for calculation of the optimum bearing support damping for flexible rotors to minimize the rotor response for linear systems, using modal mass (m_1) and modal stiffness (k_1) for the first mode.

For the linear system, the modal mass and modal stiffness are obtained by solving the homogeneous equations of motion for the undamped multi-degree-of-freedom model of the rotor.

$$\left[M_m \right] \left\{ \ddot{x} \right\} + \left[K_m \right] \left\{ x \right\} = \left\{ 0 \right\} \qquad [29]$$

Assume $\left\{ \phi_i \right\}$ is the normalized i-th mode shape of the rotor, $\omega_{cr.\,i}$ is the i-th critical speed of the rotor, modal mass m_i and modal stiffness k_i for the i-th mode is obtained by following equations :

$$m_i = \left\{\phi_i\right\}^T \left[M_m\right] \left\{\phi_i\right\} \qquad [30]$$

$$k_i = m_i \cdot \omega_{cr.\,i}^2 \qquad [31]$$

Neglecting the dead-band between the journal and the back-up bearing, the modal mass and the modal stiffness for the first mode of the rotor can be calculated. The following is the optimum support damping ratio $\xi_{opt.}$ [10] :

$$\xi_{opt.} = \frac{1 + k}{2} = \frac{1 + (k_{sup.} / k_1)}{2} \qquad [32]$$

So, the optimum support damping can be calculated as follows :

$$c_{sup.\,opt.} = \xi_{opt.} \times 2 \sqrt{m_1 \, k_{sup.}}$$

$$= (1 + k) \sqrt{m_1 \, k_{sup.}} \qquad [33]$$

Using equation 33 for the model of this project, the optimum support damping becomes 7640 N-s/m. This value agrees with the range of 5000 - 15000 N-s/m from the transient response analysis for the optimum support damping to minimize the disk response and the contact force.

Chapter 4

Conclusions and Recommendations

In this project, the influence of the support damping of the back-up bearing in AMB systems was analyzed to determine the optimum support damping to minimize the disk response and the contact force between the journal and the bearing, by applying the transient response technique to the non-linear model. The resulting computer code can provide a detailed examination of the rotor orbit and also the response and load of the back-up bearing.

The specific conclusions for the model considered in this project are :

(1) The optimum support damping ratio (C_b / C_s) is between 30 and 100. In this range, the backward whirl does not occur even though the constraint in the rotational movement of the back-up bearing exists or the coefficient of friction between the journal and the inner-race of the bearing is high.

(2) For a small support damping ratio ($C_b / C_s \leq 5$), the maximum disk response and the maximum contact force are very sensitive to the constraint in the rotational movement of the back-up bearing.

The general conclusions from this project are :

(1) Very low and high support damping causes the backward whirl of the rotor shortly after the drop on the back-up bearing. The backward whirl may lead to the large contact force between the journal and the bearing and the large response of the disk.

(2) For a small support damping, the maximum disk response and the maximum contact force are very sensitive to the constraint in the rotational movement of the bearing. Such constraint could exist for the AMB machinery, which operates for a long time without stops because the back-up bearings are inactive during the normal operation of the AMB system and they are highly pre-loaded.

(3) The backward whirl is not synchronous to the rotational speed of the rotor. Its frequency is equal to the first natural frequency of the system for the wide range of the damping ratio. When the damping ratio is excessive ($C_b / C_s \gtrsim 1000$) and the constraint in the rotational movement of the back-up bearing exists, the backward whirl rate becomes fast and exceeds the rotational speed, as well as the first natural frequency.

(4) The vibration (chattering) caused by the non-linearity at the contact point occurs for the small support damping ratio (C_b / C_s) when there is a spin constraint in the bearings.

(5) The optimum support damping for the flexible rotors without a dead-band can be applicable to design the back-up bearing support damping of AMB systems.

(6) The constraint in the rotational movement of the bearing and higher coefficient of friction between the journal and the inner-race lead to larger disk response and larger contact force, especially when the support damping ratio is small ($C_b / C_s \leq 1$).

After the examination of the results from this transient response analysis, the following recommendations are presented for future research and development :

(1) Experimental test results are necessary to verify the results presented in this paper.

(2) The influence of changing the clearance between the journal and the inner-race of the back-up bearing should be evaluated.

(3) Assuming the rotor model to be symmetric, only the first and the third mode of the rotor have been examined in this project. The second mode conical motion should be considered using an extended model.

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Appendix

Fortran Code for Transient Response after Rotor Drop

```
c***************************************************************************
c   Runge-Kutta method of four mass system using input datafile
c   and considering the resistance force in calculating bearing torque
c   and calculate the initial condition
c   'Transient Response after Rotor Drop Program' .....TRARD.FOR
c                                   for  Microsoft FORTRAN Version 5.0
c                                         by Toshiyasu Ishii 10/ 1/90
c***************************************************************************
      COMMON /CNST/ GR,YMUD,YMUS,DT,INDEX
      COMMON /DISK/  DM,DJ,DC,DCZ,DTQ,DEC
      COMMON /SHAF1/ SM,SC,SCC,SK,SKK
      COMMON /SHAF2/ SR,SJ,SCZ,SKZ
      COMMON /BEAR1/ BM,BC,BK
      COMMON /BEAR2/ BR,BJ,BCZ
      COMMON /BEAR3/ BOR,YMUZ,FNZ
      COMMON /HOUS/  HM,HCX,HCY,HKX,HKY
      COMMON /CONTACT/ RK,RC
      COMMON /BM/ BMKX,BMKY,BMCX,BMCY
      COMMON /ALMX/ FMAX,TFMAX,ALMAX,ALMIN

      CHARACTER FILE1*30,FILE2*30,FILE3*30,CHA*1,CHA2*1,
     &         CHD2*55,CHD3*55,CHD4*55,CHD5*55,CHD6*55,CHD7*55,CHD8*55,
     &         CHD9*55,CHD10*55,CHD11*55,CHD12*55,CHD13*55

      OPEN(9,FILE='ALPMAX.DAT',STATUS='NEW')
 9997 PAI=ACOS(0.)*2.
      GR=9.8

      WRITE(*,*) 'INPUT DATA-FILE NAME.'
      READ(*,750) FILE1
      OPEN(7,FILE=FILE1,STATUS='OLD')
 1002 CALL GETTIM(IHR,IMIN,ISEC,I100TH)
      WRITE(*,9998) FILE1,IHR,IMIN
 9998 FORMAT(2X,A30,'TIME = ',I2,':'I2)

      READ(7,700) CHA
      IF(CHA.EQ.'y') CHA='Y'
      IF(CHA.EQ.'n') CHA='N'
      READ(7,701) NS,NS1,NNSP,NNSP1,DT,DT2,NRPM

      IF(NNSP.GT.0) THEN
        WRITE(*,*) 'INPUT NORMAL ORBIT OUTPUT-FILENANE'
        READ(*,750) FILE2
        OPEN(1,FILE=FILE2,STATUS='NEW')
        WRITE(*,*) 'DO YOU USE CALAULATED STARTING CONDITIONS ? (Y/N)'
        READ(*,750) CHA2
        IF(CHA2.EQ.'y') CHA2='Y'
      ENDIF
```

```
      IF(NS.GT.0) THEN
        WRITE(*,*) 'INPUT DROP ORBIT OUTPUT-FILENANE'
        READ(*,750) FILE3
        OPEN(2,FILE=FILE3,STATUS='NEW')
      ENDIF

      READ(7,702) E,G,YMUD,YMUS,YMUZ,DEC,SR,SKK,BR,BOR,BK,RK,FNZ,HM,HKX,
     &            HKY,BMKX,BMKY,DTQ
      READ(7,703) DC,DCZ,SC,SCC,SCZ,BC,BCZ,RC,HCX,HCY,BMCX,BMCY


      OMEGA=2.*PAI*FLOAT(NRPM)/60.

      IF(CHA.EQ.'Y') GOTO 1000
      IF(CHA.EQ.'N') GOTO 1001
      GOTO 1002

 1001 READ(7,704) DM,DJ,SK,SKZ,SM,SJ,BM,BJ
      GOTO 1003

 1000 READ(7,705) DR,DTH,SDO,SL,ST,BT,DROU,SROU,BROU

C*****************************************************************
C     Calculate the required parameter from alternative data
C*****************************************************************
      DM=DROU*PAI*(DR**2*DTH+SDO**2*SL/8.)
      SK=48.*E*PAI/4.*(SDO/2.)**4/SL**3
      DJ=DM*DR**2/2.
      SKZ=PAI/8.*G*(SDO**4)/SL
      SM=SROU*PAI*(SR**2*ST*2.+SDO**2*SL/8.)
      SJ=SM*SR**2/2.
      BM=BROU*PAI*BT*2.*(BOR**2-BR**2)
      BJ=PAI/2.*BROU*BT*(BOR**4-BR**4)

c************************************************************
c     Write the basic data to the output files
c************************************************************
 1003 IF(NS.GT.0) THEN
        WRITE(2,600) FILE3
        WRITE(2,601) DT,DT2,NS,NNSP,NRPM
        WRITE(2,602) E,G,YMUD,YMUS,YMUZ,DEC,SR,SKK,BR,BOR,BK,RK,FNZ,HM,
     &               HKX,HKY,BMKX,BMKY,DTQ
        WRITE(2,603) DC,DCZ,SC,SCC,SCZ,BC,BCZ,RC,HCX,HCY,BMCX,BMCY
        WRITE(2,604) DM,DJ,SK,SKZ,SM,SJ,BM,BJ
        WRITE(2,605) DR,DTH,SDO,SL,ST,BT,DROU,SROU,BROU
        WRITE(2,608)
      ENDIF
```

```
      IF(NNSP.GT.0) THEN
        WRITE(1,620) FILE2
        WRITE(1,601) DT,DT2,NS,NNSP,NRPM
        WRITE(1,602) E,G,YMUD,YMUS,YMUZ,DEC,SR,SKK,BR,BOR,BK,RK,FNZ,HM,
     &              HKY,BMKX,BMKY,DTQ
        WRITE(1,603) DC,DCZ,SC,SCC,SCZ,BC,BCZ,RC,HCX,HCY,BMCX,BMCY
        WRITE(1,604) DM,DJ,SK,SKZ,SM,SJ,BM,BJ
        WRITE(1,605) DR,DTH,SDO,SL,ST,BT,DROU,SROU,BROU
        WRITE(1,606)
      ENDIF

 750  FORMAT(A)
 700  FORMAT(///,55X,A1)
 701  FORMAT( 4(55X,I8,/),2(55X,E20.12,/),55X,I6,/)
 702  FORMAT(/,19(55X,E20.12,/))
 703  FORMAT(/,12(55X,E20.12,/))
 704  FORMAT(/,8(55X,E20.12,/),///////////)
 705  FORMAT(///////////,9(55X,E20.12,/))

 600  FORMAT('  DATA OUTPUT-FILE (ROTOR DROP RESPONSE)  "',A15,'"')
 620  FORMAT('  DATA OUTPUT-FILE (NORMAL OPERATION)  "',A15,'"')
 601  FORMAT(/,'  Time Increment   DT   (sec) =',E12.4,6x,' DT2  =',
     &     E12.4,/,'  No. of Steps     NS          =',I8,12X,' NNSP =',
     &     I8,/,'  Rotaional Speed  NRPM (rpm) =',I6)
 602  FORMAT(/,6X,'E',11X,'G',11X,'YMUD',8X,'YMUS',8X,'YMUZ',8X,'DEC',
     &       9X,'SR',10X,'SKK',9X,'BR',10X,'BOR',/,10E12.4,/,
     &       6X,'BK',10X,'RK',10X,'FNZ',9X,'HM',10X,'HKX',9X,'HKY',9X,
     &       'BMKX',8X,'BMKY',8X,'DTQ',/,9E12.4)
 603  FORMAT(/,7X,'DC',12X,'DCZ',11X,'SC',12X,'SCC',11X,'SCZ',11X,'BC',
     &       /,6E14.4,/,7X,'BCZ'11X,'RC',12X,'HCX',11X,'HCY',11X,'BMCX',
     &       10X,'BMCY',/,6E14.4)
 604  FORMAT(/,7X,'DM',12X,'DJ',12X,'SK',12X,'SKZ',/,4E14.4,/,
     &      7X,'SM', 12X,'SJ',12X,'BM',12X,'BJ',/,4E14.4)
 605  FORMAT(/,7X,'DR',12X,'DTH',11X,'SDO',11X,'SL',12X,'ST',/,5E14.4,/,
     &     7X,'BT',12X,'DROU',10X,'SROU',10X,'BROU',/,4E14.4)
 606  FORMAT(//,6X,'Time',9X,'DX',11x,'DY',11X,'DZ',11X,'SX',11X,'SY',
     &        11X,'SZ')


C****************************************
C     Set the start condition
C****************************************
      READ(7,712) CHD2,RDX,CHD3,RDY,CHD4,RDZ,CHD5,RSX,CHD6,RSY,CHD7,
     &          RSZ,CHD8,RDVX,CHD9,RDVY,CHD10,RDVZ,CHD11,RSVX,CHD12,
     &          RSVY,CHD13,RSVZ
      CLOSE(7)
```

```
      IF(CHA2.EQ.'Y') THEN
        RSX=((SM+DM)*GR)/BMKX
        RDX=RSX+(DM*GR)/SKK
        RSY=0.0
        RDY=0.0
        RSZ=0.0
        RDZ=0.0
        RSVY=0.0
        RDVY=0.0
        RSVX=0.0
        RDVX=0.0
        RSVZ=OMEGA
        RDVZ=OMEGA
      ENDIF

c************************************************************
c     Calculate the initial condition and return the results
c     to the input data file
c************************************************************
      IF(NNSP.GT.0) THEN
        T2=0.0
        WRITE(1,607)T2,RDX,RDY,RDZ,RSX,RSY,RSZ,RDVX,RDVY,RDVZ,
     &              RSVX,RSVY,RSVZ
        DO 100 I=1,NNSP
           CALL INIT(DT2,RDX,RDY,RDZ,RSX,RSY,RSZ,RDVX,RDVY,RDVZ,RSVX,
     &              RSVY,RSVZ)
           T2=FLOAT(I)*DT2
           II=MOD(I,NNSP1)
           IF(II.EQ.0) THEN
              WRITE(1,607)T2,RDX,RDY,RDZ,RSX,RSY,RSZ,RDVX,RDVY,RDVZ,
     &                    RSVX,RSVY,RSVZ
           ENDIF
 100    CONTINUE

        I=NNSP+1
 101       DUM=RDY
           CALL INIT(DT2,RDX,RDY,RDZ,RSX,RSY,RSZ,RDVX,RDVY,RDVZ,RSVX,
     &              RSVY,RSVZ)
           T2=FLOAT(I)*DT2
           II=MOD(I,NNSP1)
           IF(II.EQ.0) THEN
              WRITE(1,607)T2,RDX,RDY,RDZ,RSX,RSY,RSZ,RDVX,RDVY,RDVZ,
     &                    RSVX,RSVY,RSVZ
           ENDIF
           SGN=DUM*RDY
        IF((SGN.LT.0.).AND.(RDVY.GT.0.0)) GOTO 102
        I=I+1
        GOTO 101
```

```
 102    WRITE(1,607)T2,RDX,RDY,RDZ,RSX,RSY,RSZ,RDVX,RDVY,RDVZ,
     &              RSVX,RSVY,RSVZ

        RDZ=MOD(RDZ,PAI)
        RSZ=MOD(RSZ,PAI)
        OPEN(8,FILE=FILE1,STATUS='OLD')
        READ(8,800)
        WRITE(8,801) CHD2,RDX,CHD3,RDY,CHD4,RDZ,CHD5,RSX,CHD6,
     &               RSY,CHD7,RSZ,CHD8,RDVX,CHD9,RDVY,CHD10,
     &               RDVZ,CHD11,RSVX,CHD12,RSVY,CHD13,RSVZ
      ENDIF

 712  FORMAT(/,12(A55,E20.12,/))
 800  FORMAT(68(/))
 801  FORMAT(12(A55,E13.7,/))

C***********************************************************************
C     Set the initial value to calculate the rotor orbit after drop
C***********************************************************************
      IF(NS.GT.0) THEN

       FMAX=0.
       ALMAX=-3.1415926536
       ALMIN= 3.1415926536
       BIG=0.

       RBX=0.0
       RBY=0.0
       RBZ=0.0
       RHX=0.0
       RHY=0.0
       RBVX=0.0
       RBVY=0.0
       RBVZ=0.0
       RHVX=0.0
       RHVY=0.0
        RDZ=MOD(RDZ,PAI)
        RSZ=MOD(RSZ,PAI)
       T=0.
       WRITE(2,609)T,RDX,RDY,RDZ,RSX,RSY,RSZ,RBX,RBY,RBZ,RHX,RHY,RDVX,
     &              RDVY,RDVZ,RSVX,RSVY,RSVZ,RBVX,RBVY,RBVZ,RHVX,RHVY


c*****************************************************
c     Calculate the rotor orbit after drop
c*****************************************************
```

```
      DO 110 N=1,NS+1
        CALL FRICTION(RDX,RDY,RDZ,RSX,RSY,RSZ,RBX,RBY,RBZ,RHX,RHY,RDVX,
     &         RDVY,RDVZ,RSVX,RSVY,RSVZ,RBVX,RBVY,RBVZ,RHVX,RHVY,N,NS1)

        T=FLOAT(N)*DT

        DRS=((RDX**2+RDY**2)**.5)*1000.
        BIG=MAX(BIG,DRS)

c*******************************************************
c     Output the routine 1  (displacenent & velocity)
c*******************************************************

        NN=MOD(N,NS1)
        IF(NN.EQ.0) THEN
           WRITE(2,609)T,RDX,RDY,RDZ,RSX,RSY,RSZ,RBX,RBY,RBZ,RHX,RHY,
     &          RDVX,RDVY,RDVZ,RSVX,RSVY,RSVZ,RBVX,RBVY,RBVZ,RHVX,RHVY
        ENDIF
 110  CONTINUE
      ENDIF

      ALMAX=ALMAX/3.1415926536*180.
      ALMIN=ALMIN/3.1415926536*180.
      WRITE(*,887) FMAX,ALMAX,ALMIN,BIG
      WRITE(2,888) FILE3,FMAX,ALMAX,ALMIN,BIG
      WRITE(9,888) FILE3,FMAX,ALMAX,ALMIN,BIG

      CLOSE(1)
      CLOSE(2)
      CLOSE(8)
      STOP

 607  FORMAT(/,7E13.6,/,13X,6E13.6)
 608  FORMAT(//,4X,'Time',7X,'DX',9x,'DY',9X,'DZ',9X,'SX',9X,'SY',9X,
     &     'SZ',9X,'BX',9X,'BY',9X,'BZ',9X,'HX',9X,'HY')
 609  FORMAT(/,12E11.4,/,11X,11E11.4)
 888  FORMAT(/,14X,A30,/,'FMAX(N)= ',F15.5,/,'AMX(dg)= ',F15.5,/,
     &      'AMN(dg)= ',F15.5,/,'DRS(mm)= ',F15.5)
 887  FORMAT(1X,'FMX=',F11.5,' AMAX=',F11.5,' AMIN=',F11.5,' DR=',F11.5)

      END

c**********************************************************************
c     Subroutine Friction .....4th-order Runge-Kutta algorithm
c         calculate the orbit of the rotor after drop considering
c        the effect of the friction
c**********************************************************************
```

```
      SUBROUTINE FRICTION(DX,DY,DZ,SX,SY,SZ,BX,BY,BZ,HX,HY,DVX,DVY,DVZ,
     &                    SVX,SVY,SVZ,BVX,BVY,BVZ,HVX,HVY,N,NS1)

        COMMON /CNST/ GR,YMUD,YMUS,DT,INDEX
        COMMON /ALMX/ FMAX,TFMAX,ALMAX,ALMIN

        ALPH1=FALPH(SX,SY,BX,BY)
        FN1=FNN(SX,SY,BX,BY,SVX,SVY,BVX,BVY,ALPH1)

        IF(FN1.GT.0.0) THEN
          IF(FN1.GT.FMAX) FMAX=FN1
          IF(ALPH1.GT.ALMAX) ALMAX=ALPH1
          IF(ALPH1.LT.ALMIN) ALMIN=ALPH1
        ENDIF

        CALL COEFFICIENT(SX,SY,BX,BY,DZ,DVZ,SZ,SVZ,BVZ,FN1,ALPH1,YU1)

        DZV1=DT*FDZ(DZ,SZ,DVZ,SVZ)
        DXV1=DT*FDX(DX,DZ,SX,DVX,DVZ,DZV1/DT,SVX)
        DYV1=DT*FDY(DY,DZ,SY,DVY,DVZ,DZV1/DT,SVY)

        DZ1=DT*DVZ
        DX1=DT*DVX
        DY1=DT*DVY

        SZV1=DT*FSZ(DZ,SZ,DVZ,SVZ,FN1,YU1)
        SXV1=DT*FSX(DX,SX,DVX,SVX,FN1,ALPH1,YU1)
        SYV1=DT*FSY(DY,SY,DVY,SVY,FN1,ALPH1,YU1)

        SZ1=DT*SVZ
        SX1=DT*SVX
        SY1=DT*SVY

        BZV1=DT*FBZ(BVZ,FN1,YU1)
        BXV1=DT*FBX(BX,HX,BVX,HVX,FN1,ALPH1,YU1)
        BYV1=DT*FBY(BY,HY,BVY,HVY,FN1,ALPH1,YU1)

        BZ1=DT*BVZ
        BX1=DT*BVX
        BY1=DT*BVY

        HXV1=DT*FHX(BX,HX,BVX,HVX)
        HYV1=DT*FHY(BY,HY,BVY,HVY)

        HX1=DT*HVX
        HY1=DT*HVY
```

```
      FFDZ=DZV1/DT
      FFDX=DXV1/DT
      FFDY=DYV1/DT
      FFSZ=SZV1/DT
      FFSX=SXV1/DT
      FFSY=SYV1/DT
      FFBZ=BZV1/DT
      FFBX=BXV1/DT
      FFBY=BYV1/DT
      FFHX=HXV1/DT
      FFHY=HYV1/DT

C*********************************************************************
C      Output routine 2 (acceleration, contact force, angle location,
c                        coefficient of friction)
C*********************************************************************
      NN=MOD(N-1,NS1)
      IF(NN.EQ.0) THEN
         WRITE(2,650) FFDX,FFDY,FFDZ,FFSX,FFSY,FFSZ,FFBX,FFBY,FFBZ,
     &                FFHX,FFHY
         WRITE(2,651) FN1,ALPH1,YU1
      ENDIF
 650  FORMAT(11X,11E11.4)
 651  FORMAT('       FNN = ',E15.5,8X,'ALPH= ',E15.5,5X,'YU =',E15.5)


      ALPH2=FALPH(SX+SX1/2.,SY+SY1/2.,BX+BX1/2.,BY+BY1/2.)
      FN2=FNN(SX+SX1/2.,SY+SY1/2.,BX+BX1/2.,BY+BY1/2.,SVX+SXV1/2.,
     &        SVY+SYV1/2.,BVX+BXV1/2.,BVY+BYV1/2.,ALPH2)

      CALL COEFFICIENT(SX,SY,BX,BY,DZ+DZ1/2.,DVZ+DZV1/2.,SZ+SZ1/2.,
     &                 SVZ+SZV1/2.,BVZ+BZV1/2.,FN2,ALPH2,YU2)

      DZV2=DT*FDZ(DZ+DZ1/2.,SZ+SZ1/2.,DVZ+DZV1/2.,SVZ+SZV1/2.)
      DXV2=DT*FDX(DX+DX1/2.,DZ+DZ1/2.,SX+SX1/2.,DVX+DXV1/2.,DVZ+DZV1/2.
     &           ,DZV2/DT,SVX+SXV1/2.)
      DYV2=DT*FDY(DY+DY1/2.,DZ+DZ1/2.,SY+SY1/2.,DVY+DYV1/2.,DVZ+DZV1/2.
     &           ,DZV2/DT,SVY+SYV1/2.)

      DZ2=DT*(DVZ+DZV1/2.)
      DX2=DT*(DVX+DXV1/2.)
      DY2=DT*(DVY+DYV1/2.)

      SZV2=DT*FSZ(DZ+DZ1/2.,SZ+SZ1/2.,DVZ+DZV1/2.,SVZ+SZV1/2.,FN2,YU2)
      SXV2=DT*FSX(DX+DX1/2.,SX+SX1/2.,DVX+DXV1/2.,SVX+SXV1/2.,FN2,
     &           ALPH2,YU2)
      SYV2=DT*FSY(DY+DY1/2.,SY+SY1/2.,DVY+DYV1/2.,SVY+SYV1/2.,FN2,
     &           ALPH2,YU2)
```

```
      SZ2=DT*(SVZ+SZV1/2.)
      SX2=DT*(SVX+SXV1/2.)
      SY2=DT*(SVY+SYV1/2.)

      BZV2=DT*FBZ(BVZ+BZV1/2.,FN2,YU2)
      BXV2=DT*FBX(BX+BX1/2.,HX+HX1/2.,BVX+BXV1/2.,HVX+HXV1/2.,FN2,
     &           ALPH2,YU2)
      BYV2=DT*FBY(BY+BY1/2.,HY+HY1/2.,BVY+BYV1/2.,HVY+HYV1/2.,FN2,
     &           ALPH2,YU2)

      BZ2=DT*(BVZ+BZV1/2.)
      BX2=DT*(BVX+BXV1/2.)
      BY2=DT*(BVY+BYV1/2.)

      HXV2=DT*FHX(BX+BX1/2.,HX+HX1/2.,BVX+BXV1/2.,HVX+HXV1/2.)
      HYV2=DT*FHY(BY+BY1/2.,HY+HY1/2.,BVY+BYV1/2.,HVY+HYV1/2.)

      HX2=DT*(HVX+HXV1/2.)
      HY2=DT*(HVY+HYV1/2.)

      ALPH3=FALPH(SX+SX2/2.,SY+SY2/2.,BX+BX2/2.,BY+BY2/2.)
      FN3=FNN(SX+SX2/2.,SY+SY2/2.,BX+BX2/2.,BY+BY2/2.,SVX+SXV2/2.,
     &        SVY+SYV2/2.,BVX+BXV2/2.,BVY+BYV2/2.,ALPH3)


      CALL COEFFICIENT(SX,SY,BX,BY,DZ+DZ2/2.,DVZ+DZV2/2.,SZ+SZ2/2.,
     &                 SVZ+SZV2/2.,BVZ+BZV2/2.,FN3,ALPH3,YU3)

      DZV3=DT*FDZ(DZ+DZ2/2.,SZ+SZ2/2.,DVZ+DZV2/2.,SVZ+SZV2/2.)
      DXV3=DT*FDX(DX+DX2/2.,DZ+DZ2/2.,SX+SX2/2.,DVX+DXV2/2.,DVZ+DZV2/2.
     &           ,DZV3/DT,SVX+SXV2/2.)
      DYV3=DT*FDY(DY+DY2/2.,DZ+DZ2/2.,SY+SY2/2.,DVY+DYV2/2.,DVZ+DZV2/2.
     &           ,DZV3/DT,SVY+SYV2/2.)

      DZ3=DT*(DVZ+DZV2/2.)
      DX3=DT*(DVX+DXV2/2.)
      DY3=DT*(DVY+DYV2/2.)

      SZV3=DT*FSZ(DZ+DZ2/2.,SZ+SZ2/2.,DVZ+DZV2/2.,SVZ+SZV2/2.,FN3,YU3)
      SXV3=DT*FSX(DX+DX2/2.,SX+SX2/2.,DVX+DXV2/2.,SVX+SXV2/2.,FN3,
     &           ALPH3,YU3)
      SYV3=DT*FSY(DY+DY2/2.,SY+SY2/2.,DVY+DYV2/2.,SVY+SYV2/2.,FN3,
     &           ALPH3,YU3)

      SZ3=DT*(SVZ+SZV2/2.)
      SX3=DT*(SVX+SXV2/2.)
      SY3=DT*(SVY+SYV2/2.)
```

```
      BZV3=DT*FBZ(BVZ+BZV2/2.,FN3,YU3)
      BXV3=DT*FBX(BX+BX2/2.,HX+HX2/2.,BVX+BXV2/2.,HVX+HXV2/2.,FN3,
     &            ALPH3,YU3)
      BYV3=DT*FBY(BY+BY2/2.,HY+HY2/2.,BVY+BYV2/2.,HVY+HYV2/2.,FN3,
     &            ALPH3,YU3)

      BZ3=DT*(BVZ+BZV2/2.)
      BX3=DT*(BVX+BXV2/2.)
      BY3=DT*(BVY+BYV2/2.)

      HXV3=DT*FHX(BX+BX2/2.,HX+HX2/2.,BVX+BXV2/2.,HVX+HXV2/2.)
      HYV3=DT*FHY(BY+BY2/2.,HY+HY2/2.,BVY+BYV2/2.,HVY+HYV2/2.)

      HX3=DT*(HVX+HXV2/2.)
      HY3=DT*(HVY+HYV2/2.)


      ALPH4=FALPH(SX+SX3,SY+SY3,BX+BX3,BY+BY3)
      FN4=FNN(SX+SX3,SY+SY3,BX+BX3,BY+BY3,SVX+SXV3,
     &        SVY+SYV3,BVX+BXV3,BVY+BYV3,ALPH4)


      CALL COEFFICIENT(SX,SY,BX,BY,DZ+DZ3,DVZ+DZV3,SZ+SZ3,
     &                 SVZ+SZV3,BVZ+BZV3,FN4,ALPH4,YU4)

      DZV4=DT*FDZ(DZ+DZ3,SZ+SZ3,DVZ+DZV3,SVZ+SZV3)
      DXV4=DT*FDX(DX+DX3,DZ+DZ3,SX+SX3,DVX+DXV3,DVZ+DZV3,DZV4/DT,
     &            SVX+SXV3)
      DYV4=DT*FDY(DY+DY3,DZ+DZ3,SY+SY3,DVY+DYV3,DVZ+DZV3,DZV4/DT,
     &            SVY+SYV3)

      DZ4=DT*(DVZ+DZV3)
      DX4=DT*(DVX+DXV3)
      DY4=DT*(DVY+DYV3)

      SZV4=DT*FSZ(DZ+DZ3,SZ+SZ3,DVZ+DZV3,SVZ+SZV3,FN4,YU4)
      SXV4=DT*FSX(DX+DX3,SX+SX3,DVX+DXV3,SVX+SXV3,FN4,ALPH4,YU4)
      SYV4=DT*FSY(DY+DY3,SY+SY3,DVY+DYV3,SVY+SYV3,FN4,ALPH4,YU4)

      SZ4=DT*(SVZ+SZV3)
      SX4=DT*(SVX+SXV3)
      SY4=DT*(SVY+SYV3)

      BZV4=DT*FBZ(BVZ+BZV3,FN4,YU4)
      BXV4=DT*FBX(BX+BX3,HX+HX3,BVX+BXV3,HVX+HXV3,FN4,ALPH4,YU4)
      BYV4=DT*FBY(BY+BY3,HY+HY3,BVY+BYV3,HVY+HYV3,FN4,ALPH4,YU4)
```

```
      BZ4=DT*(BVZ+BZV3)
      BX4=DT*(BVX+BXV3)
      BY4=DT*(BVY+BYV3)

      HXV4=DT*FHX(BX+BX3,HX+HX3,BVX+BXV3,HVX+HXV3)
      HYV4=DT*FHY(BY+BY3,HY+HY3,BVY+BYV3,HVY+HYV3)
      HX4=DT*(HVX+HXV3)
      HY4=DT*(HVY+HYV3)


      DVZ=DVZ+(DZV1+2.*DZV2+2.*DZV3+DZV4)/6.0
      DVX=DVX+(DXV1+2.*DXV2+2.*DXV3+DXV4)/6.0
      DVY=DVY+(DYV1+2.*DYV2+2.*DYV3+DYV4)/6.0
      DZ=DZ+(DZ1+2.*DZ2+2.*DZ3+DZ4)/6.0
      DX=DX+(DX1+2.*DX2+2.*DX3+DX4)/6.0
      DY=DY+(DY1+2.*DY2+2.*DY3+DY4)/6.0

      SVZ=SVZ+(SZV1+2.*SZV2+2.*SZV3+SZV4)/6.0
      SVX=SVX+(SXV1+2.*SXV2+2.*SXV3+SXV4)/6.0
      SVY=SVY+(SYV1+2.*SYV2+2.*SYV3+SYV4)/6.0
      SZ=SZ+(SZ1+2.*SZ2+2.*SZ3+SZ4)/6.0
      SX=SX+(SX1+2.*SX2+2.*SX3+SX4)/6.0
      SY=SY+(SY1+2.*SY2+2.*SY3+SY4)/6.0

      BVZ=BVZ+(BZV1+2.*BZV2+2.*BZV3+BZV4)/6.0
      BVX=BVX+(BXV1+2.*BXV2+2.*BXV3+BXV4)/6.0
      BVY=BVY+(BYV1+2.*BYV2+2.*BYV3+BYV4)/6.0
      BZ=BZ+(BZ1+2.*BZ2+2.*BZ3+BZ4)/6.0
      BX=BX+(BX1+2.*BX2+2.*BX3+BX4)/6.0
      BY=BY+(BY1+2.*BY2+2.*BY3+BY4)/6.0

      HVX=HVX+(HXV1+2.*HXV2+2.*HXV3+HXV4)/6.0
      HVY=HVY+(HYV1+2.*HYV2+2.*HYV3+HYV4)/6.0
      HX=HX+(HX1+2.*HX2+2.*HX3+HX4)/6.0
      HY=HY+(HY1+2.*HY2+2.*HY3+HY4)/6.0

      RETURN
      END

c*********************************************************
c     Function FALPH
c         calculate the angle location of the journal
c       in the back-up bearing
c*********************************************************
      FUNCTION FALPH(SX,SY,BX,BY)
        FALPH=ATAN2(SY-BY,SX-BX)
       RETURN
       END
```

```
c*************************************************************
c     Function FNN
c          calculate the contact force between the journal
c        and the inner-race of the back-up bearing
c*************************************************************
      FUNCTION FNN(SX,SY,BX,BY,SVX,SVY,BVX,BVY,ALPH)
       COMMON /CNST/  GR,YMUD,YMUS,DT,INDEX
       COMMON /SHAF2/ SR,SJ,SCZ,SKZ
       COMMON /BEAR2/ BR,BJ,BCZ
       COMMON /CONTACT/ RK,RC
         DIS=(SX-BX)*COS(ALPH)+(SY-BY)*SIN(ALPH)-BR+SR
         VEL=(SVX-BVX)*COS(ALPH)+(SVY-BVY)*SIN(ALPH)
         FNN=RK*DIS+RC*VEL
        IF(DIS.LE.0.0) FNN=0.0
        IF(FNN.LE.0.0) FNN=0.0
       RETURN
       END


c*************************************************************
c     Function FDZ
c          calculate the angular acceralation of the disk
c*************************************************************
      FUNCTION FDZ(DZ,SZ,DVZ,SVZ)
       COMMON /CNST/ GR,YMUD,YMUS,DT,INDEX
       COMMON /DISK/  DM,DJ,DC,DCZ,DTQ,DEC
       COMMON /SHAF2/ SR,SJ,SCZ,SKZ
         dtq1=dtq
         if(dvz.le.0.) dtq1=-dtq
         FDZ=(-SCZ*(DVZ-SVZ)-SKZ*(DZ-SZ)-DCZ*DVZ-DM*GR*DEC*SIN(DZ)-DTQ1)
     &       /DJ
       RETURN
       END

c*****************************************************************
c     Function FDX
c          calculate the X-direction acceralation of the disk
c*****************************************************************
      FUNCTION FDX(DX,DZ,SX,DVX,DVZ,DAZ,SVX)
       COMMON /CNST/ GR,YMUD,YMUS,DT,INDEX
       COMMON /DISK/  DM,DJ,DC,DCZ,DTQ,DEC
       COMMON /SHAF1/ SM,SC,SCC,SK,SKK
        FDX=(-SC*(DVX-SVX)-SK*(DX-SX)-DC*DVX)/DM+GR+DEC*(DVZ**2*COS(DZ)
     &       +DAZ*SIN(DZ))
       RETURN
       END
```

```
c******************************************************************
c     Function FDY
c          calculate the Y-direction acceralation of the disk
c******************************************************************
      FUNCTION FDY(DY,DZ,SY,DVY,DVZ,DAZ,SVY)
        COMMON /DISK/  DM,DJ,DC,DCZ,DTQ,DEC
        COMMON /SHAF1/ SM,SC,SCC,SK,SKK
          FDY=(-SC*(DVY-SVY)-SK*(DY-SY)-DC*DVY)/DM+DEC*(DVZ**2*SIN(DZ)-
     &        DAZ*COS(DZ))
        RETURN
        END

c******************************************************************
c     Function FSZ
c          calculate the angular acceralation of the journal
c******************************************************************
      FUNCTION FSZ(DZ,SZ,DVZ,SVZ,FN,YU)
          FSZ=FSZ1(DZ,SZ,DVZ,SVZ)+FSZ2(FN)*YU
        RETURN
        END

      FUNCTION FSZ1(DZ,SZ,DVZ,SVZ)
        COMMON /SHAF2/ SR,SJ,SCZ,SKZ
          FSZ1=(-SCZ*(SVZ-DVZ)-SKZ*(SZ-DZ))/SJ
        RETURN
        END

      FUNCTION FSZ2(FN)
        COMMON /SHAF2/ SR,SJ,SCZ,SKZ
          FSZ2=(-SR*FN)/SJ
        RETURN
        END

c******************************************************************
c     Function FSX
c          calculate the X-direction acceralation of the journal
c******************************************************************
      FUNCTION FSX(DX,SX,DVX,SVX,FN,ALPH,YU)
         FSX=FSX1(DX,SX,DVX,SVX,FN,ALPH)+FSX2(FN,ALPH)*YU
       RETURN
       END

      FUNCTION FSX1(DX,SX,DVX,SVX,FN,ALPH)
       COMMON /CNST/ GR,YMUD,YMUS,DT,INDEX
       COMMON /SHAF1/ SM,SC,SCC,SK,SKK
         FSX1=(-SC*(SVX-DVX)-SK*(SX-DX)-FN*COS(ALPH))/SM+GR
       RETURN
       END
```

```
      FUNCTION FSX2(FN,ALPH)
       COMMON /SHAF1/ SM,SC,SCC,SK,SKK
         FSX2=FN*SIN(ALPH)/SM
       RETURN
       END


c******************************************************************
c     Function FSY
c          calculate the Y-direction acceleralation of the journal
c******************************************************************
      FUNCTION FSY(DY,SY,DVY,SVY,FN,ALPH,YU)
         FSY=FSY1(DY,SY,DVY,SVY,FN,ALPH)+FSY2(FN,ALPH)*YU
       RETURN
       END

      FUNCTION FSY1(DY,SY,DVY,SVY,FN,ALPH)
       COMMON /SHAF1/ SM,SC,SCC,SK,SKK
         FSY1=(-SC*(SVY-DVY)-SK*(SY-DY)-FN*SIN(ALPH))/SM
       RETURN
       END

      FUNCTION FSY2(FN,ALPH)
       COMMON /SHAF1/ SM,SC,SCC,SK,SKK
         FSY2=-FN*COS(ALPH)/SM
       RETURN
       END


c****************************************************************
c     Function FBZ
c          calculate the angular acceralation of the bearing
c****************************************************************
      FUNCTION FBZ(BVZ,FN,YU)
           FBZ=FBZ1(BVZ,FN)+FBZ2(FN)*YU
         IF(FBZ.GE.0.0) GOTO 10
         IF(BVZ.LE.0.0) FBZ=0.0
  10    RETURN
        END

      FUNCTION FBZ1(BVZ,FN)
        COMMON /BEAR2/ BR,BJ,BCZ
        COMMON /BEAR3/ BOR,YMUZ,FNZ
           FBZ1=(-BCZ*BVZ-YMUZ*BOR*(FN+FNZ))/BJ
        RETURN
        END
```

```
      FUNCTION FBZ2(FN)
        COMMON /BEAR2/ BR,BJ,BCZ
          FBZ2=BR*FN/BJ
        RETURN
        END


c******************************************************************
c     Function FBX
c          calculate the X-direction acceralation of the bearing
c******************************************************************
      FUNCTION FBX(BX,HX,BVX,HVX,FN,ALPH,YU)
         FBX=FBX1(BX,HX,BVX,HVX,FN,ALPH)+FBX2(FN,ALPH)*YU
       RETURN
       END

      FUNCTION FBX1(BX,HX,BVX,HVX,FN,ALPH)
       COMMON /BEAR1/ BM,BC,BK
         FBX1=(-BC*(BVX-HVX)-BK*(BX-HX)+FN*COS(ALPH))/BM
       RETURN
       END

      FUNCTION FBX2(FN,ALPH)
       COMMON /BEAR1/ BM,BC,BK
         FBX2=-FN*SIN(ALPH)/BM
       RETURN
       END

c******************************************************************
c     Function FBY
c          calculate the Y-direction acceralation of the bearing
c******************************************************************
      FUNCTION FBY(BY,HY,BVY,HVY,FN,ALPH,YU)
         FBY=FBY1(BY,HY,BVY,HVY,FN,ALPH)+FBY2(FN,ALPH)*YU
       RETURN
       END

      FUNCTION FBY1(BY,HY,BVY,HVY,FN,ALPH)
       COMMON /BEAR1/ BM,BC,BK
         FBY1=(-BC*(BVY-HVY)-BK*(BY-HY)+FN*SIN(ALPH))/BM
       RETURN
       END

      FUNCTION FBY2(FN,ALPH)
       COMMON /BEAR1/ BM,BC,BK
         FBY2=FN*COS(ALPH)/BM
       RETURN
       END
```

```
c*****************************************************************
c     Function FHX
c          calculate the X-direction acceralation of the housing
c*****************************************************************
      FUNCTION FHX(BX,HX,BVX,HVX)
        COMMON /BEAR1/ BM,BC,BK
        COMMON /HOUS/  HM,HCX,HCY,HKX,HKY
          FHX=(-HCX*HVX-HKX*HX+BC*(BVX-HVX)+BK*(BX-HX))/HM
        RETURN
        END

c*****************************************************************
c     Function FHY
c          calculate the Y-direction acceralation of the housing
c*****************************************************************
      FUNCTION FHY(BY,HY,BVY,HVY)
        COMMON /BEAR1/ BM,BC,BK
        COMMON /HOUS/  HM,HCX,HCY,HKX,HKY
          FHY=(-HCY*HVY-HKY*HY+BC*(BVY-HVY)+BK*(BY-HY))/HM
        RETURN
        END


c*************************************************************************
c     Subroutine INIT .....4th-order Runge-Kutta algorithm
c         calculate the orbit of the rotor when it is supported by
c        the magnetic bearing
c*************************************************************************
      SUBROUTINE INIT(DT2,DX,DY,DZ,SX,SY,SZ,DVX,DVY,DVZ,SVX,SVY,SVZ)

      COMMON /CNST/ GR,YMUD,YMUS,DT,INDEX
      COMMON /DISK/  DM,DJ,DC,DCZ,DTQ,DEC
      COMMON /SHAF1/ SM,SC,SCC,SK,SKK
      COMMON /SHAF2/ SR,SJ,SCZ,SKZ
      COMMON /BEAR1/ BM,BC,BK
      COMMON /BEAR2/ BR,BJ,BCZ
      COMMON /BEAR3/ BOR,YMUZ,FNZ
      COMMON /HOUS/  HM,HCX,HCY,HKX,HKY
      COMMON /CONTACT/ RK,RC
      COMMON /BM/ BMKX,BMKY,BMCX,BMCY

       DXV1=DT2*FMDX(DX,DZ,SX,DVX,SVX,DVZ)
       DYV1=DT2*FMDY(DY,DZ,SY,DVY,SVY,DVZ)

       DX1=DT2*DVX
       DY1=DT2*DVY
```

```
      SXV1=DT2*FMSX(DX,SX,DVX,SVX)
      SYV1=DT2*FMSY(DY,SY,DVY,SVY)

      SX1=DT2*SVX
      SY1=DT2*SVY

      DXV2=DT2*FMDX(DX+DX1/2.,DZ+DT2*DVZ/2.,SX+SX1/2.,DVX+DXV1/2.,
     &             SVX+SXV1/2.,DVZ)
      DYV2=DT2*FMDY(DY+DY1/2.,DZ+DT2*DVZ/2.,SY+SY1/2.,DVY+DYV1/2.,
     &             SVY+SYV1/2.,DVZ)

      DX2=DT2*(DVX+DXV1/2.)
      DY2=DT2*(DVY+DYV1/2.)

      SXV2=DT2*FMSX(DX+DX1/2.,SX+SX1/2.,DVX+DXV1/2.,SVX+SXV1/2.)
      SYV2=DT2*FMSY(DY+DY1/2.,SY+SY1/2.,DVY+DYV1/2.,SVY+SYV1/2.)

      SX2=DT2*(SVX+SXV1/2.)
      SY2=DT2*(SVY+SYV1/2.)

      DXV3=DT2*FMDX(DX+DX2/2.,DZ+DT2*DVZ/2.,SX+SX2/2.,DVX+DXV2/2.,
     &             SVX+SXV2/2.,DVZ)
      DYV3=DT2*FMDY(DY+DY2/2.,DZ+DT2*DVZ/2.,SY+SY2/2.,DVY+DYV2/2.,
     &             SVY+SYV2/2.,DVZ)

      DX3=DT2*(DVX+DXV2/2.)
      DY3=DT2*(DVY+DYV2/2.)

      SXV3=DT2*FMSX(DX+DX2/2.,SX+SX2/2.,DVX+DXV2/2.,SVX+SXV2/2.)
      SYV3=DT2*FMSY(DY+DY2/2.,SY+SY2/2.,DVY+DYV2/2.,SVY+SYV2/2.)

      SX3=DT2*(SVX+SXV2/2.)
      SY3=DT2*(SVY+SYV2/2.)

      DXV4=DT2*FMDX(DX+DX3,DZ+DT2*DVZ,SX+SX3,DVX+DXV3,
     &             SVX+SXV3,DVZ)
      DYV4=DT2*FMDY(DY+DY3,DZ+DT2*DVZ,SY+SY3,DVY+DYV3,
     &             SVY+SYV3,DVZ)

      DX4=DT2*(DVX+DXV3)
      DY4=DT2*(DVY+DYV3)

      SXV4=DT2*FMSX(DX+DX3,SX+SX3,DVX+DXV3,SVX+SXV3)
      SYV4=DT2*FMSY(DY+DY3,SY+SY3,DVY+DYV3,SVY+SYV3)

      SX4=DT2*(SVX+SXV3)
      SY4=DT2*(SVY+SYV3)
```

```
      DVX=DVX+(DXV1+2.*DXV2+2.*DXV3+DXV4)/6.0
      DVY=DVY+(DYV1+2.*DYV2+2.*DYV3+DYV4)/6.0
      DZ=DZ+DT2*DVZ
      DX=DX+(DX1+2.*DX2+2.*DX3+DX4)/6.0
      DY=DY+(DY1+2.*DY2+2.*DY3+DY4)/6.0

      SVX=SVX+(SXV1+2.*SXV2+2.*SXV3+SXV4)/6.0
      SVY=SVY+(SYV1+2.*SYV2+2.*SYV3+SYV4)/6.0
      SZ=SZ+DT2*SVZ
      SX=SX+(SX1+2.*SX2+2.*SX3+SX4)/6.0
      SY=SY+(SY1+2.*SY2+2.*SY3+SY4)/6.0

      RETURN
      END


c*****************************************************************
c     Function FMSX
c          calculate the X-direction acceleration of the journal
c       when the rotor is supported by the magnetic bearing
c*****************************************************************
      FUNCTION FMSX(DX,SX,DVX,SVX)
        COMMON /CNST/ GR,YMUD,YMUS,DT,INDEX
        COMMON /SHAF1/ SM,SC,SCC,SK,SKK
        COMMON /BM/ BMKX,BMKY,BMCX,BMCY
          FMSX=(-SCC*(SVX-DVX)-SKK*(SX-DX)-BMCX*SVX-BMKX*SX)/SM+GR
        RETURN
        END

c*****************************************************************
c     Function FMSY
c          calculate the Y-direction acceleration of the journal
c       when the rotor is supported by the magnetic bearing
c*****************************************************************
      FUNCTION FMSY(DY,SY,DVY,SVY)
        COMMON /CNST/ GR,YMUD,YMUS,DT,INDEX
        COMMON /SHAF1/ SM,SC,SCC,SK,SKK
        COMMON /BM/ BMKX,BMKY,BMCX,BMCY
          FMSY=(-SCC*(SVY-DVY)-SKK*(SY-DY)-BMCY*SVY-BMKY*SY)/SM
        RETURN
        END


c*****************************************************************
c     Function FMDX
c          calculate the X-direction acceleration of the disk
c       when the rotor is supported by the magnetic bearing
c*****************************************************************
```

```
      FUNCTION FMDX(DX,DZ,SX,DVX,SVX,DVZ)
        COMMON /CNST/ GR,YMUD,YMUS,DT,INDEX
        COMMON /DISK/  DM,DJ,DC,DCZ,DTQ,DEC
        COMMON /SHAF1/ SM,SC,SCC,SK,SKK
          FMDX=DVZ**2*DEC*COS(DZ)-(DVX*(DC+SCC)-SVX*SCC+SKK*(DX-SX))/DM
     &        +GR
        RETURN
        END
c******************************************************************
c     Function FMDY
c          calculate the Y-direction acceleralation of the disk
c        when the rotor is supported by the magnetic bearing
c******************************************************************
      FUNCTION FMDY(DY,DZ,SY,DVY,SVY,DVZ)
        COMMON /CNST/ GR,YMUD,YMUS,DT,INDEX
        COMMON /DISK/  DM,DJ,DC,DCZ,DTQ,DEC
        COMMON /SHAF1/ SM,SC,SCC,SK,SKK
          FMDY=DVZ**2*DEC*SIN(DZ)-(DVY*(DC+SCC)-SVY*SCC+SKK*(DY-SY))/DM
        RETURN
        END


c**************************************************************************
c     Subroutine CEFFICIENT
c          calculate the coefficient of friction between the journal and
c        the inner-race of the back-up bearing
c**************************************************************************
        SUBROUTINE COEFFICIENT(RSX,RSY,RBX,RBY,DZ,DVZ,SZ,SVZ,BVZ,
     &                         FN,ALPH,YU)

         COMMON /CNST/ GR,YMUD,YMUS,DT,INDEX
         COMMON /SHAF2/ SR,SJ,SCZ,SKZ
         COMMON /BEAR2/ BR,BJ,BCZ

         IF(FN.LE.0.0) THEN
              YU=0.0
              INDEX=0
              GOTO 11
         ENDIF

         A1=DT*(SR*FSZ2(FN)-BR*FBZ2(FN))
         B1=(ALPH-FALPH(RSX,RSY,RBX,RBY))/DT
         B2=BR*(BVZ+DT*FBZ1(BVZ,FN))-SR*(SVZ+DT*FSZ1(DZ,SZ,DVZ,SVZ))
         YU=(B1*(SR-BR)+B2)/A1
```

```
      IF(YU.GT.YMUS) THEN
         YU=YMUD
         INDEX=0
         GOTO 11
      ENDIF
      YMUS1=-1.*YMUS
      IF(YU.LT.YMUS1) THEN
         YU=-1.*YMUD
         INDEX=0
         GOTO 11
      ENDIF
      IF(ABS(YU).GT.YMUD) THEN
         IF(INDEX.EQ.0) THEN
            IF(YU.LT.0.) YU=YMUD*(-1.)
            IF(YU.GT.0.) YU=YMUD
            INDEX=0
            GOTO 11
         ENDIF
      ENDIF
      INDEX=1
11    RETURN
      END
```

```
INPUT DATA-FILE    'TESTDAT.IN '
c     Sample Data File for   TRARD.FOR
*******************************************************************************
  Calculate Mass & Stiffness of the element ?          -- No
  No. of Steps for Calculation       NS                -- 90000
                   Reduced Ratio     NS1               -- 30
                   Initial Cond.     NNSP              -- 1000
                   Reduced Ratio     NNSP1             -- 1
  Time Increment for Calculation     DT   (sec)        -- 5.0 E-6
                     Initial Cond.   DT2  (sec)        -- 2.0 E-4
  Rotational Speed                   NRPM (rpm)        -- 9000

 indispensable data
  Modulus of Elasticity              E    (N/m^2)      -- 69.E9
  Shearing Modulus                   G    (N/m^2)      -- 26. E9
  Coefficient of Friction (Dynamic Sliding) YMUD       -- 0.3
                          (Static Sliding)  YMUS       -- 0.50
                          (Bearing Rolling) YMUZ       -- 1.0015
  Disk Mass Eccentricity             DEC  (m)          -- .03 E-3
  Journal Radius                     SR   (m)          -- 14.8E-3
  Shaft Stiffness (M.B. Span)        SKK  (N/m)        --  6. E6
  Bearing Inner Radius               BR   (m)          -- 15.00E-3
          Inner-race Outer Radius    BOR  (m)          -- 21.25E-3
          Support Stiffness          BK   (N/m)        -- 3. E6
          Contact Stiffness          RK   (N/m)        -- 500.E6
          Pre-Load                   FNZ  (N)          -- 1.0
  Housing Mass                       HM   (kg)         -- 5000.
          Vertical Stiffness         HKX  (N/m)        -- 1.0 E8
          Horizontal Stiffness       HKY  (N/m)        -- 1.0 E8
  Magnetic Bearing Stiffness         BMKX (N/m)        -- 2.0 E6
                                     BMKY (N/m)        -- 1.8 E6
  Disk Frictional Torque             DTQ  (N.m)        -- 0.2

 damping coefficient
  Disk Displacement     (abs)        DC   (kg/s)       -- 1.
       Torsion          (abs)        DCZ  (kg.m^2/s)   -- 2.0E-4
  Shaft Displacement    (rel. Disk)  SC   (kg/s)       -- 150.
            (rel. Disk)(M.B.Span)    SCC  (kg/s)       -- 200.
        Torsion         (rel. Disk)  SCZ  (kg.m^2/s)   -- 0.0
  Bearing Displacement  (rel. Hous)  BC   (kg/s)       -- 30.
          Torsion       (rel. Hous)  BCZ  (kg.m^2/s)   -- 0.0
          Contact       (rel. Shaft) RC   (kg/s)       -- 2000.
  Housing Vertical      (abs)        HCX  (kg/s)       -- 5.E5
          Horizontal    (abs)        HCY  (kg/s)       -- 5.E5
  Magnetic Bearing      (rel. shaft) BMCX (kg/s)       -- 1000.
                        (rel. Shaft) BMCY (kg/s)       --  900.
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basic data
  Disk Mass                             DM   (kg)         -- 4.0
       Moment of Inertia                DJ   (kg.m^2)     -- .0023
  Shaft Stiffness                       SK   (N/m)        -- 2.0E6
        Torsional Stiffness             SKZ  (N.m)        -- 17.7 E3
  Journal Mass                          SM   (kg)         -- 1.60
          Moment of Inertia             SJ   (kg.m^2)     -- .0012
  Bearing           Mass                BM   (kg)         -- .4
      Inner-race Moment of Inertia      BJ   (kg.m^2)     -- .0001

alternative basic data
  Disk Radius                           DR   (m)          -- 0.
       Thickness                        DTH  (m)          -- 0.
  Shaft Diameter                        SDO  (m)          -- 0.
        Length                          SL   (m)          -- 0.
  Journal Length                        ST   (m)          -- 0.
  Bearing Length                        BT   (m)          -- 0.
  Density of Disk                       DROU (kg/m^3)     -- 0.
             Shaft & Journal            SROU (kg/m^3)     -- 0.
             Bearing                    BROU (kg/m^3)     -- 0.

initial condition
  Disk Displacement     Vertical          RDX  (   0.)  --  .6897697E-04
                        Horizontal        RDY  (   0.)  --  .3526320E-05
                        Rotational        RDZ  (   0.)  --  .3014172E+01
  Shaft Displacement    Vertical          RSX  (   0.)  --  .5918439E-04
                        Horizontal        RSY  (   0.)  -- -.7461175E-06
                        Rotational        RSZ  (   0.)  --  .3014172E+01
  Disk Velocity         Vertical          RDVX (   0.)  -- -.4308834E-02
                        Horizontal        RDVY (   0.)  --  .3133090E-01
                        Rotational        RDVZ (OMEGA)  --  .9424778E+03
  Shaft Velocity        Vertical          RSVX (   0.)  --  .2686580E-03
                        Horizontal        RSVY (   0.)  --  .2925860E-01
                        Rotational        RSVZ (OMEGA)  --  .9424778E+03
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Vita of Toshiyasu Ishii

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Work Experience:

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Tobu District Headquarters, Customer Developing Department.
- Field engineer to design the system of the natural-gas related energy utility, including gas turbines, gas engines, and boilers.

Toshiyasu Ishii