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TRANSIENT STABILITY SIMULATION BY WAVEFORM RELAXATION METHODS

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Abstract

In this paper, a new methodology for power system dynamic response calculations is presented. The technique known as the waveform relaxation has been extensively used in transient analysis of VLSI circuits and it can take advantage of new architectures in computer systems such as parallel processors. The application in this paper is limited to swing equations of a large power system. Computational results are presented.

I. INTRODUCTION

The main trust of this paper is to introduce the waveform relaxation method (WRM) for the transient stability analysis of very large scale power systems. In recent years this method has been shown to be very effective for the transient analysis of VLSI circuits. Although the VLSI systems are technologically newer compared to electric power systems, they share many commonalities with them: the number of nodes typically exceeds several thousand on realistic systems and both circuits are sparse. Also, a typical VLSI circuit is a circuit containing mainly R-C components, while a typical power system consists primarily of R-L components (with some capacitive effects on long lines and as shunt compensation). Because of the dualism between the RC and RL circuits, it is reasonable to assume that the WR techniques can be applied to power systems as well. The implementation of WR algorithms on pipeline or parallel processors will result in enhanced computational efficiency. Hence, the results of this paper complement the recent research work [1]-[3] on applying parallel processors for solving power system problems. Both VLSI circuits and very large scale power systems (VLSP) share the common property that the system matrix is diagonally dominant, if the coupling between machines or groups of machines is weak. In the WR algorithm, this helps in speeding up the convergence. We emphasize here some of the common problems in the VLSP and VLSI systems since the intensive research efforts in the VLSI area can benefit power systems research and vice versa. Moreover, the developments in the VLSI research area are most often geared towards the newest computer technology and architectures [6]. These are important factors in on-line computer implementation or developing software for power system simulators. It is a well known fact that researchers in VLSI simulation have benefited immensely from the pioneering work on sparse matrix techniques first applied to power systems [4]. It is appropriate to emphasize such cross fertilization of ideas in terms of dynamical simulation as a healthy trend.

In particular, in this paper, we present recent results of using waveform relaxation in VLSI circuits [5]-[7] to the transient stability simulation problem with the classical model. The results are general enough to be applicable to multi-machine systems with mixed algebraic and differential equations. Use of parallel processing architectures and numerical convergence aspects are discussed. The simulation results on three, ten, and twenty-machine systems are presented. They are very encouraging and support the viewpoint that new avenues for finding more efficient numerical methods for stability transient analysis (TSA) still exist to the point that the transient stability analysis, if combined with a proper computer technology, may become feasible for real-time monitoring and security evaluation of electric power systems.

II. THE DYNAMICAL MODEL FOR ELECTRIC POWER SYSTEMS

Generally, a multi-machine dynamical model is described by the set of differential and algebraic equations of the type

$$\dot{x} = F(x,y) \ , \ x(0) = x_0 \tag{2.1a}$$

$$0 = G(x,y) \tag{2.1b}$$

where (2.1a) are equations of the generating unit and (2.1b) are those corresponding to the interface equations and the network equations.

If a classical model is assumed for the machine and all loads are converted into impedances, we have equations of the form

$$\dot{X} = F(X) \quad X(0) = X_0 \tag{2.2}$$

Swing equations with the classical model can be put in the form (2.2) as shown below.

The swing equation for i^{th} machine is

$$M_i \frac{d^2\delta_i}{dt^2} = P_{mi} - E_i^2 G_{ii} - \sum_{\substack{j=1 \\ \neq i}}^{n} (C_{ij} \sin\delta_{ij} + D_{ij} \cos \delta_{ij})$$

$$= P_i - P_{ei} (\delta_1 \dots \delta_n) \tag{2.3}$$

where

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$M_i = \frac{H_i}{\pi f}$ with H_i = inertia constant in secs.

P_{mi} = mechanical input in p.u., $P_i = P_{mi} - E_i^2 G_{ii}$

$\underline{E_i / \delta_i}$ = voltage behind the transient reactance X'_{di}

$C_{ij} = E_i E_j B_{ij}$; $D_{ij} = E_i E_j G_{ij}$

G_{ij}, B_{ij} are elements of the Y matrix at the internal nodes of the generator. Depending on the system configuration (faulted or post-fault), these will assume appropriate values.

We take δ_i as the rotor angle with respect to the synchronously rotating reference axis. Let $\dot{\delta}_i = \frac{d\theta_i}{dt} - \omega_s \triangleq \omega_i$ be the relative angular velocity where θ_i is the rotor angle w.r.t. absolute reference frame and ω_s = synchronous velocity (377 rad/sec). Introducing the state variables δ_i and ω_i ($i = 1, 2, \ldots, n$), we get the state space equations

$$\dot{\delta}_i = \omega_i$$

$$\dot{\omega}_i = \frac{1}{M_i}\left(P_i - P_{ei}\,(\delta_1, \ldots, \delta_n)\right) \qquad i = 1, 2, \ldots, n \tag{2.4}$$

The initial conditions of (2.4) are $\left(\delta_i(0),\ 0\right)$ $i = 1, 2, \ldots, n$ where $\delta_i(0)$ is computed from the load flow data and x'_{di} of the machine. The usual assumptions regarding P_{mi} = constant and E_i = constant are made during the simulations.

III. BACKGROUND ON THE WAVEFORM RELAXATION METHOD

General Overview

The dynamical model used for transient stability studies of an electric power system consisting of n machines reviewed in Section II, can be thought of as a system of 2n coupled first-order differential equations. If the unknown variables are denoted in a vector form

$$X = [\delta_1\ \omega_1 \quad \delta_2\ \omega_2 \quad \cdots \quad \delta_n\ \omega_n]^T$$

$$= [X_{11}\ X_{12} \quad X_{21}\ X_{22} \cdots \quad X_{n1}\ X_{n2}]^T \tag{3.1}$$

the power systems equations defined in Section II could be represented as

$$\dot{X}_{11} = F_{11}\,(X_{11}, X_{12}, X_{21}, X_{22}, \ldots, X_{n1}, X_{n2}) \tag{3.2a}$$

$$\dot{X}_{12} = F_{12}\,(X_{11}, X_{12}, X_{21}, X_{22}, \ldots, X_{n1}, X_{n2}) \tag{3.2b}$$

$$\vdots$$

$$\dot{X}_{n1} = F_{n1}\,(X_{11}, X_{12}, \ldots, X_{n1}, X_{n2}) \tag{3.2c}$$

$$\dot{X}_{n2} = F_{n2}\,(X_{11}, X_{12}, \ldots, X_{n1}, X_{n2}) \tag{3.2d}$$

Mathematically, the transient analysis problem is a problem of integrating for given initial conditions $X(t_0) = X_0$ equations (3.2a)-(3.2d) in time to obtain $X(t)$ for $t > t_0$ and $t \varepsilon [t_0, T]$. The time T is fixed prior to performing the integration and is generally 1-2 seconds. The efficiency of the transient analysis method is mainly measured by the necessary CPU time to generate $X(t)$ for all $t \varepsilon [t_0, T]$. Both implicit and explicit methods of integration are used in power system dynamic response calculations [8]. Unique properties of power systems like highly localized fault propagation phenomena or coherency of machines, etc., are not accounted for in the standard transient stability programs, unless coherent dynamic equivalents are established by a separate program. Research in coherency and dynamic equivalents has resulted in good production grade codes [9]-[10]. It is, however, highly desirable that results in coherency be integrated with numerical algorithms and WR algorithm precisely allows for this. Also if some variables do not change during simulation, i.e., latency phenomena, it could be accommodated in our methodology [12].

For any high order problems, direct numerical integration (explicit) methods (DNI) are more attractive than the class of so-called implicit numerical integration (INI) schemes (backward Euler formula being one of the simplest numerically stable in this group). A basic difference between these two methods is that the INI method involves the inversion of a matrix whose order is the same as the order of differential equations to be solved, e.g., 2n for the system (3.2). This is thought of as being computationally expensive for transient analysis studies since the matrix inversion needs to be done at each step in time. Algebraization of the differential equations with LU factorization with sparsity and vector array processors reduce the computation [13]. Typically, if the integration time is $[t_0, T]$ and the integration step is h, the number of inversions would be of order $\frac{[T - t_0]}{h}$, which is extremely time consuming. This reasoning should not be confused with the reasoning in load flow computations, where algebraic, rather than differential equations are solved. Here, Newton-Raphson's method with sparsity and vector array processor reduces the equations of the form AX = b is the most favored approach. In power systems transient analysis, DNI methods like Runge-Kutta are well accepted and used. The proposed Waveform Relaxation Algorithms as a new option are originally based on INI methods (like Backward Euler Formula) and then combined with the latency [12] property of diagonally dominant dynamical systems, to reduce the order of matrices which need to be inverted.

Waveform Relaxation Method

The Waveform Relaxation Algorithms are illustrated first on a simple system of two differential equations in two unknowns, X_1, X_2:

$$\dot{X}_1 = F_1(X_1, X_2) \quad X_1(t_0) = X_{10} \tag{3.3a}$$

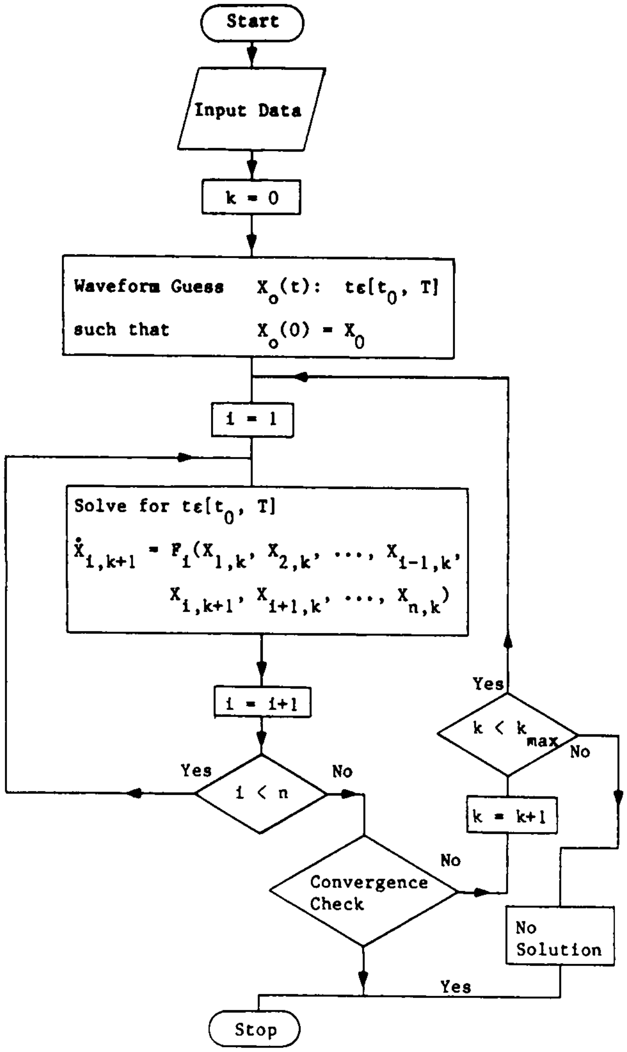
$$\dot{X}_2 = F_2(X_1, X_2) \quad X_2(t_0) = X_{20} \tag{3.3b}$$

The basic idea of the WRM is to fix the waveformn X_2 in $[t_0, T]$ and integrate equation (3.3a) as a one-dimensional differential equation in $X_1(t)$ over the whole time interval $[t_0, T]$ ("one sweep"). The solution obtained for $X_1(t)$ can be substituted in (3.3b), which will then reduce to another first-order differential equation in one variable $X_2(t)$. Equation (3.3a) is then integrated again using the new solution $X_2(t)$ and the procedure becomes iterative. A standard numerical integration problem of n differential equations in n unknowns becomes a problem of solving iteratively a sequence of n differential equations in one variable. This algorithm is analogous to the Jacobi method for solving the load flow equations. Details can be found in [14].

The basic WRM for solving a general system of nonlinear differential equations (2.2) is presented in Flow Chart 1.

FLOW CHART 1.

The Basic Waveform Relaxation Method.


Start
Input Data
k = 0
Waveform Guess $X_o(t)$: $t\epsilon[t_0, T]$
such that $X_o(0) = X_0$
i = 1
Solve for $t\epsilon[t_0, T]$
$\dot{X}_{i,k+1} = F_i(X_{1,k}, X_{2,k}, \dots, X_{i-1,k}, X_{i,k+1}, X_{i+1,k}, \dots, X_{n,k})$
i = i+1
Yes
i < n
No
Convergence Check
No
Yes
k = k+1
$k < k_{max}$
Yes
No
No Solution
Stop

k_{max} = maximum number of sweeps

A typical convergence check is

$$\max_{1\leq i\leq n} \quad \max_{t\epsilon[t_0, T]} |X_{i,k+1}(t) - X_{i,k}(t)| \leq \varepsilon \tag{3.4}$$

Important specific features of this numerical algorithm compared to conventionally used power systems transient stability schemes are:

(a) the algorithm is iterative

(b) the step where the solution in the (k+1)st sweep of differential equations is employed involves solving a differential equation in one unknown $X_{i,k+1}$ only. The other variables $X_{1,k}$, $X_{2,k}$, ..., $X_{i-1,k}$, $X_{i+1,k}$, ..., $X_{n,k}$ are known from the previous sweep k. In the VLSI literature, a typical INI method used to perform this step is the backward Euler algorithm.

We adopt the same method here, and it basically amounts to the following. Letting j correspond to the integration step i, to the ith state variable and k to the sweep

$$\dot{X}_{i,k+1} = F_i \left(X_{1,k}, X_{2,k}, \dots, X_{i-1,k}, X_{i,k+1}, X_{i+1,k}, \dots, X_{n,k}\right) \tag{3.5}$$

or

$$\frac{X^{j+1}_{i,k+1} - X^j_{i,k+1}}{h} = F_i \left(X^j_{1,k}, X^j_{2,k}, \dots, X^j_{i-1,k}, X^j_{i,k+1}, X^j_{i+1,k}, \dots, X^j_{n,k}\right)$$
$$+ \sum_{\substack{\ell=1\\ \ell\neq i}}^{n} \frac{\partial F_i}{\partial X_\ell} \left(X^j_{1,k}, \dots, X^j_{i,k+1}, \dots, X^j_{n,k}\right)\left(X^{j+1}_{\ell,k} - X^j_{\ell,k}\right)$$
$$+ \frac{\partial F_i}{\partial X_i} \left(X^j_{1,k}, \dots, X^j_{i,k+1}, \dots, X^j_{n,k}\right)\left(X^{j+1}_{i,k+1} - X^j_{i,k+1}\right) \tag{3.6}$$

which implicitly defines

$$X^{j+1}_{i,k+1} = X^j_{i,k+1} + h\left(1 - h\frac{\partial F_i}{\partial X_i}(X^j_{1,k}, \dots, X^j_{i,k+1}, \dots X^j_{n,k})\right)^{-1} \cdot \left(F_i(X^j_{1,k}, \dots, X^j_{i,k+1}, \dots, X^j_{n,k}) + \sum_{\substack{\ell=1\\ \ell\neq i}}^{n} \frac{\partial F_i}{\partial X_\ell}(X^j_{1,k}, \dots, X^j_{i,k+1}, \dots, X^j_{n,k})(X^{j+1}_{\ell,k} - X^j_{\ell,k})\right) \tag{3.7}$$

If higher order terms in the right-hand side of (3.7) are neglected, (3.7) can be approximated as

$$X^{j+1}_{i,k+1} = X^j_{i,k+1} + h\left(1 - h\frac{\partial F_i}{\partial X_i}(X^j_{1,k}, \dots, X^j_{i,k+1}, \dots, X^j_{n,k})\right)^{-1} \cdot F_i(X^j_{1,k}, \dots, X^j_{i,k+1}, X^j_{n,k}) \tag{3.8}$$

IV. CONVERGENCE CONDITIONS AND PROCEDURES FOR SPEED-UP

It has been shown that rather mild conditions on continuity of functions F_i, $i = 1, \ldots, n$ are required for the WRM to converge for any initial guess $X(t_0)$, $t\varepsilon[t_0, T]$. (Theorems 3.1, 3.1 in [5]).

For typical power systems the required conditions are always satisfied. Going back to the parallelism with the Jacobi method for load flow solution, a well known, theoretically proven fact [23] is that this method converges for a wider range of initial guesses than the standardly used Newton-Raphson method. It is the rate of convergence that makes the Jacobi method inferior for load flow calculations. Jacobi method is considered to converge linearly, while Newton-Raphson converges almost quadratically. Since the WRM technique in transient analysis is analogous to Jacobi's method for solving algebraic equations whose convergence rate is linear, this might be a point of concern. Gauss-Seidel WRM algorithm will improve the convergence rate [5].

It is not surprising, however, that the initial convergence rate obtained for VLSP systems is much higher than the convergence rate for VLSI systems. We argue in the next section that if the knowledge about coupling between machines on a typical large scale power system is used, the WRM easily becomes a superior technique, even prior to parallel processing implementation.

A true issue in comparing convergence rate of conventionally used direct integration methods for transient analysis with the WRM is the fact that the DNI methods become inefficient for extremely large problems because of the following two reasons.

(a) The sparse matrix solution time grows superlinearly with the size of the problem. Experimental evidence indicates that the point where the matrix solution time begins to dominate is when the system has over several thousand nodes, which is the case for VLSP systems.

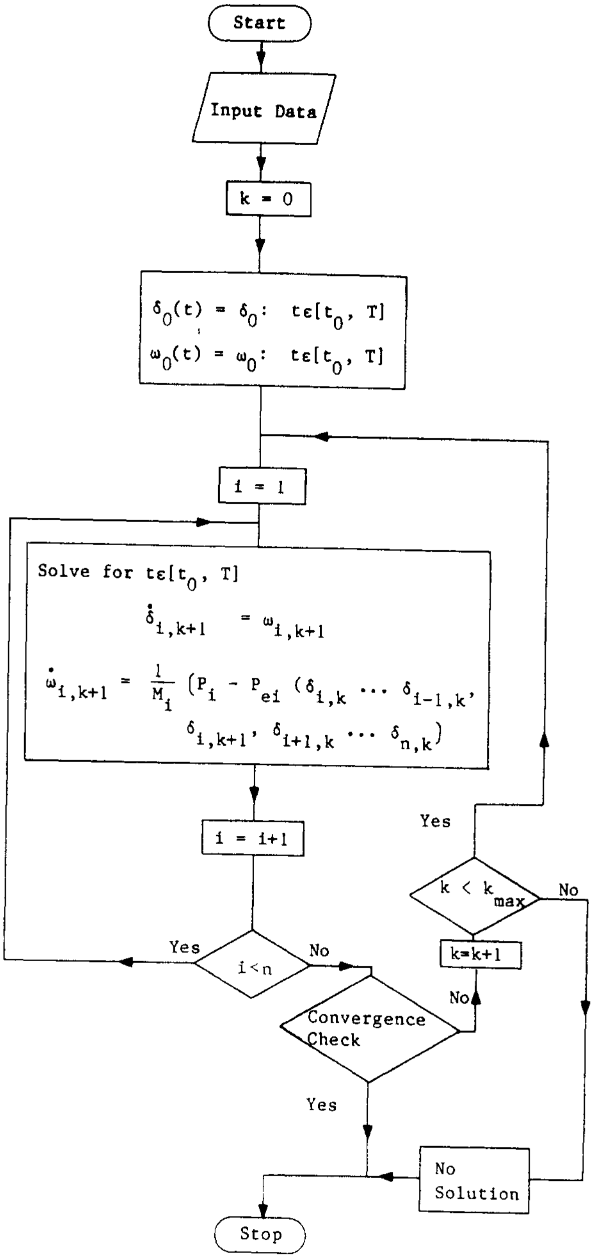
(b) The DNI methods become inefficient for large problems when the differential equations are stiff. Direct application of the integration method forces every differential equation in the system to be discretized identically, and this discretization must be small enough so that the fastest changing variable in the system is accurately represented. Gear's formula [16] overcomes this difficulty. If it were possible to pick different discretization points, or time steps, for groups of differential equations in the system so that each could use the largest time step that would accurately reflect the behavior of the associated variables, the efficiency of the simulation would be greatly improved. This is the multiple time-scale issue and has been implemented in a DNI framework [17].

Specific Techniques Employed

The partitioning of original power equations (2.4) is done in 2 x 2 blocks, i.e., the angle δ_i and speed ω_i are solved from two equations simultaneously. The notation corresponding to the power system problem is given in (3.1). If equations are solved one at a time, the WRM does not converge. So, the results reported here are for the case where two equations are solved in two unknowns by the Backward Euler Formula and the WRM adopted to this simulation is given by Flow Chart No. 2.

FLOW CHART 2.

WRM Algorithm for Transient Stability Analysis


Start
Input Data
k = 0
$\delta_0(t) = \delta_0$: $t\varepsilon[t_0, T]$
$\omega_0(t) = \omega_0$: $t\varepsilon[t_0, T]$
i = 1
Solve for $t\varepsilon[t_0, T]$
$\dot{\delta}_{i,k+1} = \omega_{i,k+1}$
$\dot{\omega}_{i,k+1} = \frac{1}{M_i}(P_i - P_{ei}(\delta_{1,k} \cdots \delta_{i-1,k}, \delta_{i,k+1}, \delta_{i+1,k} \cdots \delta_{n,k})$
i = i+1
Yes
i<n
No
Convergence Check
Yes
No
k=k+1
$k < k_{max}$
Yes
No
No Solution
Stop

k_{max} = maximum number of sweeps

Different time responses of different variables for which we are solving naturally suggest grouping based on coherency [9]-[11]. Initial experiments with this show that the convergence rate is rather sensitive to grouping. If the machines within a group are tightly coupled, then the convergence rate in terms of number of "sweeps" is better. WR algorithm can thus incorporate coherency based technique with numerical integration. A major research effort on VLSP systems with nodes of order of thousands should be based on exploring this fact together with the WRM.

The other technique for speeding up the WRM convergence rate is based on the so-called time windowing [6]. If we are interested in results on time $[t_0, T]$, the WRM could be applied on $[t_0, T_1]$ then in $[T_1, T_2]$ and $[T_2, T]$ in order to reduce the total CPU time. The effect of time windowing on the numerical examples is presented in the next section.

The grouping of differential equations to be solved simultaneously has a significant effect on very large scale systems and it should be done carefully. The "adaptive clustering" idea reported in the VLSI literature [6] could be directly implemented since grouping under the same name and meaning has been recently reported by Zaborszky et al. [18]. Again, this is what makes this numerical method very promising and currently research is in progress in this area.

V. PARALLEL PROCESSING BASED WRM

An additional property of WRM is the fact that parallel processing is a natural setup for the method. If one processor is assigned to one group of variables which are solved for simultaneously, the CPU time is reduced proportionally to the number of groups. Considerable work exists on parallel solution of ordinary differential equations [19]-[20].

A detailed discussion of the possible computer architectures for WRM methods is given in [6]. Previously reported work on parallel processing in power systems [1]-[3] exploits time-point pipelining algorithms. The shared memory computer architecture is argued to be more efficient for the WRM support of VLSI systems which, we believe, should be common for VLSP systems. The key problem in designing a parallel processor lies in the communication between the processors and is much easier here.

The main advantage of a shared memory system is that it is not necessary to explicitly transfer data from one processor to another. When a processor needs data from another processor, it simply reads from the memory location in which the other processor has written. This allows for more dynamic algorithm structures (like WRM with fault dependent grouping) because it is not necessary to determine beforehand which processors will need the results of a given calculation. There are disadvantages, however (synchronization and locking being the major), but overall the shared memory architecture appears to be better for more adaptive algorithms.

VI. NUMERICAL RESULTS

The WR algorithm discussed in Section III was implemented on the 3, 10 and 20 machine systems. The 3 machine data is taken from Ref. [21], and 10 and 20 machine data from Ref. [22]. From the numerical point of view it appears that the number of sweeps is fairly independent of the size of the system. This is probably due to the fact that the natural modes of the system lie in a fairly small range. In each case we have compared the WR algorithm with the explicit fourth order Runge-Kutta method. Both simulations were coded in PASCAL so that CPU times etc. can be compared on the same basis. The step size chosen was $\Delta t = .00375$s because the comparison was with Runge-Kutta method which becomes unstable for larger Δt. If an implicit method such as the trapezoidal method is used, Δt could have been chosen higher. In fact, in the WR algorithm, Δt can be chosen even higher because of the backward Euler method which is numerically stable.

The CPU times were computed in each case and are shown in Table I. Also shown are the number of "sweeps" for the WR algorithm and the CPU time with $\Delta t = 0.0375$ s. The progressive convergence for $\delta_6(t)$ in the 10 machine case for different sweeps is shown in Figure 1. There was no significant change in number of sweeps when Δt was increased from .00375 s to .0375 s. It can be seen that CPU time decreases for the WR algorithm when Δt is increased to .0375 s.

	3 m/c	10 m/c	20 m/c
1. CPU time (WR) Δt = .00375s	4.440	36.232	151.941
2. CPU time (WR) Δt = .0375s	0.402	3.072	21.337
3. CPU time (RK) Δt = .00375s	1.130	7.291	26.674
4. No. of Sweeps (WR) Δt = .00375s	19	18	19

TABLE I. CPU time comparison and number of "sweeps" for WR algorithm.

The effect of "windowing" in the WR algorithm is shown in Table II. Two different time "windowing" patterns were investigated. There is a significant decrease in CPU time when windowing is used. Only the results using three "windows" are presented in Table II. Comparing these results with the corresponding cases 1 and 2 in Table I, we observe a 50% reduction in CPU time. Since "windowing" is not possible in the RK method, no direct comparison is possible. At this point, it is felt that depending on the time interval of simulation $[t_0, T]$ beyond a certain number of "windows," one cannot expect a decrease in CPU time. The same experience has been reported for the VLSI systems [6]. Figure 2 shows the effect of "windowing" on $\delta_6(t)$ in the 10 machine case.

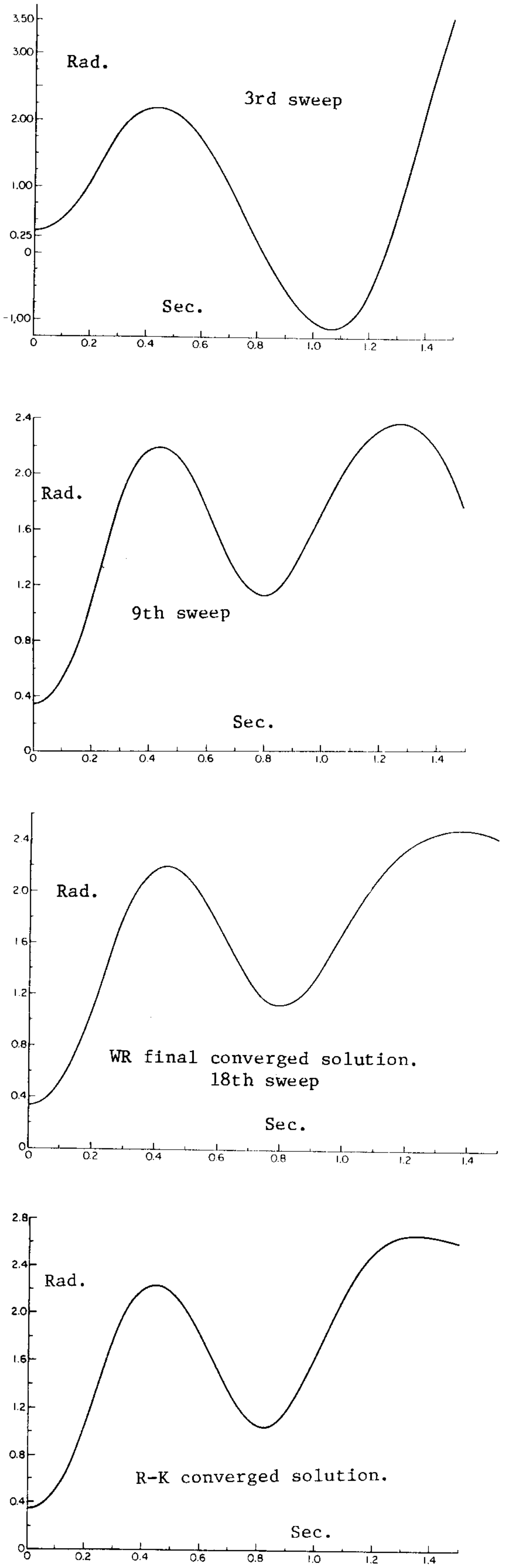
	3 m/c	10 m/c	20 m/c
1. CPU time (WR) Δt = .00375s	2.436	17.889	79.053
2. CPU time (WR) Δt = .0375s	0.220	1.614	9.605

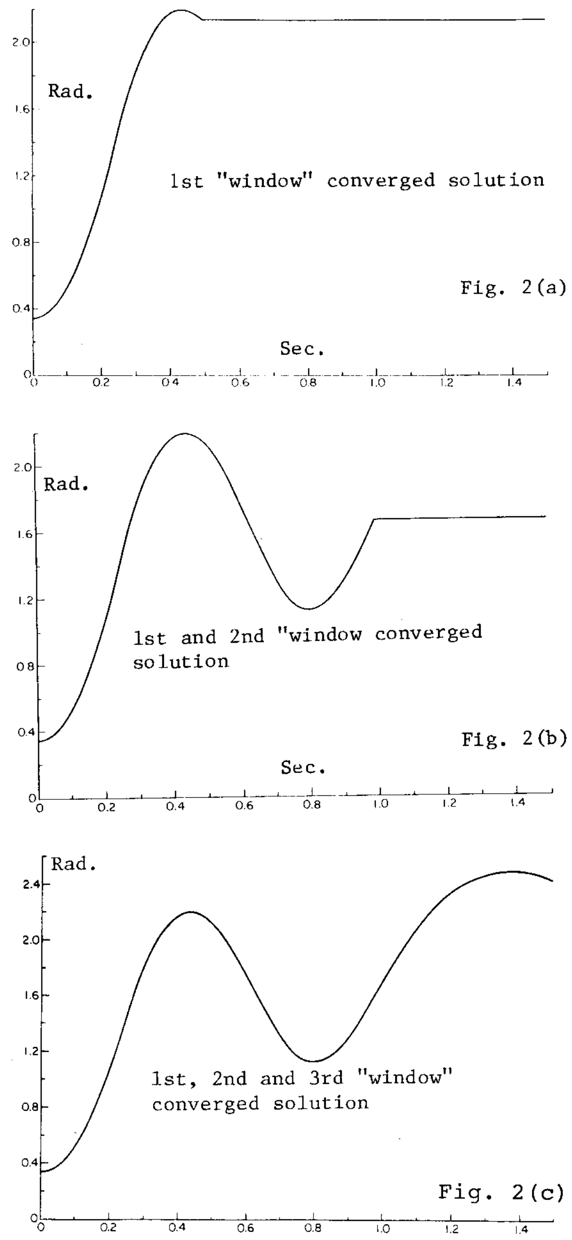
TABLE II. CPU time with "windowing" (.5, 1.0, 1.5 S)

Finally, the WR algorithm in combination with coherent grouping was investigated. In the 10 machine case there were six coherent groups as in [10]. The groups were machines (2,3), (4,6,7,8,10), (5), (9), (1) (see Figure 3). Instead of grouping the variables as in (3.1), we group all variables corresponding to a coherent group as one set. If the coupling between the areas is weak and within an area is tight, this will have the effect of improving both the CPU time and the number of sweeps. The improvement is dependent on the degree of coherency. Table III shows the effect of grouping on the 10 and 20 machine system.

	10 m/c	20 m/c
1. CPU time (WR) Δt = .00375s	29.961	105.147
2. CPU time (WR) Δt = .0375s	3.075	14.806
3. CPU time (RK) Δt = .00375s	14	13

TABLE III. Effect of grouping.





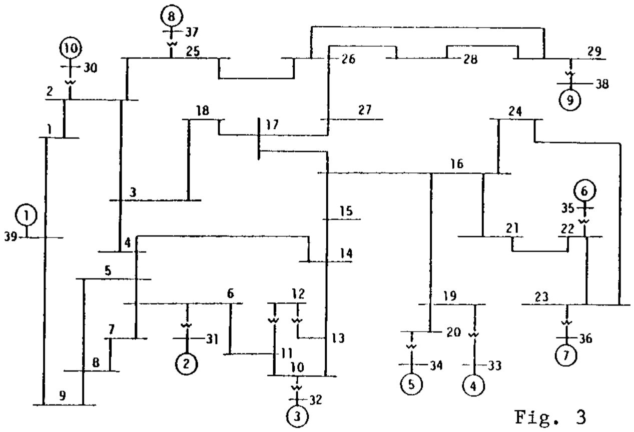


Fig. 3

Figure 1. Successive sweeps of $\delta_6(t)$ for fault at bus #35 cleared in .24 Sec. (Δt = .00375 Sec.) and solution by R-K Method

Figure 2. Converged solution by "windowing" technique (.5, 1.00, 1.5 Sec.)

Figure 3. Single line diagram of 10 machine, 39 bus system.

Note that a combination of windowing and grouping still needs to be studied. Based on the results in Tables II and III, we believe that the CPU time for solving a 20 machine case will be shorter than the shortest reported for the WRM, i.e., 9.605 CPU. It is true that as the size of the system increases, the ratio of CPU time of WR/RK increases from 2.155 for the three machine case to 2.96 for the 20 machine case for Δt = .00375 sec. However, this is offset by the possibility of taking larger Δt for the WR method in which case the same ratio varies from .1946 to .36. A comparison of the WR method with a stable method like the trapezoidal or backward Euler method indicated that the latter takes much computer time. Sparsity considerations were not used and hence a direct comparison method is not fair at this stage. Research on this aspect is continuing. If the parallel processing argument is introduced with a suggested number of processors M, the reported computing time needed for the WRM of 9.605 CPU would reduce to approximately r the (9.605/M) CPU. However, many different constraints, like cost and specific computer architecture, should determine the choice of the number of processors M.

VII. CONCLUSION

In this paper a new method for the transient stability analysis of very large scale power systems is proposed. This is the waveform relaxation method intensively used in VLSI systems. Groups of differential-algebraic equations are integrated for the "sweep" and iteratively done as in Jacobi method of solving load flow equations. The technique is very promising. Current further work in this area concerns the detailed models of machines and use of parallel processors to make the tool an effective one. Also, theoretical results on coherency are being combined with the numerical aspects of the WRM.

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