Transient Stabilization of Multimachine Power Systems With Nontrivial Transfer Conductances

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***Abstract*—In this paper, we provide a solution to the long-standing problem of transient stabilization of multimachine power systems with nonnegligible transfer conductances. More specifically, we consider the full $3n$-dimensional model of the n-generator system with lossy transmission lines and loads and prove the existence of a nonlinear static state feedback law for the generator excitation field that ensures asymptotic stability of the operating point with a well-defined estimate of the domain of attraction provided by a *bona fide* Lyapunov function. To design the control law we apply the recently introduced interconnection and damping assignment passivity-based control methodology that endows the closed-loop system with a port-controlled Hamiltonian structure with desired total energy function. The latter consists of terms akin to kinetic and potential energies, thus has a clear physical interpretation. Our derivations underscore the deleterious effects of resistive elements which, as is well known, hamper the assignment of simple "gradient" energy functions and compel us to include nonstandard cross terms. A key step in the construction is the modification of the energy transfer between the electrical and the mechanical parts of the system which is obtained via the introduction of state-modulated interconnections that play the role of multipliers in classical passivity theory.**

***Index Terms*—Nonlinear systems, passivity-based control, power systems, stability.**

I. INTRODUCTION

TRADITIONAL analysis and control techniques for power systems are undergoing a major reassessment in recent years; see [29] for an excellent tutorial account. This worldwide trend is driven by multiple factors including the adoption of new technologies, like flexible ac transmission systems, which offer improvements in power angle and voltage stability but give rise to many modeling and control issues that remain to be resolved. Also, the ever increasing utilization of power electronic converters is drastically modifying the energy consumption profile, as well as the underlying distributed generation. The new deregulated market, on the other hand, has seen the emergence of separate entities for generation that imposes more stringent requirements on the dynamic behavior of voltage regulated units and the task of coordinating a large number of (small and large) active and reactive control units in the face of significant load uncertainty. It is, by now, widely recognized that the existing methods and tools to approach power systems should be revisited to ensure reliable and secure planning—with the recent dramatic blackouts in North America and Italy providing compelling evidence of this fact.

In this paper, we study the fundamental problem of *transient stability* of power systems whose reliable assessment has become a major operating constraint, particularly in regions that rely on long distance transfers of bulk power. Transient stability is concerned with a power system's ability to reach an acceptable steady-state following a fault, e.g., a short circuit or a generator outage, that is later cleared by the protective system operation; see [2], [9], [13], [25], and the tutorial paper [4] for more details. The fault modifies the circuit topology—driving the system away from the stable operating point—and the question is whether the trajectory will remain in the basin of attraction of this (or other) equilibrium after the fault is cleared. The key analysis issue is then the evaluation of the domain of attraction of the system's operating equilibrium, while the *control objective* is the enlargement of the latter.

Transient stability *analysis* dates back to the beginning of the electric age [5] with the problem originally studied via numerical integration and, starting in the 1947 seminal paper [15], with Lyapunov-like methods. A major open problem in this area is the derivation of Lyapunov functions for transmission systems that are *not lossless*, i.e., with transfer conductances between busses.[1] While the transmission system itself can be modeled as being lossless without loss of accuracy, the classical network reduction of the load busses induces transfer conductances between the rest of the system busses, rendering the negligible transfer conductances assumption highly unsatisfactory [2], [13]. Although considerable efforts have been made to find Lyapunov functions for lossy line systems, to the best of the authors' knowledge this research has unfortunately been in vain. Actually, in [18] it is claimed that, even for the simple

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[1]More precisely, the conductances represent partial losses caused by the line and the loads in the nodes. For the sake of simplicity, in this paper we say the line is lossless or lossy if the conductances are neglected or not, respectively.

swing equation model,[2] the standard energy function of a lossless system cannot be extended (in general) to a lossy system. (See also [27] for some surprising results obtained via local stability analysis.)

Our interest in this paper is in the design of *excitation controllers* to enhance transient stability. These controllers are proposed to replace the traditional automatic voltage regulator (AVR) plus power system stabilizer (PSS) control structure, and questions about the benefits of this replacement have not yet been answered. Given the highly nonlinear nature of the power system models the applicability of linear controller design techniques for transient stability enhancement is severely restricted. On the other hand, the application of more promising nonlinear control methods has attracted much attention in the literature, with feedback linearization being one of the early strategies to be explored [8], [13], [16], [34]. The well-known robustness problems, both against parameter uncertainties and unmodeled dynamics, of these nonlinearity cancellation schemes has motivated the more recent works on *passivity-based* techniques [19], [33]. Most results along these lines are based on the application of damping injection (also called L_gV) controllers (see [7], [14], and [17].) See also [20] for an alternative passivation approach. In [3], a dynamic damping injection controller is proposed which is proven to enlarge the estimate of the region of attraction and is shown (in simulation studies) to increase critical clearing times for a single machine infinite bus (SMIB) system with lossless transmission lines.

Attention has been given also to passivity-based methods that rely on *port-controlled Hamiltonian* (PCH) descriptions of the system [10], [22], [23], and [33]. As explained in Section III, these techniques go beyond L_gV schemes endowing the closed-loop system with a PCH structure, with stability of the desired equilibrium established assigning an energy function that qualifies as Lyapunov function. In [30], we exploit the fact that the lossless SMIB open-loop system is in PCH form, to give conditions for a *constant* field control action to shape the energy function. In [28], again for this class of systems, we add an adaptive $\mathcal{L}_2$-disturbance attenuation controller (which belong to the class of L_gV controllers.) An energy function for the *multimachine* case was first suggested in [31], where a domination design is used to cope with the effect of the losses. In [35], the existence of a static state feedback that assigns a PCH structure—using the same energy function and still retaining the lossless assumption—is established. This result is important because it paves the way to additionally apply, in the spirit of [28], an $\mathcal{L}_2$-disturbance attenuation controller to this "PCH-ized" system. Unfortunately, in neither one of these papers it is possible to prove that the energy function qualifies as a Lyapunov function, hence the stability of the desired equilibrium, which is the issue of main concern in transient stability studies, is left unclear.[3]

To the best of our knowledge, even in the lossless case, the problem of designing a state-feedback controller that ensures asymptotic stability of the desired equilibrium point for multimachine systems remains open. The *main contribution* of this paper is to provide an affirmative answer to this problem for the lossy case. Our work is the natural extension of [11] where, for the lossless SMIB system, we propose a state-feedback controller that effectively shapes the total energy function and enlarges the domain of attraction. As in [11], the control law is derived applying the recently introduced Interconnection and damping assignment passivity-based control (IDA-PBC) methodology [23], [24]. (See also the recent tutorials [22], [24] and the closely related work [10].) The parameterization of the energy function is motivated by our previous works on mechanical [21] and electromechanical systems [26], and consists of terms akin to kinetic and potential energies, thus has a clear physical interpretation. Our derivations underscore the deleterious effects of *transfer conductances* which, as is well-known, hamper the assignment of simple "gradient" energy functions [4], [18] and compel us to include nonstandard cross terms. As usual in IDA-PBC designs, a key step in the construction is the modification of the energy transfer between the electrical and the mechanical parts of the system which is obtained via the introduction of state-modulated interconnections that play the role of multipliers in classical passivity theory [6]. As a by-product of our derivations we also present the first "globally"[4] asymptotically stabilizing law for the lossy SMIB system.

The remaining of the paper is organized as follows. The problem is formulated in Section II. In Section III, we briefly review the IDA-PBC technique. We give the slightly modified version presented in [24] where the open-loop system is not in PCH form, as is the case for the problem at hand. In Section IV, we apply the method to the lossy SMIB system where we show that a separable Lyapunov function that ensures "global" stability can be assigned. In Section V we show that, already for the two-machines case, the presence of transfer conductances hampers the assignment of a separable Lyapunov function, therefore, cross-term must be included. The stabilization mechanism of the control is analyzed in Section VI, where we provide a passive subsystem decomposition that reveals the underlying stabilization mechanism. The general n-machines case is treated in Section VII. Two simulation studies are presented in Section VIII showing the enlargement of the domains of attraction and its effect on the enhancement of critical clearing times for a SMIB and a two-machines system. We conclude with some final remarks in Section IX.

II. Model and Problem Formulation

We consider in this paper the problem of transient stabilization of a large-scale power system consisting of n generators interconnected through a transmission network which we assume is lossy, that is, we explicitly take into account the presence of *transfer conductances*. The dynamics of the ith machine with

[2]The situation for the more realistic flux-decay model [2], [13], that we consider in this paper, is of course much more complicated.

[3]In [35], this claim is established if we make the assumption that the load angles remain within $(-\pi, \pi)$. Even though this is true for the open loop system, which lives in the torus, this structure is destroyed by the control, rendering the assumption *a priori* unverifiable.

[4]The qualifier "global" is used here in the sense that we provide an estimate of the domain of attraction that contains all the operating region of the system.

excitation is represented by the classical three-dimensional flux decay model [4, eq. (9)] or [13, eq. (6.47)]

$$\begin{aligned}\dot{\delta}_i &= \omega_{i0}\omega_{Mi}\\ M_i\dot{\omega}_{Mi} &= -D_{Mi}\omega_{Mi} + P_{mi} - G_{Mii}E'^2_{qi} - E'_{qi}\\ &\quad\times\sum_{j=1,j\neq i}^{n} E'_{qj}\{G_{Mij}\cos(\delta_i-\delta_j)+B_{Mij}\sin(\delta_i-\delta_j)\}\\ T_{di}\dot{E}'_{qi} &= -\left[1-B_{Mii}\left(x_{di}-x'_{di}\right)\right]E'_{qi} - \left(x_{di}-x'_{di}\right)\\ &\quad\times\sum_{j=1,j\neq i}^{n} E'_{qj}\{G_{Mij}\sin(\delta_i-\delta_j)-B_{Mij}\cos(\delta_i-\delta_j)\}\\ &\quad + E_{fsi} + u_{fi}\end{aligned}.$$

The state variables of this subsystem are the rotor angle δ_i, the rotor speed ω_{Mi} and the quadrature axis internal voltage E'_{qi}, hence, the overall system is of dimension $3n$.[5] The control input is the field excitation signal u_{fi}. The parameters $G_{Mij} = G_{Mji}$, $B_{Mij} = B_{Mji}$ and G_{Mii} are, respectively, the conductance, susceptance and self-conductance of the generator i. E_{fsi} represents the constant component of the field voltage and P_{mi} the mechanical power, which is assumed to be constant. The parameters x_{di}, x'_{di}, ω_{i0} and D_{Mi} represent the direct-axis—synchronous and transient—reactances, the synchronous speed and damping coefficient, respectively. We note that all parameters are positive and $x_{di} > x'_{di}$. See [13] and [25] for further details on the model.

Using the following identities:

$$\begin{aligned}G\cos\delta + B\sin\delta &= Y\sin(\delta+\alpha)\\ G\sin\delta - B\cos\delta &= -Y\cos(\delta+\alpha)\end{aligned} \tag{1}$$

with $Y^2 = G^2 + B^2$ and $\alpha = \arctan(G/B)$, and introducing the parameters

$$\begin{aligned}&E_{fi} \triangleq \frac{E_{fsi}}{T_{di}} \quad u_i \triangleq \frac{u_{fi}}{T_{di}} \quad D_i \triangleq \frac{D_{Mi}}{M_i} \quad P_i \triangleq \frac{P_{mi}\omega_{i0}}{M_i}\\ &G_{ij} \triangleq \frac{G_{Mij}\omega_{i0}}{M_i} \quad B_{ij} \triangleq \frac{B_{Mij}\omega_{i0}}{M_i} \quad \omega_i \triangleq \omega_{i0}\omega_{Mi}\end{aligned}.$$

We can rewrite the system in the more compact form

$$\begin{aligned}\dot{\delta}_i &= \omega_i\\ \dot{\omega}_i &= -D_i\omega_i + P_i - G_{ii}E_i^2 - E_i\sum_{j=1,j\neq i}^{n} E_jY_{ij}\sin(\delta_i-\delta_j+\alpha_{ij})\\ \dot{E}_i &= -a_iE_i + b_i\sum_{j=1,j\neq i}^{n} E_j\cos(\delta_i-\delta_j+\alpha_{ij}) + E_{fi} + u_i\end{aligned} \tag{2}$$

where, in order to simplify the notation, we use E_i instead of E'_{qi}, and we have defined

$$\begin{aligned}&Y_{ij} \triangleq \sqrt{G_{ij}^2 + B_{ij}^2} \quad \alpha_{ij} \triangleq \arctan\frac{G_{ij}}{B_{ij}}\\ &a_i \triangleq \frac{1}{T_{di}}\left[1-B_{Mii}\left(x_{di}-x'_{di}\right)\right] \quad b_i \triangleq \frac{x_{di}-x'_{di}}{T_{di}}Y_{ij}.\end{aligned} \tag{3}$$

[5]In the sequel, we will consider that the full system consists of the interconnection of n subsystems of dimension 3, and talk about the ith subsystem only.

Observe that $a_i, b_i > 0$, $\alpha_{ij} = \alpha_{ji}$ and that, *if* $M_i = M_j$, $Y_{ij} = Y_{ji}$. This assumption will be made throughout the rest of the paper to simplify the derivations; see Remark 6.

We underscore that if $G_{ij} = 0$ then $\alpha_{ij} = 0$. As shown later, this ubiquitous assumption considerably simplifies the stabilization problem. (It is convenient at this point to make the following clarification: even in the case when the line is lossless, the classical reduction of transmission lines and *load buses* will lead to a reduced model with $G_{ij} \neq 0$. To simplify the notation we will refer to the case $G_{ij} = 0\ (\neq 0)$ as lossless (respectively, lossy) network cases.)

Problem Formulation: Assume the model (2), with $u_i = 0$ has a stable equilibrium point at $[\delta_{i*}, 0, E_{i*}]$, with $E_{i*} > 0$.[6] Find a control law u_i such that in closed-loop

— an operating equilibrium is preserved;
— we have a Lyapunov function for this equilibrium;
— it is asymptotically stable with a well-defined domain of attraction.

Two additional requirements are that the domain of attraction of the equilibrium is enlarged by the controller and that the Lyapunov function has an energy-like interpretation.

A detailed analysis of the equilibria of (2) is clearly extremely involved—even in the two-machines case. To formulate our claims we will make some assumptions on these equilibria that will essentially restrict $|\delta_{i*} - \delta_{j*}|$ to be small, a scenario which is reasonable in practical situations. We will also sometimes assume that the line conductances are sufficiently small.

Remark 1: In [27], it is shown that, contrary to a widely held opinion, equilibrium solutions of the lossy swing equations with $|\delta_{i*} - \delta_{j*} - \alpha_{ij}| < (\pi/2)$ may be unstable, while it is known that with transfer conductances neglected they are stable.

III. Interconnection and Damping Assignment Control

To solve this problem, we will use the IDA-PBC methodology proposed in [23]; see also [22]. IDA-PBC is a procedure that allows us to design a static state feedback that stabilizes the equilibria of nonlinear systems of the form

$$\dot{x} = f(x) + g(x)u \tag{4}$$

where $x \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}^m$, $m < n$ is the control action, endowing the closed-loop with a port-controlled Hamiltonian (PCH) structure[7]

$$\dot{x} = [J_d(x) - \mathcal{R}_d(x)]\nabla H_d \tag{5}$$

where the matrices $J_d(x) = -J_d^\top(x)$ and $\mathcal{R}_d(x) = \mathcal{R}_d^\top(x) \geq 0$, which represent the desired interconnection structure and dissipation, respectively, are selected by the designer—hence the name IDA—and $H_d : \mathbb{R}^n \to \mathbb{R}$ is the desired total stored energy. If the latter has an isolated minimum at the desired equilibrium $x_\star \in \mathbb{R}^n$, that is if

$$x_\star = \arg\min H_d(x) \tag{6}$$

[6]Because of physical constraints E_i is restricted to be positive.

[7]We note that all vectors defined in the paper are *column* vectors, even the gradient of a scalar function. We use the notation $\nabla_x = (\partial/\partial x)$, when clear from the context the subindex in ∇ will be omitted.

then $x_\star$ is stable with Lyapunov function $H_d(x)$. As stated in the simple proposition that follows, whose prove may be found in [24], the admissible energy functions are characterized by a parameterized partial differential equation (PDE).

Proposition 1: Consider the system (4). Assume there exists matrices $J_d(x) = -J_d^\top(x)$, $\mathcal{R}_d(x) = \mathcal{R}_d^\top(x) \geq 0$ and a function $H_d(x)$, which satisfies (6), such that the PDE

$$g^\perp(x)f(x) = g^\perp(x)\left[J_d(x) - \mathcal{R}_d(x)\right]\nabla H_d \tag{7}$$

is solved, where $g^\perp(x)$ is a left annihilator of $g(x)$, i.e., $g^\perp(x)g(x) = 0$. Then, the closed-loop system (4) with

$$u = \left[g^\top(x)g(x)\right]^{-1} g^\top(x)\left\{\left[J_d(x) - \mathcal{R}_d(x)\right]\nabla H_d - f(x)\right\}$$

will be a PCH system with dissipation of the form (5) with $x_\star$ a (locally) *stable* equilibrium. It will be *asymptotically* stable if, in addition, the largest invariant set under the closed-loop dynamics (5) contained in

$$\left\{x \in \mathbb{R}^n \mid (\nabla H_d)^\top \mathcal{R}_d(x)\nabla H_d = 0\right\} \tag{8}$$

equals $\{x_\star\}$. An estimate of its *domain of attraction* is given by the largest *bounded* level set $\{x \in \mathbb{R}^n | H_d(x) \leq c\}$.

A large list of applications of this method may be found in the recent tutorial paper [24]. In particular, it has been applied in [11] for transient stabilization of SMIB systems fixing $J_d(x)$ and $\mathcal{R}_d(x)$ to be constant and solving the PDE (7) for $H_d(x)$. Inspired by [10], where the energy function is fixed and then a set of algebraic equations for $J_d(x)$ and $\mathcal{R}_d(x)$ are solved, we applied this variant of the method for a class of electromechanical systems in [26] with a quadratic in increments desired energy function. In [21], these two extremes were combined for application in mechanical systems. Namely, we fixed $H_d(x)$ to be of the form

$$H_d(q,p) = \frac{1}{2}p^\top M_d^{-1}(q)p + V_d(q)$$

where the state $x = (q,p)$ consists of the generalized position and momenta, and $M_d(q) = M_d^\top(q) > 0$ and $V_d(q)$ represent the (to be defined) closed-loop inertia matrix and potential energy function, respectively. PDEs for $M_d(q)$ and $V_d(q)$ were then established, and solved in several examples selecting suitable functions for $J_d(q,p)$.

In this paper, we also proceed as in [21] for (2), fix the kinetic energy of the total energy function as $(1/2)\sum_{i=1}^n \omega_i^2$, and choose a quadratic function in the electrical coordinates E_i, which similarly to the matrix $M_d(q)$ above, is parameterized by a function of the angular positions δ_i. The proposed energy function is completed adding a potential-energy-like function of δ_i.

For the sake of clarity of presentation we will illustrate the procedure first with the simplest SMIB case. Interestingly, we obtain in this case an IDA-PBC that ensures the largest attainable domain of attraction—namely, the whole normal operating region.

Remark 2: We have concentrated here in the design of the energy shaping term of the controller. As usual in IDA-PBC, it is possible to add a damping injection term of the form $-K_{di}g^\top(x)\nabla H_d$, where $K_{di} = K_{di}^\top > 0$, that enforces the seminegativity of $\dot{H}_d$. As shown in the simulations of Section VIII this term may improve the transient performance of the system.

IV. Single Machine System With Lossy Network

In the case $n = 1$, the model (2) reduces to the well-known SMIB system

$$\begin{aligned}\dot{\delta} &= \omega\\ \dot{\omega} &= -D\omega + P_m - GE^2 - EY\sin(\delta+\alpha)\\ \dot{E} &= -aE + b\cos(\delta+\alpha) + E_f + u\end{aligned} \tag{9}$$

where we have introduced some obvious simplifying notation. We underscore the fact that, due to the presence of losses in the line, there appears a quadratic term GE^2 and a phase α in the trigonometric functions. Compare with [11, eq. (1)].

We are interested in the behavior of the system in the set $\mathcal{D} = \{(\delta,\omega,E) | \delta \in [-(\pi/2),(\pi/2)], E > 0\}$. Mimicking the derivations in [11] it is possible to show that inside this set there is a stable equilibrium that we denote $(\delta_\star, 0, E_\star)$, and the corresponding Lyapunov function provides estimates of its domain of attraction which are strictly contained in $\mathcal{D}$. The objective is to design a control law that assigns a Lyapunov function which provides larger estimates of the domain of attraction of this equilibrium.

As will be shown later, in this particular case we will be able to assign a separable energy function, more precisely a function of the form

$$H_d(\delta,\omega,E) = \frac{1}{2}\omega^2 + \psi(\delta) + \frac{\gamma}{2}(E - E_\star)^2 \tag{10}$$

where $\gamma > 0$ is some weighting coefficient and $\psi(\delta)$ is a potential-energy-like function that should satisfy $\delta_\star = \arg\min \psi(\delta)$. Our choice of energy function candidate (10), together with the first equation of (9) fixes the first row, and consequently the first column, and the (2, 2) term of the matrix $J_d(\delta,E) - \mathcal{R}_d$.[8] Hence, we propose

$$J_d(\delta,E) = \begin{bmatrix} 0 & 1 & 0\\ -1 & 0 & J_{23}(\delta,E)\\ 0 & -J_{23}(\delta,E) & 0\end{bmatrix}$$
$$\mathcal{R}_d = \begin{bmatrix} 0 & 0 & 0\\ 0 & D & 0\\ 0 & 0 & r\end{bmatrix}$$

where $J_{23}(\delta,E)$ is a function to be determined and $r > 0$ is a constant damping injection gain.

According to Proposition 1, the PDE to be solved is

$$\begin{aligned}-\nabla_\delta H_d - D\nabla_\omega H_d + J_{23}(\delta,E)\nabla_E H_d\\ = -D\omega + P_m - GE^2 - EY\sin(\delta+\alpha)\end{aligned}$$

which, upon replacement of (10), yields the ordinary differential equation

$$-\nabla\psi + \gamma J_{23}(\delta,E)(E - E_\star) = P_m - GE^2 - EY\sin(\delta+\alpha). \tag{11}$$

[8]Similar to [21], and motivated by the form of the proposed energy function, we make J_d dependent only on (δ, E) and take $\mathcal{R}_d$ to be constant.

Evaluating (11) at $E = E_\star$, we obtain

$$\nabla\psi = -P_m + GE_\star^2 + E_\star Y \sin(\delta+\alpha).$$

Replacing this expression back in (11) we can compute $J_{23}(\delta, E)$ as

$$J_{23}(\delta, E) = -\frac{1}{\gamma}\left[(E+E_\star)G + Y\sin(\delta+\alpha)\right]. \qquad (12)$$

From the previous construction, it is clear that $\nabla\psi(\delta_\star) = 0$, therefore to ensure the minimum condition (6), and to estimate the domain of attraction, it only remains to study the second derivative of $\psi(\delta)$, which is given by

$$\begin{aligned}\nabla^2\psi &= E_\star Y\cos(\delta+\alpha)\\ &= E_\star(B\cos\delta - G\sin\delta)\end{aligned}$$

where we have used the trigonometric identities (1) and the definitions (3) to obtain the last equation. Some simple calculations prove that the function $\psi(\delta)$ is strictly convex in the interval $(-(\pi/2), \arctan(B/G))$, where we remark that the right-hand term of the interval approaches $\pi/2$ as the line resistance G tends to 0.[9]

We have established the following proposition.

Proposition 2: The SMIB system with losses (9) in closed-loop with the control law

$$u = -b\left[\cos(\delta+\alpha) - \cos(\delta_\star+\alpha)\right] - J_{23}(\delta, E)\omega$$

where $J_{23}(\delta, E)$ is given in (12) and γ is an arbitrary positive constant, ensures asymptotic stability of the desired equilibrium ($\delta_\star$,0,$E_\star$) with the Lyapunov function

$$\begin{aligned}H_d(\delta,\omega,E) = \frac{1}{2}\omega^2 + \frac{\gamma}{2}(E-E_\star)^2 \\ + \left(GE_\star^2 - P_m\right)\delta - E_\star Y\cos(\delta+\alpha)\end{aligned}$$

and a domain of attraction containing the largest connected component inside the set

$$\left\{(\delta,\omega,E) \mid H_d(\delta,\omega,E) \le \max_{\delta\in\left(-\frac{\pi}{2},\arctan\frac{B}{G}\right)} H_d(\delta,0,E_\star)\right\}$$

that contains the equilibrium ($\delta_\star$,0,$E_\star$).

In particular, if the system is lossless, then the control takes the simpler proportional-plus-derivative form

$$u = -\left(b + \frac{1}{\tilde\gamma}p\right)\left[\cos(\delta) - \cos(\delta_\star)\right]$$

where $p \triangleq (d/dt)$, $\tilde\gamma > 0$ is a free parameter, and the estimate of the domain of the attraction above *coincides* with $\mathcal{D}$.

Proof: The proof of stability is completed computing the control law as suggested in Proposition 1, selecting the free coefficients to satisfy $r\gamma = a$, and using the equilibrium equations to simplify the controller expression. The Lyapunov function is obtained evaluating the integral of $\nabla\psi$ and replacing in (10). Asymptotic stability follows also from Proposition 1 noting, first, that the closed-loop system lives in the set[10] $[-\pi,\pi]\times\mathbb{R}\times\mathbb{R}$ and the energy function $H_d(\delta,\omega,E)$ is positive definite and proper throughout this set. Second, since $\dot H_d \le 0$, we have that all solutions are bounded. Finally, we have that $\dot H_d = 0 \Rightarrow E = E_\star$, $\omega = 0$, and this in its turn implies $\delta = \delta_\star$.

The claim for the lossless transmission line is established noticing that, in this case, $G = \alpha = 0$ reducing the control law to the expression given in the proposition (with $\tilde\gamma \triangleq (\gamma/B)$) and enlarging the estimate of the domain of attraction. ◃

Remark 3: The construction proposed above should be contrasted with the one given in [11]. In the latter, $J_{23}(\delta, E)$ is fixed to be a constant and no particular structure is assumed *a priori* for $H_d(\delta,\omega,E)$. It turns out that, in this case, the energy function obtained from the solution of the PDE contains quadratic terms in the full state plus trigonometric functions of δ. A nice feature of this approach is that the resulting controller is linear. On the other hand, the estimate of the domain of attraction does not cover the whole operating region as proven here.

Remark 4: It has been mentioned before that, because of physical constraints, $E > 0$. The controller of Proposition 2 does not guarantee that this bound is satisfied, however, it can easily be modified to do so. For, we propose instead of (10) a function that grows unbounded as E approaches zero, for instance

$$H_d(\delta,\omega,E) = \frac{1}{2}\omega^2 + \psi(\delta) + \gamma\left[E - E_\star\log(E)\right].$$

Mimicking the previous derivations, we obtain the same function $\psi(\delta)$, but a new interconnection term

$$J_{23}(\delta,E) = -\frac{E}{\gamma}\left[G(E+E_\star) + Y\sin(\delta+\alpha)\right]$$

that accordingly modifies the control. Another interesting variation that allow us to ensure *global asymptotic stability* consists of selecting

$$H_d(\delta,\omega,E) = \frac{1}{2}\omega^2 + \psi(\delta) + \frac{\gamma}{2}\left[E - \lambda(\delta)E_\star\right]^2$$

with $\lambda(\delta)$ a function to be defined. The PDE, evaluated at $E = E_*\lambda(\delta)$, becomes

$$GE_*^2\lambda^2(\delta) + E_*Y\sin(\delta+\alpha)\lambda(\delta) - \nabla\psi - P_m = 0.$$

If we now fix

$$\psi(\delta) = \frac{P_m}{2}\log\left[1 + (\delta-\delta_*)^2\right]$$

which is positive definite and radially unbounded, then the quadratic equation shown previously can be solve globally (for $\lambda(\delta)$.)

V. Necessity of Nonseparable Energy Functions

In the previous section, we have shown that for the single machine case it is possible to assign a separable Lyapunov function

[9] Actually, in the case $n = 1$, we have that $B \gg G$ in most practical scenarios. Therefore, the interval is typically of the form $(-(\pi/2), (\pi/2) - \epsilon]$, with $\epsilon > 0$ a small number.

[10] Notice that, in contrast to [34], the proposed controller is periodic in δ, hence, it leaves the system living in the torus.

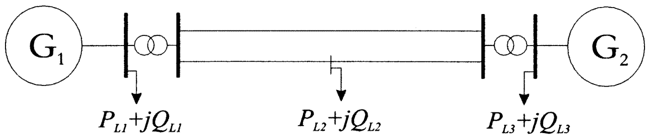

Fig. 1. Two machines system with G_i the generators, and P_{Li}, Q_{Li} the active and reactive power of the loads.

of the form (10). Before considering the n-machines case we will show now that, already for the two machines case, separable Lyapunov functions are not assignable via IDA-PBC, thus it is necessary to include cross-terms in the energy function. We then propose a procedure to add this cross term for the two-machines system.

Caveat: In the two-machines case, the dynamics can be considerably simplified defining a new coordinate $\delta_1-\delta_2$. This simplification is, however, of little interest in the general case. Since the derivations in this section will help to setup the notation that will be used for the general n-machines problem we avoid this simplification and consider all the equations of the system.

A. Separable Lyapunov Function for the Lossless Case

We consider a power system represented by two machines connected via a lossy transmission line as depicted in Fig. 1. The dynamics of this system are obtained from (2) resulting in the sixth-order model[11]

$$\begin{aligned}\dot{\delta}_1 &= \omega_1\\ \dot{\omega}_1 &= -D_1\omega_1 + P_1 - G_{11}E_1^2 - YE_1E_2\sin(\delta_1-\delta_2+\alpha)\\ \dot{E}_1 &= -a_1E_1 + b_1E_2\cos(\delta_1-\delta_2+\alpha) + E_{f1} + u_1\\ \dot{\delta}_2 &= \omega_2\\ \dot{\omega}_2 &= -D_2\omega_2 + P_2 - G_{22}E_2^2 + YE_1E_2\sin(\delta_1-\delta_2-\alpha)\\ \dot{E}_2 &= -a_2E_2 + b_2E_1\cos(\delta_2-\delta_1+\alpha) + E_{f2} + u_2. \end{aligned} \quad (13)$$

We want to investigate the possibility of assigning, via IDA-PBC, an energy function of the form

$$H_d(\delta,\omega,E) = \psi(\delta) + \frac{1}{2}|\omega|^2 + \frac{1}{2}(E-E_*)^\top\Gamma(E-E_*)$$

with $\delta = [\delta_1,\delta_2]^\top$, $\omega = [\omega_1,\omega_2]^\top$, $E = [E_1,E_2]^\top$, $E_\star = [E_{1*},E_{2*}]^\top$, $|\cdot|$ the Euclidean norm and $\Gamma = \mathrm{diag}\{\gamma_1,\gamma_2\} > 0$. Reasoning as in the single machine case we propose (14), as shown at the bottom of the page, where the $J_{ij}(\delta,E)$ are functions to be defined and $r_i > 0$.

The system of PDEs to be satisfied is

$$\begin{aligned} &-\nabla_{\delta_1}\psi + J_{23}(\delta,E)\gamma_1(E_1-E_{1*})\\ &\quad + J_{26}(\delta,E)\gamma_2(E_2-E_{2*}) = F_1(\delta,E)\\ &-\nabla_{\delta_2}\psi - J_{35}(\delta,E)\gamma_1(E_1-E_{1*})\\ &\quad + J_{56}(\delta,E)\gamma_2(E_2-E_{2*}) = F_2(\delta,E) \end{aligned} \quad (15)$$

where we introduced the functions

$$F(\delta,E) = \begin{bmatrix}F_1(\delta,E)\\ F_2(\delta,E)\end{bmatrix} \triangleq \begin{bmatrix}P_1-G_{11}E_1^2-YE_1E_2\sin(\delta_1-\delta_2+\alpha)\\ P_2-G_{22}E_2^2-YE_1E_2\sin(\delta_2-\delta_1+\alpha)\end{bmatrix}. \quad (16)$$

Now, evaluating (15) at $E = E_\star$, we get

$$\nabla\psi = -\tilde{F}^{E_\star}(\delta)$$

where we have defined $\tilde{F}^{E_\star}(\delta) \triangleq F(\delta,E_\star)$. Recalling Poincare's Lemma,[12] we see that there exists a scalar function $\psi:\mathbb{R}^2\to\mathbb{R}$ such that the previous equation holds if and only if

$$\nabla\tilde{F}^{E_\star} = (\nabla\tilde{F}^{E_\star})^\top. \quad (17)$$

Now

$$\begin{aligned}\nabla_{\delta_1}\tilde{F}_2^{E_\star} &= YE_{1*}E_{2*}\cos(\delta_1-\delta_2-\alpha)\\ \nabla_{\delta_2}\tilde{F}_1^{E_\star} &= YE_{1*}E_{2*}\cos(\delta_1-\delta_2+\alpha)\end{aligned}$$

where $\cos(x) = \cos(-x)$ was used now to invert the arguments in the first equation. These two functions are equal if and only if $\alpha = 0$—that is, if the line is lossless—concluding then that

- a separable Lyapunov function is assignable via IDA-PBC *if and only if* the line is lossless.

In this case, we can integrate $\nabla\psi$ to get

$$\psi(\delta) = -YE_{1*}E_{2*}\left[\sin(\delta_{1*}-\delta_{2*})(\delta_1-\delta_2) + \cos(\delta_1-\delta_2)\right]$$

where we invoked again the equilibrium equations to simplify the expression. The Hessian of $\psi(\delta)$ is positive semidefinite in the interval $|\delta_1-\delta_2| \le (\pi/2)$. Hence, the proposed energy function will have a minimum at the desired equilibrium provided $|\delta_{1*}-\delta_{2*}| \le (\pi/2)$, and the IDA-PBC will ensure its asymptotic stability.

[11]Using the property $\sin(x) = -\sin(-x)$ we have inverted the sign of the last term in the fifth equation to underscore the nice antisymmetry property that appears if $\alpha = 0$, which is alas lost in the lossy case. As will be proven later, this implies a "loss of integrability" which constitutes the main stumbling block for the assignment of energy functions.

[12]Poincare's Lemma: Given $f:\mathbb{R}^n\to\mathbb{R}^n$, $f\in\mathcal{C}^1$ in $\mathcal{S}\subset\mathbb{R}^n$. There exists $\psi:\mathbb{R}^n\to\mathbb{R}$ such that $\nabla\psi = f$ if and only if $\nabla f = (\nabla f)^\top$.

$$J_d(\delta,E)-\mathcal{R}_d = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0\\ -1 & -D_1 & J_{23}(\delta,E) & 0 & 0 & J_{26}(\delta,E)\\ 0 & -J_{23}(\delta,E) & -r_1 & 0 & J_{35}(\delta,E) & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & -J_{35}(\delta,E) & -1 & -D_2 & J_{56}(\delta,E)\\ 0 & -J_{26}(\delta,E) & 0 & 0 & -J_{56}(\delta,E) & -r_2 \end{bmatrix} \quad (14)$$

B. Lossy Line

If the line is not lossless we have to introduce a cross-term in the energy function and, reasoning like in [21] and [26], propose to include a function $\lambda : \mathbb{R}^2 \to \mathbb{R}$ as

$$H_d(\delta,\omega,E) = \psi(\delta) + \frac{1}{2}|\omega|^2 + \frac{1}{2}\left[E - \lambda(\delta)E_*\right]^\top \Gamma \left[E - \lambda(\delta)E_*\right] \quad (18)$$

where $\lambda(\delta_\star) = 1$. The PDEs now take the form

$$\begin{aligned} F_1(\delta,E) =& -\nabla_{\delta_1}\psi + \gamma_1\left[E_1 - E_{1*}\lambda(\delta)\right]E_{1*}\nabla_{\delta_1}\lambda \\ &+ \gamma_2\left[E_2 - E_{2*}\lambda(\delta)\right]E_{2*}\nabla_{\delta_1}\lambda \\ &+ J_{23}(\delta,E)\gamma_1\left[E_1 - E_{1*}\lambda(\delta)\right] \\ &+ J_{26}(\delta,E)\gamma_2\left[E_2 - E_{2*}\lambda(\delta)\right] \\ F_2(\delta,E) =& -\nabla_{\delta_2}\psi + \gamma_1\left[E_1 - E_{1*}\lambda(\delta)\right]E_{1*}\nabla_{\delta_2}\lambda \\ &+ \gamma_2\left[E_2 - E_{2*}\lambda(\delta)\right]E_{2*}\nabla_{\delta_2}\lambda \\ &- J_{35}(\delta,E)\gamma_1\left[E_1 - E_{1*}\lambda(\delta)\right] \\ &+ J_{56}(\delta,E)\gamma_2\left[E_2 - E_{2*}\lambda(\delta)\right] \end{aligned} \quad (19)$$

which, evaluated at $E = \lambda(\delta)E_*$, yields

$$\nabla\psi = -F^{E_\star}(\delta) \quad (20)$$

with[13]

$$F^{E_\star}(\delta) \triangleq F\left(\delta, \lambda(\delta)E_\star\right). \quad (21)$$

The problem is now to prove the existence of a function $\lambda(\delta)$ such that the integrability conditions of Poincare's Lemma, i.e.,

$$\nabla_{\delta_1}F_2^{E_\star} = \nabla_{\delta_2}F_1^{E_\star}$$

are satisfied. This identity defines a PDE for $\lambda(\delta)$, that we could try to solve. With an eye on the general case, $n > 2$, when we have to deal with a set of PDEs that becomes extremely involved, we will proceed in an alternative way and directly construct this function as follows. First, we postulate that $\psi(\delta)$ is a function of $\delta_1 - \delta_2$, in which case

$$\nabla_{\delta_1}\psi + \nabla_{\delta_2}\psi = 0$$

and, consequently, invoking (20), we also have that

$$F_1^{E_\star}(\delta) + F_2^{E_\star}(\delta) = 0.$$

Evaluating the latter [from (16) and (21)] we get

$$P_1 + P_2 - \lambda^2(\delta)\left\{G_{11}E_{1*}^2 + G_{22}E_{2*}^2 + YE_{1*}E_{2*}\left[\sin(\delta_1 - \delta_2 + \alpha) + \sin(\delta_2 - \delta_1 + \alpha)\right]\right\} = 0.$$

The expression in brackets evaluated at δ_* equals $P_1 + P_2$. This implies, on one hand, that $\lambda(\delta_\star) = 1$ as desired while, on the other hand, it ensures the existence of a neighborhood of $\delta_{1*} - \delta_{2*}$ where this expression is bounded away from zero, thus, we can define

$$\lambda(\delta) = +\sqrt{\frac{P_1 + P_2}{P_1 + P_2 + 2G_{12}E_{1*}E_{2*}\left[\cos(\delta_1 - \delta_2) - \cos(\delta_{1*} - \delta_{2*})\right]}} \quad (22)$$

which is obtained using the equilibrium equations and (3). It is clear from (22) that the neighborhood increases as the line conductance G_{12} (and α) approaches zero, and in the limit of a lossless line it becomes the whole real axis and we recover the previous derivations with $\lambda(\delta) = 1$.

Some lengthy, but straightforward, calculations prove that $F(\delta, \lambda(\delta)E_\star)$—with $\lambda(\delta)$ given by (22)—satisfies the integrability conditions of Poincare's Lemma, ensuring the existence of the function $\psi(\delta)$ that solves (20). The design of the IDA-PBC is completed evaluating the functions $J_{ij}(\delta, E)$ from (19) and computing the control law[14] according to Proposition 1. The result is summarized in Proposition 3.

Proposition 3: Consider the two machines system with losses (13) and stable operating equilibrium satisfying.

Assumption A.1

$$|\delta_{1*} - \delta_{2*}| \le \frac{\pi}{2}. \quad (23)$$

Assumption A.2

$$P_1 + P_2 > 4G_{12}E_{1*}E_{2*}. \quad (24)$$

Assumption A.3

$$(P_1 + P_2)B_{12}\cos(\delta_{1*} - \delta_{2*}) + (P_1 - P_2)G_{12}\sin(\delta_{1*} - \delta_{2*}) > 0. \quad (25)$$

Then, the system in closed-loop with the control law

$$\begin{aligned} u_1 =& a_1E_1 - b_1E_2\cos(\delta_1 - \delta_2 + \alpha) - E_{f1} - J_{23}(\delta,E)\omega_1 \\ &- r_1\gamma_1\left[E_1 - E_{1*}\lambda(\delta)\right] + J_{35}(\delta,E)\omega_2 \\ u_2 =& a_2E_2 - b_2E_1\cos(\delta_2 - \delta_1 + \alpha) - E_{f2} - J_{26}(\delta,E)\omega_1 \\ &- r_2\gamma_2\left[E_2 - E_{2*}\lambda(\delta)\right] - J_{56}(\delta,E)\omega_2 \end{aligned}$$

where $\lambda(\delta)$ is given in (22) and

$$\begin{aligned} J_{23}(\delta,E) =& -\frac{YE_{2*}}{\gamma_1}\lambda(\delta)\sin(\delta_1 - \delta_2 + \alpha) \\ &- \frac{G_1}{\gamma_1}\left[E_1 + E_{1*}\lambda(\delta)\right] - E_{1*}\nabla_{\delta_1}\lambda \\ J_{26}(\delta,E) =& -\frac{Y}{\gamma_2}E_1\sin(\delta_1 - \delta_2 + \alpha) + E_{2*}\nabla_{\delta_1}\lambda \\ J_{35}(\delta,E) =& \frac{YE_{2*}}{\gamma_1}\lambda(\delta)\sin(\delta_2 - \delta_1 + \alpha) + E_{1*}\nabla_{\delta_1}\lambda \\ J_{56}(\delta,E) =& -\frac{Y}{\gamma_2}E_1\sin(\delta_2 - \delta_1 + \alpha) \\ &- \frac{G_2}{\gamma_2}\left[E_2 + E_{2*}\lambda(\delta)\right] + E_{2*}\nabla_{\delta_1}\lambda \end{aligned}$$

[13]Due to the presence of $\lambda(\delta)$, the functions $\tilde{F}^{E_\star}(\delta)$ and $F^{E_\star}(\delta)$ are obviously nonequal.

[14]We should underscore that the controller depends on $\nabla\psi$, hence we do not require the knowledge of the function $\psi(\delta)$ itself.

and

$$\nabla_{\delta_1}\lambda=\frac{\sqrt{P_1+P_2}G_{12}E_{1*}E_{2*}\sin(\delta_1-\delta_2)}{2\left\{P_1+P_2+2G_{12}E_{1*}E_{2*}\left[\cos(\delta_1-\delta_2)-\cos(\delta_{1*}-\delta_{2*})\right]\right\}^{\frac{3}{2}}}$$

has an asymptotically stable equilibrium at $(\delta_\star,0,E_\star)$. Further, a Lyapunov function for this equilibrium is given by (18) with (22) and

$$\psi(\delta)=-P_1(\delta_1-\delta_2)+\int_0^{\delta_1-\delta_2}\left\{\frac{(P_1+P_2)\left[G_{11}E_{1*}^2+YE_{1*}E_{2*}\sin(\tau+\alpha)\right]}{P_1+P_2+2G_{12}E_{1*}E_{2*}[\cos(\tau)-\cos(\delta_{1*}-\delta_{2*})]}\right\}d\tau$$

and an estimate of its *domain of attraction* is the largest *bounded* level set

$$\left\{(\delta,\omega,E)\in\mathbb{R}^6 \mid H_d(\delta,\omega,E)\le c\right\}.$$

Proof: First, note that (23) and (24) assure that $\lambda(\delta)$, given by (22), is well defined (in some neighborhood of the desired equilibrium). Now, we will compute from (19) the functions $J_{ij}(\delta,E)$. For, with some obvious abuse of notation, define the vector function

$$F^{E_1}(\delta)\triangleq F\left(\delta,E_1,\lambda(\delta)E_{2*}\right)$$

that is, the value of the function $F(\delta,E)$, given in (16), at $E_2=\lambda(\delta)E_{2*}$. Evaluating the first equation of (19) at this point and using the notation just defined, we get

$$F_1^{E_1}(\delta)=-\nabla_{\delta_1}\psi+\gamma_1\left[E_1-E_{1*}\lambda(\delta)\right]E_{1*}\nabla_{\delta_1}\lambda+J_{23}(\delta,E)\gamma_1\left[E_1-E_{1*}\lambda(\delta)\right].$$

Now, from (20) we have that $\nabla_{\delta_1}\psi=-F_1^{E_\star}(\delta)$, and doing some calculations we get

$$F_1^{E_1}(\delta)-F_1^{E_\star}(\delta)=-\left[E_1-E_{1*}\lambda(\delta)\right]\times\left\{G_{11}\left[E_1+E_{1*}\lambda(\delta)\right]+YE_{2*}\lambda\left[\sin(\delta_1-\delta_2+\alpha)\right]\right\}.$$

Plugging this expression back in the first of equations (19) and eliminating the common factor $[E_1-E_{1*}\lambda(\delta)]$ yields, after the calculation of $\nabla_{\delta_1}\lambda$ from (22), the expression of $J_{23}(\delta,E)$ in (26). With this definition of $J_{23}(\delta,E)$ substituted in (19) we immediately obtain $J_{26}(\delta,E)$.

Proceeding in exactly the same way, with the second set of equations (19), we calculate $J_{35}(\delta,E)$ and $J_{56}(\delta,E)$.

It remains only to prove that the proposed energy function has indeed a minimum at the desired equilibrium point. Introduce a partial change of coordinates

$$z=E-\lambda(\delta)E_* \tag{26}$$

and—recalling that $\lambda(\delta_\star)=1$, with $\delta_\star=(\delta_{1\star},\delta_{2\star})$—we look at the minima of the function

$$\tilde{H}_d(\delta,\omega,z)=\psi(\delta)+\frac{1}{2}|\omega|^2+\frac{1}{2}z^\top\Gamma z$$

which are obviously determined solely by $\psi(\delta)$. From (16) and (20), we have that $\nabla\psi(\delta_\star)=0$. Some lengthy calculations establish that $\nabla^2\psi(\delta_\star)\ge 0$ if and only if

$$P_2\cos(\delta_1-\delta_2+\alpha)+P_1\cos(\delta_1-\delta_2-\alpha)>0$$

which, using (1), can be shown to be equivalent to (25), completing the proof. ◃

Remark 5: Assumption A.1 captures the practically reasonable constraint that the normal operating regime of the system should not be "overly stressed." (See also [27]). Notice that, as indicated in the proof, the assumption provides a sufficient condition ensuring that $\lambda(\delta)$, given by (22), is well defined, but it is far from being necessary. The last two assumptions are obviated in the lossless case, but may be satisfied even for large values of the conductance G_{12}. In particular, Assumption A.2—which is given in this way for ease of presentation—can clearly be relaxed restricting our analysis to the set

$$\left\{(\delta,\omega,E)\in\mathbb{R}^6 \middle| \left|P_1+P_2+2G_{12}E_{1*}E_{2*}\left[\cos(\delta_1-\delta_2)-\cos(\delta_{1*}-\delta_{2*})\right]\right|>0\right\}$$

that covers the whole $\mathbb{R}^6$ as G_{12} tends to zero.

Remark 6: If the rotor inertias of the two machines, M_i, are different we have that $Y_{12}\neq Y_{21}$. However, retracing the derivations above we can easily derive the new control law taking into account this fact.

VI. Passivity Interpretation

The action of the controller described above has a nice interpretation in terms of a passivity property. Indeed, as discussed in [24], the terms in the interconnection matrix, $J_{ij}(\delta,E)$, play a role similar to multipliers in passive subsystems interconnection. To better perceive their effect let us write the equations of the closed-loop system (5), (14), (18) using the coordinate z introduced in (26) and take, for simplicity, $\Gamma=I$. Noting that $z=\nabla_E H_d$, and after some simple calculations we get

$$\begin{aligned}\dot{\delta}&=\omega\\ \dot{\omega}&=-D\omega+\alpha(\delta,E)z-\nabla\psi\\ \dot{z}&=-Rz-\alpha^\top(\delta,E)\omega\end{aligned}$$

where we have defined the matrices

$$D\triangleq\operatorname{diag}\{D_1,D_2\}$$

$$R\triangleq\operatorname{diag}\{r_1,r_2\}$$

$$\alpha(\delta,E)\triangleq\begin{bmatrix}J_{23}(\delta,E)+E_{1*}\nabla_{\delta_1}\lambda & J_{26}(\delta,E)+E_{2*}\nabla_{\delta_1}\lambda\\ -J_{35}(\delta,E)+E_{1*}\nabla_{\delta_2}\lambda & J_{56}(\delta,E)+E_{2*}\nabla_{\delta_2}\lambda\end{bmatrix}.$$

A block diagram representation of the system is given in Fig. 2. Clearly, the transfer matrices

$$(sI+D)^{-1}\quad (sI+R)^{-1}$$

are strictly positive real—hence, strictly output passive. Also, it is well known (see [6, Ch. 6.4, ex. 5]), that pre- and postmultiplying a passive operator by arbitrary (possibly unbounded) nonsingular matrices does not destroy its passivity property. Consequently, the (lower) feedback operator is also passive and

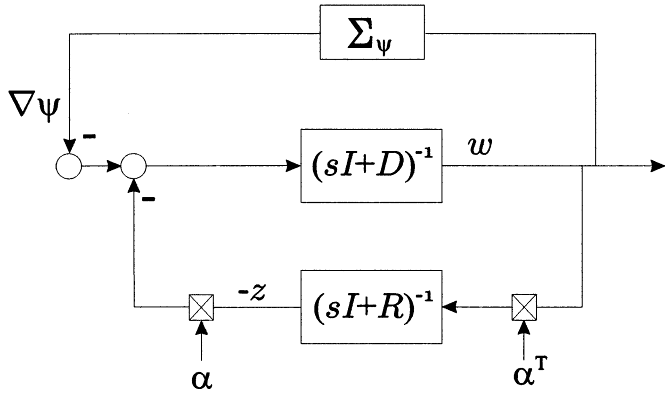

Fig. 2. Passive subsystems decomposition of the closed-loop.

the inner feedback interconnection is passive. The outer feedback operator Σ_ψ maps ω into $\nabla\psi$. From the relationship $\dot{\psi} = (\nabla\psi)^\top \omega$, and the fact that ψ is (locally) a positive definite function, we conclude (with some obvious abuse of notation) that Σ_ψ is also "locally passive," and stability is ensured.

VII. n-Machine Case

For the n-machines case with losses, the obstacle of integrability discussed in the previous section cannot be overcome with a scalar function $\lambda(\delta)$ and we need to consider a vector function. Therefore, we propose a total-energy function of the form

$$H_d(\delta, \omega, E) = \psi(\delta) + \frac{1}{2}|\omega|^2 + \frac{1}{2}\left[E - \operatorname{diag}\{\lambda_i(\delta)\} E_*\right]^\top \Gamma \left[E - \operatorname{diag}\{\lambda_i(\delta)\} E_*\right] \quad (27)$$

where $\delta = [\delta_1, \ldots, \delta_n]^\top$, $\psi(\delta) = [\psi_1(\delta), \ldots, \psi_n(\delta)]^\top$, $\omega = [\omega_1, \ldots, \omega_n]^\top$, $E = [E_1, \ldots, E_n]^\top$, with $\lambda_i, \psi : \mathbb{R}^n \to \mathbb{R}$, functions to be defined.

Consider now the matrix shown in (28) at the bottom of the page, where, for simplicity, we have omitted the arguments and use an obvious—though awkward—notation for the subindexes of the J_{ij}'s. Applying the IDA-PBC procedure we get, for each $2i$th row, $i = 1, \ldots, n$, of the previous matrix, a PDE of the form

$$-\nabla_{\delta_i}\psi + \left[E - \operatorname{diag}\{\lambda_i(\delta)\} E_*\right]^\top \Gamma \nabla_{\delta_i} \operatorname{diag}\{\lambda_i(\delta)\} E_* + \sum_{j=1}^{n} \gamma_j J_{(3i-1)(3j)} \left[E_j - E_{j*}\lambda_j(\delta)\right] = F_i(\delta, E) \quad (29)$$

TABLE I
Critical Clearing Time (m/s)

u	u_N	u_O
t_{cr}	250	200

TABLE II
System Parameters for Different Values of G

G	Y	a	b	$\delta_\star$	$E_\star$
0.01771	34.3046	0.02915	0.1490	0.9122	0.9824
0.0885	34.6526	0.1448	0.1506	0.4946	1.0815
0.1771	35.7184	0.2838	0.1552	0.0870	1.1528

TABLE III
Critical Clearing Time (m/s)

G	$u = u_N$	$u = u_O$
0.01771	170	130
0.0885	180	160
0.1771	300	50

TABLE IV
Parameters of the Post-Fault System

Parameter	Gen 1	Gen 2
a	16.7255	14.2937
b	11.1059	9.4147
Y	51.2579	36.6127
G_{ii}	28.9008	20.3936
α	0.5430	0.5430
E_f	5.8103	7.9279
P	52.2556	48.4902

$$J_d - \mathcal{R}_d = \left[\begin{array}{ccc|ccc|c|ccc}
0 & 1 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 \\
-1 & -D_1 & J_{23} & 0 & 0 & J_{26} & \cdots & 0 & 0 & J_{2(3n)} \\
0 & -J_{23} & -r_1 & 0 & J_{35} & 0 & & 0 & J_{3(3n-1)} & 0 \\
\hline
0 & 0 & 0 & 0 & 1 & 0 & & 0 & 0 & 0 \\
0 & 0 & -J_{35} & -1 & -D_2 & J_{56} & & 0 & 0 & J_{5(3n)} \\
0 & -J_{26} & 0 & 0 & -J_{56} & -r_2 & & 0 & J_{6(3n-1)} & 0 \\
\hline
 & \vdots & & & \vdots & & \ddots & & \vdots & \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & & 0 & 1 & 0 \\
0 & 0 & -J_{3(3n-1)} & 0 & 0 & -J_{6(3n-1)} & \cdots & -1 & -D_n & J_{(3n-1)3n} \\
0 & -J_{2(3n)} & 0 & 0 & -J_{5(3n)} & 0 & & 0 & -J_{(3n-1)3n} & -r_n
\end{array}\right] \quad (28)$$

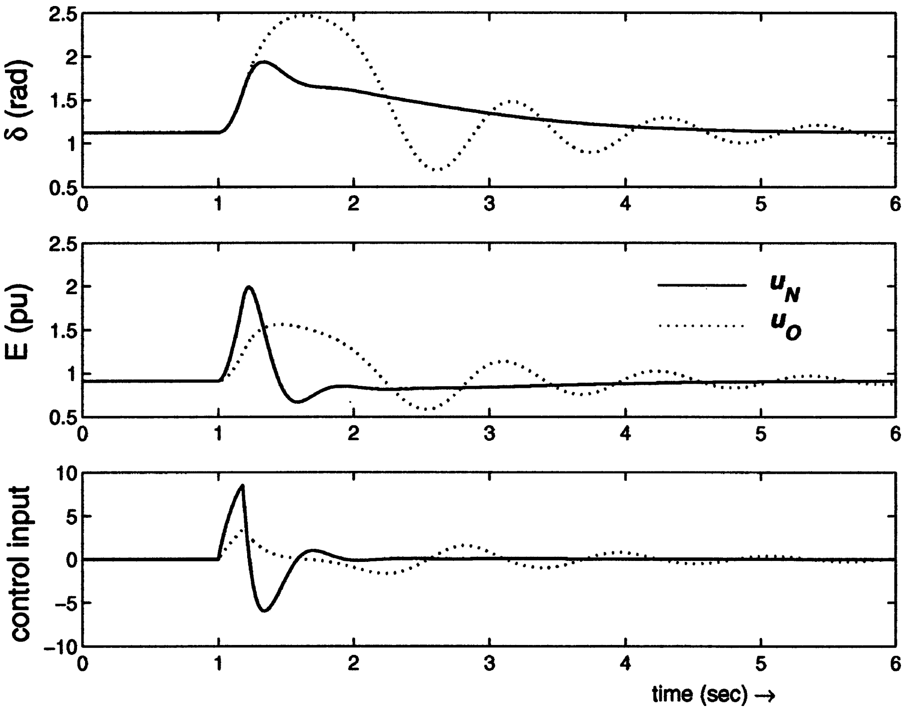

Fig. 3. SMIB system, with $G = 0$ and $t_{cl} = 200\ \mathrm{m\,sec}$, in closed-loop with u_N (continuous line) and u_O (dotted line). Behavior of load angle, internal voltage, and control input.

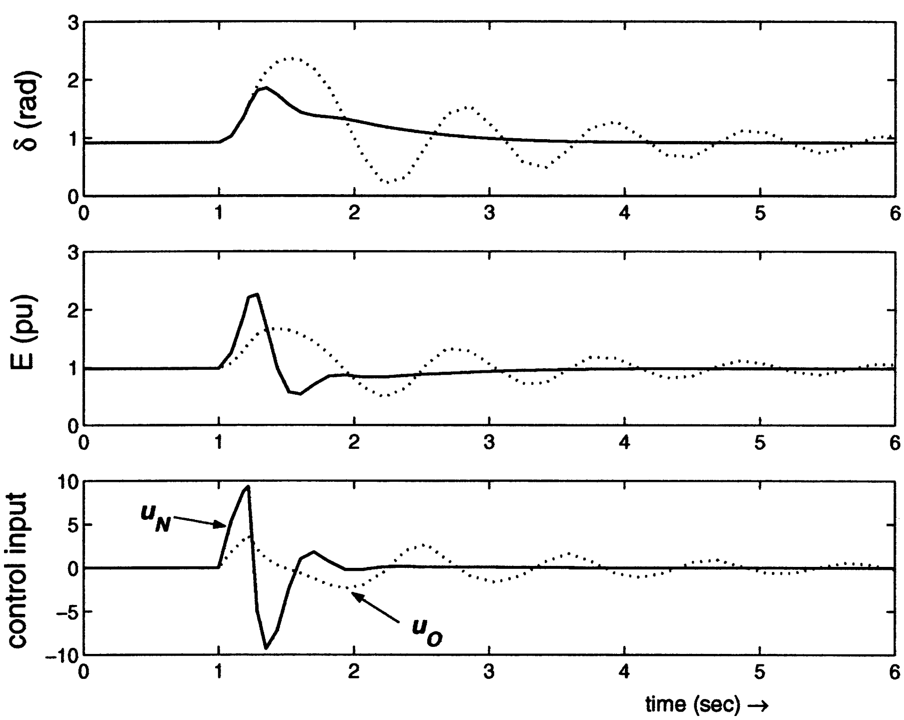

Fig. 4. SMIB system, with $G = 0.017\,71\ \mathrm{S}$, $t_{cl} = 130\ \mathrm{m/s}$, in closed-loop with u_N (continuous line) and u_O (dotted line). Behavior of load angle, internal voltage, and control input.

with our now familiar notation

$$F_i(\delta, E) \triangleq P_i - G_{ii}E_i^2 - E_i \sum_{j=1, j\neq i}^{n} E_j Y_{ij} \sin(\delta_i - \delta_j + \alpha_{ij})$$

and the fact that $J_{ij} = -J_{ji}$. Setting $E = \mathrm{diag}\{\lambda_i(\delta)\}E_\star$ in (29), and piling up all the elements in a vector, we obtain

$$\nabla\psi = -F^{E_\star}(\delta) \tag{30}$$

where, as before

$$F^{E_\star}(\delta) \triangleq F\left(\delta, \mathrm{diag}\{\lambda_i(\delta)\}E_\star\right) = \begin{bmatrix} F_1\left(\delta, \mathrm{diag}\{\lambda_i(\delta)\}\, E_\star\right) \\ \vdots \\ F_n\left(\delta, \mathrm{diag}\{\lambda_i(\delta)\}\, E_\star\right) \end{bmatrix}.$$

As pointed out before, it is possible to prove that

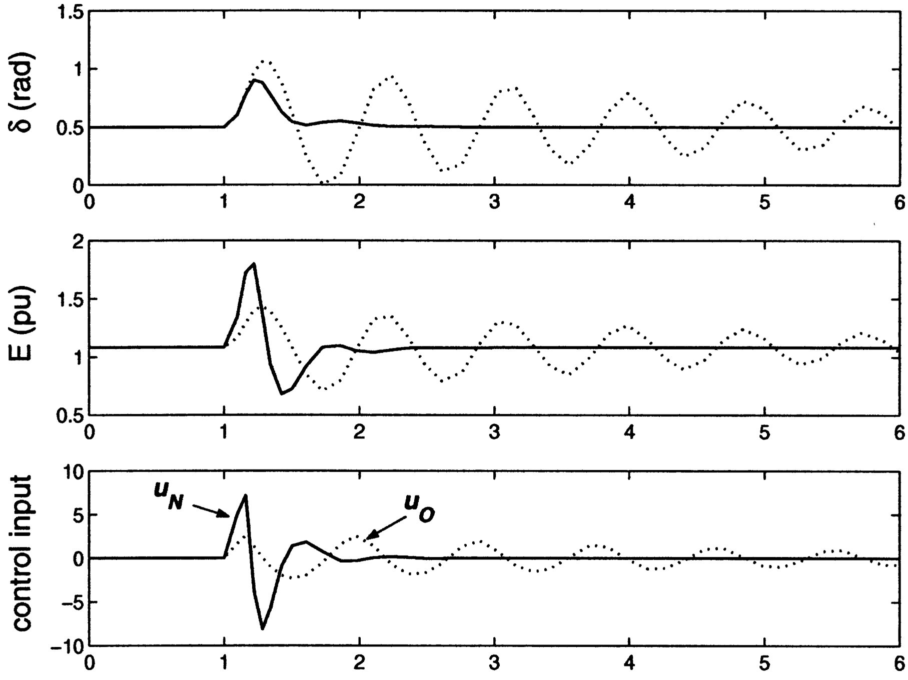

Fig. 5. SMIB system, with $G = .0885$ S, $t_{cl} = 160$ m/s, in closed-loop with u_N (continuous line) and u_O (dotted line). Behavior of load angle, internal voltage, and control input.

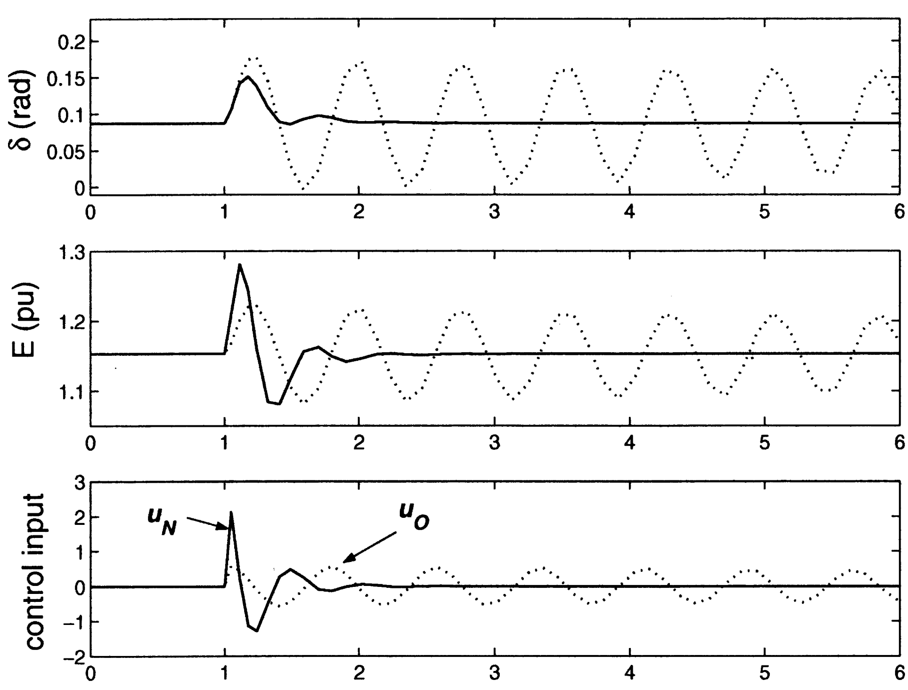

Fig. 6. SMIB system, with $G = .1771$ S, $t_{cl} = 50$ m/s, in closed-loop with u_N (continuous line) and u_O (dotted line). Behavior of load angle, internal voltage, and control input.

- there does not exist a scalar function $\tilde{\lambda} : \mathbb{R}^n \to \mathbb{R}$ such that $\tilde{F}^{E_\star}(\delta) \triangleq F(\delta, \tilde{\lambda}(\delta)E_\star)$ satisfies the integrability conditions imposed by Poincare's Lemma (17).

Hence, the construction proposed in Section V-B is not feasible. Further, the task of solving the associated system of PDEs for a vector function, already for the three machines system, seems formidable. Therefore, we proceed in an alternative way and apply the Implicit Function Theorem [1] to prove the existence of a vector function that, for arbitrary $\psi(\delta)$, (locally) "inverts" (30)—provided the system conductances are sufficiently small.

The result is summarized in the next proposition whose proof requires the following simple fact.

Fact 1: Let $M = S + D$ be an $n \times n$ matrix with $S = -S^\top$ and $D = \mathrm{diag}\{d_i\} > 0$. Then, M is full rank.

Proposition 4: Consider the n-machines system with losses (2) and assume the line conductances are sufficiently small.

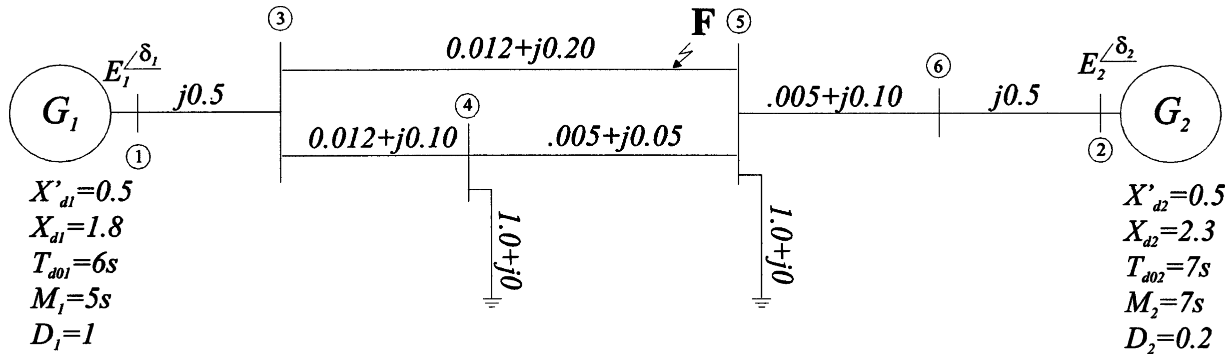

Fig. 7. Two machines system.

More precisely, for all $i \neq j$, $G_{ij} \leq \epsilon$ for some sufficiently small $\epsilon > 0$.

Then, for any arbitrary $\mathcal{C}^1$ function $\psi(\delta)$ such that $\delta_\star = \arg\min \psi$, there exists a (locally-defined) IDA-PBC that ensures asymptotic stability of the equilibrium ($\delta_\star$,0,$E_\star$) with a Lyapunov function of the form (27) and estimate of its *domain of attraction* the largest *bounded* level set

$$\left\{(\delta,\omega,E) \in \mathbb{R}^{3n} \mid H_d(\delta,\omega,E) \leq c\right\}.$$

Proof: From our previous derivations it is clear that the key step is to prove the existence of functions $\psi : \mathbb{R}^n \to \mathbb{R}$, with $\delta_\star = \arg\min \psi$, and $\lambda_i : \mathbb{R} \to \mathbb{R}^n$, with $\lambda_i(\delta_\star) = 1$, such that (30) is satisfied. For, we first factor the function $F^{E_\star}(\delta)$ as

$$F^{E_\star}(\delta) = P - \text{diag}\left\{\lambda_i(\delta)\right\} M(\delta)\lambda(\delta) \tag{31}$$

where $P \triangleq [P_1,\ldots,P_n]^\top$, $\lambda(\delta) \triangleq [\lambda_1(\delta),\ldots,\lambda_n(\delta)]^\top$ and $M(\delta) = \{m_{ij}(\delta)\} \in \mathbb{R}^{n\times n}$ with elements

$$m_{ii} \triangleq G_{ii}E_{i*}^2 \quad m_{ij}(\delta) \triangleq Y_{ij}E_{i*}E_{j*}\sin(\delta_i - \delta_j + \alpha_{ij}), \qquad i \neq j.$$

We make at this point two important observations.

P.1) From (31) and the definition of equilibria of (2) we have

$$F^{E_\star}(\delta_\star) = 0.$$

P.2) If $\alpha_{ij} = 0$, then

$$m_{ij}(\delta) = -m_{ji}(\delta), \qquad i \neq j$$

and the matrix $M(\delta)$ is the sum of a full rank diagonal matrix and a skew symmetric matrix. Hence, invoking Fact 1 above, and a continuity argument we conclude that $M(\delta)$ is *full rank* for sufficiently small α_{ij}.

Now, we fix an arbitrary $\mathcal{C}^1$ function $\psi(\delta)$, denote $\phi \triangleq \nabla\psi$, and define a parameterized function $Q_\delta : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ as

$$Q_\delta(\lambda,\phi) \triangleq \phi - P + \text{diag}\{\lambda_i\}M(\delta)\lambda$$

where the subindex $(\cdot)_\delta$ is included to underscore that the function is parameterized in δ. With this notation we can write (30) and (31) in the alternative form $Q_\delta(\lambda,\phi) = 0$. We will now verify the conditions of the Implicit Function Theorem to prove the (local) existence of a function $\chi : \mathbb{R}^n \to \mathbb{R}^n$ such that $Q_\delta(\chi(\phi),\phi) = 0$. Toward this end, we define the point of interest as

$$\lambda_\star \triangleq \lambda(\delta_\star) = [1,1,\ldots,1]^\top \quad \phi_\star \triangleq \nabla\psi(\delta_\star) = 0.$$

From P.1), we have that $Q_{\delta_\star}(\lambda_\star,\phi_\star) = 0$ as required by the theorem. To check the rank condition, we compute

$$\begin{aligned}\nabla_\lambda & Q_{\delta_\star}(\lambda_\star,\phi_\star) \\ &= M(\delta_\star) + \text{diag}\left\{\sum_{i=1}^n M_{1i}(\delta_\star),\cdots,\sum_{i=1}^n M_{ni}(\delta_\star)\right\} \\ &= M(\delta_\star) + \text{diag}\{P_1,\ldots,P_n\}\end{aligned}$$

where the second right-hand term, which is obviously full rank, stems from the fact that $F^{E_\star}(\delta_\star) = 0$ and (31). Property P.2), together with the smallness assumption of the G_{ij}'s, (consequently of the α_{ij}'s), yields the claim.

Summarizing, we have proven that for any $\mathcal{C}^1$ function $\psi(\delta)$, with $\delta_\star = \arg\min\psi$, there exists a function $\chi : \mathbb{R}^n \to \mathbb{R}^n$ such that $\lambda(\delta) = \chi(\nabla\psi(\delta))$ solves (30) in a neighborhood of $\delta_\star$.

The design of the IDA-PBC is completed proceeding as in Proposition 3. That is, given $\psi(\delta)$ and $\lambda(\delta)$, we compute from (29) the functions $J_{ij}(\delta,E)$ and subsequently the control law. ◁

Remark 7: Unlike the two-machines case considered before, Proposition 4 does not explicitly impose an assumption on the operating equilibrium of the form $|\delta_{i*} - \delta_{j*}| \leq (\pi/2)$. In view of the results of [27], it is expected that a similar condition will be needed to ensure stability of the open-loop equilibrium. Also, as in the two-machines case it will appear in the definition of the admissible domain of operation. Furthermore, Proposition 4 allows to assign *arbitrary* potential energy functions $\psi(\delta)$, of course, at the price of only proving the existence of the controller.

VIII. Simulations

In this section, we present numerical simulations of the proposed controller for the SMIB and the two-machines systems.

A. Single Machine Infinite Bus

We carried out simulations for the SMIB with and without losses, and compared the behavior of two controllers: the control

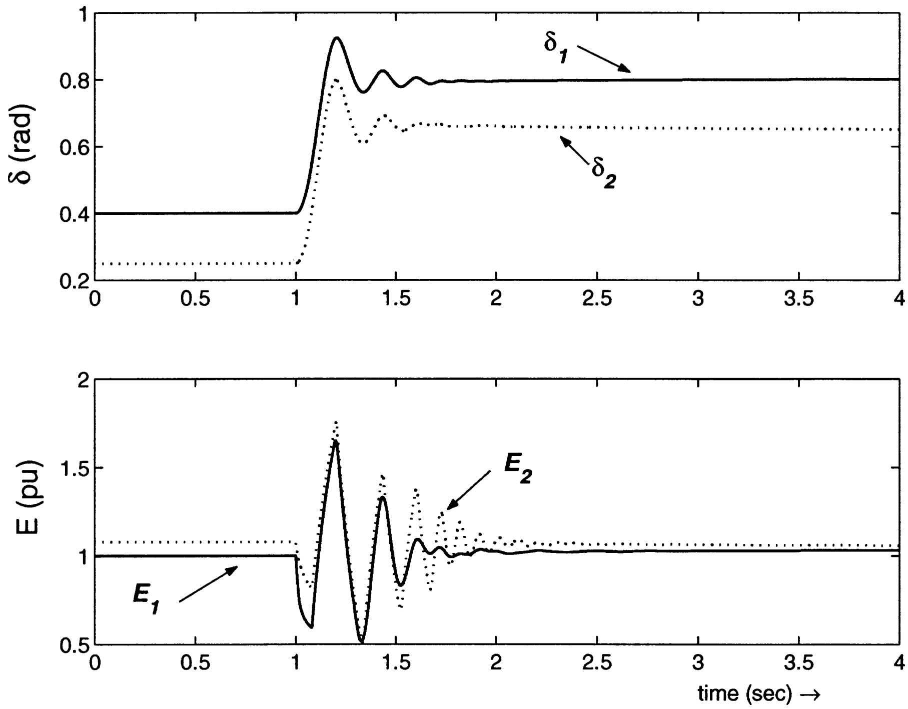

Fig. 8. Two machines system in closed-loop with the proposed IDA-PBC and $t_{cl} = 80\ \mathrm{m/s}$. Behavior of load angles and internal voltages.

law of Proposition 2 with added damping injection, that we call u_N, and is given by

$$u_N = -b\left[\cos(\delta+\alpha) - \cos(\delta_\star+\alpha)\right] + \frac{1}{\gamma}\left[(E+E_\star)G + Y\sin(\delta+\alpha)\right]\omega - k(E-E_\star)$$

and the control law proposed in [11, eq. (7)], called u_O, that we rewrite here for ease of reference

$$u_O = -k_v b_1(\cos\delta_\star - \cos\delta) - \alpha_1\alpha_2\left(\frac{b_3}{b_1} + k_v\right)(\delta-\delta_\star) - \alpha_1\omega - \left(\frac{b_3}{b_1}\alpha_2 - b_4 + k_v\alpha_2\right)(E-E_\star)$$

with the tuning parameters $k_v \geq 0$, $\alpha_2 > 0$, and $\alpha_1 < 0$, verifying

$$\alpha_2 \geq \frac{b_1 b_4}{b_3} \quad \alpha_1 < -\frac{b_1}{\alpha_2}.$$

The parameters of the SMIB model (9) are taken from [3] as $Y = 34.29$, $a = 0.3341$, $b = 0.1490$, $P = 28.22$, and $E_f = 0.2405$. The parameters of the controllers are set for u_N as $k = 4$, $\gamma = 15$, and for u_O as $\alpha_1 = -0.8$, $\alpha_2 = 76.88$, $k_v = 0.01$.

We analyze the response of (9) *without losses*, that is, $G = 0$, to a short circuit which consists of the temporary connection of a small impedance between the machine's terminal and ground. The fault is introduced at $t = 1$ s and removed after a certain time (called the clearing time and denoted t_{cl}), after which the system is back to its pre-disturbance topology. During the fault the trajectories diverge, the largest time interval "before instability,"[15] called the *critical* clearing time (t_{cr}), is determined via simulation. Table I shows the critical clearing times for the two controllers, as we see from the table the proposed controller, u_N, effectively increases the critical clearing time. Moreover, the new control also improves the transient performance as shown in Fig. 3, which presents the system's response to a fault with $t_{cl} = 200\ \mathrm{m/s}$ for both controllers.

We then consider the *effect of the losses* for both controllers reacting to the fault explained above. Simulations were carried out for increasing values of G, which changes the system parameters and the equilibrium point as indicated in Table II.

The parameters of the controllers are set for u_N as $k = 10$, $\gamma = 10$ and for u_O as $\alpha_1 = -0.8$, $\alpha_2 = 76.88$, and $k_v = 0.1$.

Table III presents the critical clearing time obtained for each value of G. Similar to the lossless case, the proposed controller increases the critical clearing time and enhances the transient performance. (The dramatic improvement for large values of G is consistent with the fact that the controller u_O was designed neglecting the losses, while u_N explicitly takes them into account.) Figs. 4–6 present the transient behavior of the system for different values of G and the critical clearing time obtained for u_O.

B. Two Machines System

This subsection presents simulations of the two machines system depicted in Fig. 7. In this case, the disturbance is a three-phase fault in the transmission line that connects buses 3 and 5, cleared by isolating the faulted circuit simultaneously at both ends, which modifies the topology of the network an consequently induces a change in the equilibrium point.

[15]This is practically detected observing the evolution of the signals that should remain within physically reasonable values, e.g., $E > 0$ and $u \leq U_M$.

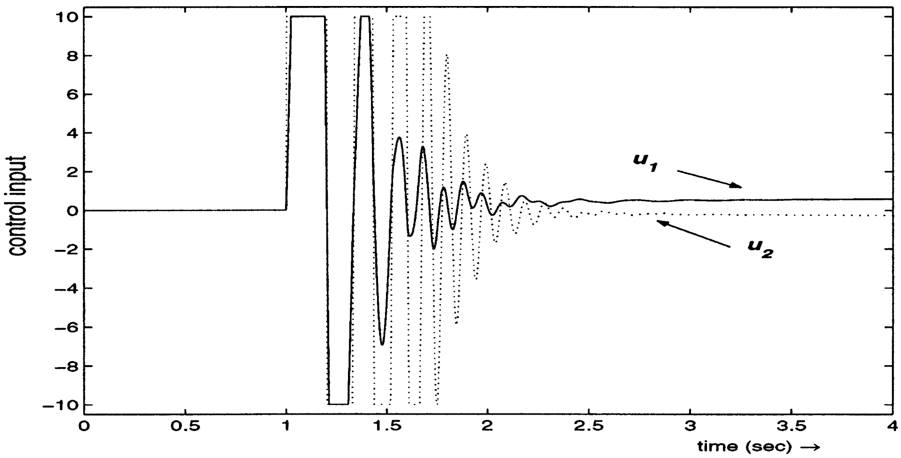

Fig. 9. Two machines system in closed-loop with the proposed IDA-PBC and $t_{cl} = 80\ \mathrm{m/s}$. Behavior of the control inputs.

The parameters of the model (13) are given in Table IV, and the equilibrium point after the fault is $(\delta_{1\star}, \omega_{1\star}, E_{1\star}, \delta_{2\star}, \omega_{2\star}, E_{2\star}) = (0.6481, 0, 1.0363, 0.8033, 0, 1.0559)$, which verifies the conditions of Proposition 3. We remark that the rotor inertias of the two machines, M_i, are different thus we have that $Y_{12} \neq Y_{21}$ and, consequently, the control law of Proposition 3 has to be slightly modified as indicated in Remark 6.

The simulation scenario corresponds to an overly stressed system. Without control the system is highly sensitive to the fault and the critical clearing time is almost zero. Although this scenario may be practically unrealistic we decided to present it to show that, even in this extreme case, the proposed controller enhances performance. Fig. 8 presents the system's response to a short-circuit with clearing time $t_{cr} = 80\ \mathrm{m/s}$. The control inputs are depicted in Fig. 9, which were clipped at ± 10.

IX. Conclusion

We have presented a static state feedback controller that ensures asymptotic stability of the operating equilibrium for multimachine power systems with lossy transmission lines. The controller is derived using the recently developed IDA-PBC methodology, hence endows the closed loop system with a PCH structure with a Hamiltonian function akin to a true total energy of an electromechanical system. A key step in the procedure is the inclusion of an interconnection between the electrical and the mechanical dynamics that may be interpreted as multipliers of the classical passivity theory.

Unfortunately, because of the use of the Implicit Function Theorem, in the general n-machines case only *existence* of the IDA-PBC is ensured and we need to rely on a "sufficiently small" transfer conductances assumption. On the other hand, for the single and two-machines problems we give a complete constructive solution. Some preliminary calculations for the three-machines system suggest that, with a suitable selection of the "potential energy" term, it is possible to obtain an explicit expression for the controller in the general case. In any case, the complexity of the resulting control law certainly stymies its practical application and the result must be understood only as a proof of assignability of a suitable energy function. Alternative routes must be explored to come out with a practically feasible design—probably trying other parameterizations for the energy function. In this respect it is interesting to note that the proposed function (27) differs from the ones used in mechanical and electromechanical systems [[21, eq. (2.3)] and [26, eq. (5)], respectively]. It is easy to see that neither one of these forms is suitable for the power systems problem at hand.

Simulations were carried out to evaluate, in academic examples, the performance of the proposed scheme. Currently, we are working on the development of a realistic simulation example for the two-machines problem where the performance of the proposed scheme will be compared with the classical AVR-PSS configuration. The outcome of this research will be reported in the near future.

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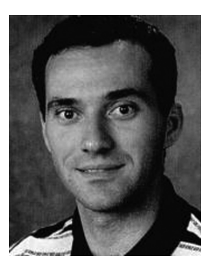

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