Transition form factors in π^0, η, and η' couplings to $\gamma\gamma^*$

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Recent measurements of the transition form factors for the $P\gamma\gamma^*$ vertices, with $P=\pi^0$, η, and η', are compared with different models. These include vector-meson dominance, constituent-quark loops, the QCD-inspired interpolation by Brodsky-Lepage, and chiral perturbation theory. General agreement is observed and differences—due to SU(3) breaking—are stressed and discussed.

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Experimental data for the two-photon transitions $\gamma\gamma^*\to\pi^0$, η, and η' have been recently obtained and discussed [1,2]. They involve (at least) one spacelike photon γ^* with squared four-momentum $q^2=-Q^2<0$. This completes and confirms older results concerning timelike photons ($q^2>0$) obtained from $\eta,\eta'\to\gamma\gamma^*\to\gamma\mu^+\mu^-$ decays [3,4] and solves the chaotic situation related to the $\pi^0\gamma\gamma^*$ vertex [4,5]. One usually fits the observed q^2 dependence in the different $P\gamma\gamma^*$ transitions by means of a normalized, single-pole term with an associated mass Λ_P, i.e.,

$$F_P(q^2)=F(\Lambda_p,q^2)/F(\Lambda_p,0)$$
$$=(1-q^2/\Lambda_P^2)^{-1}\simeq 1+q^2/\Lambda_P^2\equiv 1+b_P q^2\ , \quad (1)$$

where in the last steps (for small q^2) we have introduced the slope $b_P\equiv 1/\Lambda_P^2=\langle r_P^2\rangle/6$ related to the size of the pseudoscalar meson P. The available experimental data [1–3] for $\Lambda_{\pi^0,\eta,\eta'}$ and their averaged values [2] are summarized in Table I. The amplitude for a generic $P\leftrightarrow\gamma\gamma^*$ process is then

$$A(P\leftrightarrow\gamma\gamma^*)=\pm iF(\Lambda_P,q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon^\mu k^\nu \varepsilon^{*\alpha}q^\beta \quad (2)$$

TABLE I. Experimental values for the pole mass Λ_P (in GeV) in the transition form factors of pseudoscalar mesons $P=\pi^0$, η, and η'.

	Λ_{π^0} (GeV)	Λ_η (GeV)	$\Lambda_{\eta'}$ (GeV)
Lepton-G [3]		0.72 ± 0.09	0.77 ± 0.18
TPC/2γ [1]		0.70 ± 0.08	0.85 ± 0.07
CELLO [2]	0.75 ± 0.03	0.84 ± 0.06	0.79 ± 0.04
Average [2]	0.75 ± 0.03	0.77 ± 0.04	0.81 ± 0.04

with $k^2=0$ ($q^2\neq 0$) for the real (virtual) photon with polarization ε (ε^*).

Theoretically, $P\gamma\gamma$ transitions involving on-mass-shell photons, $k^2=q^2=0$, contain valuable information on the quark content (or mixing) of the η,η' mesons. Concerning this point, the situation is quite satisfactory and general agreement has been achieved [2,4,6]. This implies

$$\eta=\cos\theta\eta_8-\sin\theta\eta_1=\cos\beta(u\bar u+d\bar d)/\sqrt2-\sin\beta s\bar s\ ,$$
$$\eta'=\sin\theta\eta_8+\cos\theta\eta_1=\sin\beta(u\bar u+d\bar d)/\sqrt2+\cos\beta s\bar s\ , \quad (3)$$
$$\theta=\beta-\arctan\sqrt2\simeq-\text{arccot}2\sqrt2\simeq-19.5^\circ\ .$$

The q^2 dependence observed in $P\gamma\gamma^*$ transitions can then be viewed as a tool for understanding light-quark dynamics. To this aim several models have been discussed. The purpose of this note is to compare the experimental measurements of Λ_P quoted in Table I with the predictions of the most successful and/or traditional models. These include conventional ideas related to vector-meson dominance (VMD) or constituent-quark loops (QL) and QCD-inspired approaches such as the Brodsky-Lepage (BL) interpolation formula or chiral perturbation theory (ChPT).

Using VMD one immediately obtains [7,8]

$$F^{\text{VMD}}(\Lambda_P,q^2)=\sum_V \frac{g_{PV\gamma}}{f_V}\frac{M_V^2}{M_V^2-q^2}\ , \quad (4)$$

where the sum includes the three lightest vector mesons $V=\rho^0$, ω, and φ with SU(3)-symmetric couplings to the photon (f_V) and to $P\gamma$ ($g_{VP\gamma}$). Λ_V is then related to the vector-meson masses M_V, thus introducing the only source of SU(3) breaking (apart from mixing) through [5] $M_\rho\simeq M_\omega\simeq\lambda M_\varphi$, with $1/\lambda\simeq1.30$. More explicitly, one obtains

$$\Lambda_\pi^2 \simeq M_{\rho,\omega}^2 , \quad \Lambda_\pi = 0.78 \text{ GeV} ,$$
$$\Lambda_\eta^2 = \frac{5\cos\beta - \sqrt{2}\sin\beta}{5\cos\beta - \sqrt{2}\lambda\sin\beta} M_{\rho,\omega}^2 ,$$
$$\Lambda_\eta = 0.96\Lambda_\pi = 0.75 \text{ GeV} , \tag{5}$$
$$\Lambda_{\eta'}^2 = \frac{5\sin\beta + \sqrt{2}\cos\beta}{5\sin\beta + \sqrt{2}\lambda\cos\beta} M_{\rho,\omega}^2 ,$$
$$\Lambda_{\eta'} = 1.06\Lambda_\pi = 0.83 \text{ GeV} ,$$

where the numerical values follow from Eq. (3) and Ref. [5] and have been collected in Table II.

The QL predictions for the $P\gamma\gamma^*$ form factors are easily obtained computing the q^2 dependence generated by a triangle loop of constituent quarks of masses m_q and charges e_q. One obtains [7,8]

$$F^{\rm QL}(\Lambda_P,q^2) = \sum_q \frac{g_{Pq\bar{q}}}{m_q} e_q^2 \left[\frac{1}{\lambda_q} \arcsin\lambda_q \right]^2 , \quad \lambda_q^2 \equiv \frac{q^2}{4m_q^2} , \tag{6}$$

where the $Pq\bar{q}$ couplings are SU(3) symmetric and breaking appears only through the constituent quark masses $m_u = m_d = \lambda' m_s$, with $1/\lambda' \simeq 1.40$. More explicitly, one has

$$\Lambda_\pi^2 = 12 m_{u,d}^2 , \quad \Lambda_\pi = 0.80 \text{ GeV} ,$$
$$\Lambda_\eta^2 = \frac{5\cos\beta - \sqrt{2}\lambda'\sin\beta}{5\cos\beta - \sqrt{2}\lambda'^3\sin\beta} 12 m_{u,d}^2 ,$$
$$\Lambda_\eta = 0.96\Lambda_\pi = 0.77 \text{ GeV} , \tag{7}$$
$$\Lambda_{\eta'}^2 = \frac{5\sin\beta + \sqrt{2}\lambda'\cos\beta}{5\sin\beta + \sqrt{2}\lambda'^3\cos\beta} 12 m_{u,d}^2 ,$$
$$\Lambda_{\eta'} = 1.06\Lambda_\pi = 0.84 \text{ GeV} ,$$

where we have used Eq. (3) and a somewhat small constituent mass ($m_{u,d} \simeq 0.23$ GeV) in order to agree reasonably with the data and also with the VMD results [5].

The latter agreement is a manifestation of the old idea of quark-hadron or Q^2 duality already checked in [7,8] for $\eta \to \gamma\gamma^*$. Here, we have extended its validity to the SU(3)-breaking contributions exploiting the approximate equalities $\lambda \simeq \lambda'$ and $M_V^2 \simeq 12 m_q^2$ between VMD and QL parameters.

The Brodsky-Lepage (BL) interpolation formula [9] for these transition form factors is extremely simple, namely,

$$F_P^{\rm BL}(\Lambda_P,q^2) = \frac{2\sqrt{2}\alpha}{\Lambda_P}(1 - q^2/\Lambda_P^2)^{-1} , \tag{8}$$

where $\Lambda_P = 2\pi f_P$ is related to the pseudoscalar-meson decay constant f_P. It is an elegant expression interpolating two theoretically well-rooted results valid at the extreme energies $q^2 \to 0$ and $Q^2 \to \infty$. In the first case, current algebra (CA) unambiguously predicts $F(\Lambda_P, q^2 \to 0) = \sqrt{2}\alpha/\pi f_P$, whereas, QCD leads to $F(\Lambda_P, Q^2) = 4\pi\alpha\sqrt{2} f_P/Q^2$, in the opposite and reliable region of asymptotically large Q^2. Our normalization is such that the pion decay constant $f_{P=\pi} = \sqrt{2}\times 93$ MeV $= 132$ MeV and, therefore, one has $\Lambda_\pi = 2\pi f_\pi = 0.83$ GeV in the correct range of the experimental values. SU(3) breaking now proceeds exclusively through $f_\pi \neq f_\eta \neq f_{\eta'}$. The two latter decay constants are not directly measurable (in contrast with $f_{K^\pm}$ or $f_{\pi^\pm} = f_{\pi^0} = f_\pi$, by isospin) but can be deduced from $\eta, \eta' \to \gamma\gamma$ decays into real photons. One has

$$\frac{1}{f_\eta} = \frac{1}{\sqrt{3}} \left[\frac{\cos\theta}{f_8} - \frac{\sqrt{8}\sin\theta}{f_1} \right] = \frac{0.914}{f_\pi} ,$$
$$\frac{1}{f_{\eta'}} = \frac{1}{\sqrt{3}} \left[\frac{\sin\theta}{f_8} + \frac{\sqrt{8}\cos\theta}{f_1} \right] = \frac{1.25}{f_\pi} , \tag{9}$$

where the numerical values follow from Eqs. (3) and the averaged data [5] for π^0, η, and $\eta' \to \gamma\gamma$ decays. Indeed, several analyses [6,10,11] lead to the values

$$\theta = -20^\circ , \quad f_8 \simeq (0.25 - 1.30) f_\pi , \quad f_1 \simeq 1.1 f_\pi \tag{10}$$

and, then, to those quoted in Eq. (9). Therefore, one predicts

$$\Lambda_\pi = 2\pi f_\pi = 0.83 \text{ GeV} ,$$
$$\Lambda_\eta = 1.10\Lambda_\pi = 0.91 \text{ GeV} , \tag{11}$$
$$\Lambda_{\eta'} = 0.80\Lambda_\pi = 0.66 \text{ GeV} ,$$

as quoted in Table II. The qualitative relation $\Lambda_\eta > \Lambda_{\eta'}$ seems unavoidable and contrasts with the experimental data (Table I) which tend to prefer $\Lambda_{\eta'} > \Lambda_\eta$. This discrepancy is already present in the analysis of Ref. [1], where the values $f_\eta = 91 \pm 6$ MeV and $f_{\eta'} = 78 \pm 5$ MeV are deduced from the decay widths into two real photons contrasting with the values $f_\eta = 79 \pm 9$ MeV and $f_{\eta'} = 96 \pm 8$ MeV also deduced in [1] from the observed q^2 dependence.

ChPT is particularly appropriate for dealing with $P\gamma\gamma^*$ processes. It is a QCD-inspired model with a Lagrangian written in terms of the pseudoscalar meson fields, which are assumed to be the pseudo-Goldstone-boson fields appearing in the process of dynamical break-

TABLE II. Values for $\Lambda_{\pi^0,\eta,\eta'}$ predicted by vector-meson dominance (VMD), quark model loops (QL), the Brodsky-Lepage interpolating formula, and chiral perturbation theory (ChPT).

	Λ_{π^0} (GeV)	Λ_η (GeV)	$\Lambda_{\eta'}$ (GeV)
VMD [7,8]	$M_{\rho,\omega} = 0.78$	$0.96\Lambda_\pi = 0.75$	$1.06\Lambda_\pi = 0.83$
QL ($m_u = m_s/1.4 = 0.23$ GeV)	$\sqrt{12} m_u = 0.80$	$0.96\Lambda_\pi = 0.77$	$1.06\Lambda_\pi = 0.84$
Brodsky-Lepage [9]	$2\pi f_\pi = 0.83$	$1.10\Lambda_\pi = 0.91$	$0.80\Lambda_\pi = 0.66$
ChPT ($M_{\bar{V}} = 0.828$ GeV)	$(b_L + b_V)^{-1/2} = 0.75$	$1.03\Lambda_\pi = 0.77$	$1.06\Lambda_\pi = 0.79$

ing of the chiral symmetry of massless QCD. The Lagrangian is the most general one reproducing the symmetries of the original QCD Lagrangian. It is expanded in powers of p^2/Λ^2 and m^2/Λ^2, where p is a typical momentum, m is the quark mass and $\Lambda \sim 4\pi f_\pi$ is the scale of chiral symmetry breaking. The relevant lowest-order terms of the action are

$$S=\int d^4x\, L_2 - N_c S_{\rm WZ}\ ,\quad N_c=3\ , \tag{12}$$

with

$$L_2=\tfrac{1}{8}f^2\,{\rm tr}(D_\mu\Sigma\, D^\mu\Sigma^\dagger+\chi^\dagger\Sigma+\Sigma^\dagger\chi)\ ,$$
$$S_{\rm WZ}=\frac{i}{48\pi^2}\int d^4x\, \epsilon^{\mu\nu\alpha\beta}Z_{\mu\nu\alpha\beta}+\cdots\ , \tag{13}$$

where the ellipsis refer to nonphotonic terms of no relevance here and

$$Z_{\mu\nu\alpha\beta}=-ieA_\mu\,{\rm tr}[Q(\partial_\nu\Sigma\,\partial_\alpha\Sigma^\dagger\,\partial_\beta\Sigma\,\Sigma^\dagger-\partial_\nu\Sigma^\dagger\,\partial_\alpha\Sigma\,\partial_\beta\Sigma^\dagger\,\Sigma)]$$
$$+2e^2(\partial_\mu A_\nu)A_\alpha\,{\rm tr}[Q^2\,\partial_\beta\Sigma\,\Sigma^\dagger+Q^2\Sigma^\dagger\,\partial_\beta\Sigma+\tfrac{1}{2}Q\Sigma Q\Sigma^\dagger\partial_\beta\Sigma\Sigma^\dagger+\tfrac{1}{2}Q\Sigma^\dagger Q\Sigma\,\partial_\beta\Sigma^\dagger\,\Sigma]\ . \tag{14}$$

The covariant derivative $D_\mu\Sigma=\partial_\mu\Sigma+ie[Q,\Sigma]A_\mu$ contains the photon field and the quark charge matrix $Q={\rm diag}(\frac{2}{3},-\frac{1}{3},-\frac{1}{3})$. The pseudoscalar meson fields are contained in a nonlinear form in Σ,

$$\Sigma=\exp\left[\frac{2i}{f}M\right]\ , \tag{15}$$

with

$$M=\begin{bmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}}+\frac{\eta_1}{\sqrt{3}} \end{bmatrix} \tag{16}$$

and f is a free constant that, at lowest order, can be identified with the pion-decay constant f_π. Under chiral $U(3)_L\times U(3)_R$, Σ transforms as $\Sigma\to U_L\Sigma U_R^\dagger$. The Lagrangian L_2 in Eq. (13) introduces a small spontaneous chiral-symmetry breaking through the quark-mass matrix M, contained in $\chi=BM+\cdots$. where B is a free constant that can be fixed relating the quark masses to the pseudoscalar masses.

The term containing two photons in S_{WZ} is the only one contributing at lowest order to the amplitude for $P\leftrightarrow\gamma\gamma^*$. The contribution turns out to be q^2 independent:

$$F^{\rm ChPT}(\Lambda_P,q^2)=\frac{\sqrt{2}C_P\alpha}{\pi f_P} \tag{17}$$

with $C_\pi=1$, $C_{\eta_8}=1/\sqrt{3}$ and $C_{\eta_1}=2\sqrt{2}/\sqrt{3}$. It should be noticed that, since the only source of U(3) breaking in Eqs. (12) and (13) are the quark masses, all the f_P are the same at this order. As expected from the nonrenormalization of the anomaly and explicitly shown in Refs. [10,12,13], loop corrections for real photons do not modify the lowest-order result and only amount to the introduction of U(3) breaking in the values of f_P. The π^0, η, and $\eta'\to\gamma\gamma$ decay widths are, then, well understood in terms of the parameters in Eq. (10). Their finite parts can be calculated from the assumption that they are saturated by vector-meson contributions [13]. As a result, one obtains ($\sin\theta=-\frac{1}{3}$)

$$F_\pi(q^2)=1+(b_L+b_V)q^2\ ,$$
$$F_\eta(q^2)=1+\left[\frac{2f_1+f_8}{2f_1+2f_8}b_L+b_V\right]q^2\ , \tag{18}$$
$$F_{\eta'}(q^2)=1+\left[\frac{f_1-4f_8}{f_1-8f_8}b_L+b_V\right]q^2\ ,$$

where the finite part of the loop correction to the slope is given by

$$b_L=-\frac{1}{24\pi^2f^2}[1+\ln(m_K m_\pi/\mu^2)]=+0.32\ {\rm GeV}^{-2} \tag{19}$$

for $\mu^2\equiv M_{\bar V}^2\simeq(9M_\rho^2+M_\omega^2+2m_\varphi^2)/12=0.69\ {\rm GeV}^2$, which is the relevant mean vector-meson mass for our processes. This same mean mass fixes the contribution dominated by vector mesons, namely,

$$b_V=1/\mu^2=1.46\ {\rm GeV}^{-2}\ , \tag{20}$$

which (at the present order) is common to π^0, η, and η'. The only sources of SU(3) breaking are, therefore, $f_1\neq f_8\neq f_\pi$ and the fact that the loop correction for π^0

and η_8 (b_L) is twice as large as for η_1 ($b_L/2$) leading to the different coefficients of b_L in Eqs. (18). From these equations one gets

$$\Lambda_\pi=(b_L+b_V)^{-1/2}=0.75 \text{ GeV} ,$$
$$\Lambda_\eta=1.03\Lambda_\pi=0.77 \text{ GeV} , \quad (21)$$
$$\Lambda_{\eta'}=1.06\Lambda_\pi=0.79 \text{ GeV} .$$

In summary, all the models considered agree in the correct value for a mean Λ_P, but differ in the breaking pattern when $P=\pi^0$, η, or η'. The VMD and QL approaches lead to $\Lambda_\eta<\Lambda_\pi<\Lambda_{\eta'}$, in agreement with the data of Refs. [1,3]. The BL interpolation formula, instead, implies $\Lambda_{\eta'}<\Lambda_\pi<\Lambda_\eta$, in disagreement with the experimental data. Finally, ChPT predicts $\Lambda_\pi<\Lambda_\eta<\Lambda_{\eta'}$ in agreement with the averaged data. At this stage, it seems reasonable to conclude that accurate experiments (with precision of the order of a few percent) are required in order to decide on the correct scheme accounting for the $P\gamma\gamma^*$ transition form factors.

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