THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 E. 47th St., New York, N.Y. 10017

95-GT-241

TRANSITION LENGTH PREDICTION FOR FLOWS WITH RAPIDLY CHANGING PRESSURE GRADIENTS

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ABSTRACT

A new method for calculating intermittency in transitional boundary layers with changing pressure gradients is proposed and tested against standard turbomachinery flow cases. It is based on recent experimental studies which show the local pressure gradient parameter to have a significant effect on turbulent spot spreading angles and propagation velocities (and hence transition length). This can be very important for some turbomachinery flows. On a turbine blade suction surface for example, it is possible for transition to start in a region of favorable pressure gradient and finish in a region of adverse pressure gradient. Calculation methods which estimate the transition length from the local pressure gradient parameter at the start of transition will seriously overestimate the transition length under these conditions. Conventional methods based on correlations of zero pressure gradient transition data are similarly inaccurate. The new calculation method continuously adjusts the spot growth parameters in response to changes in the local pressure gradient through transition using correlations based on data given in the companion paper by Gostelow, Melwani and Walker (1995). Recent experimental correlations of Gostelow, Blunden and Walker (1994) are used to estimate the turbulent spot generation rate at the start of transition. The method has been incorporated in a linear combination integral computation and tested with good results on cases which report both the intermittency and surface pressure distribution data. It has resulted in a much reduced sensitivity to errors in predicting the start of the transition zone, and can be recommended for engineering use in calculating boundary layer development on axial turbomachine blades.

NOMENCLATURE

aU	spot leading edge velocity
bU	spot trailing edge velocity
C_f	skin-friction coefficient
$F(\gamma)$	Narasimha intermittency function, $[-\ln(1-\gamma)]^{1/2}$
H	velocity profile shape factor, δ^*/θ
N	non-dimensional breakdown rate parameter, $n\sigma\theta_t^3/\nu$
Re_{xt}	transition start Reynolds number, $x_t U/\nu$
Re_λ	transition length Reynolds number, $\lambda U/\nu$
Re_θ	momentum thickness Reynolds number, $\theta U/\nu$
U	local free-stream velocity
n	spot generation rate, $\mathrm{m}^{-1}\mathrm{s}^{-1}$
q_t	free-stream turbulence level at x_t, %
u	local velocity in boundary layer
x	streamwise distance from stagnation point
x_t	transition onset point
y	distance normal to surface
γ	intermittency factor
δ	boundary layer thickness
δ^*	displacement thickness, $\int_0^\delta (1-u/U)\,dy$
λ	characteristic transition length, $\lambda = x\|_{\gamma=0.75} - x\|_{\gamma=0.25}$
λ_θ	pressure gradient parameter, $(\theta^2/\nu)(dU/dx)$
$\lambda_{\theta t}$	pressure gradient parameter at x_t
σ	spot propagation parameter (dimensionless)
θ	momentum thickness, $\int_0^\delta (1-u/U)\,u/U\,dy$
q	local free-stream turbulence level, %

INTRODUCTION

The importance of laminar-turbulent transition in determining the aerodynamic characteristics of immersed bodies is well known. The transition behavior has a dominant effect on the evolution of losses, the appearance of separation and stall, and other factors of practical significance such as the distributions of wall shear stress and surface heat transfer. A comprehensive discussion of boundary layer transition phenomena and transition zone modeling has been given by Narasimha (1985). The practical significance of transition in relation to gas turbine engines has recently been surveyed by Mayle (1991).

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*Member ASME.

Presented at the International Gas Turbine and Aeroengine Congress & Exposition
Houston, Texas - June 5-8, 1995
This paper has been accepted for publication in the Transactions of the ASME
Discussion of it will be accepted at ASME Headquarters until September 30, 1995

For turbomachine blades operating at relatively low Reynolds number, a useful method of analysis must give reasonable estimates for the transition onset point, the length of the transition zone (i.e. region of intermittently turbulent flow) and the conditions of the turbulent boundary layer at the end of transition. Other important effects which need to be considered include separation, re-laminarization, free stream turbulence effects and unsteadiness associated with periodic wake passing. The present paper is principally concerned with the problem of predicting transition length.

Many factors can influence the length of the transition zone. Dhawan and Narasimha (1958) introduced a correlation for Re_λ in terms of Re_{xt}, based mainly on zero pressure gradient data. Potter and Whitfield (1962) and more recently Clark et al. (1994) have demonstrated the effects of Mach number on transition. Effects of pressure gradient on spot development have been studied by Gostelow et al. (1994a), Clark et al. (1994) and others; freestream turbulence by workers such as Blair (1982); surface roughness by Feiereisen and Acharya (1986); and surface curvature by Kim and Simon (1991).

The early transition length correlation of Dhawan and Narasimha (1958) was based on a limited range of data and essentially represents the transition behavior in constant pressure flows. Chen and Thyson (1971) subsequently used the turbulent spot theory of Emmons (1951) to develop a transitional flow model which purported to allow for the influence of pressure gradient on the intermittency distribution and transition length. This is incorporated in the popular boundary layer computation method of Cebeci and Smith (1974).

Walker et al. (1988) reported a complete breakdown in low Reynolds number airfoil computations with the Cebeci-Smith method due to the Chen-Thyson model predicting excessive transition lengths for flows with laminar separation. It was pointed out that the Chen-Thyson method predicted a transition length little different from that in zero pressure gradient, whereas the theoretical study of Walker (1989) had suggested the transition zone should be almost an order of magnitude shorter in positive pressure gradient situations.

The observations of Walker and Gostelow (1990) confirmed the markedly different nature of transition in decelerating flow. Experiments by Gostelow et al. (1994a) subsequently produced transition length correlations covering a wide range of positive pressure gradients and free stream turbulence levels. The new correlations gave reliable predictions of transition length for strongly decelerating flows typical of turbomachine blade operation in cases where the pressure gradient did not alter significantly over the transition zone. However, as shown in the present paper they are still inadequate for cases such as a turbine airfoil suction surface where the pressure gradient changes markedly throughout the region of intermittently turbulent flow.

The latter problems have been ascribed to an unexpectedly large influence of local pressure gradient on turbulent spot propagation, as revealed by recent experiments of Gostelow et al. (1995). The authors have used these results, together with correlations for spot inception rates from Gostelow et al. (1994a) and Fraser et al. (1994), to produce a modified Chen-Thyson model for the intermittency distribution in flows with rapidly changing pressure gradient.

The present paper commences with a brief review of transition zone modeling. The new method of determining intermittency distributions is then outlined and compared with experiments for which reliable intermittency and surface pressure distribution data is available. Finally, the new transition zone model is incorporated in a linear combination integral boundary layer computation code and tested against flow cases typical of turbine airfoil suction surfaces.

TRANSITION ZONE MODELS

Introduction

Both integral and differential boundary layer calculation techniques can be applied to transitional flows for a given distribution of turbulent intermittency γ through the transition zone. In the integral codes, laminar and turbulent boundary layer calculations are combined according to the intermittency. In the differential codes the eddy viscosity in the transitional zone is modified according to the intermittency.

Three transition zone models, all based on the turbulent spot theory of Emmons (1951) and the concentrated breakdown hypothesis of Narasimha (1957), are considered in this paper. The first model, referred to here as 'Narasimha, Dey' uses Narasimha's universal intermittency distribution with essentially zero pressure gradient transition length correlations. The second method, 'Gostelow', also uses the universal intermittency distribution, but with a transition length correlation which is related to the local pressure gradient parameter at transition onset. The new proposal discards the universal intermittency distribution and allows the spot propagation characteristics to react to changes in local pressure gradient through the transition zone.

As well as transition length correlations, all of these models require a specification of the transition onset location, x_t. The transition zone models should all be started at the point where γ first exceeds zero. At least one commonly used transition onset correlation (Abu-Ghannam and Shaw, 1980) gives predictions for x_t which more closely correspond to the point where $\gamma = 0.25$ (Dey and Narasimha, 1988). To avoid such sources of bias, all the test cases shown here have used (where possible) manually selected onset points which closely match the experimental intermittency distribution.

Narasimha, Dey

Narasimha (1957) showed that the transition zone model of Emmons (1951), when modified by the concentrated breakdown hypothesis, gave a good description of experimental streamwise intermittency distributions. The expression obtained by Narasimha was

$$\gamma = \begin{cases} 1 - \exp[-(x - x_t)^2 n\sigma/U] & (x \geq x_t) \\ 0 & (x < x_t) \end{cases} \qquad (1)$$

where x_t is the transition onset point, n is the spot generation rate (the number spots per unit length, per unit time generated at x_t) and σ is the spot propagation parameter.

Defining the transition length in terms of $\lambda = x|_{\gamma=0.75} - x|_{\gamma=0.25}$ leads to Narasimha's universal intermittency distribution

$$\gamma = \begin{cases} 1 - \exp[-0.412(x - x_t)^2/\lambda^2] & (x \geq x_t) \\ 0 & (x < x_t) \end{cases} \qquad (2)$$

Eqn. (2) generally agrees well with experimental results, even in flows with non-zero pressure gradient, provided the pressure gradient parameter remains nearly constant through the transition zone and λ and x_t are chosen correctly (Gostelow et al., 1994a). It also leads to a linear plot of $F(\gamma) \sim x$ which has been used by Narasimha, Gostelow and co-workers to provide a consistent basis for determining the transition onset point x_t from experimental data.

Narasimha (1985) demonstrates that the most appropriate non-dimensional breakdown parameter is of the form

$$N = n\sigma\theta_t^3/\nu \tag{3}$$

This expression is obtained from Eqn. (1) and Eqn. (2) using the Blasius boundary layer relationship, $Re_{\theta t} = 0.664\sqrt{Re_{xt}}$, and a transition length correlation of the form $Re_\lambda = 9Re_{xt}^{3/4}$.

Dey and Narasimha (1988) tested several correlations for λ with experimental data from various sources. Their study was hampered by the different definitions for the start and end of transition used by different authors, but useful suggestions for converting between definitions were made. For zero pressure gradient flows with levels of free-stream turbulence above around 0.3%, N was found to be constant. Pressure gradient effects were only considered for favorable pressure gradients and correlated with the pressure gradient parameter at transition onset, $\lambda_{\theta t}$. The correlation proposed was

$$N = \begin{cases} 0.7 \times 10^{-3} & (\lambda_{\theta t} \leq 0) \\ 0.7 \times 10^{-3} + 0.24(\lambda_{\theta t})^2 & (0 < \lambda_{\theta t} \leq 0.10) \end{cases} \tag{4}$$

This expression was used with Eqn. (1) and Eqn. (3) at a given x_t to obtain the intermittency distribution and the resulting transition length.

For the cases considered in this paper where x_t occurred in a region of favorable pressure gradient, $\lambda_{\theta t}$ was generally small and the transition length values predicted by this procedure differed little from those for zero pressure gradient flow.

Gostelow

An extensive set of boundary layer transition length measurements in adverse and zero pressure gradients has been reported by Gostelow et al. (1994a) for a range of freestream turbulence levels. Correlations of these measurements are variously presented as functions of boundary layer thickness, predicted minimum transition length and spot formation rate. The spot formation rate correlation was obtained using the expression

$$N = 0.412R_{\theta t}^3/Re_\lambda^2 \tag{5}$$

which is an alternative form of Eqn. (3) for zero pressure gradients. The resulting correlation was

$$N = 0.86 \times 10^{-3}\exp(2.134\lambda_{\theta t}\ln(q_t) - 59.23\lambda_{\theta t} - 0.564\ln(q_t)) \tag{6}$$

Gostelow et al. (1994b) have proposed a calculation method based on the linear combination integral boundary layer technique of Dey and Narasimha (1988), but using Eqn. (6) in place of Eqn. (4). However, Eqn. (6) is not valid for flows which have a transition onset point in a region of favorable pressure gradient and some modification is required to cover this case. This has been done here on the basis of correlations presented by Fraser et al. (1994) who suggest

$$N = N_0 \times \exp(-10\sqrt{\lambda_{\theta t}}) \tag{7}$$

where N_0 is the value of N at $\lambda_{\theta t} = 0$.

For the current work, therefore, N for the 'Gostelow' method is found from Eqn. (6) for $\lambda_{\theta t} < 0$ and Eqn. (7) for $\lambda_{\theta t} > 0$. The full correlation for N is plotted in Fig. 1. The corresponding intermittency distribution and transition length are then determined from Eqn. (1) and Eqn. (3) as for the 'Narasimha, Dey' method.

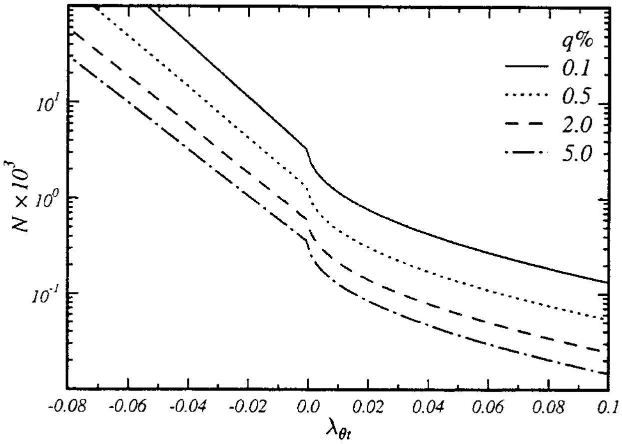

FIGURE 1: Non-dimensional breakdown rate parameter as a function of freestream turbulence and pressure gradient parameter at transition onset. Data of Gostelow et al. (1994a) for $\lambda_{\theta t} \leq 0$; Fraser et al. (1994) for $\lambda_{\theta t} > 0$.

New method

Turbulent spot propagation characteristics. The development of the new transition model has been prompted by recent experiments of Gostelow and co-workers which provide new data on turbulent spot propagation characteristics in adverse pressure gradients. The variation of spot spreading angle α and propagation parameter σ with pressure gradient parameter λ_θ, as compiled by Gostelow et al. (1995), is shown in Fig. 2 and Fig. 3. Both parameters are seen to vary markedly for $\lambda_\theta < 0$. This unexpectedly large variation with adverse pressure gradients clearly invalidates the widely held assumption (e.g. Chen and Thyson (1971) and Mayle (1992)) that spot propagation characteristics should not vary significantly with pressure gradient through the transition zone.

The values of spot propagation parameter shown in Fig. 3 have been derived from experimental data for the leading and trailing edge celerities, aU and bU, and the spreading half-angle α using the relation

$$\sigma = \tan\alpha(b^{-1} - a^{-1}) \tag{8}$$

This relation implies a triangular spot planform as assumed by various workers (McCormick, 1968; Chen and Thyson, 1971).

The following tentative correlations for the variation of α and σ with λ_θ have been made from the data assembled by Gostelow et al. (1995):

$$\alpha = 4 + (22.14/(0.79 + 2.72\exp(47.63\lambda_\theta))) \tag{9}$$

$$\sigma = 0.03 + (0.37/(0.48 + 3.0\exp(52.9\lambda_\theta))) \tag{10}$$

Ideally the correlations of α and σ should also allow for the influence of freestream turbulence q and Reynolds number Re_θ. However, the available data is too sparse to do this at present.

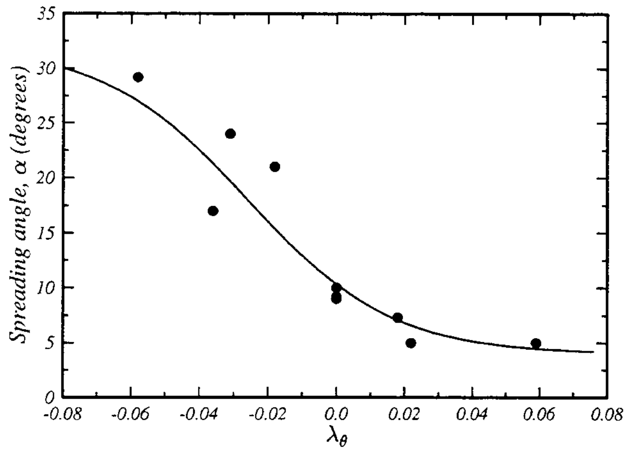

FIGURE 2: Fit to turbulent spot spreading data from various sources compiled by Gostelow et al. (1995).

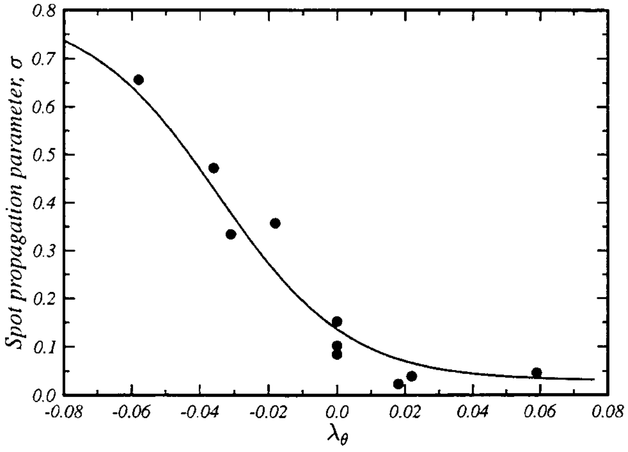

FIGURE 3: Fit to turbulent spot propagation data from various sources compiled by Gostelow et al. (1995).

Development of the new model. Various hypotheses have been used to explain departures from the universal intermittency distribution (or 'subtransitions') observed when the pressure gradient changes significantly through the transition zone. Narasimha (1985) has suggested that boundary layer stability may be a factor. Mayle (1991) has proposed a variation in spot generation function through the transition zone.

Based on the new data of Gostelow et al. (1995), we abandon these ideas and propose that subtransitions are essentially caused by the influence of local pressure gradient on turbulent spot propagation characteristics. The basic features of the new model are as follows:

(a) the concentrated breakdown hypothesis of Narasimha is retained;
(b) the spot inception rate is assumed to depend only on the local conditions at transition onset, x_t;
(c) the spreading rate of turbulent spots is allowed to vary continuously through the transition zone in response to changes in the local pressure gradient parameter λ_θ.

The new model can easily be implemented by simple modifications to the original model of Chen and Thyson (1971)). For two-dimensional flow Chen and Thyson give

$$\gamma = 1 - \exp\left[-G(x - x_t)\int_{x_t}^{x}\left(\frac{dx}{U}\right)\right] \qquad (11)$$

for the intermittency distribution in the transition zone, where $G = n(b^{-1} - a^{-1})\tan\alpha = n\sigma$ and σ has been assumed constant through the transition zone. This reduces to Narasimha's universal distribution, Eqn. (1), if U is constant.

Chen and Thyson's model, Eqn. (11), should give slightly better results than Eqn. (1) since it does account for effects of changing U on the spot convection velocities. But as pointed out by Narasimha et al. (1984) and Walker et al. (1988) this effect is small and is unable to account for the large changes in transition length with pressure gradient which are actually observed. The major problem is that the Chen–Thyson formulation for G does not allow for any variations in spot inception rate, or relative spreading celerities with pressure gradient.

Incorporating the new model assumptions in the original Chen-Thyson formulation leads to the modified intermittency distribution

$$\gamma = 1 - \exp\left[-n\int_{x_t}^{x}\frac{\sigma}{\tan(\alpha)}\left(\frac{dx}{U}\right)\int_{x_t}^{x}\tan\alpha\,dx\right] \qquad (12)$$

In the present paper this is applied by using a value of the spot inception rate n inferred from Eqns. (6), (7) and (3) by using the local value of λ_θ at transition onset x_t. The values of α and σ are obtained from Eqn. (9) and Eqn. (10) using the local value of pressure gradient parameter from the laminar boundary layer component in the linear combination integral computation. This implies that the spot propagation parameters respond instantaneously to changes in pressure gradient, although some lag must be expected in practice.

RESULTS AND DISCUSSION

Boundary Layer Code

Dey and Narasimha (1988) present a simple linear-combination integral boundary layer method which is a convenient test-bed for the new intermittency distribution. (A version of this code was also used by Gostelow et al. (1994b)). The method uses a modified Thwaites method for the laminar boundary layer and a lag-entrainment method (Green et al., 1973) for the turbulent boundary layer. Within the transition zone the laminar and turbulent solutions are combined as follows:

$$\delta^* = (1-\gamma)\delta_L^* + \gamma\delta_T^* \qquad (13)$$

$$\theta = \gamma(1-\gamma)\int_0^{\delta}[u_L(1-u_T)+u_T(1-u_L)]dy(1-\gamma)^2\theta_L+\gamma^2\theta_T \qquad (14)$$

$$C_f = (1-\gamma)C_{fL} + \gamma C_{fT} \qquad (15)$$

where the subscripts L and T refer to laminar and turbulent values respectively. The value of intermittency γ is determined from one of the transition models above.

It was necessary to continue calculation of the laminar boundary layer component through separating flow for a few of the test cases.

This was done crudely by fixing $H_L = 3.70$, $C_{fL} = 0$ and continuing with evaluation of the momentum integral equation to determine θ_L. Any errors arising from this approach will only have a small effect on the final solution provided the intermittency is reasonably large at the laminar separation point. An obvious imperfection in the linear combination method is the assumption of instantaneous switching between the laminar and turbulent velocity profiles. It is well known that there is a significant lag in recovery of the laminar profile following the passage of a turbulent spot. The linear combination method nevertheless gives useful results and is certainly no less realistic than using a turbulent boundary layer calculation method with modified eddy viscosity in the transition zone.

General Test Cases - Narasimha et al. (1984)

The experimental cases of Devasia reported by Narasimha et al. (1984) provide convenient benchmarks for testing transition models under conditions of changing pressure gradients. Since intermittency is reported, it is possible to make direct comparisons of the predicted intermittency with experiment. There is also little uncertainty in specifying the transition onset point. In the absence of intermittency data it is necessary to infer the performance of the transition model from comparisons of predicted and measured boundary layer parameters; that introduces additional uncertainty because of imperfections in boundary layer modeling which appear in combination with errors in intermittency prediction.

For this work three cases designated DFU1, DFU3 and DAU1 will be presented. The surface velocity distributions and corresponding variations in pressure gradient parameter for the laminar boundary layer component in the transition zone are shown in Fig. 4. Comparisons of measured intermittency with the predictions of the three transitional flow models are shown in Figs. 5, 6 and 7 respectively.

Case DFU1 involves an increasing acceleration over the forward part of the transition region. This is subsequently relaxed so that the pressure gradient is near zero at the end of transition. As indicated by Narasimha et al. (1984) this produces a relatively mild subtransition (as indicated by a discontinuity in slope of the $F(\gamma)$ distribution) around $x = 1.15$ m. In this case the Narasimha-Dey transition length is much too short; however, the shape of the predicted intermittency distribution is seen to be a reasonable match near the end of the transition zone where the conditions of near zero pressure gradient approximate those of the model. The Gostelow model, which is based on the local pressure gradient parameter at the start of transition, gives a good prediction of the transition length; but this is somewhat fortuitous, as the predicted and measured intermittency diverge significantly with increasing acceleration towards the center of the transition zone and subsequently approach each other again as the pressure gradient is removed. The new model clearly performs best overall, giving creditable predictions for both transition length and intermittency distribution; the minor systematic deviations from the measured intermittency which are still evident may arise partly from uncertainties in the correlations of spot properties for the accelerating regime and partly from neglecting the influence of Reynolds number on spot propagation.

Case DFU3 involves a change from accelerating flow over the forward part of the transition zone to a slightly decelerating flow towards the end of transition. This produces a much more marked subtransition effect around $x = 1.05$ m. The Narasimha-Dey model is even less accurate in this case; the transition length is again underestimated, and the shape of the intermittency distribution is not matched in any part of the transition zone. The Gostelow model captures the shape of the intermittency distribution quite well in the forward part of the transition zone where the pressure gradient parameter is essentially constant; but the predicted intermittency diverges considerably and the transition length is significantly overpredicted as the pressure gradient subsequently becomes slightly negative. The new model again performs creditably, albeit with the same slight systematic deviations in predicted intermittency exhibited in case DFU1.

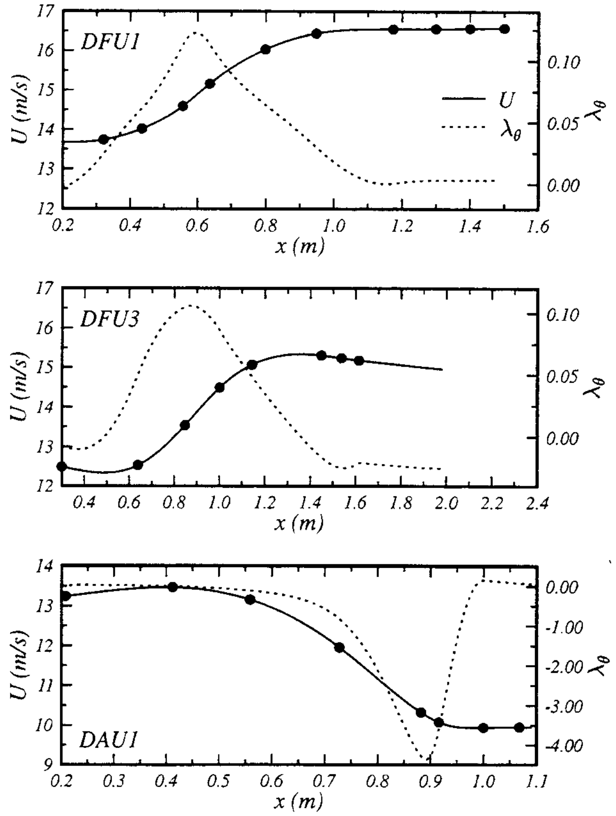

FIGURE 4: Velocity and laminar pressure gradient parameter distributions for the Devasia test cases (data scaled from figures of Narasimha (1985))

Case DAU1 involves an even more marked subtransition as the pressure gradient changes from mildly favorable to strongly adverse. The Narasimha-Dey model performs rather better here due to the steepening of the experimental intermittency distribution towards the end of transition. In these circumstances, however, the Gostelow model fails quite spectacularly; its divergence from experiment clearly illustrates the subtransition around $x = 0.7$ m. The new model again gives a very good prediction of transition length, although the divergence from measurement is somewhat greater around the center of the transition zone. The latter effect may be partly due to experimental uncertainty, as the scatter in measured intermittency values appears rather greater in this case.

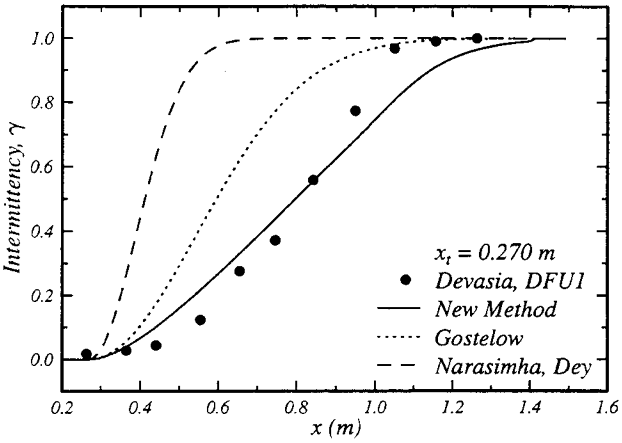

FIGURE 5: Devasia case DFU1. Intermittency distributions

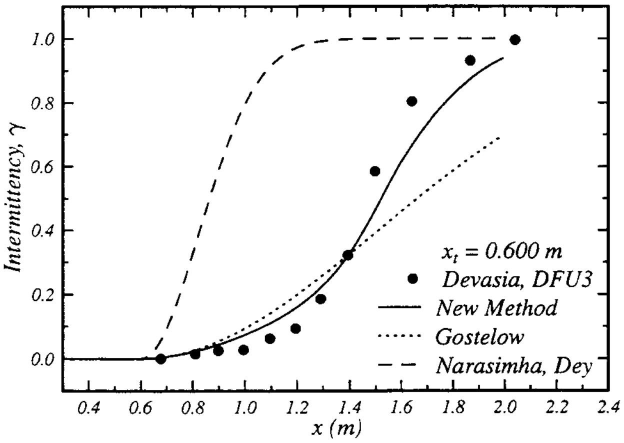

FIGURE 6: Devasia case DFU3. Intermittency distributions

Forward-Loaded Turbine Airfoil - Sharma et al. (1982)

Sharma et al. (1982) have presented detailed measurements of the boundary layer on a plate subjected to a pressure distribution typical of a forward-loaded (or 'squared-off') turbine airfoil design. As indicated in Fig. 8 this has a region of nearly constant pressure up to about $x = 0.5$ m followed by a region of decelerating flow. The inlet turbulence level for this case is 2.4%, and the Reynolds number based on exit velocity and test section length is 8×10^5. Intermittency data from flush-mounted hot-film probes indicate that transition commences in the region of decelerating flow between $x = 0.45$ m and $x = 0.50$ m, with intermittency values closely following Narasimha's universal distribution (i.e. there is no significant subtransition evident in this case).

Fig. 9 compares the experimental data of Sharma et al. (1982) with the results of boundary layer calculations using the three different transition zone models of the present paper. All computations assume the same transition onset point of $x = 0.45$ m, which is slightly upstream of the experimentally observed location. In this case the Gostelow correlation gives the best prediction of the intermittency distribution through the transition zone. The Narasimha-Dey method, being based essentially on zero pressure gradient data gives too long a transition length. The current method gives a transition length which is a little short, the intermittency again rising a little too rapidly with increasing deceleration as was noted for the general test cases of Narasimha et al. above. It should be borne in mind, however, that the intermittency indicated by wall gages will be lower than the near wall values in the boundary layer under adverse pressure gradient conditions.

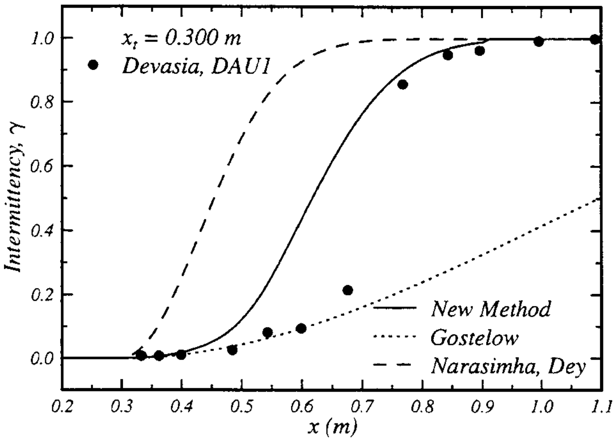

FIGURE 7: Devasia case DAU1. Intermittency distributions

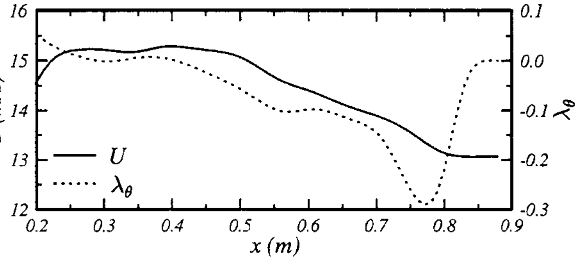

FIGURE 8: Sharma et al. (1982) squared-off turbine blade. Velocity and laminar pressure gradient parameter distributions

The decay in experimental shape factor values through transition is rather more protracted than indicated by the intermittency distributions. This is probably due to:

(a) the influence of free-stream turbulence on the laminar boundary layer prior to transition onset, which produces a noticeable decrease in H below the predicted values for a steady laminar flow; and

(b) a lag in response of the boundary layer velocity profile to the introduction of turbulent mixing, which is not incorporated in the calculation.

The influence of free-stream turbulence on the boundary layer is also reflected by the somewhat higher values of measured skin friction coefficient prior to transition.

The predicted momentum thickness, as indicated by the values

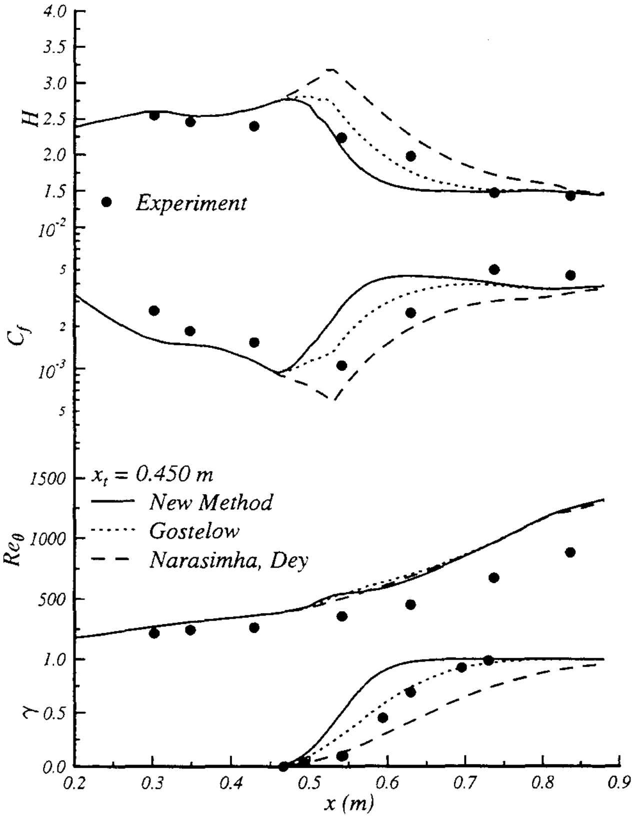

FIGURE 9: Sharma et al. (1982) squared-off turbine blade. Comparison of methods, all started at the same transition onset point close to the experimentally observed location.

of Re_θ, is consistently high throughout the calculation. There is evidently some initial deviation for the laminar values which propagates through the whole calculation. Matching of Re_θ values for the fully turbulent region can only be achieved by imposing a totally unrealistic delay in the transition onset.

Aft-Loaded Turbine Blade - ERCOFTAC Case T3C2

The ERCOFTAC Special Interest Group on Transition uses a set of test cases from the Rolls Royce Applied Science Laboratory described by Coupland (1993). Case T3C2 corresponds to a pressure distribution typical of the suction surface on an aft-loaded turbine blade, as shown in Fig. 10. This consists of a long region of acceleration followed by an increasing deceleration which is strong enough to promote separation of the laminar boundary layer component in the transition zone. The situation is similar to that of Devasia DAU1, and a strong subtransition effect is again expected. The upstream turbulence level for the T3C2 case is 3% and the Reynolds number based on exit velocity and test section length is 6.56×10^5.

No intermittency data have been published for this case. There is therefore some uncertainty about the transition onset location, and the performance of transition zone models can only be checked by comparison with measured values of integral properties. For this reason, a parametric approach is adopted and the sensitivity of calculation methods to variations in the specified transition onset point is investigated. The results are shown in Figs. 11, 12 and 13. Values of H, C_f and Re_θ are compared; the predicted intermittency distributions are also presented to indicate the assumed extent of the transition zone.

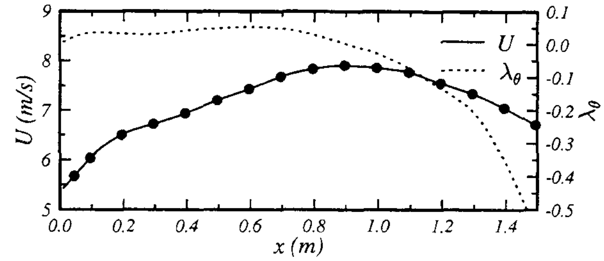

FIGURE 10: ERCOFTAC case T3C2, aft-loaded turbine blade. Velocity and laminar pressure gradient parameter distributions

Fig. 11 shows the computed boundary layer development for $x_t = 0.60$, 0.75 and 0.95 m using the Narasimha-Dey transition model. The first two assumed values of x_t lie in the accelerating flow region, while the latter falls slightly after the suction peak. The transition length, being derived essentially from constant pressure flow data, varies only slightly with the boundary layer properties at x_t and is quite insensitive to the value of x_t chosen; however, the predicted variation of boundary layer properties through the transition zone is markedly affected. For $x_t = 0.60$ m the chosen transition onset is clearly too early; $x_t = 0.75$ m gives the best overall agreement; $x_t = 0.95$ m is clearly too late for the transition onset, and the adverse pressure gradient causes the laminar component to separate in the transition zone giving values of H and C_f which are too high and too low respectively. In all cases, there is a noticeable tendency for the predicted shape factor to be too high prior to transition onset; this is again ascribed to the influence of free-stream turbulence on the laminar boundary layer, as for the squared-off turbine airfoil case above.

As seen from Fig. 12, the predicted boundary layer behavior with the Gostelow transition length correlation is far more sensitive to the assumed transition onset. For x_t prior to the suction peak, the estimated transition length is much too long; the predicted boundary layer still has a major laminar component in the decelerating flow region and is thus very prone to separation. Locating x_t after the suction peak gives better agreement for the distribution of C_f, but still allows separation to occur.

Computations using the current method of intermittency prediction are shown in Fig. 13. The results are surprisingly insensitive to variations in the assumed transition onset position. The wall shear stress distribution is particularly good. Best overall agreement is obtained for $x_t = 0.60$ m; however, the predictions remain quite good for values of x_t up to 0.95 m. The end of transition is dominated by the effects of decelerating flow from about $x = 0.9$ m onwards; transition is completed by around $x = 1.2$ m regardless of the assumed onset point.

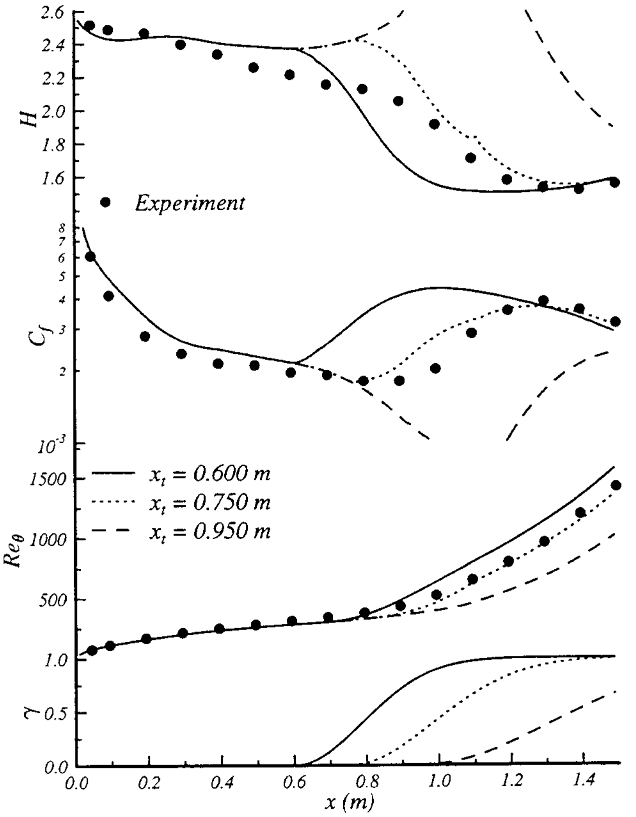

FIGURE 11: ERCOFTAC case T3C2. Solutions using the 'Narasimha, Dey' transition model; various onset positions

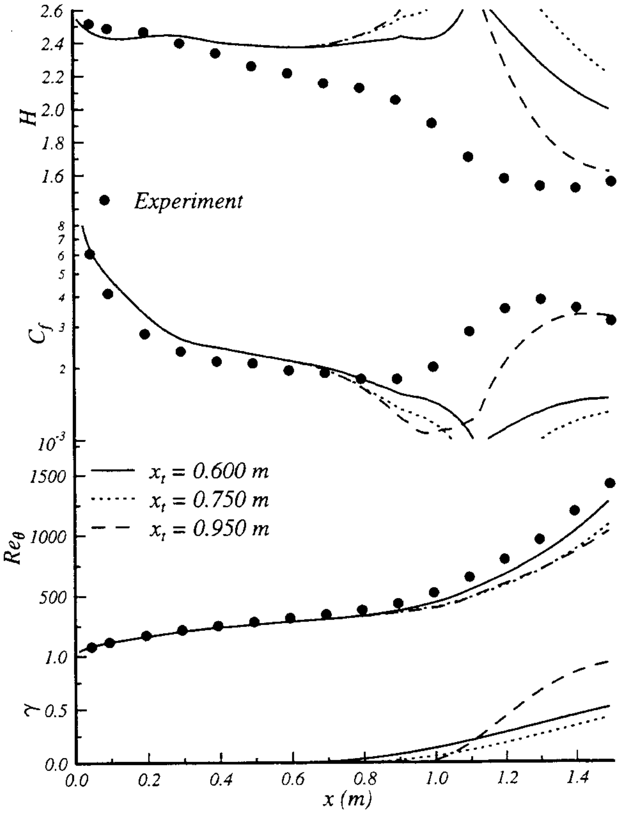

FIGURE 12: ERCOFTAC case T3C2. Solutions using the 'Gostelow' transition model; various onset positions

CONCLUSIONS

The length of transitional flow in regions of rapidly changing pressure gradient is not adequately predicted by correlations of data from flows in which the pressure gradient parameter remains essentially constant through the transition zone. The turbine airfoil suction surface, on which transition may commence in a region of accelerating flow and end in a region of decelerating flow, provides a particularly severe test of calculation methods. Here the transition length predicted from constant pressure gradient correlations may vary greatly depending on the location of the assumed transition onset in relation to the pressure minimum. This may cause significant variations in computed skin friction and surface heat transfer distributions; the stability of iterative methods used to couple viscous and inviscid flow solutions might also be threatened if the transition onset moves between favorable and adverse pressure gradient regions on successive iterations.

A new method of computing transitional flow length, based on recent experimental results for the variation of turbulent spot spreading rates with local pressure gradient, has been successfully demonstrated for typical turbine airfoil test cases. It is considerably more robust in relation to errors in predicting transition onset, and is recommended for immediate engineering use.

The present computations have also shown that rapid changes in pressure gradient may cause significant deviations from the universal intermittency distribution of Narasimha through the transition zone. This provides an explanation for the anomalous behavior generally referred to as 'subtransition'. For flows involving subtransition, the start of the transition zone inferred by assuming a standard intermittency distribution may differ significantly from the true onset point. A re-examination of existing transition data in the light of this observation and the new computation method is considered desirable.

In particular, it would be useful to reprocess the data of Gostelow et al. (1994a) with the new model to produce revised values of transition length and spot inception rates. The data processing techniques adopted by Gostelow et al. ignored the slight subtransitions which were evident in those experiments; thus the values of transition length obtained from the resulting correlations such as Eqn. (6) may be a little low. To that extent, there may have been some double accounting for subtransition effects through using the original correlations of Gostelow et al. with the new model. This could partially explain the tendency of the new method to give slight underestimates of the transition length where strong subtransitions occur.

The present work has produced results of immediate practical value despite the fact that it has been mainly confined to studying the effects of changing pressure gradient. In the longer term, however, the new transitional flow model could benefit from extensions to incorporate additional factors such as:

(a) the influence of changing turbulent spot shape on the values

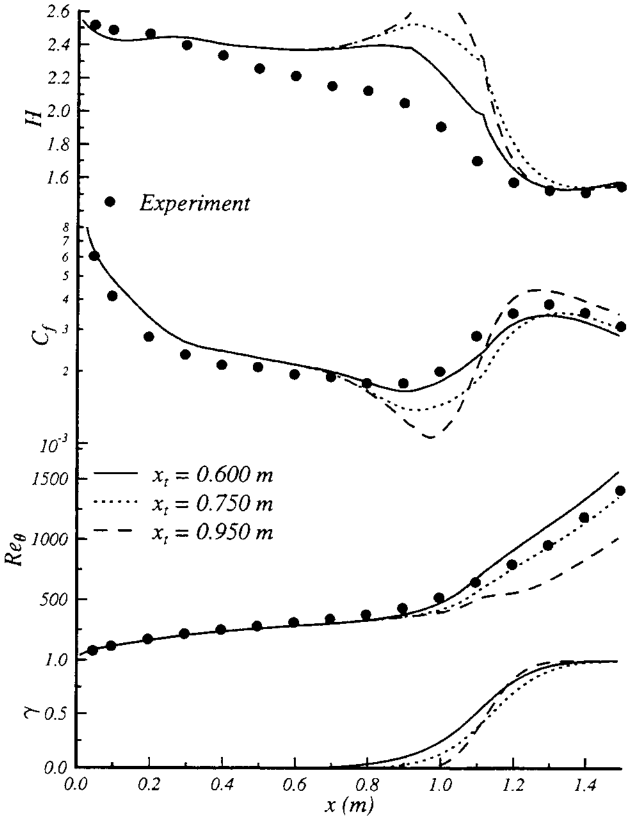

FIGURE 13: ERCOFTAC case T3C2. Solutions using the new transition length method; various onset positions

of spot propagation parameter σ;

(b) a possible replacement of the concentrated breakdown hypothesis with a Gaussian source density function for turbulent spots;

(c) the influence of Reynolds number, free-stream turbulence and periodic unsteadiness on spot propagation characteristics;

(d) improved modeling of the emerging turbulent boundary layer and an allowance for relaxation effects in the laminar layer following the passage of a turbulent spot; and

(e) the influence of free-stream turbulence on the laminar boundary layer prior to transition inception.

ACKNOWLEDGMENTS

The authors wish to acknowledge the support of the Australian Research Council and Rolls-Royce plc.

REFERENCES

Abu-Ghannam, B. J., and Shaw, R., 1980, "Natural Transition of Boundary Layers - the Effects of Pressure Gradient and Flow History," *Journal of Mechanical Engineering Science*, Vol. 22, No. 5, pp. 213–228.

Blair, M. F., 1982, "Influence of Free-Stream Turbulence on Boundary Layer Transition in Favorable Pressure Gradients," *ASME Journal of Engineering for Power*, Vol. 104, pp. 743–750.

Cebeci, T., and Smith, A.M.O., 1974, *Analysis of Turbulent Boundary Layers*, Academic Press.

Chen, K. K., and Thyson, N. A., 1971, "Extension of Emmons' Spot Theory to Flows on Blunt Bodies," *AIAA Journal*, Vol. 9, No. 5, pp. 821–825.

Clark, J. P., Jones, T. V., and LaGraff, J. E., 1994, "On the Propagation of Naturally-Occurring Turbulent Spots," *Journal of Engineering Mathematics*, Vol. 28, pp. 1–19.

Coupland, J., 1993, *Private Communication Concerning Rolls-Royce Applied Science Laboratory Flat Plate Transitional Boundary Layer Results*, Test Cases used by the ERCOFTAC SIG on Transition.

Dey, J., and Narasimha, R., 1988, "An Integral Method for the Calculation of 2-D Transitional Boundary Layers," Fluid Mechanics Report 88 FM 7, Indian Institute of Science, Bangalore.

Dhawan, S., and Narasimha, R., 1958, "Some Properties of Boundary Layer Flow During the Transition from Laminar to Turbulent Motion," *Journal of Fluid Mechanics*, Vol. 3, pp. 418–436.

Emmons, H. W., 1951, "The Laminar-Turbulent Transition in a Boundary Layer - Part I," *Journal of Aerospace Sciences*, Vol. 18, No. 7, pp. 490–498.

Feiereisen, W. J., and Acharya, M., 1986, "Modeling of the Transition and Surface Roughness Effects in Boundary-Layer Flows," *AIAA Journal*, Vol. 24, pp. 1642–1649.

Fraser, C. J., Higazy, M. G., and Milne, J. S., 1994, "End-Stage Boundary Layer Transition Models for Engineering Calculations," *Proceedings of the Institution of Mechanical Engineers, Part C*, Vol. 208, pp. 47–58.

Gostelow, J. P., Blunden, A. R., and Walker, G. J., 1994a, "Effects of Free-Stream Turbulence and Adverse Pressure Gradients on Boundary Layer Transition," *ASME Journal of Turbomachinery*, Vol. 116, pp. 392–404.

Gostelow, J. P., Hong, G., Walker, G. J., and Dey, J., 1994b, "Modeling of Boundary Layer Transition in Turbulent Flows by Linear Combination Integral Method," ASME Paper 94-GT-358, International Gas Turbine Congress, The Hague.

Gostelow, J. P., Melwani, N., and Walker, G. J., 1995, "Effects of a Streamwise Pressure Gradient on Turbulent Spot Development," ASME Paper 95-GT-XXX, International Gas Turbine Congress, Houston.

Green, J. E., Weeks, D. J., and Brooman, J. W. F., 1973, "Prediction of Turbulent Boundary Layers and Wakes in Compressible Flow by a Lag-Entrainment Method," Tech. Report 72231, RAE, Farnborough.

Kim, J., and Simon, T. W., 1991, "Freestream Turbulence and Concave Curvature Effects on Heated Transitional Boundary Layers," NASA CR187150.

Mayle, R. E., 1991, "The Role of Laminar-Turbulent Transition in Gas Turbine Engines," *ASME Journal of Turbomachinery*, Vol. 113, pp. 509–537, The 1991 IGTI Scholar Lecture.

Mayle, R. E., 1992, "Unsteady Multimode Transition in Gas Turbine Engines," *AGARD PEP 80.*

McCormick, M. E., 1968, "An Analysis of the Formation of Turbulent Patches in the Transition Boundary Layer," *ASME Journal of Applied Mechanics*, pp. 216–219.

Narasimha, R., 1957, "On the Distribution of Intermittency in the Transition Region of a Boundary Layer," *Journal of Aerospace Sciences*, Vol. 24, No. 9, pp. 711–712.

Narasimha, R., 1985, "The Laminar-Turbulent Transition Zone in the Boundary Layer," *Progress in Aerospace Science*, Vol. 22, pp. 29–80.

Narasimha, R., Devasia, K. J., Gururani, G., and Badri Narayanan, M. A., 1984, "Transitional Intermittency in Boundary Layers Subjected to Pressure Gradient," *Experiments in Fluids*, Vol. 2, pp. 171–176.

Potter, J. L., and Whitfield, J. D., 1962, "Effects of Slight Nose Bluntness and Roughness on Boundary-Layer Transition in Supersonic Flows," *Journal of Fluid Mechanics*, Vol. 12, pp. 501–535.

Sharma, O. P., Wells, R. A., Schlinker, R. H., and Bailey, D. A., 1982, "Boundary Layer Development on Turbine Airfoil Suction Surfaces," *ASME Journal of Engineering for Power*, Vol. 104, pp. 698–706.

Walker, G. J., 1989, "Transitional Flow on Axial Turbomachine Blading," *AIAA Journal*, Vol. 27, No. 5, pp. 595–602.

Walker, G. J., and Gostelow, J. P., 1990, "Effects of Adverse Pressure Gradients on the Nature and Length of Boundary Layer Transition," *ASME Journal of Turbomachinery*, Vol. 112, pp. 196–205.

Walker, G. J., Subroto, P. H., and Platzer, M. F., 1988, "Inviscid Interaction Analysis of Low Reynolds Number Airfoil Flows Involving Laminar Separation Bubbles," ASME Paper 88-GT-32, International Gas Turbine Congress, Amsterdam.