Transition Modeling and Econometric Convergence Tests

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With

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Time Varying Common Factor Representation

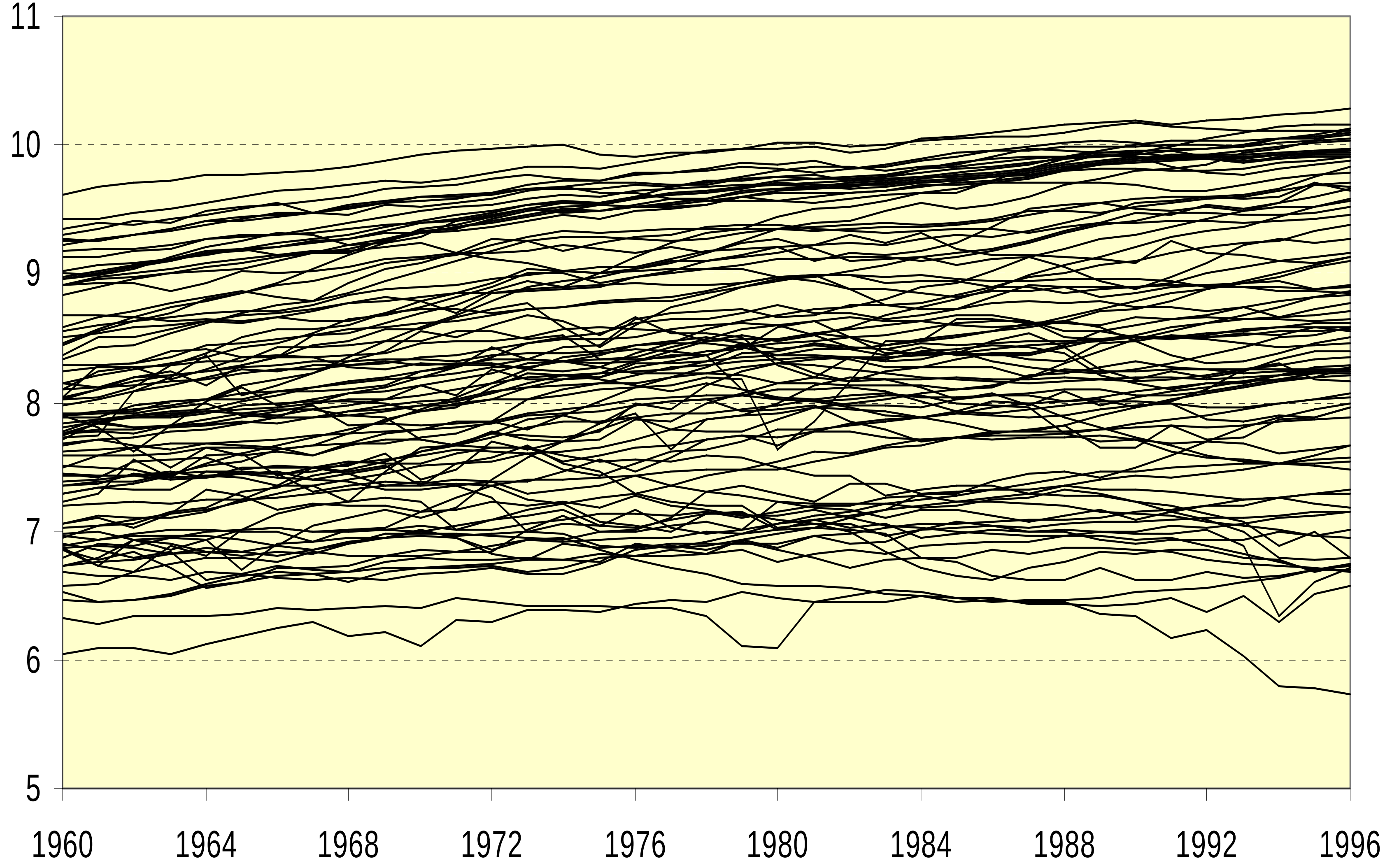
No. I am not covering this

Panel Data:

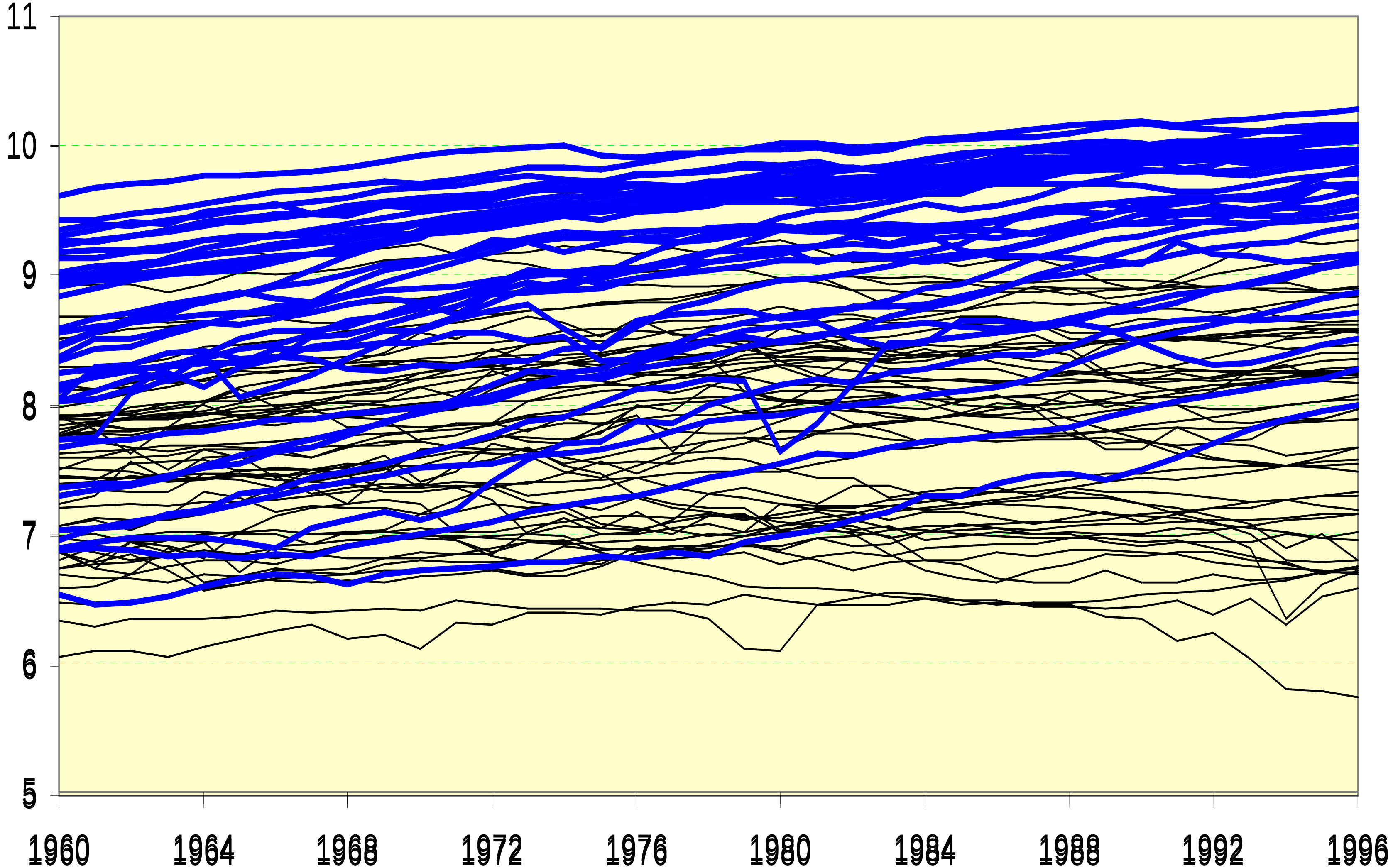
Yes, no index

	person A	person B	person C	person D
1960	0.52	0.21	0.47	0
1961	0.65	0.78	0.24	1
1962	0.25	0.47	0.36	1
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

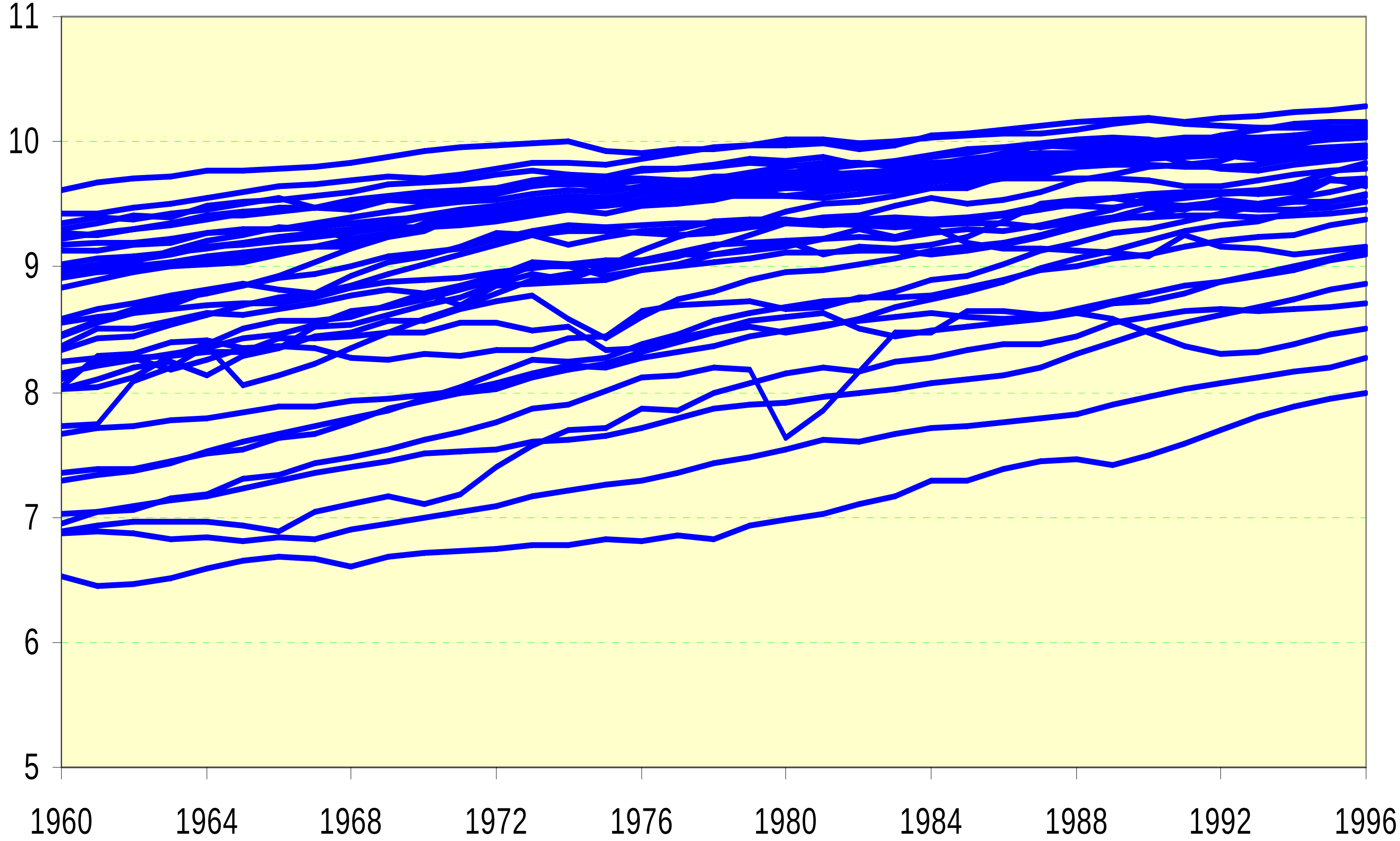
PWT 6.1 Countries log per capita real income



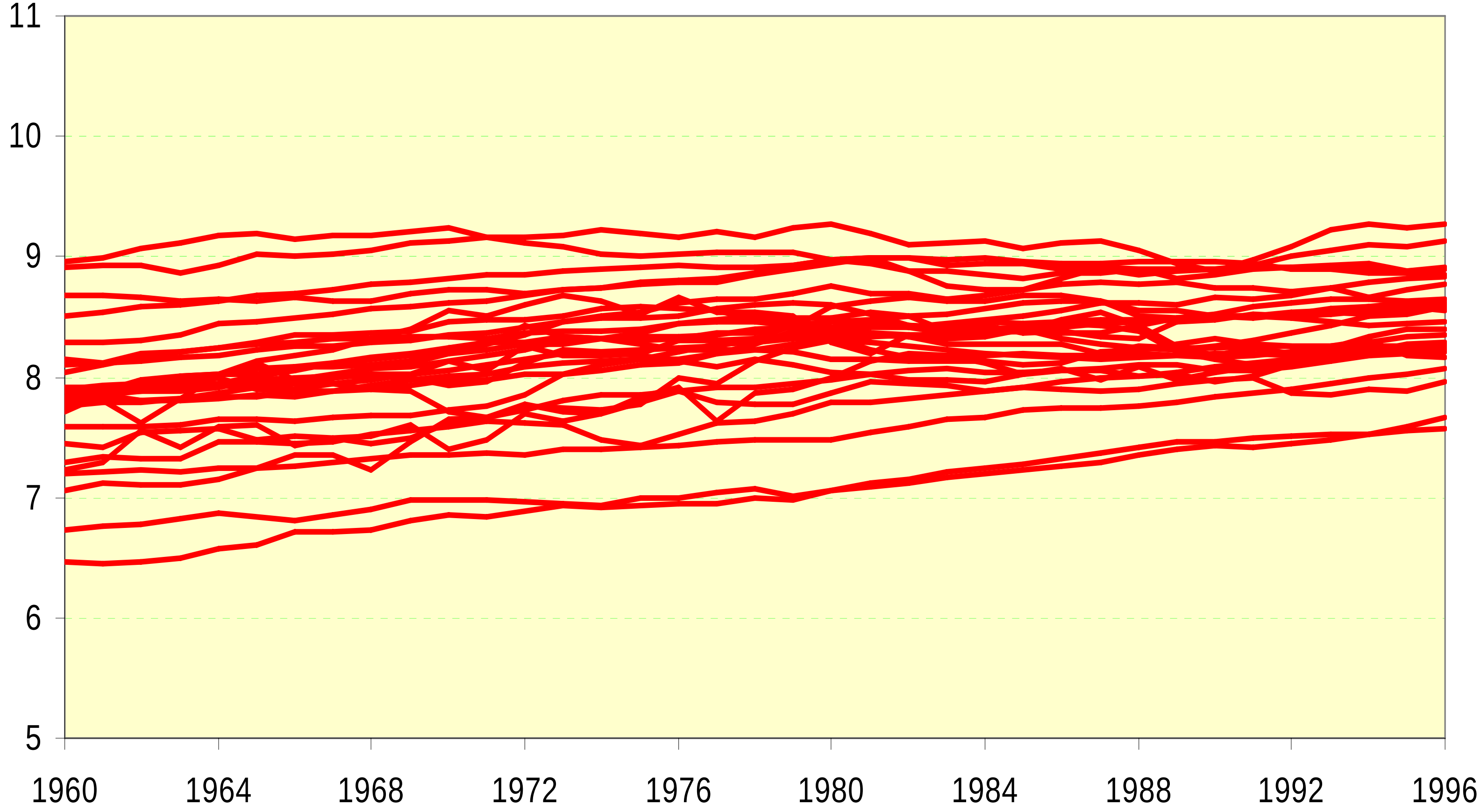
PWT 6.1 Countries log per capita real income



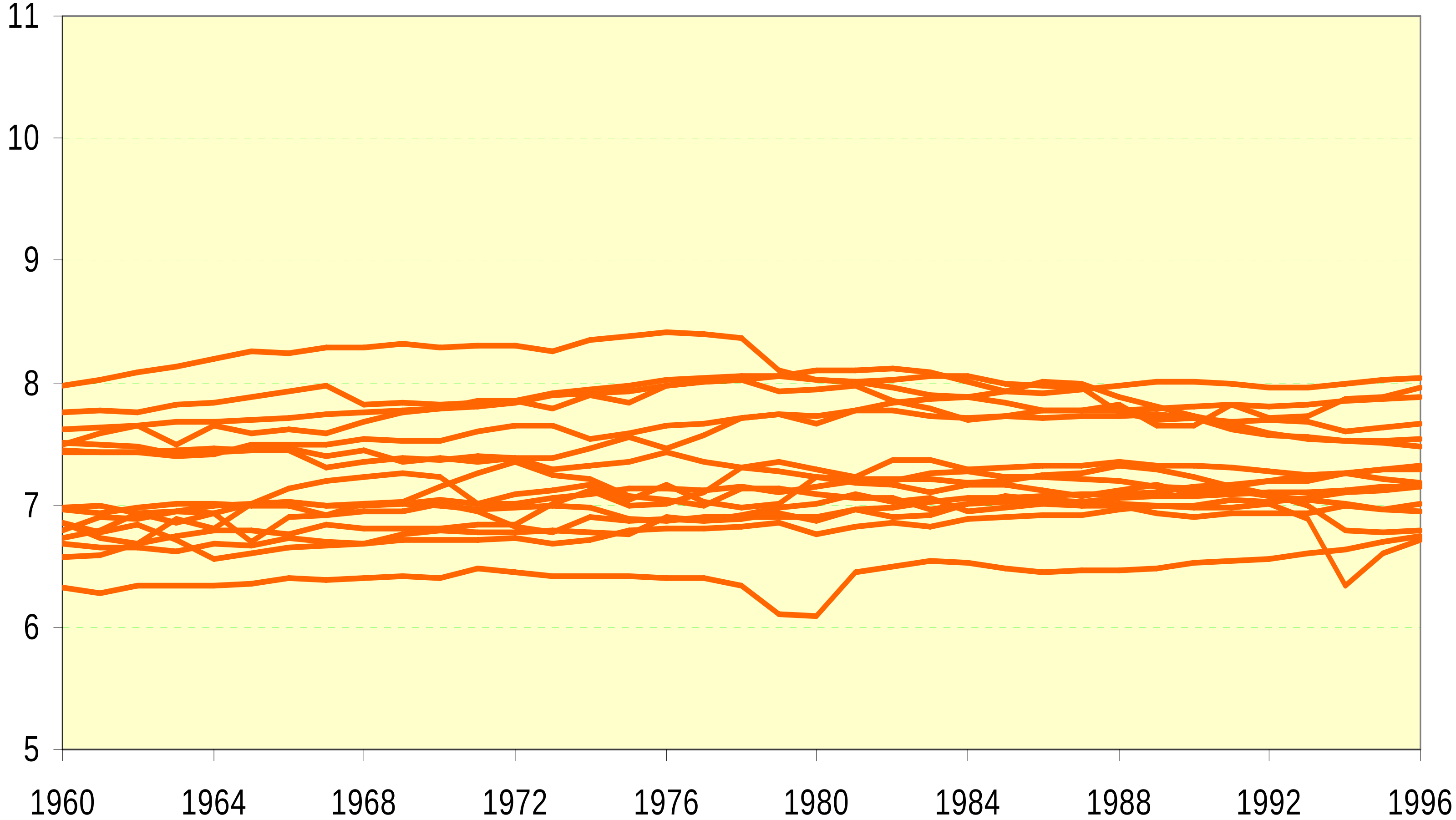
The first convergence club



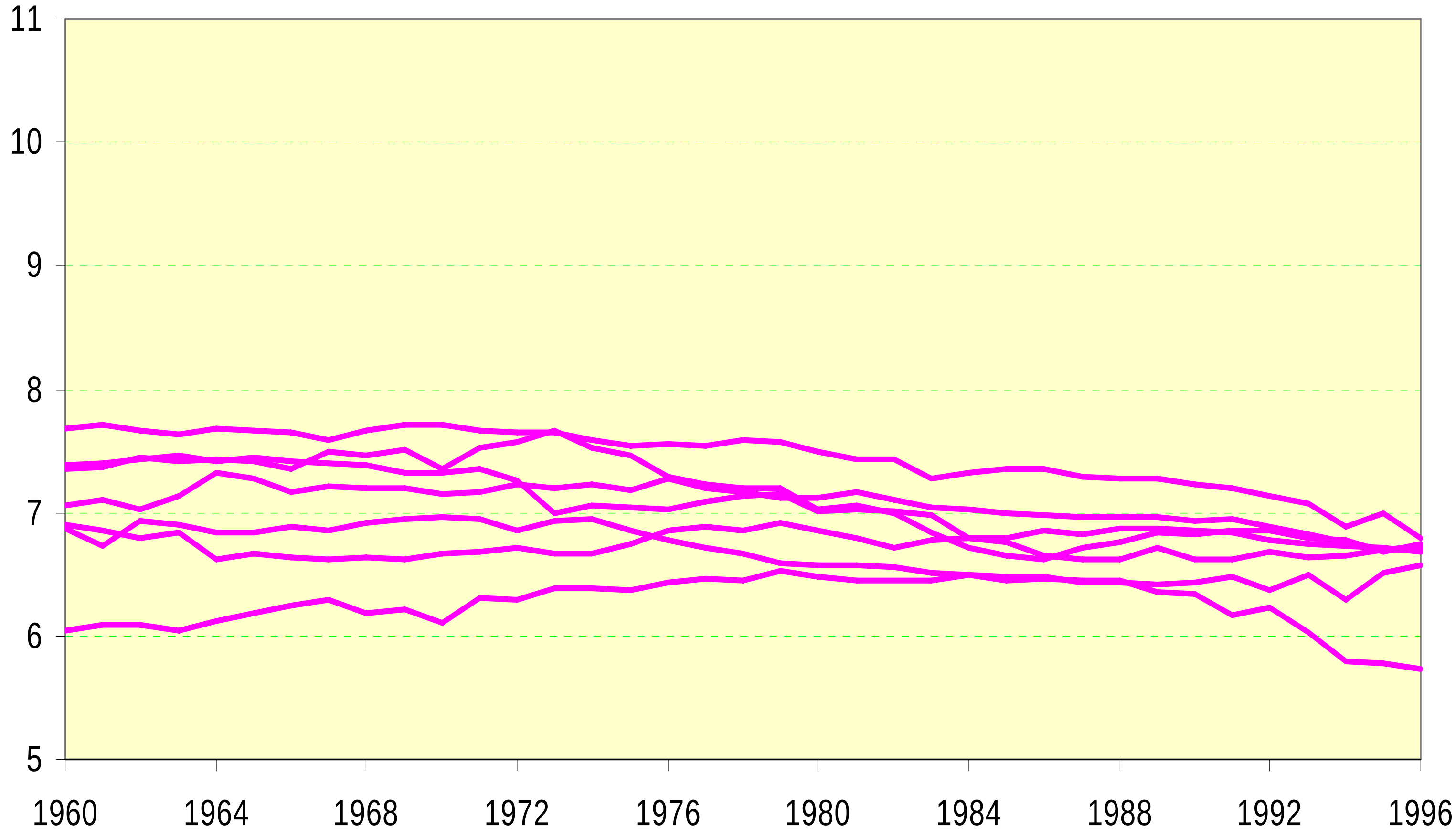
The second convergence club



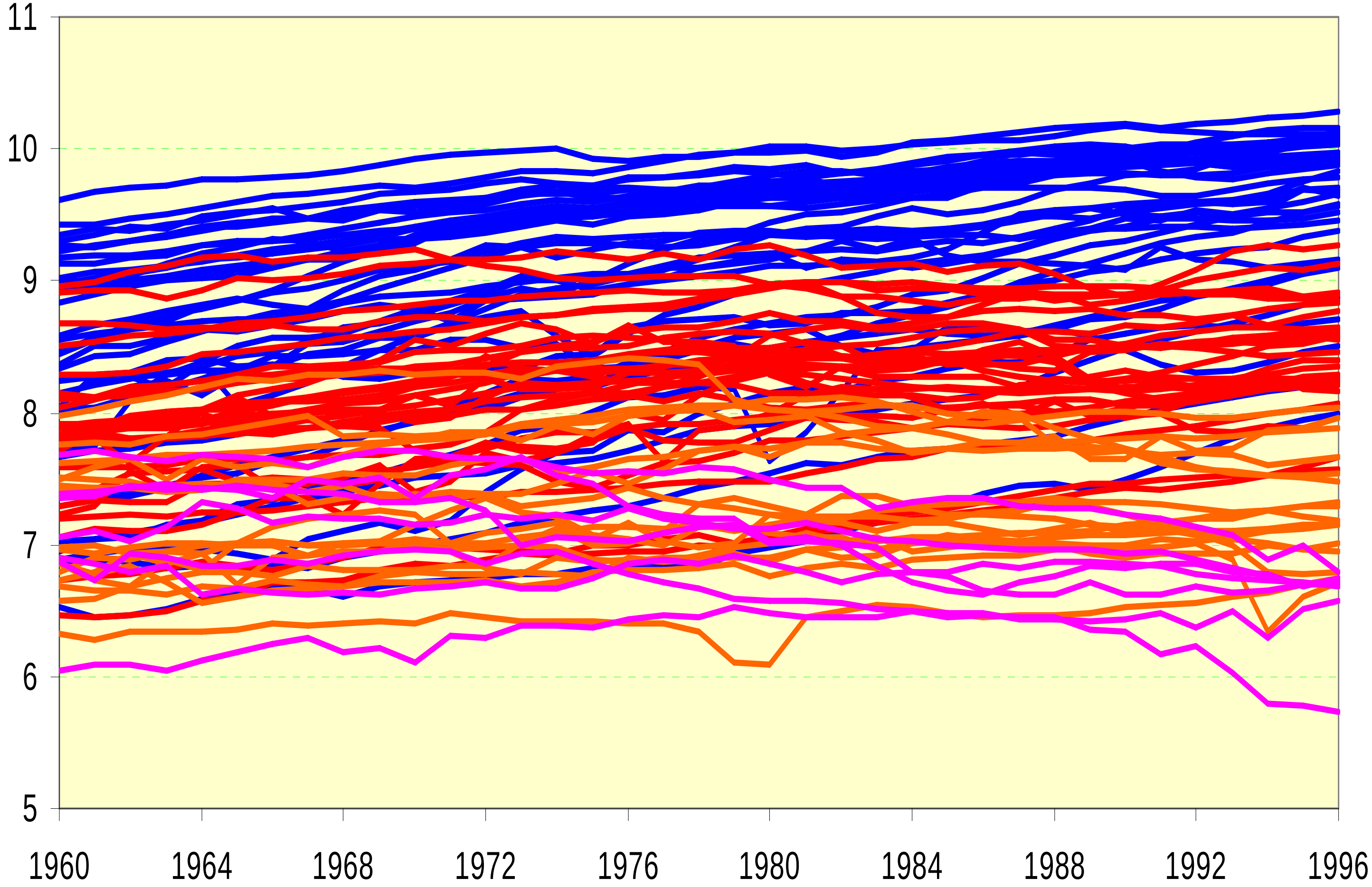
The third convergence club



The fourth group



PWT 6.1 Countries log per capita real income data



$\log t$ Regression

$$\log\left(\frac{H_1}{H_t}\right) - 2\log L(t) = a + b\log t + u_t$$

$$\text{for } t = rT, rT+1, \ldots, T \text{ with } r = 1/3$$

$$\text{where } L(t) = \log t$$

$$H_t = \frac{1}{N}\sum_{i=1}^{N}(h_{it} - 1)^2$$

$$h_{it} = \frac{\log X_{it}}{\frac{1}{N}\sum_{i=1}^{N}\log X_{it}}$$

Contents

Time Varying Common Factor Representation

$$\ln X_{it} = \delta_{it}\mu_t$$

$i = 1, \ldots, N$; individuals

$t = 1, \ldots, T$; time

Time Varying Common Factor Representation

$$\ln X_{it} = \delta_{it}\mu_t$$

Common Factor:

Common behavior,

Representative economic agent's behavior

Time Varying Common Factor Representation

$$\ln X_{it} = \delta_{it} \mu_t$$

Factor loading coefficients

-idiosyncratic behavior

-Economic distance between an individual and representative economic agent

Time Varying Common Factor Representation

$$\ln X_{it} = \delta_{it}\mu_t$$

Parameter of interest $= \delta_{it}$

Especially convergent behavior of δ_{it}

Time Varying Common Factor Representation: Comparison

Conventional Common Factor Model	Time Varying Common Factor Model
$\ln X_{it} = \delta_i \mu_t + \varepsilon_{it}$	$\ln X_{it} = \delta_{it} \mu_t$

Time Varying Common Factor Representation: Comparison

Conventional Common Factor Model	Time Varying Common Factor Model
$\ln X_{it} = \delta_i \mu_t + \varepsilon_{it}$	$\ln X_{it} = \delta_{it} \mu_t$
Two un-identified Idiosyncratic components	One un-identified Idiosyncratic component

Time Varying Common Factor Representation: Comparison

$$\ln X_{it} = \delta_i \mu_t + \varepsilon_{it}$$

$$= \left(\delta_i + \frac{\varepsilon_{it}}{\mu_t}\right)\mu_t$$

$$= \delta_{it}\mu_t$$

$$\delta_{it} \to_p \delta_i \text{ as } t \to \infty$$

$$\text{since } \varepsilon_{it}/\mu_t = o_p(1)$$

Time Varying Common Factor Representation

$$\ln X_{it} = b \ln z_{1t} + c \ln z_{2t} + u_{it}$$

$$\ln X_{it} = b \ln z_{1t} + u_{it}, \ \ i \in G_1$$

$$\ln X_{it} = c \ln z_{2t} + u_{it}, \ \ i \in G_2$$

$$\ln X_{it} = b_1 \ln z_{1t} + u_{it}, \ \ i \in G_1$$

$$\ln X_{it} = b_2 \ln z_{1t} + u_{it}, \ \ i \in G_2$$

Time Varying Common Factor Representation

$$\ln X_{it} = b \ln z_{1t} + c \ln z_{2t} + u_{it}$$

$$\ln X_{it} = b \ln z_{1t} + u_{it}, \ \ i \in G_1$$

$$\ln X_{it} = c \ln z_{2t} + u_{it}, \ \ i \in G_2$$

$$\ln X_{it} = b_1 \ln z_{1t} + u_{it}, \ \ i \in G_1$$

$$\ln X_{it} = b_2 \ln z_{1t} + u_{it}, \ \ i \in G_2$$

$$\ln X_{it} = \delta_{it} \mu_t = \begin{cases} \delta_{it} \to \delta_1 \text{ if } i \in G_1 \\ \delta_{it} \to \delta_2 \text{ if } i \in G_2 \end{cases}$$

Time Varying Common Factor Representation: Examples

Ex 1: Transitional Growth Path under Heterogeneous Technology Progress: Phillips and Sul (2006)

$$\begin{aligned}\log y_{it} &= \log y_i^* + (\log y_{i0} - \log y_i^*)e^{-\beta_{it}} + \log A_{it} \\ &= a_{it} + \log A_{it} \\ &= \left(\frac{a_{it} + \log A_{it}}{\mu_t}\right)\mu_t \\ &= \delta_{it}\mu_t\end{aligned}$$

$$\log y_{it} - \log y_{jt} = (\delta_{it} - \delta_{jt})\mu_t$$

Time Varying Common Factor Representation: Examples

Ex 2. Heterogeneous economic agents model

$$\frac{1}{1+r_t} = E_t\left[\beta_{it}\frac{U(C_{it+1})'}{U(C_{it})'}\right]$$

$$\beta_{it}\left[\frac{C_{it}}{C_{it+1}}\right] = \frac{1}{1+r_t}$$

$$\beta_{it}\left[\frac{C_{it}}{C_{it+1}}\right] = \beta_{Rt}\left[\frac{C_{Rt}}{C_{Rt+1}}\right] = \mu_t,$$

$$X_{it} = \left[\frac{C_{it}}{C_{it+1}}\right] = \frac{\beta_{Rt}}{\beta_{it}}\left[\frac{C_{Rt}}{C_{Rt+1}}\right] = \delta_{it}\mu_t$$

Time Varying Common Factor Representation: Examples

Ex 2. Heterogeneous economic agents model

$$X_{it} = \left[\frac{C_{it}}{C_{it+1}} \right] = \frac{\beta_{Rt}}{\beta_{it}} \left[\frac{C_{Rt}}{C_{Rt+1}} \right] = \delta_{it} \mu_t$$

$$\beta_{it} \to \beta \ \ (?)$$

Relationship b.t β_{it} and Q_t, TB_t

Time Varying Common Factor Representation: Examples

Ex 3. Stock Return

$$X_{it} = \delta_{1i}\mu_{1t} + \delta_{2i}\mu_{2t} + \delta_{3i}\mu_{3t} + \epsilon_{it}$$

$$X_{it} = \delta_{1,it}\mu_{1t} + \delta_{2,it}\mu_{2t} + \delta_{3,it}\mu_{3t} + \epsilon_{it}$$

$$X_{it} = \left(\delta_{1,it} + \delta_{2,it}\frac{\mu_{2t}}{\mu_{1t}} + \delta_{3,it}\frac{\mu_{3t}}{\mu_{1t}} + \frac{\epsilon_{it}}{\mu_{1t}}\right)\mu_{1t}$$

$$\mu_{jt} = m_j t + \sum_{s=1}^{t} e_{js}, \text{ for } j = 1,2,3, \text{ with } m_1 \neq 0,$$

$$\frac{\mu_{jt}}{\mu_{1t}} = \frac{m_j t + \sum_{s=1}^{t}\epsilon_{js}}{m_1 t + \sum_{s=1}^{t}\epsilon_{1s}} = \frac{m_j}{m_1} + o_p(1),$$

Time Varying Common Factor Representation: Examples

Ex 3. Stock Return

$$X_{it} = \delta_{1,it}\mu_{1t} + \delta_{2,it}\mu_{2t} + \delta_{3,it}\mu_{3t} + \epsilon_{it}$$

$$X_{it} = \delta_{it}\mu_{t}$$

$$\delta_{it} = \delta_{1,it} + \left\{\delta_{2,it}\frac{m_2}{m_1} + \delta_{3,it}\frac{m_3}{m_1}\right\}\{1 + o_p(1)\}$$

$$\mu_t = \mu_{1t}$$

Time Varying Common Factor Representation: Examples

Ex 3. Earning and Wage in Labor Economics

$$\log Y_{it} = \delta_{1,it}\mu_{1t} \quad \text{if } i = \text{male}$$

$$\log Y_{it} = \delta_{2,it}\mu_{2t} \quad \text{if } i = \text{female}$$

$$\log Y_{it} = \delta_{1,it}\mu_{1t} + \delta_{2,it}\mu_{2t}$$

$$= \delta_{it}\mu_t$$

$$\delta_{1,it} \to \delta_1, \delta_{2,it} \to \delta_2$$

$$\delta_{it} \to \begin{cases} \delta_1 \text{ if } i = \text{male} \\ \delta_2 \text{ if } i = \text{femal} \end{cases}$$

Time Varying Common Factor Representation: Examples

Ex 3. Earning and Wage in Labor Economics

$$\log Y_{it} = \sum_{s=1}^{k} \delta_{sit}\mu_{st} \quad \text{if } i \in s$$

$$= \delta_{it}\mu_t$$

$$\delta_{sit} \to \delta_s$$

$$\delta_{it} \to \delta_s \qquad \text{if } i \in s$$

Convergence Concept

Absolute Convergence $(p)\lim_{k\to\infty}(\ln X_{it+k} - \ln X_{jt+k}) = 0$

Relative Convergence $(p)\lim_{k\to\infty}\frac{\ln X_{it+k}}{\ln X_{jt+k}} = 1$

RC nests AC.

When AC holds, RC should hold.

Even when RC holds, AC may not hold.

Convergence Concept: Absolute Convergence

$$\ln X_{it} = \delta_{it}\mu_t$$

$$\ln X_{it} - \ln X_{jt} = (\delta_{it} - \delta_{jt})\mu_t$$

$$\text{Let } \mu_t = t,$$

$$\delta_{it} - \delta_{jt} = \delta t^{-\alpha},\ \alpha > 0$$

$$\delta_{it} - \delta_{jt} \to 0$$

$$\ln X_{it} - \ln X_{jt} \to \begin{cases} \infty \text{ if } \alpha < 1 \\ \delta \text{ if } \alpha = 1 \\ 0 \text{ if } \alpha > 1 \end{cases}$$

Convergence Concept: Relative Convergence

$$\ln X_{it} = \delta_{it}\mu_t$$

$$\ln X_{it} - \ln X_{jt} = (\delta_{it} - \delta_{jt})\mu_t$$

$$\text{Let } \mu_t = t,$$

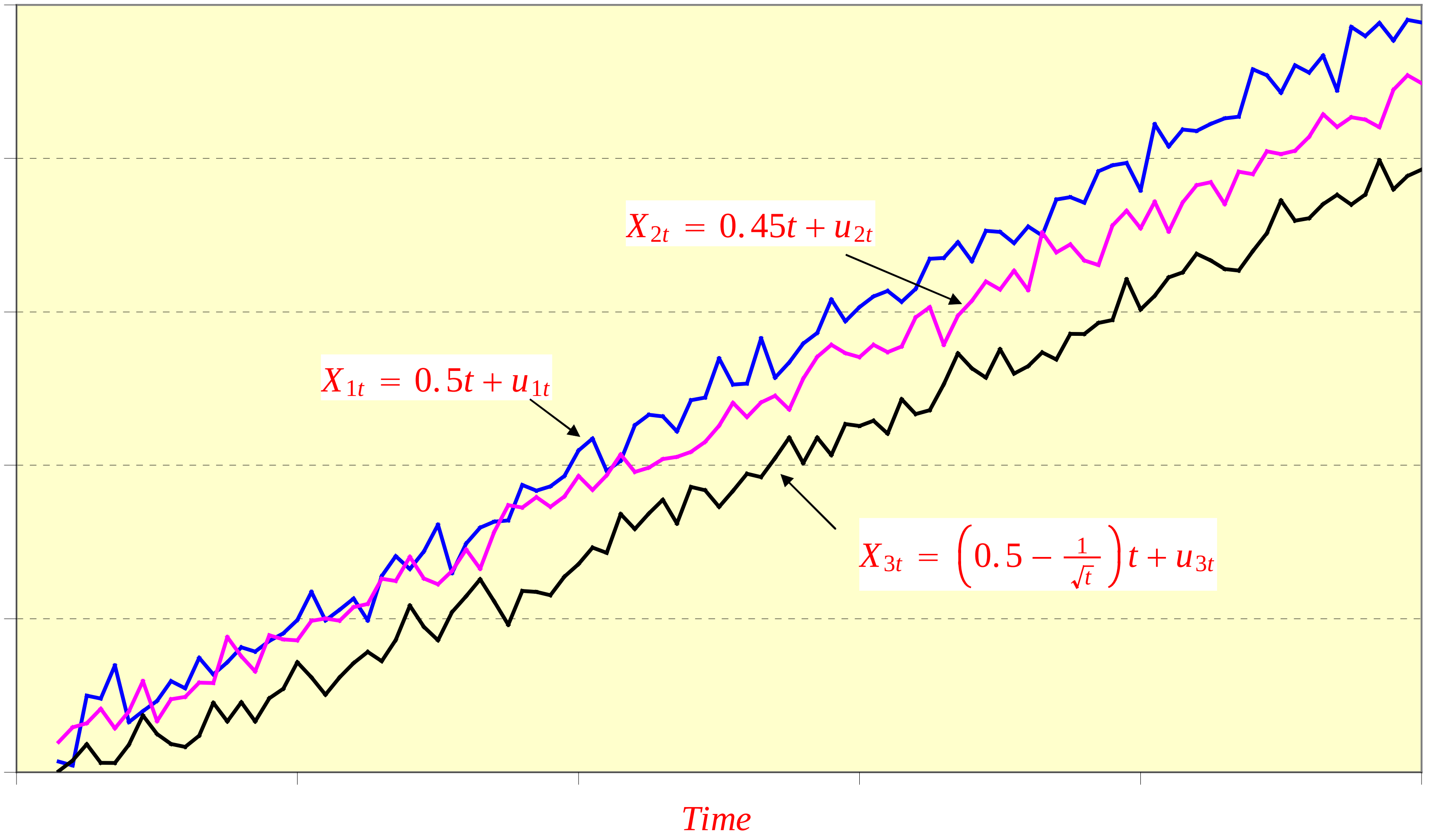
$$\delta_{it} - \delta_{jt} = \delta t^{-\alpha},\ \alpha > 0$$

$$\delta_{it} - \delta_{jt} \to 0$$

$$\ln X_{it} - \ln X_{jt} \to \begin{cases} \infty \text{ if } \alpha < 1 \\ \delta \text{ if } \alpha = 1 \\ 0 \text{ if } \alpha > 1 \end{cases}$$

$$\ln X_{it}/\ln X_{jt} = \delta_{it}/\delta_{jt} \to 1$$

Convergence Concept: Relative v.s. Absolute



Convergence Concept: Relative v.s. Absolute

$$\ln X_{it} - \ln X_{jt} = \delta t^{1-\alpha}$$

$$\text{Let } \delta = 1, \alpha = 1/2$$

$$\ln X_{it} - \ln X_{jt} = \sqrt{t}$$

$$\Delta \ln X_{it} - \Delta \ln X_{jt} = \sqrt{t} - \sqrt{t-1} \rightarrow 0$$

Relative convergence => the same growth rate in the long run

Comparison: RC v.s. Cointegration

$$\ln X_{it} = \delta_{it}\mu_t$$

$$\mu_t = \mu + \mu_{t-1} + \epsilon_t$$

$$\delta_{1t} = \delta$$

$$\delta_{2t} = 0.9\delta + t^{-\alpha} \to 0.9\delta$$

$$\delta_{3t} = \delta + t^{-\alpha} \to \delta$$

In finite sample

$\ln X_{1t}$ & $\ln X_{2t}$: Cointegrated

$\ln X_{1t}$ & $\ln X_{3t}$: No Co.

ln long run

$$\ln X_{1t} - \ln X_{2t} = I(1)$$

$$\ln X_{1t} - \ln X_{3t} = I(0)$$

Relative Transition Parameter: Approximation of factor loading coefficients

$$\ln X_{it} = \delta_{it}\mu_t$$

1. Total number of obs. = NxT
2. Total number of unknowns = NxT + Tx1
3. Parametric estimation: Requires restrictions: Example

Parametric Restriction

$$\delta_{it} = \delta_i + \rho\delta_{it-1} + \epsilon_{it}$$

$$\mu_t = \mu + \mu_{t-1} + e_t$$

Relative Transition Parameter: Approximation of factor loading coefficients

$$\ln X_{it} = \delta_{it}\mu_t$$

Parametric Restriction

$$\delta_{it} = \delta_i + \rho\delta_{it-1} + \epsilon_{it}$$

$$\mu_t = \mu + \mu_{t-1} + e_t$$

Problem:

1. How to justify this restriction?
2. What if the common factor is I(0) or has a linear trend
3. Is it possible to examine the convergent behavior of δ_{it} ?

Relative Transition Parameter

Solution: Don't estimate them but approximate them.

How?

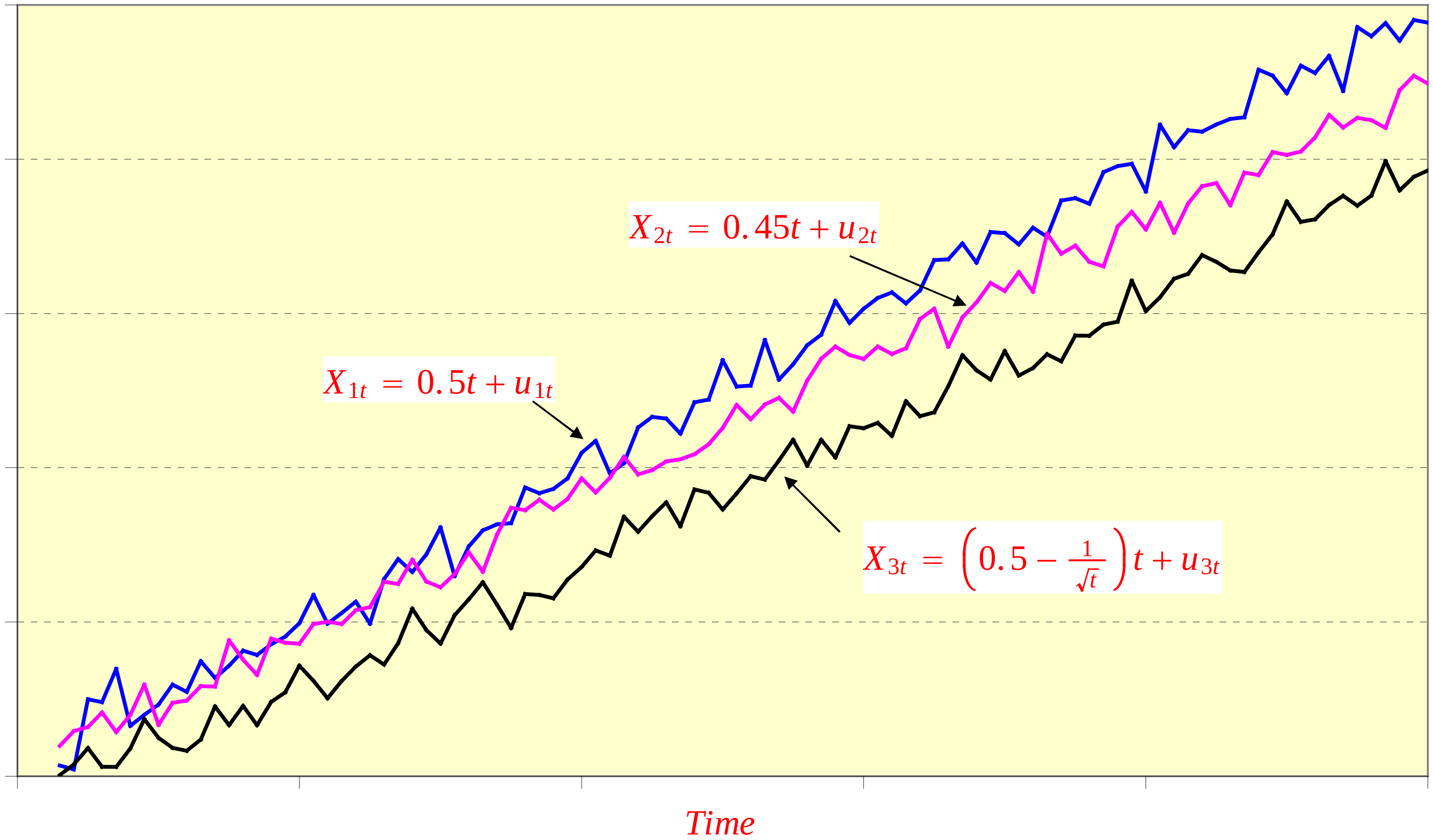
$$h_{it} = \frac{X_{it}}{\frac{1}{N}\sum_{i=1}^{N} X_{it}} = \frac{\delta_{it}}{\frac{1}{N}\sum_{i=1}^{N} \delta_{it}}$$

working with

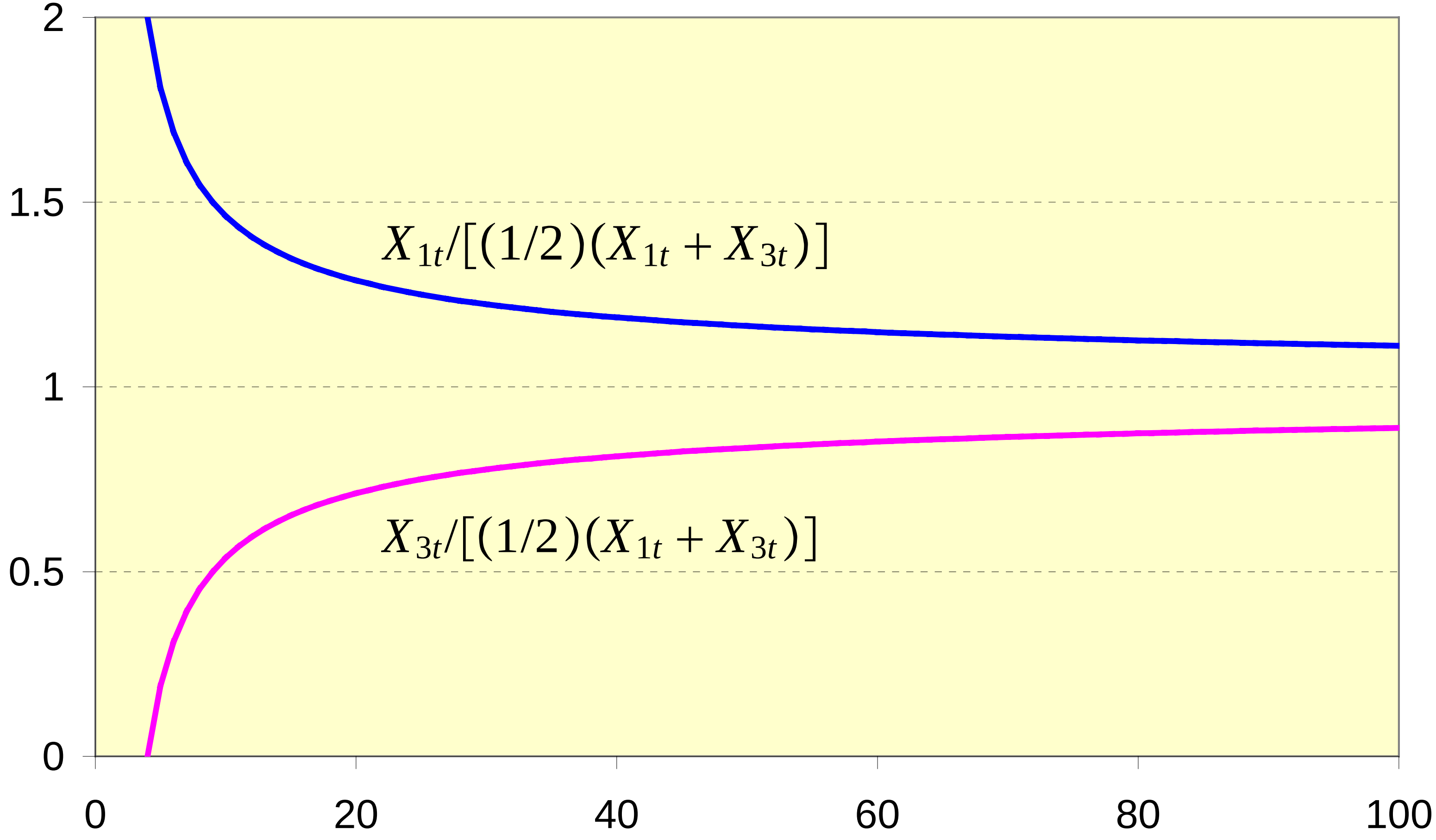
$$H_t = \frac{1}{N}\sum_{i=1}^{N}(h_{it} - 1)^2$$

If the common factor is a constant, then don't need to approximate.

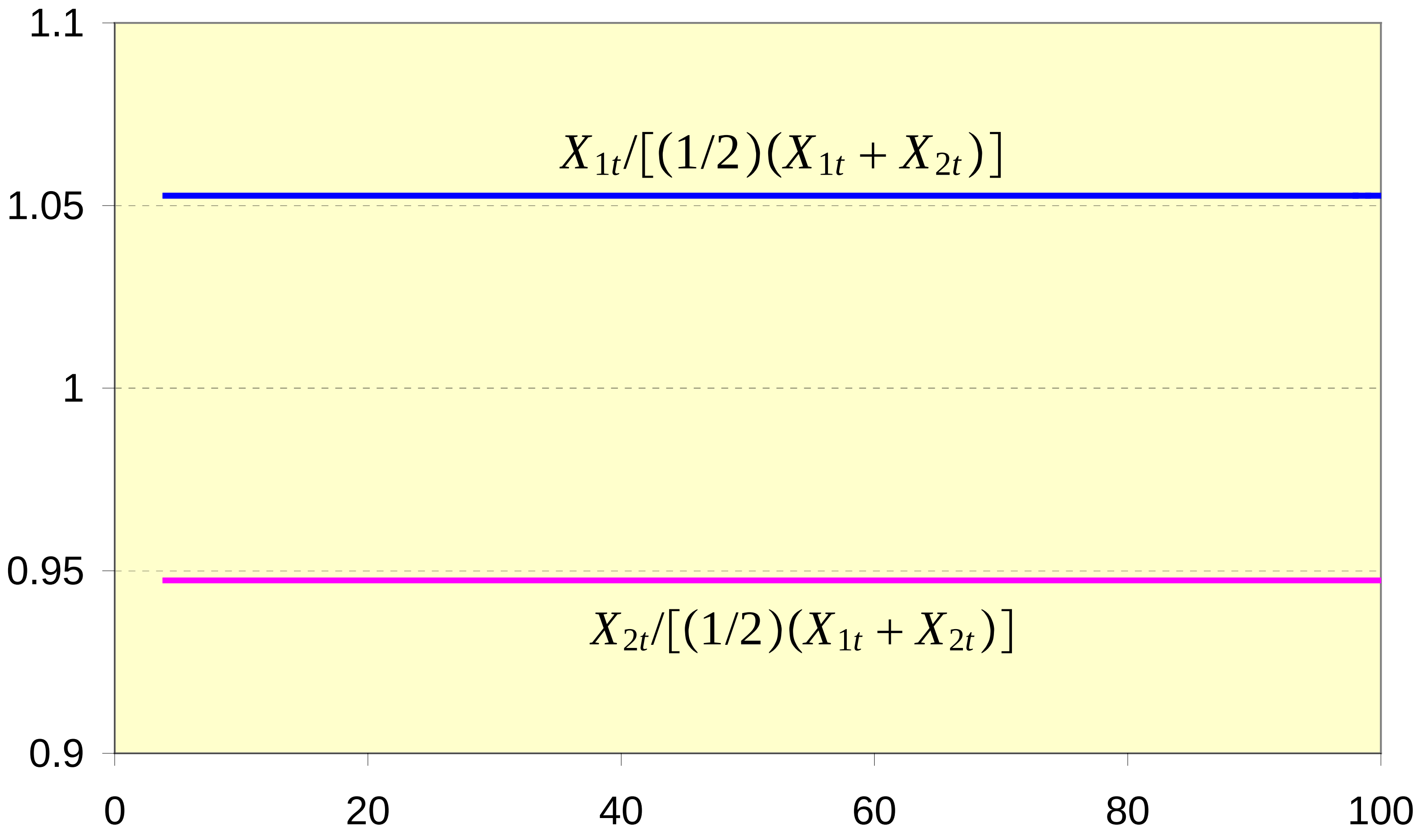
Relative Transition Parameter: Example



Relative Transition Parameter: Example



Relative Transition Parameter: Example



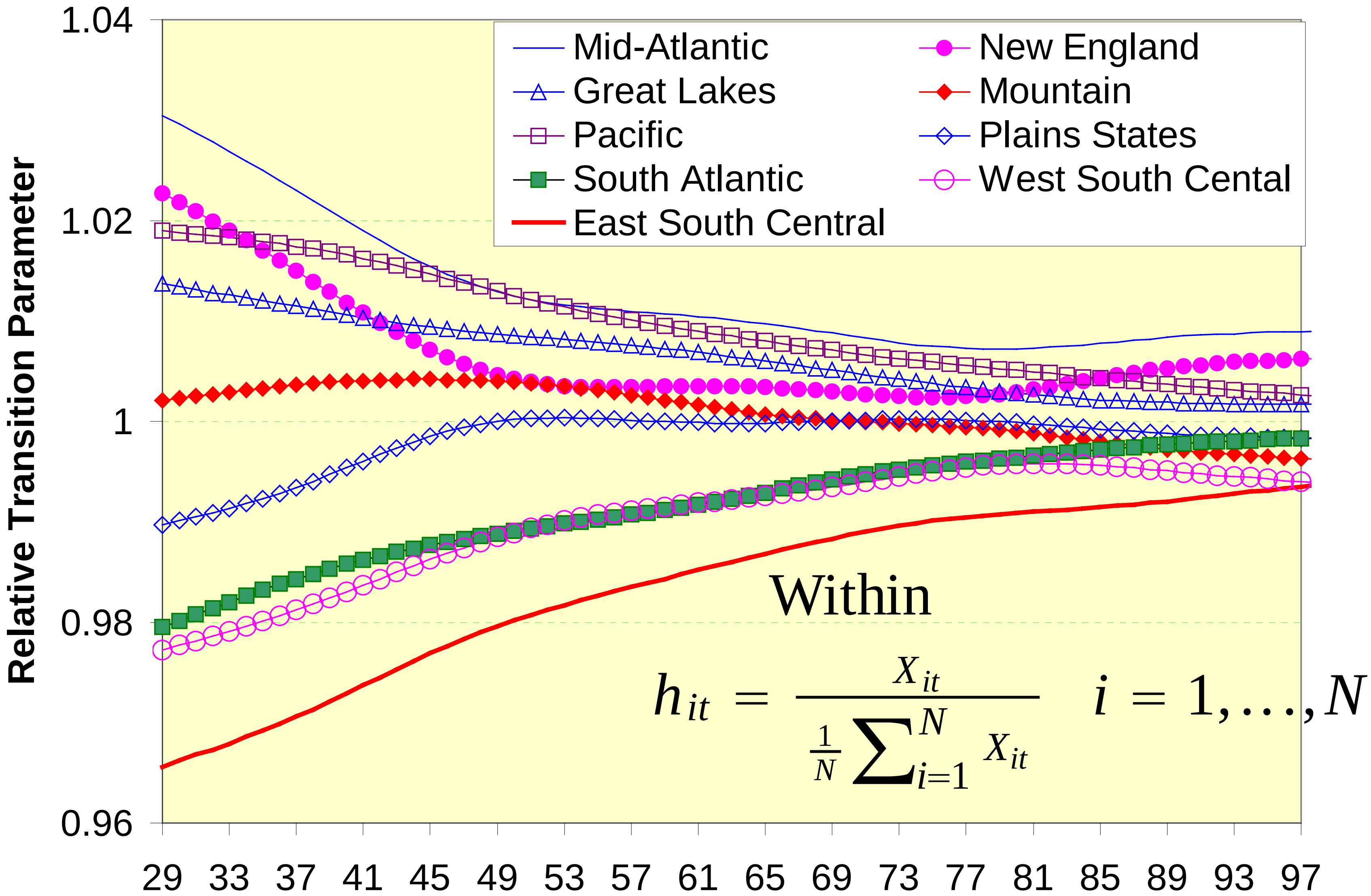
Relative Transition Parameter: How to Use

Within

$$h_{it} = \frac{X_{it}}{\frac{1}{N}\sum_{i=1}^{N} X_{it}} \quad i = 1, \ldots, N$$

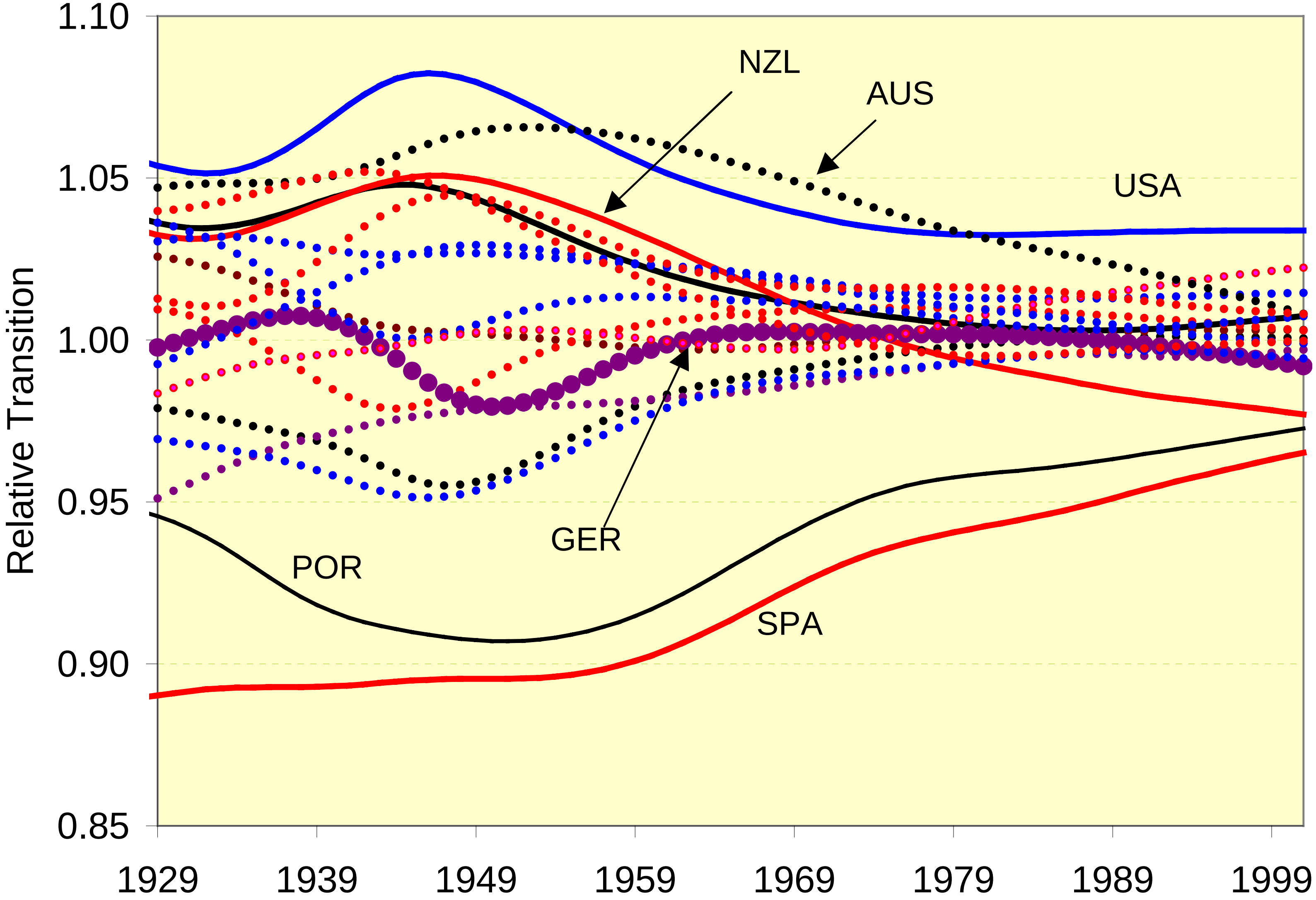
Transition Paths for Regional Groups

of the Contiguous US States



OECD Transition Paths:

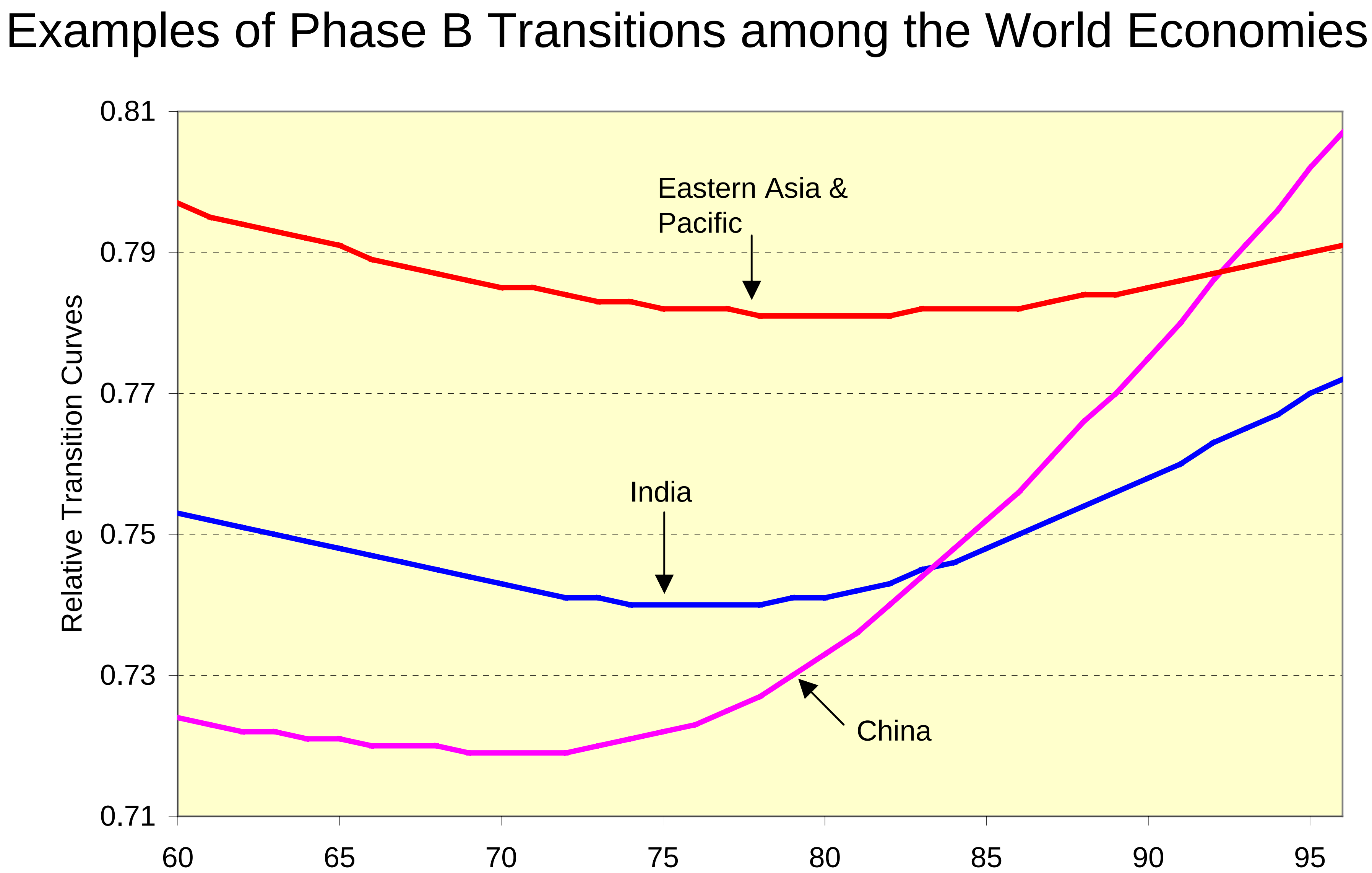
18 Western OECD countries from 1929-2001



Relative Transition Parameter: How to Use

Relative 1: (Cross) $j \notin G^*$

$$h_{jt} = \frac{X_{jt}}{\frac{1}{G}\sum_{j \neq i}^{G} X_{jt}} \qquad G = 1, \ldots, G^*, j$$


Examples of Phase B Transitions among the World Economies
Relative Transition Curves
0.81
0.79
0.77
0.75
0.73
0.71
60
65
70
75
80
85
90
95
Eastern Asia & Pacific
India
China

Relative Transition Parameter: How to Use

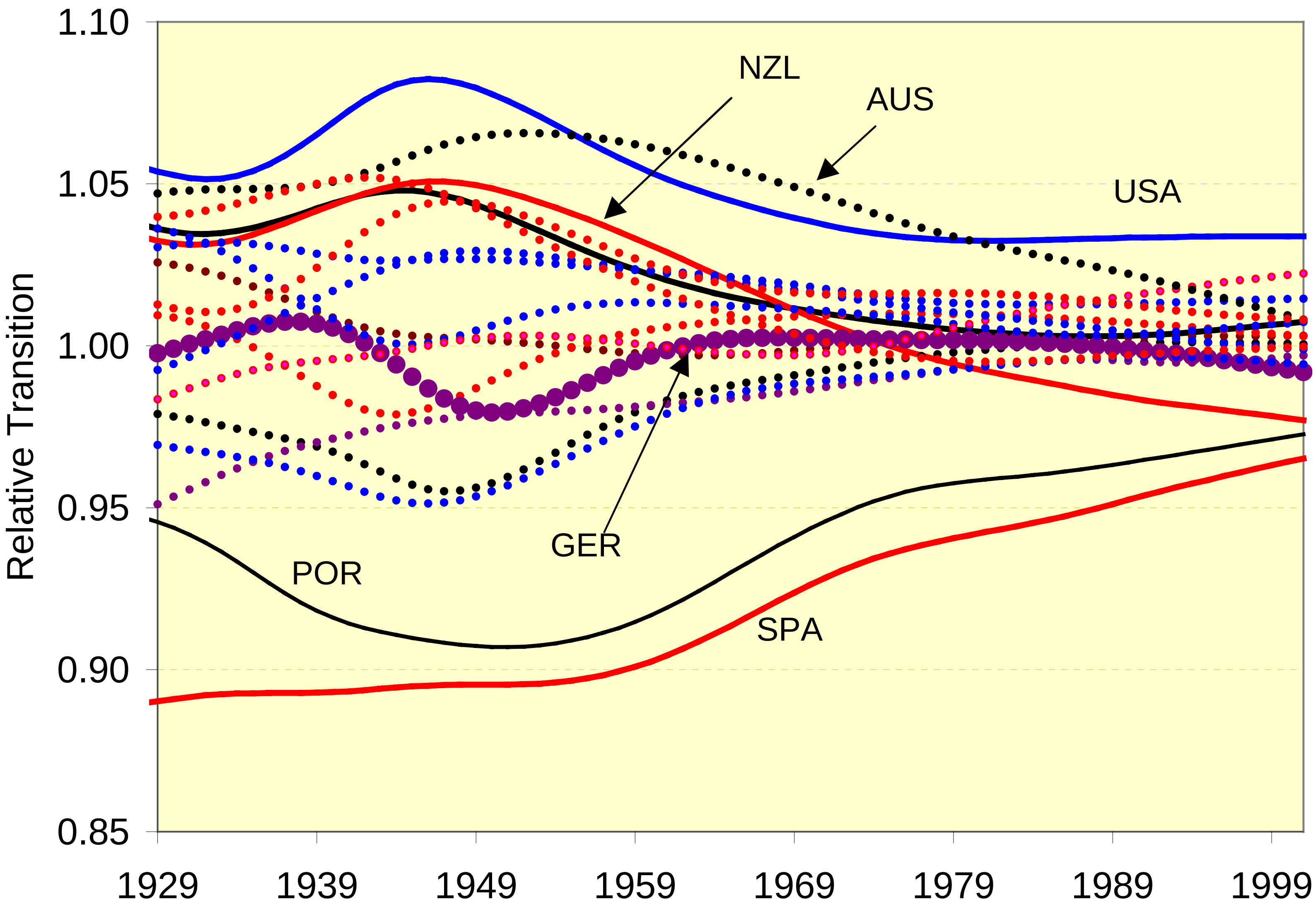
Relative 2: (Cross) $j \notin G^*$

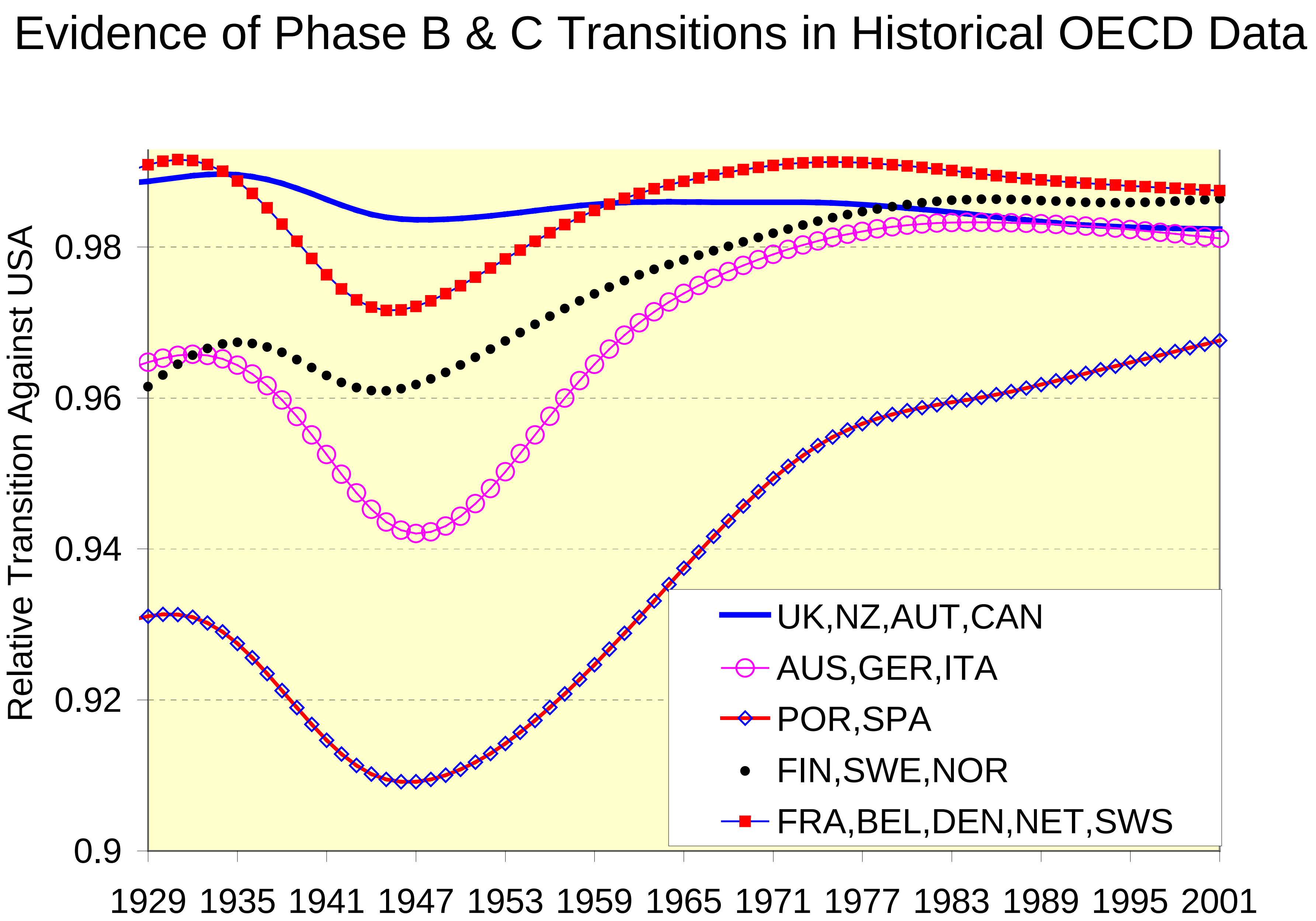
$$h_{gt} = \frac{X_{gt}}{\frac{1}{G}\sum_{g \neq j}^{G} X_{gt}} \qquad G = 1, \ldots, G^*, j$$

$$h_{gt}^* = \frac{1}{G^*}\sum_{g=1}^{G^*} h_{gt}$$

OECD Transition Paths:

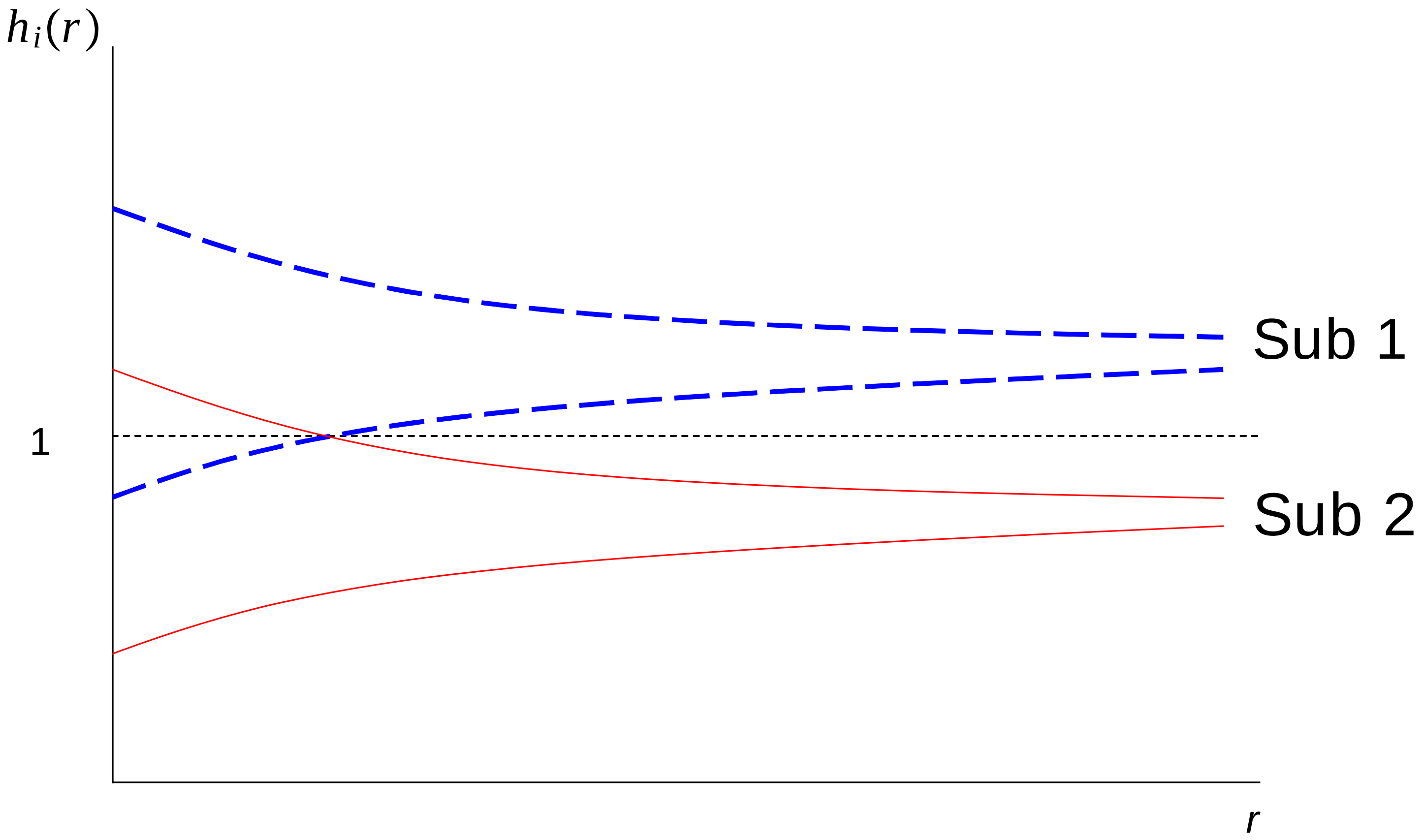
18 Western OECD countries from 1929-2001





Evidence of Phase B & C Transitions in Historical OECD Data
Relative Transition Against USA
0.98
0.96
0.94
0.92
0.9
1929
1935
1941
1947
1953
1959
1965
1971
1977
1983
1989
1995
2001
UK,NZ,AUT,CAN
AUS,GER,ITA
POR,SPA
FIN,SWE,NOR
FRA,BEL,DEN,NET,SWS

Modeling and Testing Convergence



Modeling and Testing Convergence

$$\ln X_{it} = \delta_{it}\mu_t$$

$$\delta_{it} = \delta_i + \sigma_{it}\xi_{it}, \quad \sigma_{it} = \frac{\sigma_i}{L(t)t^{\alpha}}, \quad t \geq 1, \sigma_i > 0 \text{ for all } i.$$

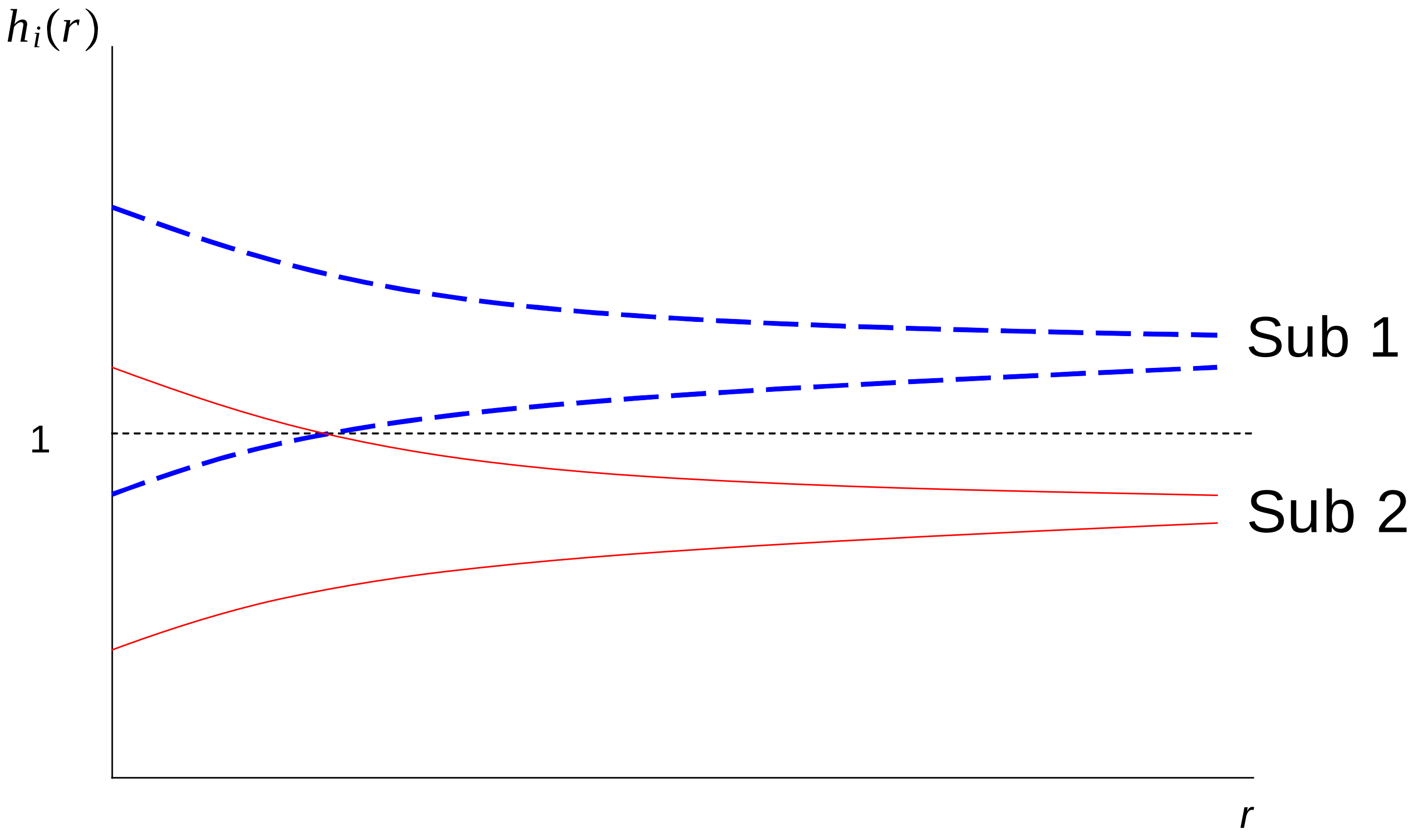
$L(t)$ is a slowlying varying function.

$L(aT)/L(T) \to 1$ as $T \to \infty$ for all $a > 0$.

Ex: $\log(T)$.

$\log(aT)/\log T = (\log a + \log T)/\log T = \frac{\log a}{\log T} + 1 \to 1$

Modeling and Testing Convergence



Modeling and Testing Convergence

$$\delta_{it} = \delta_i + \sigma_{it}\xi_{it}, \ \sigma_{it} = \frac{\sigma_i}{L(t)t^{\alpha}}$$

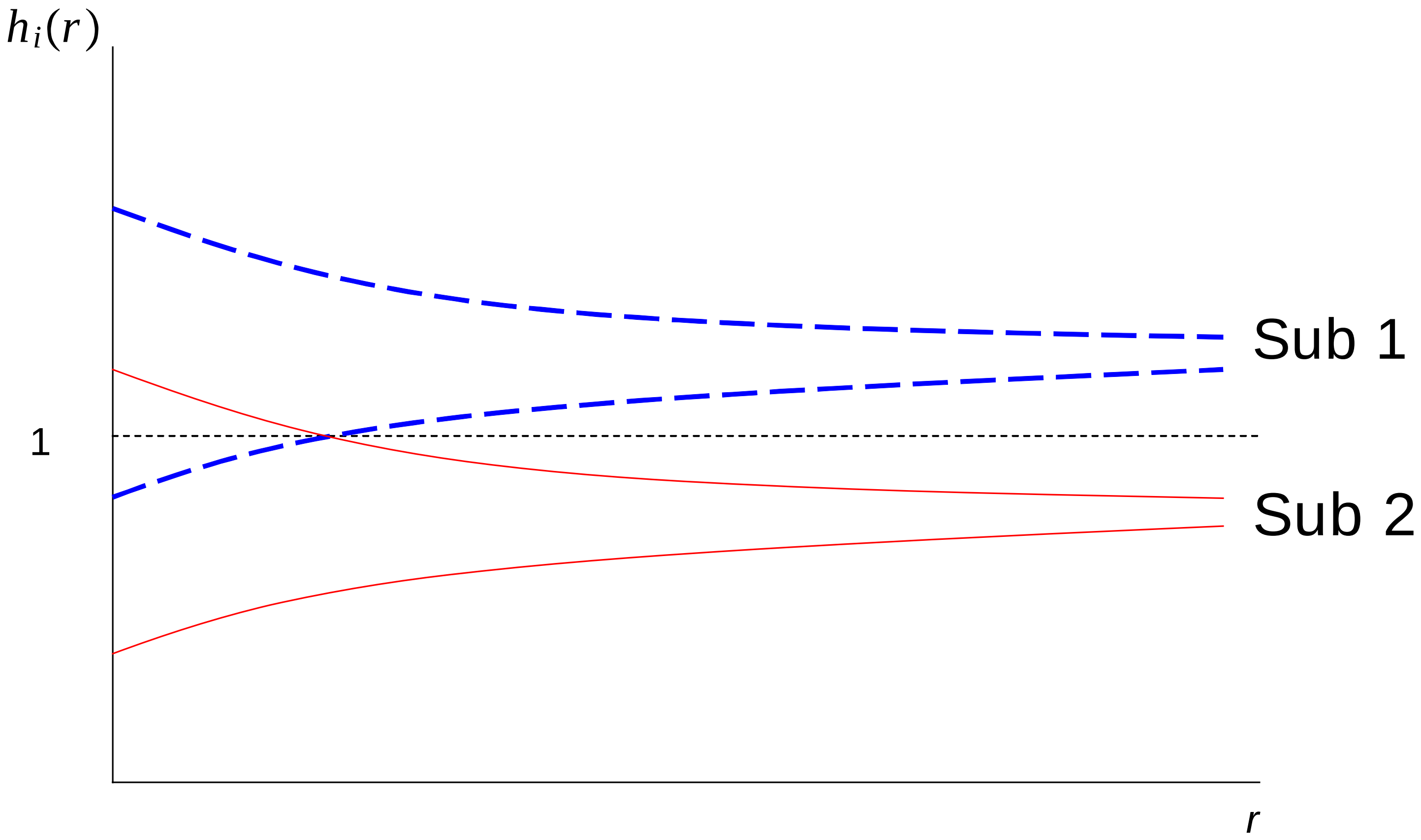
$$\mathcal{H}_0 : \delta_i = \delta \text{ and } \alpha \geq 0$$

$$\mathcal{H}_A : \delta_i \neq \delta \text{ or } \alpha < 0$$

$$\mathcal{H}_0 : \text{ Convergence for all } i$$

$$\mathcal{H}_A : \text{ Divergence for some } i$$

Modeling and Testing Convergence



Testing Convergence

Step 1 $H_t = \frac{1}{N}\sum_{i=1}^{N}(h_{it}-1)^2, \quad h_{it} = \frac{X_{it}}{N^{-1}\sum_{i=1}^{N} X_{it}}$

Step 2 $\log\frac{H_1}{H_t} - 2\log(\log t) = a + b\log t + u_t$

Step 3 Test the null of $\mathcal{H}_0 : b \geq 0$

$t_{\hat{b}}$ with HAC estimator

$$\mathcal{H}_0 : \delta_i = \delta \text{ and } \alpha \geq 0 \Leftrightarrow \mathcal{H}_0 : b \geq 0$$

Asymptotic properties of the logt regression

DGP: $\delta_{it} = \delta_i + \sigma_{it}\xi_{it}, \ \sigma_{it} = \frac{\sigma_i}{L(t)t^{\alpha}}$

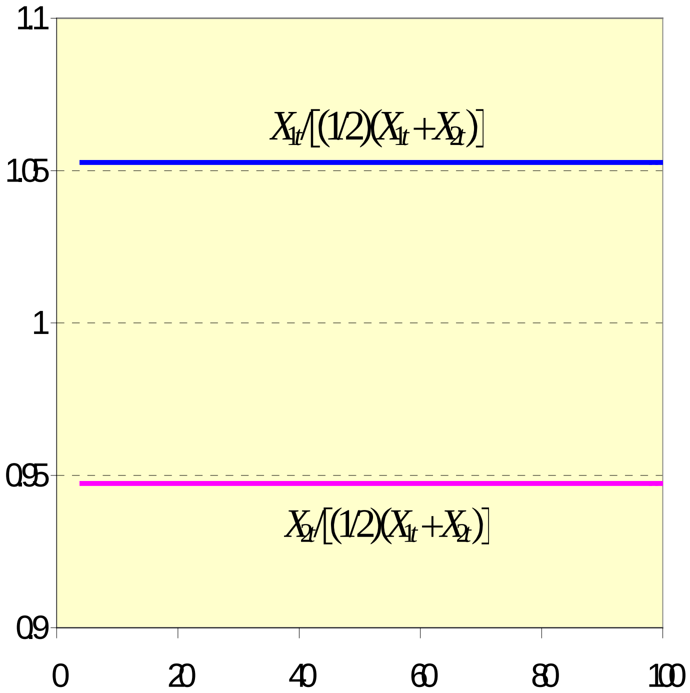
Reg: $\log \frac{H_1}{H_t} - 2\log(\log t) = a + b\log t + u_t$

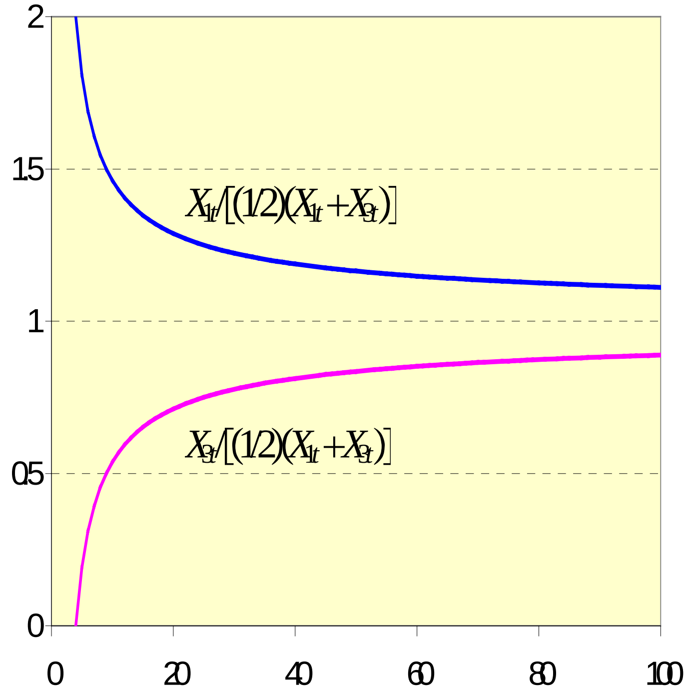
Under $\mathcal{H}_0, b = 2\alpha$

Under $\mathcal{H}_A, b \to 0,$ but $t_{\hat{b}} \to -\infty$

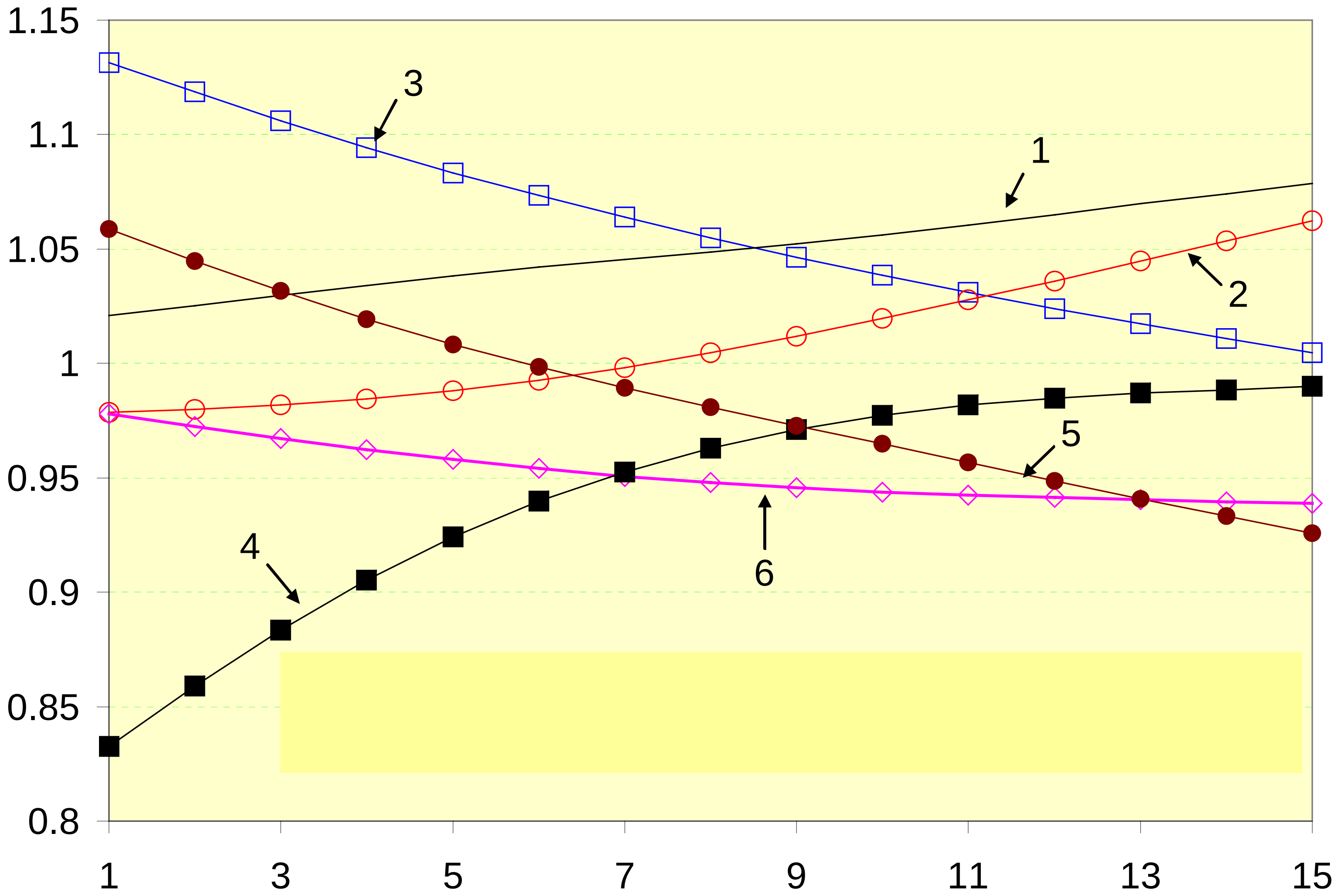
Why? Under $\mathcal{H}_A, \ h_{it}$ does not converge at all.

Asymptotic properties of the logt regression





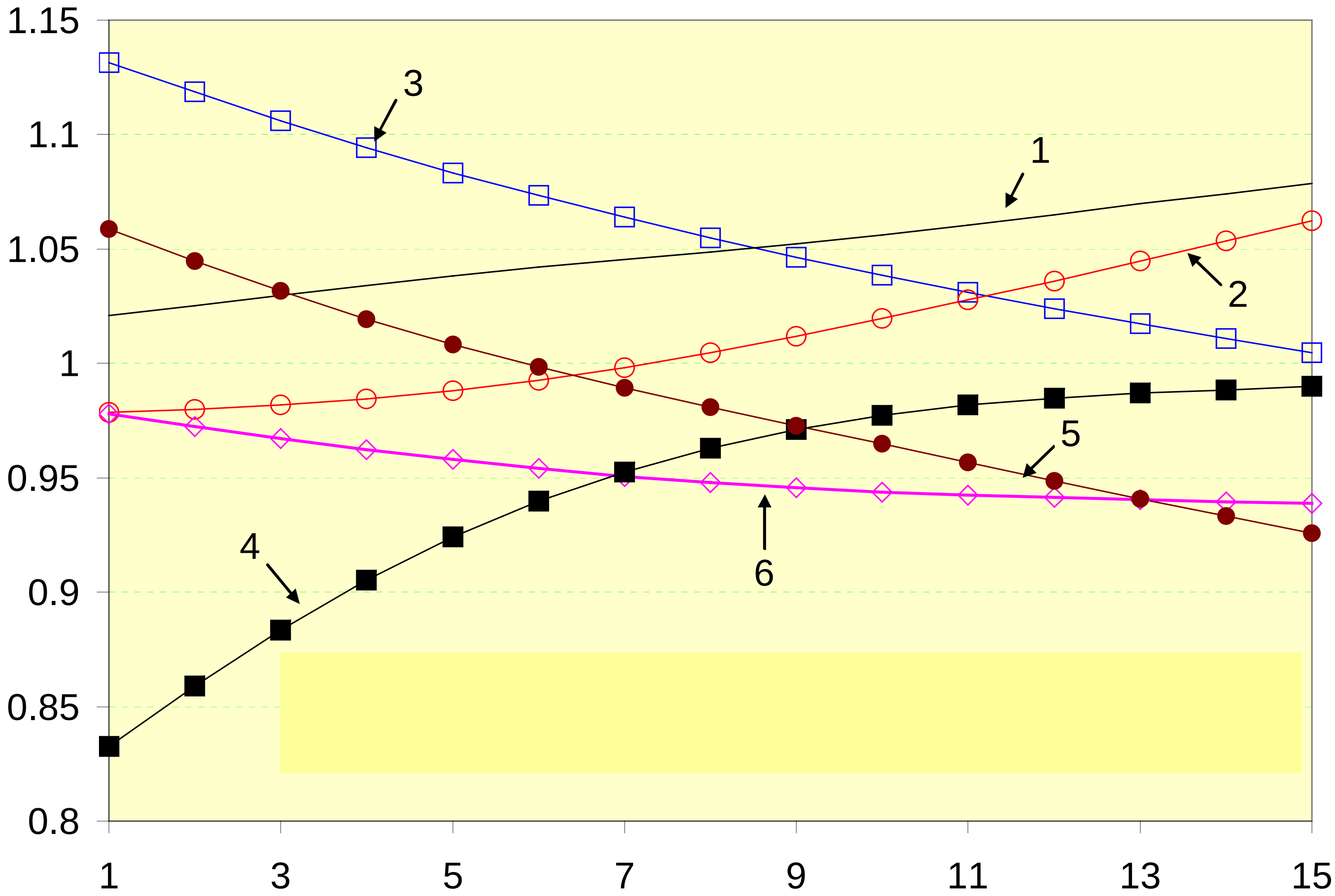
Clustering



Clustering

Step 1 Last observation ordering

Clustering



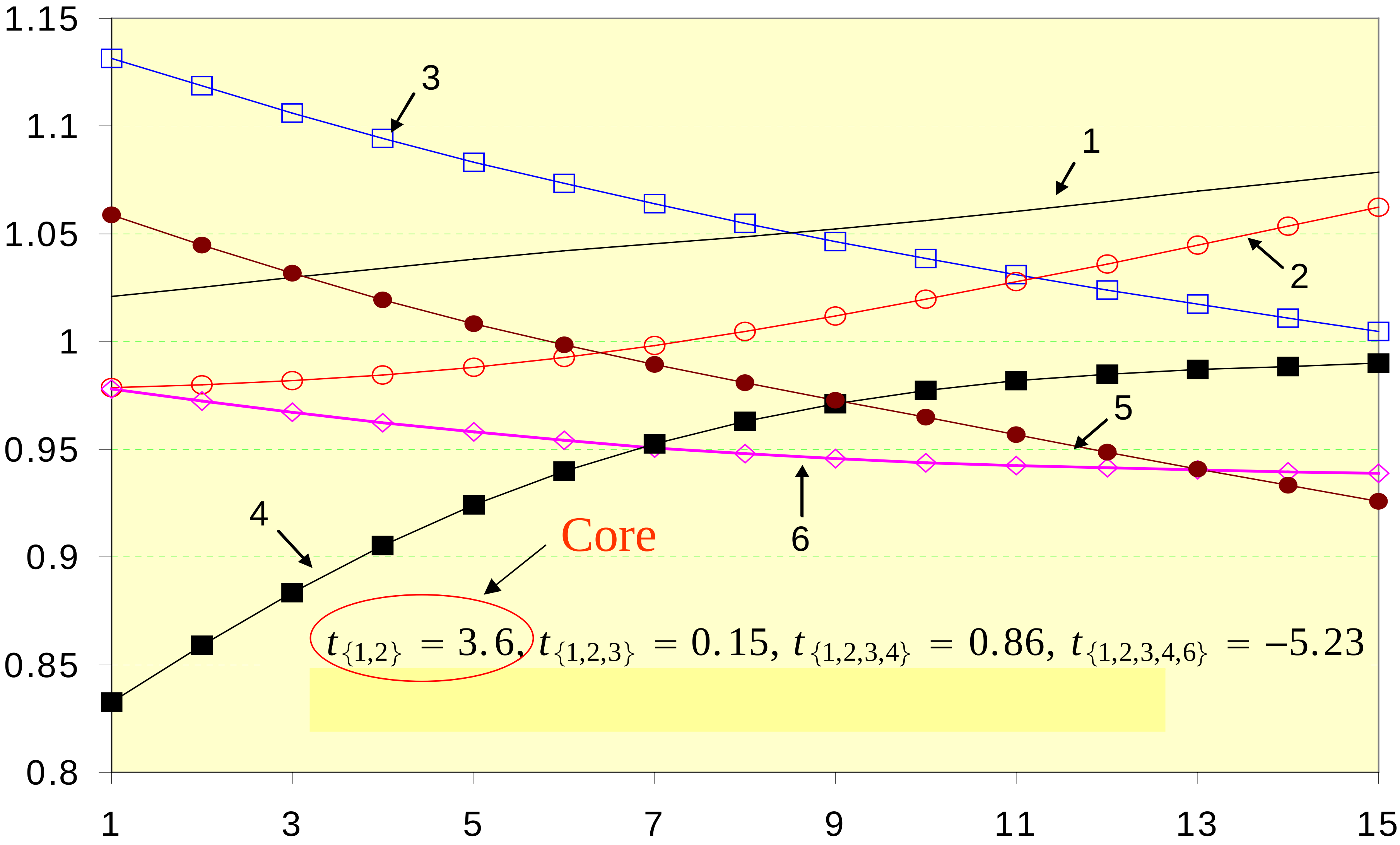
Clustering

Step 1 Last observation ordering

Step 2 Find core members, run the logt with $i = 1, \ldots, k$,

$$\left.\begin{array}{c} k^* = \arg\max_k \{t_k\} \\ \text{subject to } \min\{t_k\} > -1.65 \end{array}\right. , \; k \uparrow \text{ until } t_k < -1.65$$

Clustering



$t_{\{1,2\}} = 3.6,\ t_{\{1,2,3\}} = 0.15,\ t_{\{1,2,3,4\}} = 0.86,\ t_{\{1,2,3,4,6\}} = -5.23$

Clustering

Step 1 Last observation ordering

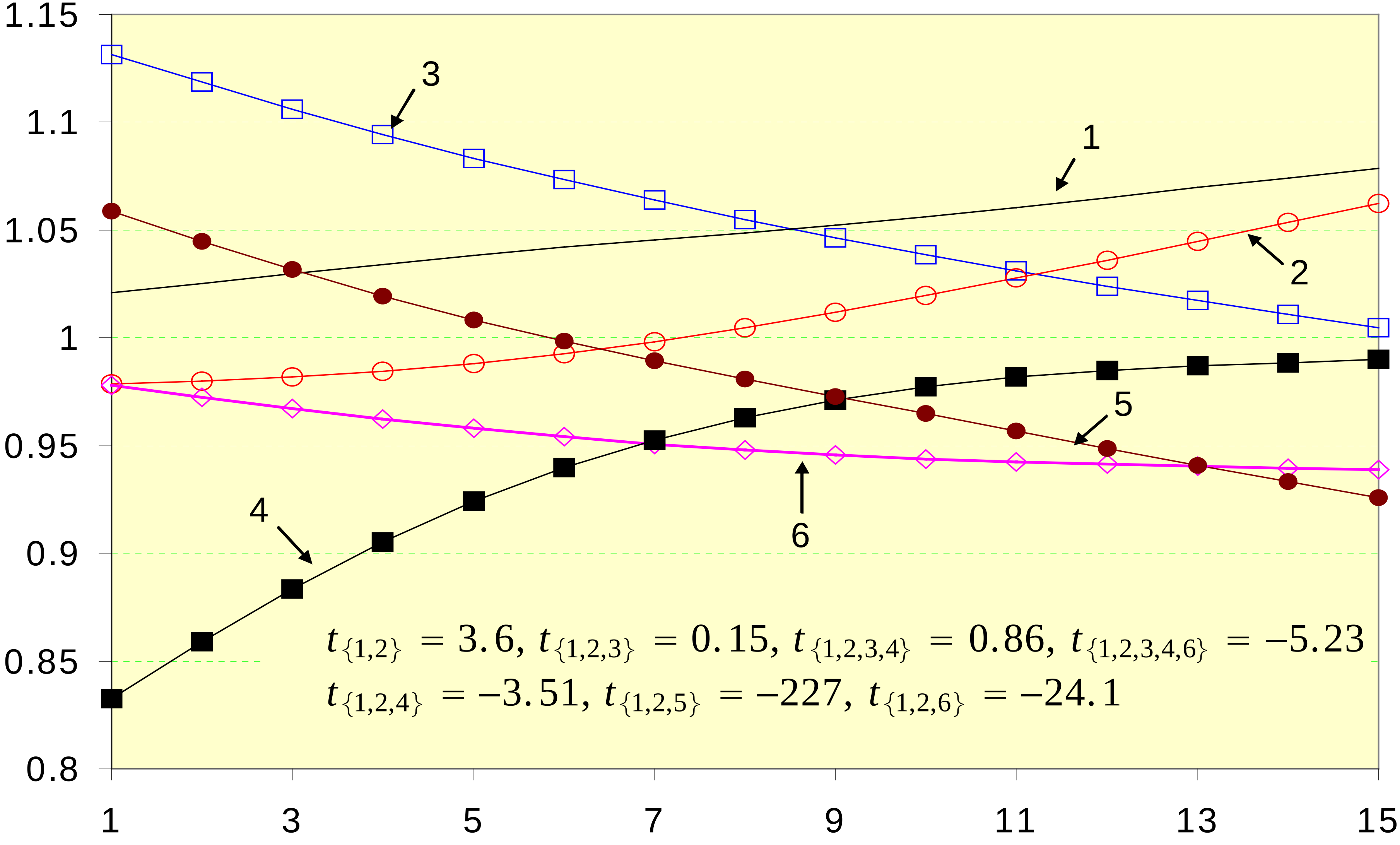
Step 2 Find core members, run the logt with $i = 1, \ldots, k$,

$$\left.\begin{array}{c} k^* = \arg\max_k \{t_k\} \\ \text{subject to } \min\{t_k\} > -1.65 \end{array}\right. , \; k \uparrow \text{ until } t_k < -1.65$$

Step 3 Run the $\log t$ reg with adding/dropping an individual

$$i \in \begin{cases} G_1 \text{ if } t_{\hat{b},i} \geq 0 \\ G_1^c \text{ if } t_{\hat{b},i} < 0 \end{cases}$$

Clustering



1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
1
3
5
7
9
11
13
15
1
2
3
4
5
6
$t_{\{1,2\}} = 3.6,\ t_{\{1,2,3\}} = 0.15,\ t_{\{1,2,3,4\}} = 0.86,\ t_{\{1,2,3,4,6\}} = -5.23$
$t_{\{1,2,4\}} = -3.51,\ t_{\{1,2,5\}} = -227,\ t_{\{1,2,6\}} = -24.1$

Clustering

Step 1 Last observation ordering

Step 2 Find core members

$$k^* = \arg\max_k \{t_k\}$$

$$\text{subject to } \min\{t_k\} > -1.65$$

Step 3 Run the $\log t$ reg with adding/dropping an individual at a time.

$$i \in \begin{cases} G_1 \text{ if } t_{\hat{b},i} \geq 0 \\ G_1^c \text{ if } t_{\hat{b},i} < 0 \end{cases}$$

Step 4 Run the $\log t$ test with G_1^c.

If $t_{G_1^c} < -1.65$, repeat Step 2 & 3 until $t_{G_s^c} \geq -1.65$

Absolute v.s. Relative

Relative	$\mathcal{H}_0 : b \geq 0$
Absolute	$\mathcal{H}_0 : b \geq 2\alpha$

α?

depending on the growth rate of μ_t

$$\alpha = \begin{cases} 1 \text{ if } \mu_t \text{ is } I(1) \text{ with a drift} \\ 0.5 \text{ if } \mu_t \text{ is } I(1) \text{ without a drift} \end{cases}$$

Repeat everything with $t_{\hat{b}} = \frac{\hat{b}-2\alpha}{s.e.}$

Monte Carlo Studies: Extreme Case

Finite Sample Performance of the log t test

Size: 5%

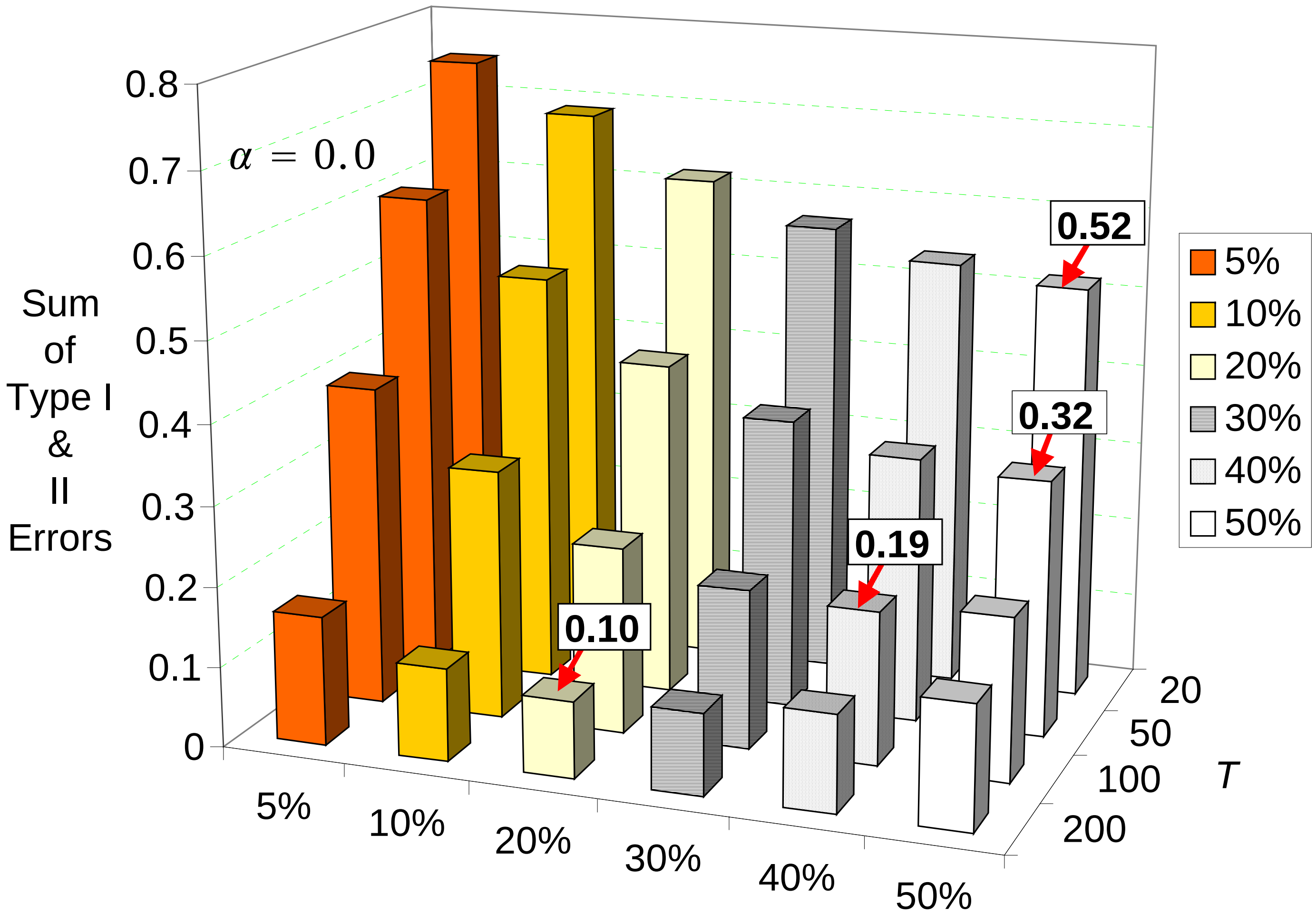
		$\rho \in [0,0.5]$				$\rho \in [0,0.9]$			
T	N	α=0.01	α=0.05	α=0.1	α=0.2	α=0.01	α=0.05	α=0.1	α=0.2
10	50	0.30	0.21	0.13	0.04	0.25	0.18	0.10	0.03
10	100	0.40	0.26	0.12	0.02	0.32	0.22	0.10	0.01
10	200	0.56	0.32	0.12	0.01	0.41	0.24	0.08	0.00

Nominal Power

				δ_1=1		
T	N	α	$\delta_i - U[1,2]$	δ_2=1.5	δ_2=1.2	δ_2=1.1
10	50	0.01	1.00	1.00	0.93	0.57
10	100	0.01	1.00	1.00	0.99	0.77
10	200	0.01	1.00	1.00	1.00	0.92
10	50	0.05	1.00	1.00	0.93	0.59
10	100	0.05	1.00	1.00	1.00	0.77
10	200	0.05	1.00	1.00	1.00	0.91

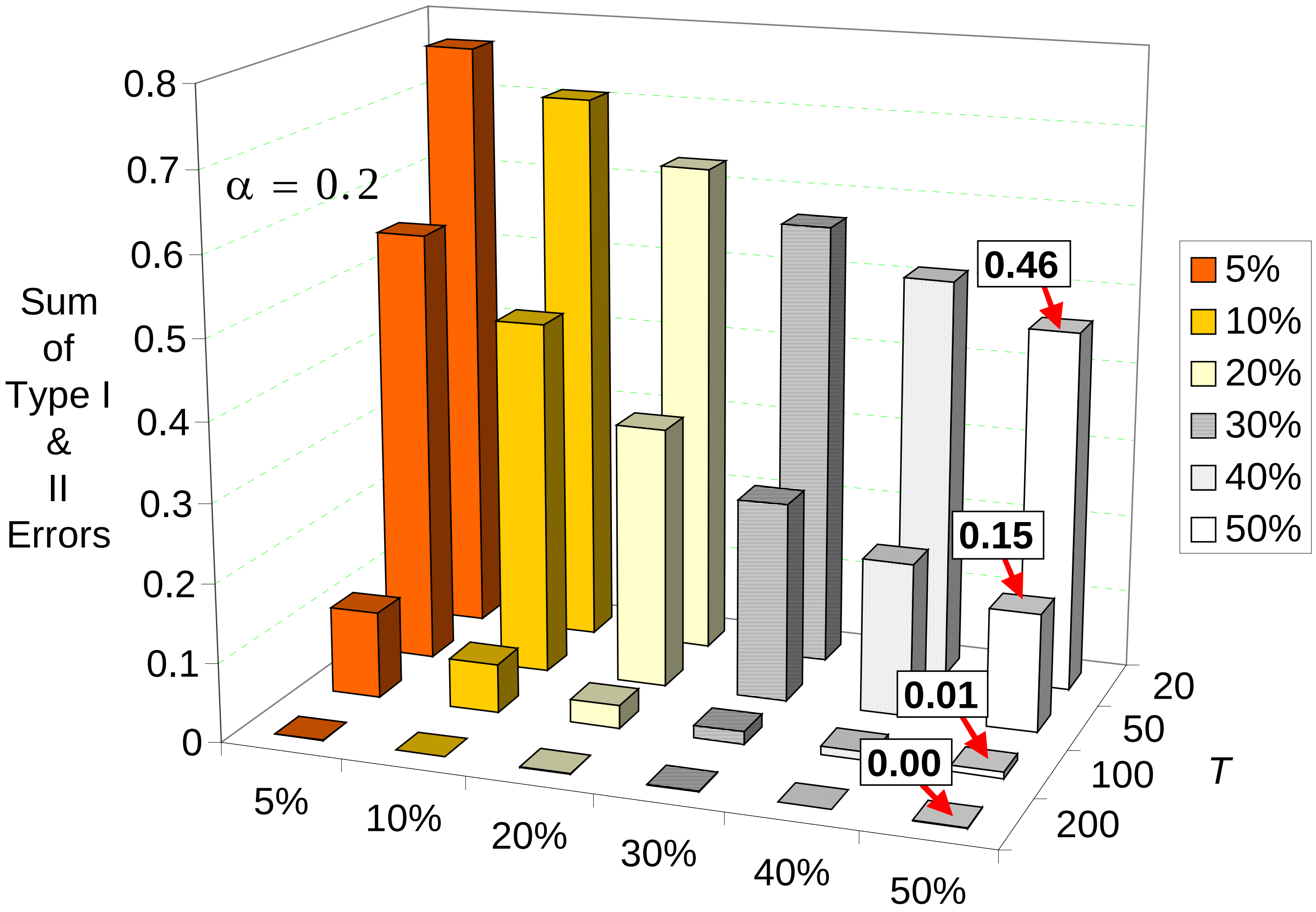
Monte Carlo Studies: Extreme Case (Clustering)

Failure Rates with $\alpha = 0.0$

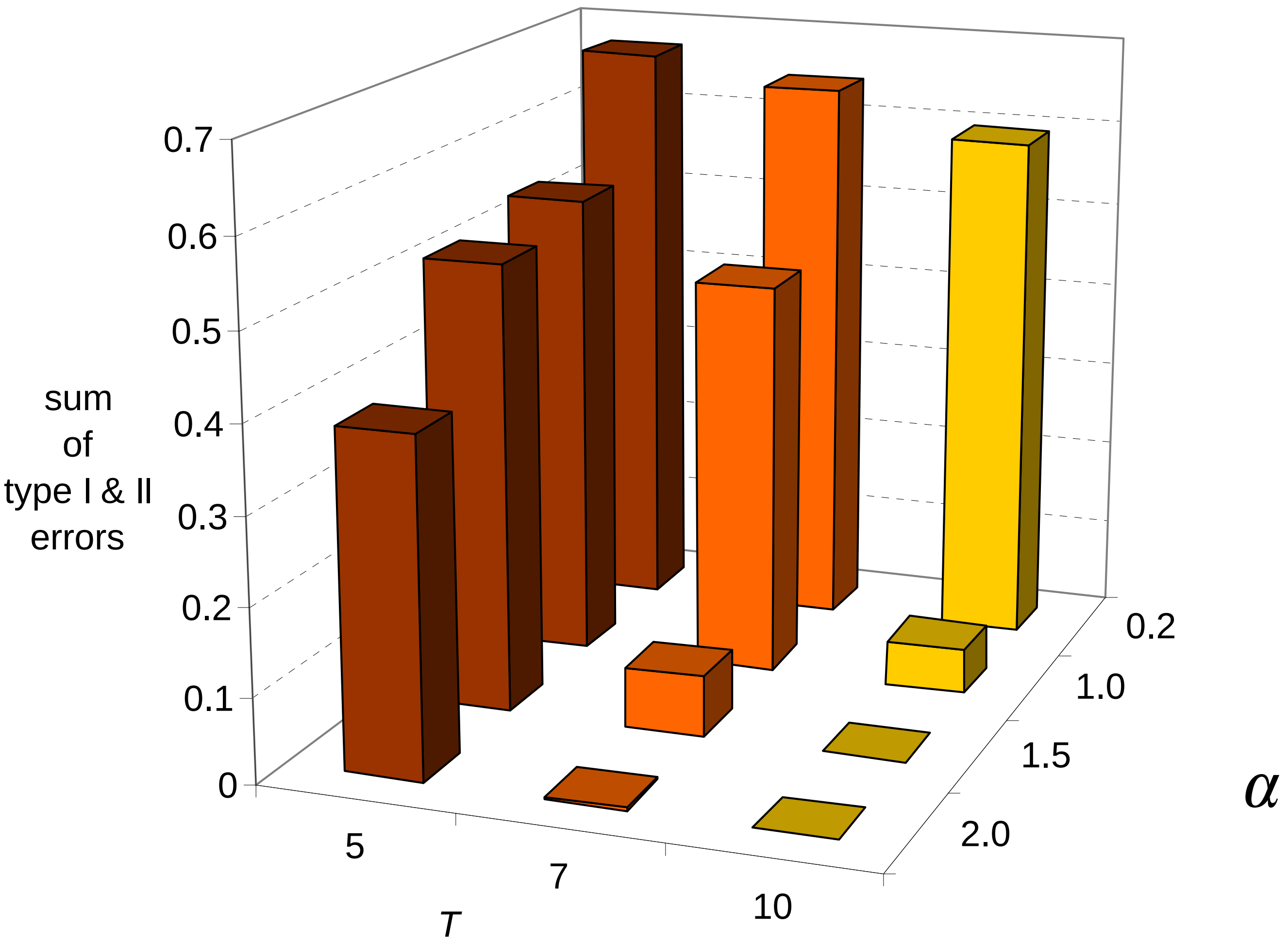


Monte Carlo Studies: Extreme Case (Clustering)

Failure Rates with $\alpha = 0.2$



Monte Carlo Studies: Moderate Case (Clustering): Sign Test: 50% significance level



Example 1: Cost of Living (COL) Index

1. Data: 19 Metropolitan U.S. Cities' CPI indices
2. CPI cannot be used to compare the COL across cities due to the base year problem.

$$\log P_{it}^{o} = \delta_{it}^{o} \log P_{t}^{o} + e_{it}$$

$$\log P_{it} = \log(P_{it}^{o}/P_{i1}^{o}) = \log P_{it}^{o} - \log P_{i1}^{o}$$

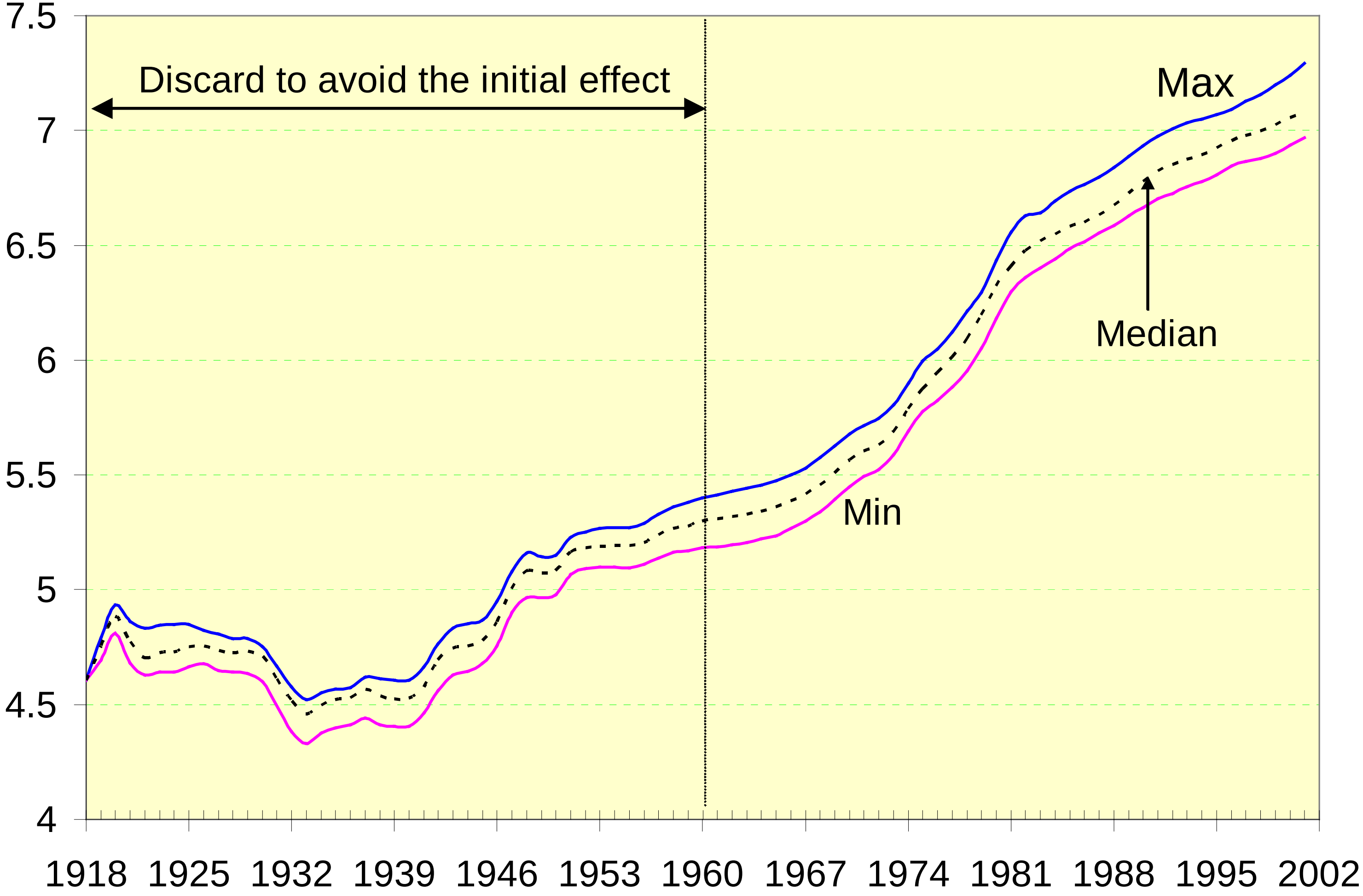
$$= \delta_{it}^{o} \log P_{t}^{o} - \delta_{i1}^{o} \log P_{1}^{o} + (e_{it} - e_{i1})$$

$$= \left(\delta_{it}^{o} - \delta_{i1}^{o} \frac{\log P_{1}^{o}}{\log P_{t}^{o}} + \frac{e_{it} - e_{i1}}{\log P_{t}^{o}} \right) \log P_{t}^{o}$$

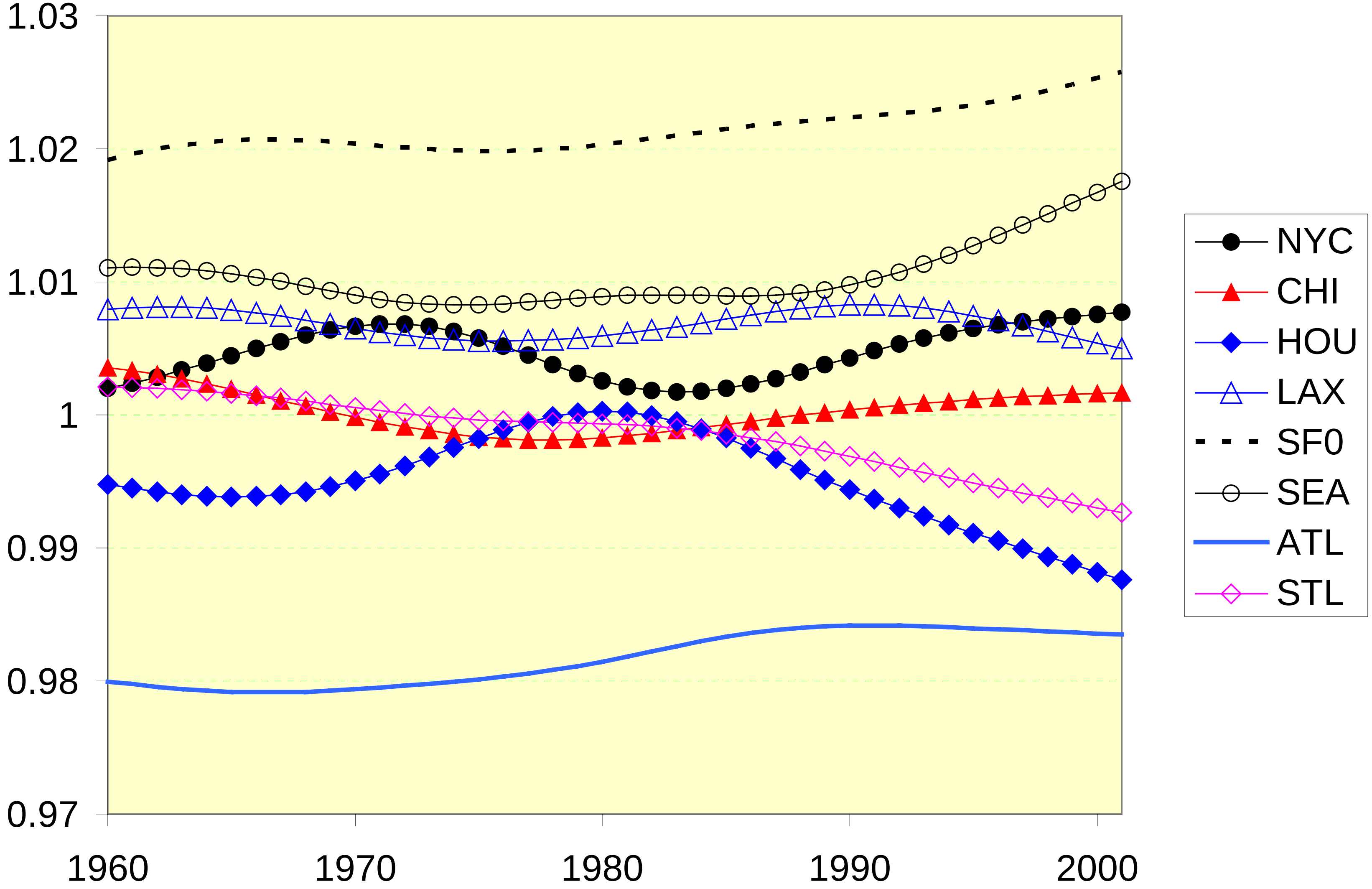
$$= \delta_{it} \log P_{t}^{o}$$

Example 1: Cost of Living (COL) Index

Min, Max, Median of 19 Consumer Price Indices



Example 1: Cost of Living (COL) Index

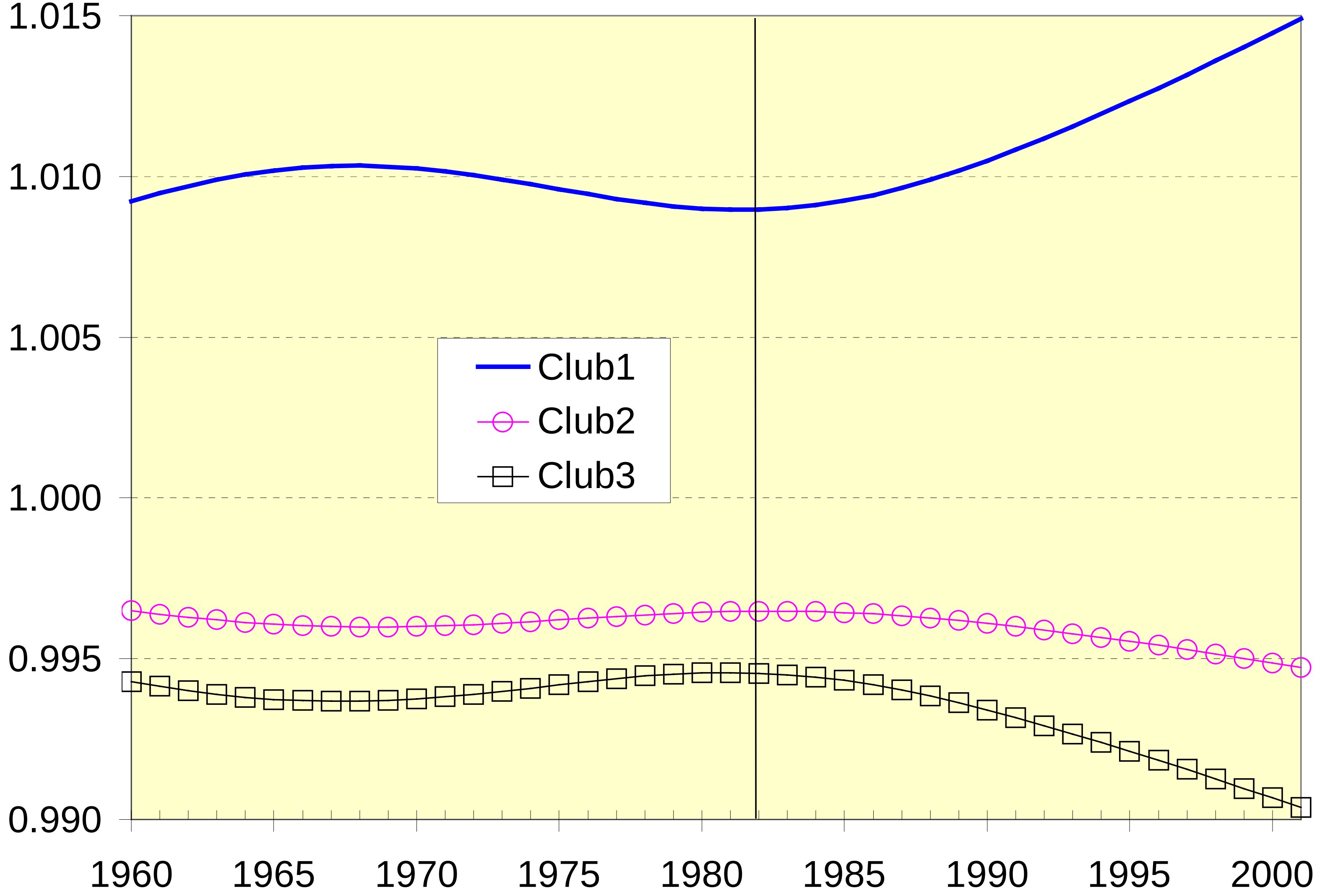


Example 1: Cost of Living (COL) Index

$$\log \frac{H_1}{H_t} - 2 \log \log t = \underset{(14.3)}{0.904} - \underset{(-51.4)}{0.98} \log t,$$

Last T Order	Name	t value Step1	t value Step2/3							
1	SF0	base	**core**	S_1	t_{S_1}					
2	SEA	**6.1**	**core**	S_1	=					
4	NYC	1.4	0.7	S_1	0.71		Step1	Step2/3		
3	CLE	-0.7	-0.7			CLE	base	**core**	S_2	t_{S_2} =
5	MIN	-7.8	-51.0			MIN	**1.0**	**core**	S_2	8.18
6	LAX		-12.2			LAX	-1.7	-1.7		
7	POR		-2.4			POR		5.3	S_2	
8	BOS		-3.7			BOS		13.9	S_2	
9	CHI		-14.9			CHI		6.1	S_2	
10	BAL		-28.8		$t_{S_1^c}$	BAL		-19.9		
11	PHI		-12.0		=	PHI		7.6	S_2	
12	PIT		-35.6		−54.6	PIT		-1.6		
13	CIN		-46.9			CIN		-18.1		
14	STL		-50.3			STL		-34.6		$t_{S_2^c}$ =
15	DET		-124.4			DET		-4.9		−0.68
16	WDC		-16.7			WDC		-12.3		
17	HOU		-134.6			HOU		-28.0		
18	KCM		-116.5			KCM		-14.1		
19	ATL		-20.7			ATL		-67.2		

Relative Transition Curves across Clubs



Example 2: International Risk Sharing

1. Data: WDI, 1975 to 2002, 38 annual obs. For 66 countries.
2. Model: Cochrane(1991); Kalemli-Ozcan, et al.(2003)

$$\log C_{it} = \delta_{it} \log C_t + e_{it}$$

$$H_0 : \delta_i = \delta, \forall i, \text{ and } \alpha \geq 0$$

$$\Delta \log C_{it} - \Delta \log C_{jt} =$$

$$(\Delta\delta_{it} - \Delta\delta_{jt}) \log C_t + (\delta_{it-1} - \delta_{jt-1}) \Delta \log C_t$$

$$+(\Delta e_{it} - \Delta e_{jt})$$

Example 2: International Risk Sharing

Club 1
-0.03(-0.70)

Club A
0.05(1.27)

Australia, *Austria* , Belgium, Canada, *Denmark* , Finland, France, *Hong Kong* , Ireland, Italy, *Japan* , Korea, Netherlands, *Norway* , Singapore, Sweden, *Switzerland* , *UK, USA*

China

Club B
0.04(0.67)

Chile, *Greece, New Zealand* , *Portugal, Spain* , Thailand

Club 2: 0.20(2.51)

Brazil, Colombia, Costa Rica, Dominican Rep., El Salvador, Gabon, Guatemala, Indonesia, Malaysia, Mexico, Morocco, Paraguay, Trinidad & Tobago, Tunisia, Uruguay

Club 3: 0.17(4.21)

Peru, Ecuador, Philippines, Algeria, Pakistan, Honduras, India, Lesotho

Club 4: 0.18(3.01)

Bangladesh, Benin, Burkina Faso, Cameroon, Congo, Rep., Cote d'Ivoire, Ghana, Kenya, Malawi, Nicaragua, Rwanda, Senegal, Togo, Zambia, Zimbabwe

Burundi,
Madagascar

Example 3:Growth Convergence

1. Data: PWT, 1960 to 1996, 37 annual obs. For 88 countries.
2. Model: Phillips and Sul (2006)

1960-1980: Club Convergence

Club 1

Israel, Barbados, Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Italy, Japan, Netherlands, NZ, Norway, Portugal, Spain, Sweden, Swiss, U.K, USA, Hong Kong, Korea, Singapore, Taiwan

Club 3

Chile, Thailand

Colombia, Costa Rica, Dominican Rep. El Salvador, Fiji, Guatemala, Iran, Jordan, Paraguay, Peru, Syria, Zimbabwe

Club 2

Botswana, Cyprus, Ireland, Malaysia, Mauritius, Romania, Trinidad &Tobago

Algeria, Argentina, Brazil, Ecuador, Mexico, Panama, South Africa, Turkey, Uruguay, Venezuela

Group 4

China, Indonesia

C. African Rep., Congo, D.R. Malawi, Mali, Mozambique, Niger, Zambia

Egypt, India, Jamaica, Pakistan, P.N.G, Sri Lanka, Bangladesh, Benin, Bolivia, Cameroon, Gambia, Ghana, Guyana, Honduras, Lesotho, Nepal, Nicaragua, Philippines, Rwanda, Senegal, Togo, Uganda

1960-1996: Evolution of Convergence Clubs

Club 1

Israel, Barbados, Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Italy, Japan, Netherlands, NZ, Norway, Portugal, Spain, Sweden, Swiss, U.K, USA, Hong Kong, Korea, Singapore, Taiwan

Club 2

Botswana, Cyprus, Ireland, Malaysia, Mauritius, Romania, Trinidad &Tobago

Algeria, Argentina, Brazil, Ecuador, Mexico, Panama, South Africa, Turkey, Uruguay, Venezuela

Club 3

Chile, Thailand

Colombia, Costa Rica, Dominican Rep. El Salvador, Fiji, Guatemala, Iran, Jordan, Paraguay, Peru, Syria, Zimbabwe

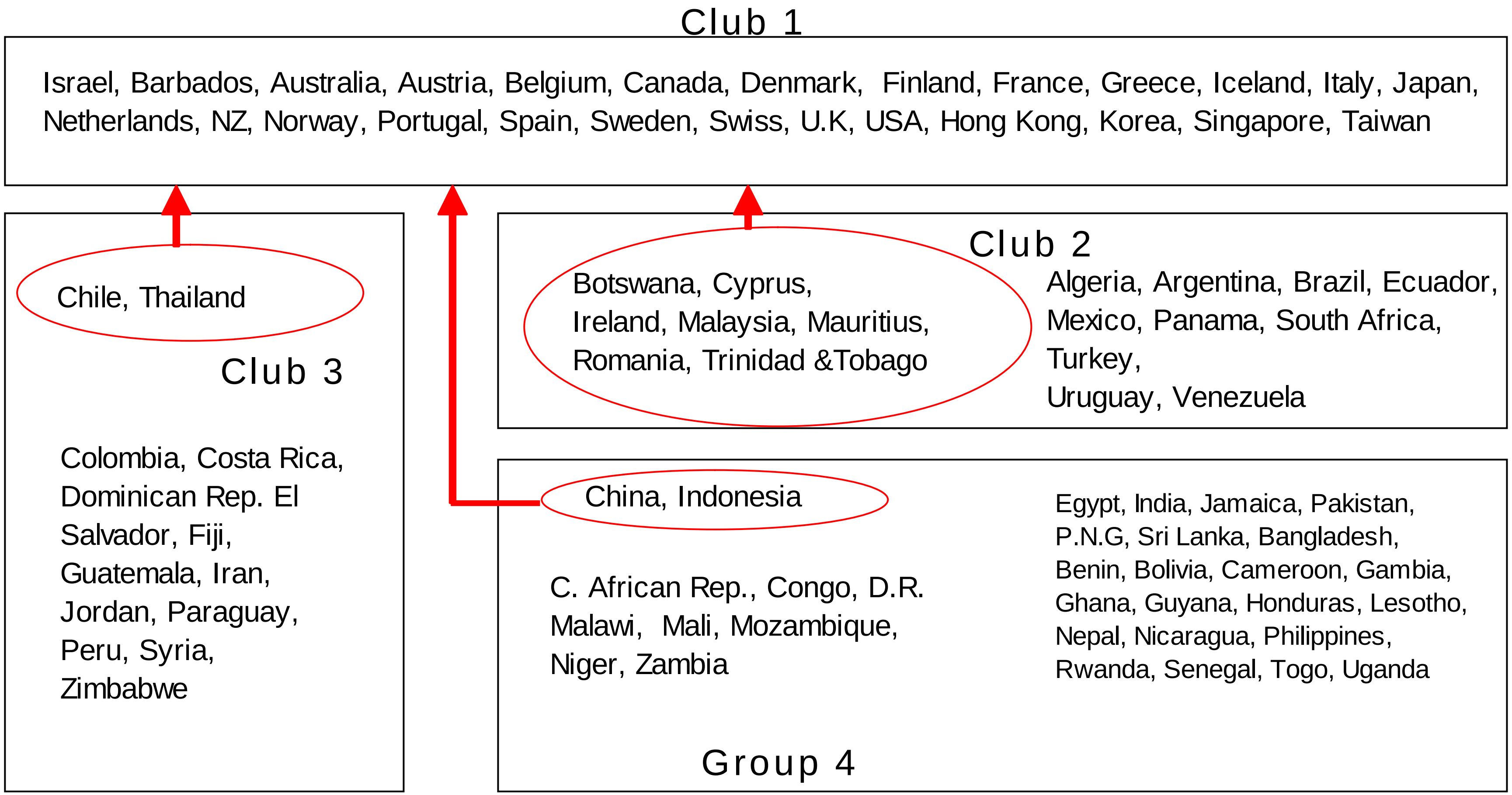
Group 4

China, Indonesia

C. African Rep., Congo, D.R. Malawi, Mali, Mozambique, Niger, Zambia

Egypt, India, Jamaica, Pakistan, P.N.G, Sri Lanka, Bangladesh, Benin, Bolivia, Cameroon, Gambia, Ghana, Guyana, Honduras, Lesotho, Nepal, Nicaragua, Philippines, Rwanda, Senegal, Togo, Uganda

1960-1996: Evolution of Convergence Clubs



1960-1996: Evolution of Convergence Clubs

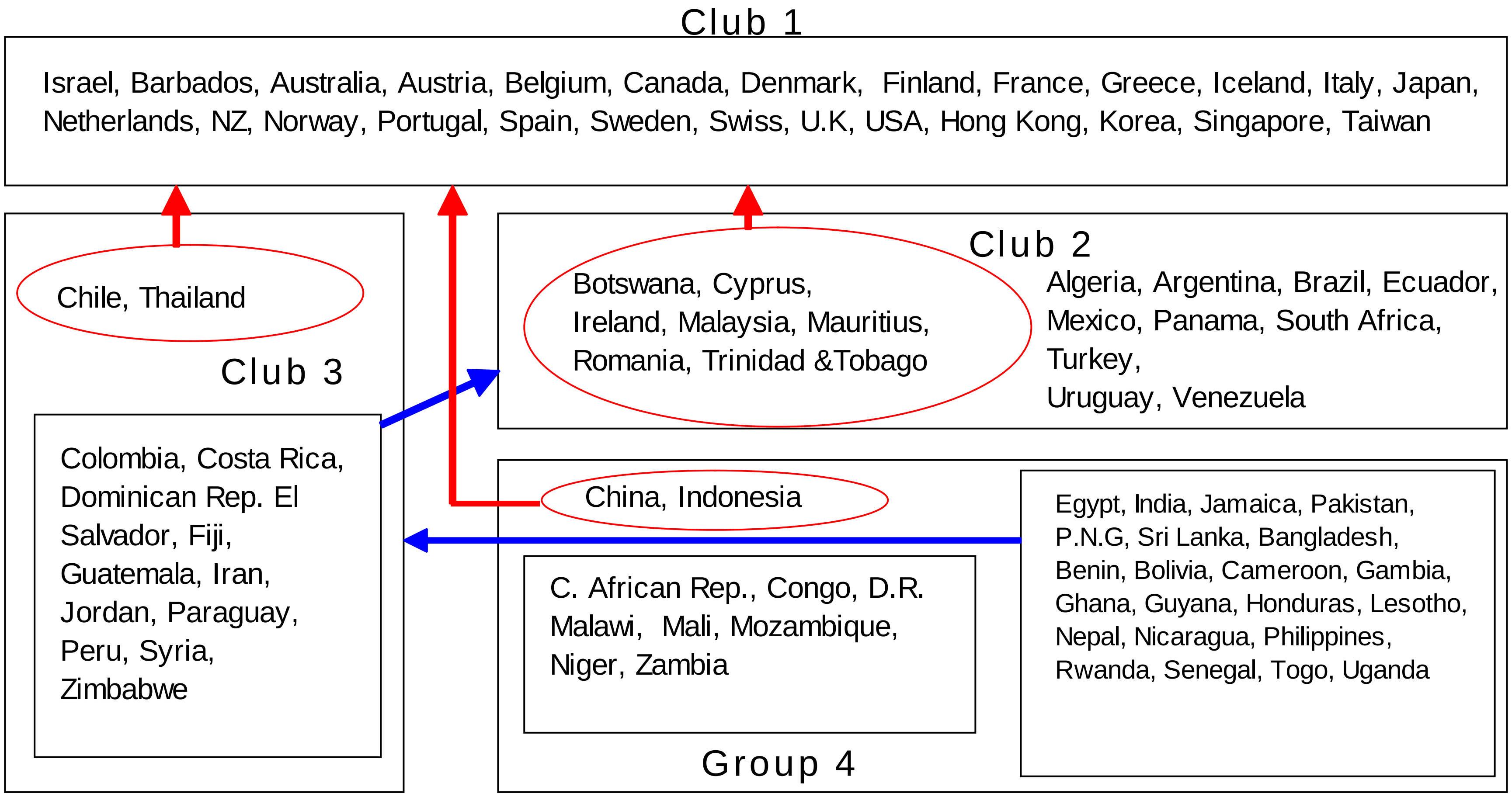


Table 2: **Convergence Clubs and Convergence between Clubs**

1960-96		1960-1980	
Club 1 (37)[1]	0.195(0.040)	Club 1 (26)	0.182(0.055)
Club 2 (28)	0.144(0.052)	Club 2 (17)	0.275(0.074)
Club 3 (16)	-0.028(0.075)	Club 3 (14)	0.224(0.095)
Group 4 (7)	-0.182(0.087)*	Group 4 (31)[2]	-0.688(0.022)**
Club 1+2(19+14)	0.013(0.013)	Club 1+2(13+8)	0.109(0.043)
Club 2+3(14+8)	0.122(0.058)	Club 2+3(9+7)	0.330(0.080)

Notes: 1) The numbers in parentheses are the number of countries.

2) This residual group shows evidence of several tiny convergent subgroups.

3) The affix '*' (respectively '**') indicates rejection of the null hypothesis of the convergence at the 5% (1%) level.

Summary

1. Data Dependent Clustering method: Simple but Powerful
2. Can use when the common factor has a growing component.
3. Form (AC or RC) subgroups easily.

μ_t