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Transmission capacity for dual-hop relaying in wireless ad hoc networks

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Abstract

To account for *randomly* distributed nodes in a wireless ad hoc network, the transmission capacity is defined as the number of successful transmissions taking place in the network per unit area under an outage constraint. In this paper, we analyze the transmission capacity for dual-hop relaying in a wireless ad hoc network in the presence of *both* cochannel interference and thermal noise, where interferers are spatially distributed following a Poisson distribution. Specifically, we first present the exact outage probability for amplify-and-forward and decode-and-forward protocols in a Poisson field of interferers without neglecting noise at all nodes. We then derive the transmission capacity of such networks, which determines the maximum allowable density of transmitting nodes for each relay strategy at a specified outage probability and data rate. Numerical results demonstrate that the dual-hop relaying is still beneficial in terms of the transmission capacity in wireless ad hoc Poisson networks.

Keywords: amplify-and-forward (AF), decode-and-forward (DF), dual-hop relay, interference, outage probability, Poisson network, transmission capacity

1 Introduction

Cooperative relay communication has recently drawn considerable attention to increase network coverage and reliability [1,2]. For a decode-and-forward (DF) strategy, a relay node decodes the received signal and transmits it after re-encoding, whereas the relay simply retransmits a scaled version of the received signal in amplify-and-forward (AF) mode [3,4]. Most of the previous work has focused on noise-limited fading environments, such as Rayleigh fading with additive white Gaussian noise (AWGN). While such work made great strides toward understanding the potential of relay networks, it dealt mainly with ideal configurations with no interference. However, network interference is inevitable in practical wireless networks due to spectral reuse. Motivated by this, relay networks have been recently studied in the presence of cochannel interference [5-7]. In Krikidis et al. [5], the asymptotic performance was analyzed under the interference scenario only at relays. The fixed-gain AF relaying in an interference network was investigated in Zhong et al. [6], where interference at the relay and

thermal noise at the destination were ignored for analytical tractability. The exact outage probability for DF relaying was further derived in Si et al. [7] accounting for multiple interferers and noise at both the relay and the destination. In all these work, the locations of network nodes are *deterministic*–without spatial randomness.

To treat the capacity of a decentralized ad hoc wireless network, the transport capacity, defined as the product of the end-to-end sum throughput and distance, has been introduced due to the difficulty in determining the capacity region of a large ad hoc network [8]. More recently, using the stochastic geometry framework, the transmission capacity has been proposed as the maximum density of active transmitting nodes per unit area to satisfy an outage constraint at a given data rate when interferers are randomly scattered and uncoordinated [9]. For a variety of scenarios, this notion of transmission capacity has been used successfully to characterize the physical layer on the ad hoc network [10-14]. A new metric akin to the transmission capacity, called the random access transport capacity, has been also developed for the end-to-end throughput in multi-hop transmission over some distance [15].

In this paper, we consider dual-hop relaying with DF and AF protocols in a wireless ad hoc network in the

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presence of both interference and noise. Each interfering node in the network independently transmits data and is randomly distributed in a Poisson law over a plane. The motivation behind imposing Poisson interference and noise is threefold: (i) many previous works neglected either interference or noise in analysis of a relay network; (ii) previous work on relay networks with interference considered only a limited number of interferers (iii) randomly distributed nodes(i.e., interferers) allowing a Poisson distribution are suitable for realistic communication model. To the best of our knowledge, relay network to qualify interference using stochastic geometry has not been addressed before. Hence, we analyze exact outage probability of a dual-hop relay with both noise and interference in a wireless ad hoc network. Unlike previous work in a relay network, when the node locations are distributed as a Poisson point process (PPP), we need to analyze relay networks using a metric for decentralized wireless network, termed transmission capacity. Furthermore, as transmission capacity has considered single-hop transmission without noise, we focus on the transmission capacity of dual-hop relay with noise and specific relaying protocols. It is worth of finding maximum successful transmitting nodes per unit area to satisfy outage probability and data rate from transmission capacity of dual-hop relay. Although both noise and interference with Poisson distribution are considered, it is noted that dual-hop relay is still beneficial in terms of transmission capacity.

The remainder of the paper is organized as follows. Section 2 presents the system model based on location of nodes, channel models, and distribution of interference. We derive outage probability analysis of DF and AF strategies in Section 3. Based on the outage probability of dual-hop relay, transmission capacity is derived in Section 4. Section 5 compares simulation results with analytical results. Finally, Section 6 concludes the paper.

2 System model

We consider a wireless ad hoc network consisting of a source, a relay, and a destination with no direct source-to-destination link. All nodes have a single antenna operating in a common frequency band and are in half-duplex mode.

2.1 Node locations

The sets of nodes in a relay network are denoted as $\mathcal{S} := \{Sx_n, n \in \mathbb{N}\}$ for sources, $\mathcal{R} := \{Rx_n, n \in \mathbb{N}\}$ for relays, and $\mathcal{D} := \{Dx_n, n \in \mathbb{N}\}$ for destinations. Now, allow the set of nodes to be divided into two different pairs for each hop transmission: (Sx_n, Rx_n) and (Rx_n, Dx_n), where the distance between two nodes in a pair is d_{SR} and d_{RD}, respectively. In a dual-hop relay network,

sources and relays are also interferers in each hop. In particular, all nodes independently transmit and distribute their locations following a Poisson distribution. Let the location of Sx_n be S_n, Rx_n be R_n, and Dx_n be D_n. The source set $\Phi_S := \{S_n\}$ generating interference at the first time slot is modeled as a homogeneous PPP on the plane of intensity λ_1. Since R_n is randomly located at a fixed distance d_{SR} from S_n, the set $\Upsilon_{\mathcal{R}} := \{R_n\}$ also follows a homogeneous PPP on a two-dimensional plane with intensity λ_2. As destination node D_n is placed at d_{RD} away from the relay node R_n, $\Psi_{\mathcal{D}} := \{D_n\}$ is also a homogeneous PPP with intensity λ_3.

In this paper, we consider reference source Sx_0, reference relay Rx_0, and reference destination Dx_0 in a decentralized wireless network that transmits desirable data in the Φ_S, Υ_R, and Ψ_D. According to each hop transmission, reference nodes can be bounded as (Sx_0, Rx_0) and (Rx_0, Dx_0). For the (Sx_0, Rx_0) pair in the first hop, the relay node Rx_0 is located at the origin and source node Sx_0 is placed at d_{SR} meters away from the relay node Rx_0. For the (Rx_0, Dx_0) pair in the second hop, we place the relay node Rx_0 d_{RD} meters away from the destination node Dx_0, where Dx_0 is located at the origin.

2.2 Channel model

In this paper, we consider a channel model with small-scale fading and large-scale path loss for a practical situation. Therefore, the square of instantaneous channel response between any pair of nodes located at x_1 and x_2 can be rewritten as

$$|h_{12}|^2 = F_{12} \cdot l(\|x_1 - x_2\|), \qquad (1)$$

where F_{12} captures the small-scale fading which obeys a Rayleigh fading model and $l(\|x_1 - x_2\|) = \|x_1-x_2\|^{-\alpha}$ characterizes the effect of large-scale path loss following the power law with path loss exponent α. We assume that channel responses for all nodes are independent and quasi-static.

2.3 Distribution of aggregate interference

In this section, we characterize the total interference power measured at the origin, as follows

$$I = \sum_{i \in \Phi} |h_i|^2 P_i \qquad (2)$$

where h_i is a instantaneous channel response of interferer i, Φ is a homogenous PPP with intensity λ, and P_i is interference power.

The moment generating function (*MGF*) of total interference power using method in Haenggi and Ganti [16], denoted by $\phi_I(s) \triangleq \bullet \{e^{-sI}\}$, can be written as

$$\phi_I(s) = \exp\left(-\lambda c_d \Gamma(1+\delta)\gamma(1-\delta, r)P_i^\delta s^\delta\right) \quad (3)$$

where λ is intensity of interferers, c_d is the volume of d-dimensional unit ball, r is the radius of finite area located interferers, and $\delta \triangleq d/\alpha$. Also, $\Gamma(x)$ is the Gamma function $\int_0^\infty t^{x-1}e^{-t}dt$ and $\gamma(s,x)$ is the lower incomplete gamma function $\int_0^x t^{s-1}e^{-t}dt$.

3 Outage performance analysis

3.1 Decode-and-forward relaying

We consider a dual-hop relay system that consists of reference source, relay, and destination nodes in a Poisson network as shown in Figure 1. Other nodes except the reference nodes are regarded as interferers. We consider interferers in a finite area with specific radius, because nodes located far from a receiver cannot be an interferer. In the first hop, relay nodes receive signal transmitted from source nodes and the received signal y_R at a reference relay node Rx_0 can be expressed as

$$y_R = \mathrm{h}_{SR}\mathrm{s}_0 + \sum_{i \in \Phi_S} \mathrm{g}_i\mathrm{s}_i + \mathrm{n}_R \quad (4)$$

where s_0 is the transmitted symbol with $E[|s_0|^2] = P_S$, h_{SR} is the channel between the reference source and the reference relay, and n_R is an additive white Gaussian noise with an average power of σ_R^2. In addition, the relay node Rx_0 of the reference pair is affected by its interferes s_i located in a homogenous PPP Φ_S. Average transmit power of an interferer is $E[|s_i|^2] = P_{iR}$, and $\{g_i\}_{i \in \Phi_S}$ are the channels from the interferer i to the reference relay Rx_0. Thus, the received signal to interference-plus-noise ratio (SINR) at the relay can be written as

$$\gamma_R^{\mathrm{DF}} = \frac{P_S|\mathrm{h}_{SR}|^2}{\sum_{i \in \Phi_S} |\mathrm{g}_i|^2 P_{iR} + \sigma_R^2}. \quad (5)$$

On the other hand, at the reference destination Dx_0, a transformed copy of the received signal at the relay, $\mathcal{T}(\gamma_R)$, is then transmitted to the destination node, which suffers from the other relay's transmission $\{w_j\}_{j \in \Upsilon_R}$ located in a homogeneous PPP Υ_R, each average transmitting power of an interferer j is $E[|w_j|^2] = P_{jD}$. Thus, the received signal at the destination Dx_0 is given by

$$y_D = \mathrm{h}_{RD}\mathcal{T}(\gamma_R) + \sum_{j \in \Upsilon_R} \mathrm{I}_j\mathrm{w}_j + \mathrm{n}_D \quad (6)$$

where h_{RD} is the channel between the reference relay and the reference destination, n_D is an additive white Gaussian noise with an average power of σ_D^2, and $\{l_j\}_{j \in \Phi}$ are the channels from the interferer j to the reference destination. In DF, the relay fully decodes the message s_0 based on the received signal, y_R, and then transmits the re-encoded message $\mathcal{T}(\gamma_R) = s_r$ with a power of P_R

to the destination terminal. Thus, the received SINR at the destination can be written as

$$\gamma_D^{\mathrm{DF}} = \frac{P_R|\mathrm{h}_{RD}|^2}{\sum_{j \in \Upsilon_R} |\mathrm{I}_i|^2 P_{jD} + \sigma_D^2}. \quad (7)$$

First, to compute cumulative density function (CDF) and probability density function (PDF) of γ_R^{DF}, we define $\mathrm{P} \sim P_S|h_{SR}|^2$ and $\mathrm{Q} \sim \sum_{\Phi_S} |g_i|^2 P_{iR}$. Then, the CDF and PDF of P can be written as

$$F_{\mathrm{P}}(p) = 1 - \exp\left(-\frac{d_{SR}^\alpha}{P_S}p\right), \quad p \geq 0$$

$$p_{\mathrm{P}}(p) = \frac{d_{SR}^\alpha}{P_S}\exp\left(-\frac{d_{SR}^\alpha}{P_S}p\right), \quad p \geq 0. \quad (8)$$

where d_{SR} is distance between reference source and reference relay and α is path loss exponent. As the main physical range of interest for path loss exponent in wireless networks is $2 < \alpha < 5$, we consider path loss exponent $\alpha = 4$ for a relatively lossy environment in an urban area. From (3), the MGF of total interference power $\mathrm{Q} \sim \sum_{\Phi_S} |g_i|^2 P_{iR}$ with two-dimensional plane ($\delta = \frac{1}{2}$ case and $c_d = \pi$) can be written as

$$\phi_Q(s) = \exp\left(-\lambda_1\pi\Gamma\left(\frac{3}{2}\right)\gamma\left(\frac{1}{2}, x\right)\sqrt{P_{iR}}s^{\frac{1}{2}}\right) = \exp\left(-\frac{\lambda_1\pi^2\mathrm{erf}\left(\sqrt{r_1}\right)\sqrt{P_{iR}}}{2}s^{\frac{1}{2}}\right) \quad (9)$$

where $\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$ and $\gamma\left(\frac{1}{2}, x\right) = \sqrt{\pi}\,\mathrm{erf}\left(\sqrt{x}\right)$ with error function $\mathrm{erf}(x) \triangleq \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$. The CDF and PDF of Q from (9) using the Levy distribution [16] with $c = \frac{\lambda_1^2\pi^4\left(\mathrm{erf}\left(\sqrt{r_1}\right)\right)^2 P_{iR}}{8}$ and $M = 0$ can be expressed as
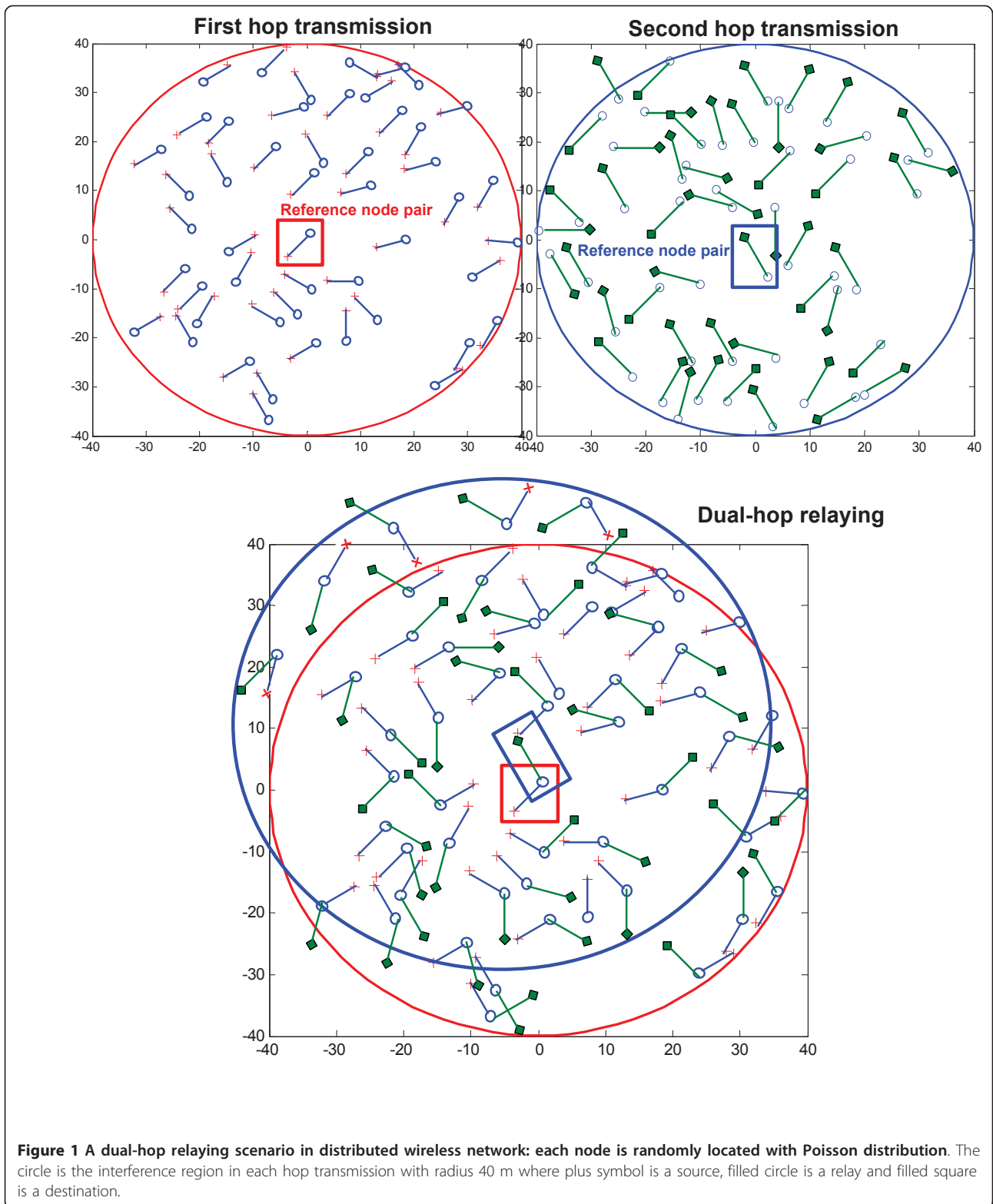
$$F_Q(q) = 1 - \mathrm{erf}\left(\frac{\pi^2\mathrm{erf}\left(\sqrt{r_1}\right)\lambda_1 P_{iR}^{\frac{1}{2}}}{4\sqrt{q}}\right), \quad q \geq 0 \quad (10)$$

$$p_Q(q) = \frac{\mathrm{erf}\left(\sqrt{r_1}\right)\lambda_1 P_{iR}^{\frac{1}{2}}}{4}\left(\frac{\pi}{q}\right)^{3/2}\exp\left(-\frac{\left[\mathrm{erf}\left(\sqrt{r_1}\right)\right]^2\pi^4\lambda_1^2 P_{iR}}{16q}\right), \quad q \geq 0 \quad (11)$$

where λ_1 is intensity of source nodes and r_1 denotes the radius of interference area.

Theorem 1 (CDF and PDF) *The CDF and PDF of received SINR at the relay* $\mathrm{X} \sim \gamma_R^{\mathrm{DF}}$ *are given by*

$$F_{\mathrm{X}}(x) = 1 - \exp\left[-\left(\frac{d_{SR}^\alpha\sigma_R^2}{P_S}x + \frac{\mathrm{erf}\left(\sqrt{r_1}\right)\pi^2\lambda_1}{2}\sqrt{\frac{d_{SR}^\alpha P_{iR}}{P_S}}x\right)\right], \quad x \geq 0$$

$$p_{\mathrm{X}}(x) = \left(\frac{d_{SR}^\alpha\sigma_R^2}{P_S} + \frac{\mathrm{erf}\left(\sqrt{r_1}\right)\pi^2\lambda_1\sqrt{\frac{d_{SR}^\alpha P_{iR}}{P_S}}}{4\sqrt{P_S x}}\right)$$

$$\times \exp\left[-\left(\frac{d_{SR}^\alpha\sigma_R^2}{P_S}x + \frac{\mathrm{erf}\left(\sqrt{r_1}\right)\pi^2\lambda_1}{2}\sqrt{\frac{d_{SR}^\alpha P_{iR}}{P_S}}x\right)\right], \quad x \geq 0$$

Figure 1 A dual-hop relaying scenario in distributed wireless network: each node is randomly located with Poisson distribution. The circle is the interference region in each hop transmission with radius 40 m where plus symbol is a source, filled circle is a relay and filled square is a destination.

Proof See Appendix A

Corollary 1 *Using the same method, the CDF and PDF of the received SINR at the destination* $Y \sim \gamma_D^{DF}$ *can be written as*

$$F_Y(\gamma) = 1 - \exp\left[-\left(\frac{d_{RD}^\alpha \sigma_D^2}{P_R} \gamma + \frac{\mathrm{erf}(\sqrt{r_2}) \pi^2 \lambda_2}{2} \sqrt{\frac{d_{RD}^\alpha P_{iD}}{P_R}} \gamma \right) \right], \quad \gamma \geq 0 \quad (12)$$

$$p_Y(\gamma) = \left(\frac{d_{RD}^\alpha \sigma_D^2}{P_R} + \frac{\mathrm{erf}(\sqrt{r_2}) \pi^2 \lambda_2 d_{RD}^\alpha P_{iD}}{\sqrt{4 P_R \gamma}} \right)$$
$$\times \exp\left[-\left(\frac{d_{RD}^\alpha \sigma_D^2}{P_R} \gamma + \frac{\mathrm{erf}(\sqrt{r_2}) \pi^2 \lambda_2}{2} \sqrt{\frac{d_{RD}^\alpha P_{iD}}{P_R}} \gamma \right) \right], \quad \gamma \geq 0 \quad (13)$$

where d_{RD} *is the distance between relay and destination and* r_2 *is radius of finite area with intensity of interferers* λ_2.

The end-to-end *SINR* of dual-hop DF relaying γ_{eq} can be written as

$$\gamma_{eq} - \min\left[\gamma_R^{DF}, \gamma_D^{DF} \right]. \quad (14)$$

Then, the outage probability can be represented as

$$P_{out}^{DF}(R) = \bullet\left\{ \frac{1}{2} \log_2(1 + \gamma_{eq}) < R \right\} \quad (15)$$

where R is end-to-end spectral efficiency.

Finally, the outage probability of DF relaying can be expressed as

$$P_{out}^{DF}(\beta) = F_X(\beta) + F_Y(\beta) - F_X(\beta) F_Y(\beta)$$
$$= 1 - \exp\left[-\left\{ \left(\frac{d_{SD}^\alpha \sigma_R^2}{P_S} + \frac{d_{RD}^\alpha \sigma_D^2}{P_R} \right) \beta \right.\right.$$
$$\left.\left. + \frac{\pi^2}{2} \left(\mathrm{erf}(\sqrt{r_1}) \lambda_1 \sqrt{\frac{d_{SR}^\alpha P_{iR}}{P_S}} + \mathrm{erf}(\sqrt{r_2}) \lambda_2 \sqrt{\frac{d_{RD}^\alpha P_{iD}}{P_R}} \right) \sqrt{\beta} \right\} \right], \quad \beta \geq 0 \quad (16)$$

where β is target SINR $\beta = 2^{2R} - 1$

3.2 Amplify-and-forward relaying

In the first hop, the received signal at the reference relay of AF relaying is identical to (4). The received signal of AF relaying at the destination can be written as

$$y_D = h_{RD} G y_R + \sum_{j \in \Upsilon_R} I_j w_j + n_D \quad (17)$$

where h_{RD} is the channel between the reference relay and the reference destination, n_D is an additive white Gaussian noise with an average power of σ_D^2 and $\{I_j\}_{j \in \Upsilon}$ are the channels from the interferer j to the reference destination. The amplification factor G of AF relaying with interference [5] can be expressed as

$$G = \sqrt{\frac{P_R}{\|y_R\|^2}} = \sqrt{\frac{P_R}{P_S |h_{SR}|^2 + \sum_{i \in \Phi_s} |g_i|^2 P_{iR} + \sigma_R^2}}. \quad (18)$$

Combining (4) and (17), y_D can be rewritten as

$$y_D = h_{RD} G\left(h_{SR} s_0 + \sum_{i \in \Phi_s} g_i s_i + n_R \right) + \sum_{j \in \Upsilon_R} I_j w_j + n_D \quad (19)$$

$$= h_{SR} h_{RD} G s_0 + h_{RD} G \sum_{i \in \Phi_s} g_i s_i + \sum_{i \in \Upsilon_R} I_j w_j + h_{RD} G n_R + n_D \quad (20)$$

End-to-end *SINR* of AF relaying at the destination can be given by

$$\gamma_D^{AF} = \frac{|h_{SR}|^2 |h_{RD}|^2 G^2 P_S}{|h_{RD}|^2 G^2 \sum_{i \in \Phi_s} |g_i|^2 P_{iR} + \sum_{j \in \Upsilon_R} |I_j|^2 P_{iD} + |h_{RD}|^2 G^2 \sigma_R^2 + \sigma_D^2}. \quad (21)$$

We define new random variables R and S, and we use the random variables P and Q in the previous section as follows:

$$P \sim |h_{SR}|^2 P_S, \quad Q \sim \sum_{i \in \Phi_s} |g_i|^2 P_{iR}, \quad R \sim |h_{RD}|^2 P_R, \quad S \sim \sum_{j \in \Upsilon_R} |I_j|^2 P_{iD}. \quad (22)$$

We rearrange the end-to-end *SINR* (21) to make the Macdonald random variable form in Barua et al. [4] as

$$\gamma_D^{AF} = \frac{PR}{SP + aS + RQ + bQ + aR + bP + ab + QS}$$
$$= \frac{PR}{P(R+b) + S(P+a) + (P+a)(R+b) - PR + QS}$$
$$= \frac{PR}{S(P+Q+a) + (P+Q+a)(R+b) - PR}$$
$$= \frac{PR}{(P+Q+a)(R+S+b) - PR} = \frac{1}{\left(\frac{P+Q+a}{P}\right)\left(\frac{R+S+b}{R}\right) - 1} \quad (23)$$
$$= \frac{1}{\frac{Q+a}{P} + \frac{S+b}{R} + \frac{(Q+a)(S+b)}{PR}} = \frac{\frac{P}{Q+a} \cdot \frac{R}{S+b}}{\frac{P}{Q+a} + \frac{R}{S+b} + 1}$$

where $a = \sigma_R^2$ and $b = \sigma_D^2$.

Hence, end-to-end *SINR* of AF relaying can be written as

$$\gamma_D^{AF} = \frac{\frac{|h_{SR}|^2 P_S}{\sum_{i \in \Phi_s} |g_i|^2 P_{iR} + \sigma_R^2} \cdot \frac{|h_{RD}|^2 P_R}{\sum_{j \in \Upsilon_S} |I_j|^2 P_{iD} + \sigma_D^2}}{\frac{|h_{SR}|^2 P_S}{\sum_{i \in \Phi_s} |g_i|^2 P_{iR} + \sigma_R^2} + \frac{|h_{RD}|^2 P_R}{\sum_{j \in \Upsilon_R} |I_j|^2 P_{iD} + \sigma_D^2} + 1} = \frac{XY}{X+Y+1} \quad (24)$$

where

$$X \sim \frac{|h_{SR}|^2 P_S}{\sum_{i \in \Phi_S} |g_i|^2 P_{iR} + \sigma_R^2}, \quad Y \sim \frac{|h_{RD}|^2 P_R}{\sum_{j \in \Upsilon_R} |I_j|^2 P_{iD} + \sigma_D^2} \quad (25)$$

The random variable X and Y were defined in DF relaying, and the *PDF* and *CDF* of them are the same as in Theorem 1 and Corollary 1.

Theorem 2 (Outage probability of AF relaying) *From the* PDF *and* CDF *of* X *and* Y, *we can compute outage probability of AF relaying as*

$$P_{\text{out}}^{\text{AF}}(\beta) = 1 - B \int_0^\infty \exp\left[-\left(A\left(\frac{\beta(u+\beta+1)}{u}\right) + B(u+\beta)\right.\right.$$
$$\left.\left. + C\sqrt{\frac{\beta(u+\beta+1)}{u}} + D\sqrt{u+\beta}\right)\right] du$$
$$- \frac{D}{2} \int_0^\infty \frac{1}{\sqrt{u+\beta}} \exp\left[-\left(A\left(\frac{\beta(u+\beta+1)}{u}\right) + B(u+\beta)\right.\right.$$
$$\left.\left. + C\sqrt{\frac{\beta(u+\beta+1)}{u}} + D\sqrt{u+\beta}\right)\right] du \qquad (26)$$

where

$$\beta = 2^{2R} - 1, \quad A = \frac{d_{SR}^\alpha \sigma_R^2}{P_S}, \quad B = \frac{d_{RD}^\alpha \sigma_D^2}{P_R},$$
$$C = \frac{\text{erf}(\sqrt{r_1})\,\pi^2\lambda_1}{2}\sqrt{\frac{d_{SR}^\alpha P_{iR}}{P_S}}, \quad D = \frac{\text{erf}(\sqrt{r_2})\,\pi^2\lambda_2}{2}\sqrt{\frac{d_{RD}^\alpha P_{iD}}{P_R}}. \qquad (27)$$

Proof See Appendix B

4 Transmission capacity analysis

In this section, we compute transmission capacity $C(\epsilon)$ based on outage probability of a dual-hop relay. Transmission capacity was defined as the maximum density of the transmitting node to satisfy outage probability and data rate on medium access control (MAC) layer performance. That is, it is efficient to find the maximum available transmitting source and relay nodes to satisfy a given outage probability and data rate (i.e., Quality of Service (QoS)) from the transmission capacity of a dual-hop relay. As transmission capacity in Weber et al. [13] considers single-hop transmission without noise, we focus on the transmission capacity of a dual-hop relay with noise and specific relaying protocols. We assume that the sets of Φ_S, Υ_R, and Ψ_D are homogeneous PPPs on the two-dimensional plane with the same intensity λ due to the random translation invariance property of PPP [17].

4.1 Direct transmission
Transmission capacity of a single hop $C(\epsilon)$ [13,14] is defined as

$$C(\varepsilon) = Rq^{-1}(\varepsilon)(1-\varepsilon) \quad \text{(bps/Hz/m}^2) \qquad (28)$$

where ϵ is outage probability for target SINR $\beta = 2^R - 1$ with spectral efficiency R, $q^{-1}(\epsilon)$ is the spatial intensity of attempted transmission associated with outage probability ϵ and it is always greater than 0. The transmission capacity $C(\epsilon)$ is thinned by the probability of success $1 - \epsilon$.

We derive transmission capacity of direct (single hop) transmission with noise in this section. For computing outage probability of direct transmission between source and destination, the received SINR at the destination is represented as

$$\gamma_D = \frac{P_S|h_{SD}|^2}{\sum_{k\in\Phi_S}|f_k|^2 P_{kD} + \sigma_D^2} \qquad (29)$$

where h_{SD} is channel between source and destination, $\{f_k\}_{k\in\Phi_s}$ are the channels from the interferer k to the reference destination, and P_{kD} is each average transmitting power of interferer k to destination. Similar to Theorem 1, the outage probability of direct transmission between source and destination can be given by

$$P_{\text{out}}^{\text{DT}}(\beta) = 1 - \exp\left[-\left(\frac{d_{SD}^\alpha \sigma_D^2}{P_S}\beta + \frac{\text{erf}(\sqrt{r})\,\pi^2\lambda}{2}\sqrt{\frac{d_{SD}^\alpha P_{kD}}{P_S}}\beta\right)\right], \quad \beta > 0. \quad (30)$$

where r is radius of finite area with the intensity of each interferer λ, and d_{SD} is the distance between source and destination.

From the outage probability (30), the spatial intensity of transmitting nodes $q_{DT}^{-1}(\varepsilon)$ can be obtained as

$$q_{DT}^{-1}(\varepsilon) = \frac{1}{\pi^2\sqrt{\beta}\text{erf}(\sqrt{r})}\sqrt{\frac{P_S}{d_{SD}^\alpha P_{iD}}}\left[\ln(1-\varepsilon)^{-1} - \left(\frac{d_{SD}^\alpha \sigma_D^2}{P_S}\right)\beta\right]. \quad (31)$$

From the above equation, unlike transmission capacity in Weber et al. [13], we can find that $q^{-1}(\epsilon)$ can be a negative number due to thermal noise. Therefore, as density of transmitting nodes cannot be a negative number, transmission capacity with noise can be redefined as follows

$$C_{DT}(\varepsilon) = \begin{cases} 0, & \text{if } q^{-1}(\varepsilon) \leq 0 \\ Rq^{-1}(\varepsilon)(1-\varepsilon), & \text{if } q^{-1}(\varepsilon) > 0 \end{cases} \quad \text{(bps/Hz/m}^2). $$

Finally, the transmission capacity of direct transmission with noise when $q^{-1}(\epsilon) > 0$ can be written as

$$C_{DT}(\varepsilon) = \frac{2R(1-\varepsilon)}{\pi^2\sqrt{\beta}\text{erd}(\sqrt{r})}\sqrt{\frac{P_S}{d_{SD}^\alpha P_{iD}}}\left[\ln(1-\varepsilon)^{-1} - \left(\frac{d_{SD}^\alpha \sigma_D^2}{P_S}\right)\beta\right]. \quad (32)$$

4.2 Dual-hop relaying
Since we are concerned with the outage probability of the relay network after two-hop transmission, the transmission capacity of dual-hop relay is identical to that of single hop except for the target SINR β.

Meanwhile, the transmission capacity of AF relaying cannot be obtained directly due to no closed-form of outage probability. Hence, the equivalent end-to-end SINR with Macdonald r.v. for AF strategy (24) can be written as

$$\frac{XY}{X+Y+1} < \frac{XY}{X+Y} < \min[X, Y] \qquad (33)$$

where $\min[X, Y]$ is end-to-end SINR of DF relaying (14). As we focus on the information theoretical capacity, the fact that DF can be the upper bound of AF is

established in it. Using this relation, we note that the transmission capacity of DF is the upper bound of AF relaying as follows

$$C_{AF}(\varepsilon) < C_{DF}(\varepsilon) = C_R(\varepsilon) = \begin{cases} 0, & \text{if } q^{-1}(\varepsilon) \leq 0 \\ Rq^{-1}(\varepsilon)(1-\varepsilon), & \text{if } q^{-1}(\varepsilon) > 0 \end{cases} \quad (\text{bps/Hz/m}^2)$$

where ϵ is the outage probability for the target $SINR$ β = 2^{2R}-1 with spectral efficiency R. In this paper, we consider a single transmission capacity of dual-hop relay C_R (ϵ) regardless of its protocols. From outage probability (16), we can obtain transmission capacity of dual-hop relay $C_R(\epsilon)$ as

$$C_R(\varepsilon) = \frac{2R(1-\varepsilon)}{\pi^2 \sqrt{\beta} \text{erf}(\sqrt{r}) \left(\sqrt{\frac{d_{SR}^\alpha P_{iR}}{P_S}} + \sqrt{\frac{d_{RD}^\alpha P_{iD}}{P_R}} \right)}$$
$$\times \left[\ln(1-\varepsilon)^{-1} - \left(\frac{d_{SR}^\alpha \sigma_R^2}{P_S} + \frac{d_{RD}^\alpha \sigma_D^2}{P_R} \right) \beta \right], \quad \text{for } q^{-1}(\varepsilon) > 0$$

(34)

where ϵ is outage probability of dual-hop relaying for target $SINR$ β.

5 Numerical and simulation results

In this section, we present some numerical examples of the outage probability and transmission capacity for a dual-hop relaying with both interference and noise in a Poisson network. We consider Rayleigh fading channel and path loss exponent α = 4 to illustrate our analytic and simulated results.

5.1 Outage probability

Figures 2 and 3 show the analytic and simulated outage probability as a function of transmission power of a source and a relay P_S = P_R for DF and AF strategies considering both noise and interference in a Poisson network, respectively. Both figures show that outage probability with end-to-end spectral efficiency R = 1 bps/Hz (target SINR β = 3) and noise variance $\sigma_R^2 = \sigma_D^2 = 1$ for various system parameters. Figures 2a and 3a depict the outage probability as a function of transmission power with different intensity of interferers for DF and AF strategies, respectively. Both figures reveal significant decrease of outage performance and have an error floor with increasing intensity of interferes λ. Figure 2b shows that outage probability of DF relaying with different distance pair (d_{SR}, d_{RD}) = (1.5,1.5), (1.5, 2) and (2, 2). It is noted that the outage performance of dual-hop relaying is degraded by increasing their each hop distance d_{SR} and d_{RD}, respectively. For AF relaying, increasing interference power is greatly influenced by outage probability performance in Figure 3b. In addition, we can see that the analysis agrees exactly with the simulation results from Figures 2 and 3.
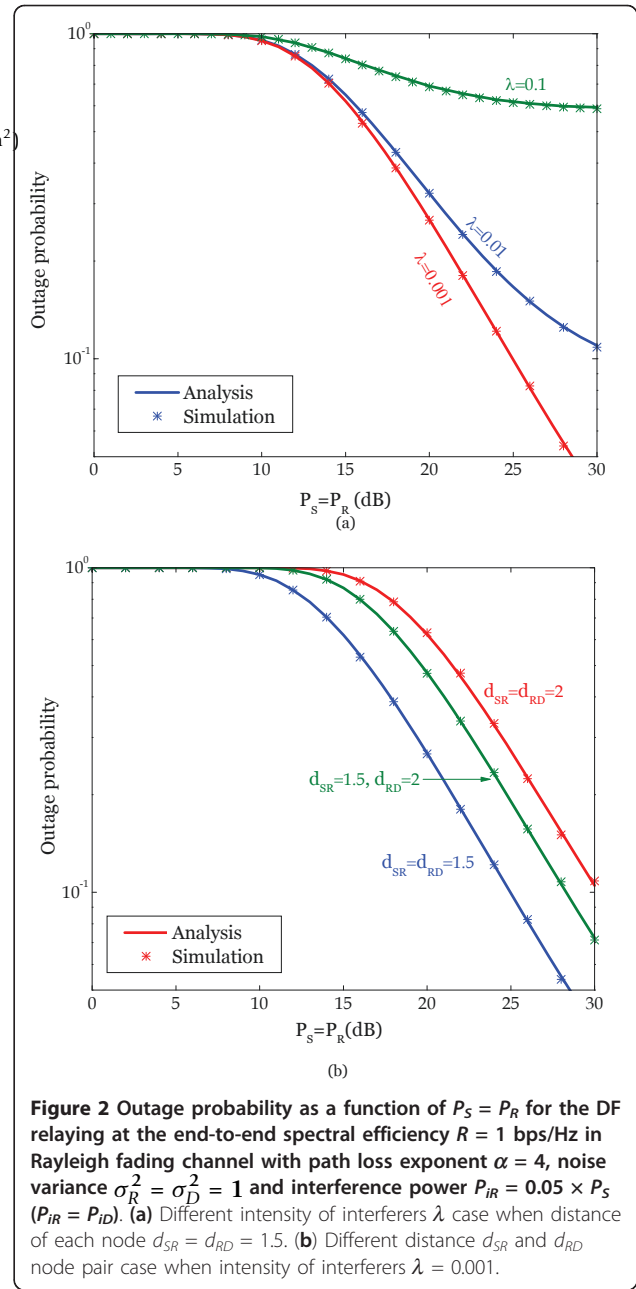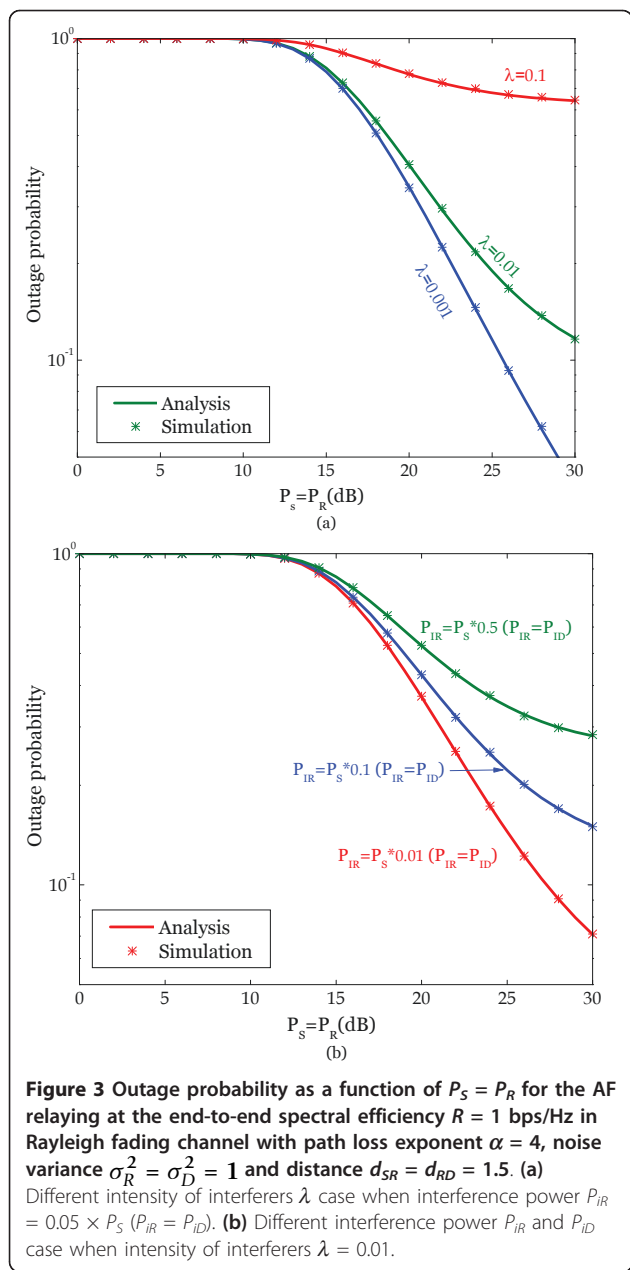

Figure 2 Outage probability as a function of P_S = P_R for the DF relaying at the end-to-end spectral efficiency R = 1 bps/Hz in Rayleigh fading channel with path loss exponent α = 4, noise variance $\sigma_R^2 = \sigma_D^2 = 1$ and interference power P_{iR} = 0.05 × P_S (P_{iR} = P_{iD}). (a) Different intensity of interferers λ case when distance of each node d_{SR} = d_{RD} = 1.5. **(b)** Different distance d_{SR} and d_{RD} node pair case when intensity of interferers λ = 0.001.

5.2 Transmission capacity

We assume that the destination node is located $d_{SD} = \sqrt{3} \times d_{SR}$ away from source node for a fair comparison, when d_{SR} is the same as d_{RD} in transmission capacity. Figure 4 shows the spatial intensity of attempted transmission associated with outage probability $q^{-1}(\epsilon)$ as a function of outage probability with noise variance $\sigma_R^2 = \sigma_D^2 = 1$ and interference power P_{iR} = P_S × 0.05 at the end-to-end spectral efficiency R = 1 bps/Hz. We compare $q^{-1}(\epsilon)$ of a dual-hop relaying with direct transmission for different transmission power P_S = P_R = 15 and 20 dB. As we have seen

Figure 3 Outage probability as a function of $P_S = P_R$ for the AF relaying at the end-to-end spectral efficiency $R = 1$ bps/Hz in Rayleigh fading channel with path loss exponent $\alpha = 4$, noise variance $\sigma_R^2 = \sigma_D^2 = 1$ and distance $d_{SR} = d_{RD} = 1.5$. (a) Different intensity of interferers λ case when interference power P_{iR} = 0.05 × P_S ($P_{iR} = P_{iD}$). **(b)** Different interference power P_{iR} and P_{iD} case when intensity of interferers $\lambda = 0.01$.
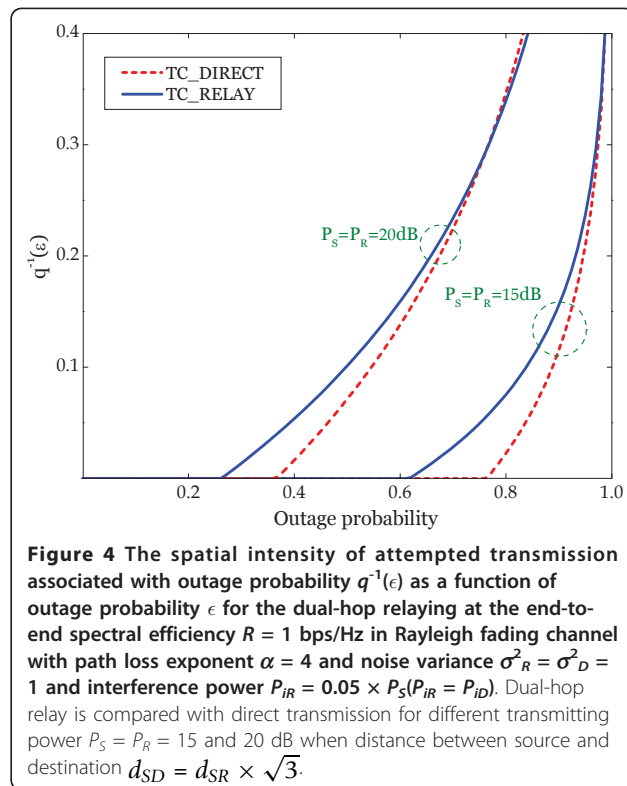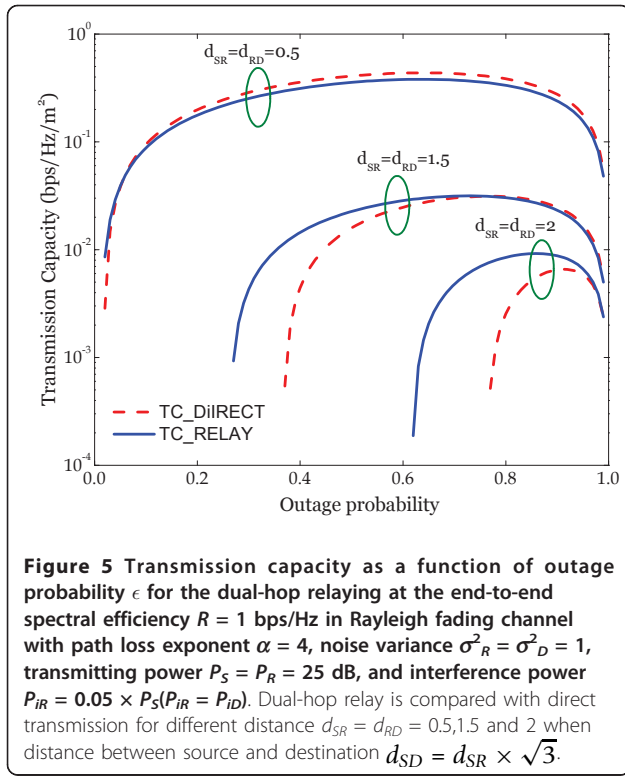
from (32) and (34), this figure shows that $q^{-1}(\epsilon)$ can be zero in relatively low-outage probability regime due to thermal noise. On the contrary, in high-outage probability regime, the curves in Figure 4 have a crossing point at $P_S = Pr = 20$ dB due to a loosen outage probability constraint, and it means that the advantage of using relay can be decreasing at communication system which permits low QoS. However, the dual-hop relaying has more transmitting nodes to satisfy outage probability than does direct transmission at most of outage probability regime.

Transmission capacity as a function of outage probability with noise variance $\sigma_R^2 = \sigma_D^2 = 1$ and interference
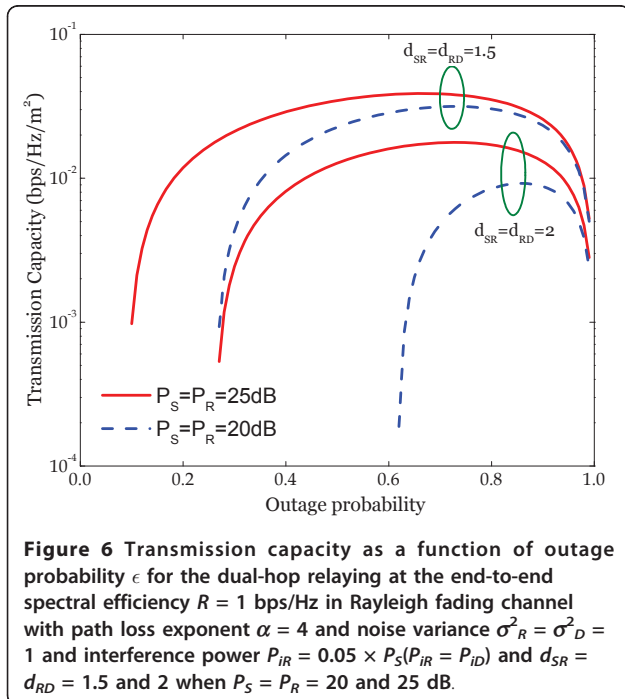
power $P_{iR} = P_S \times 0.05$ at the end-to-end spectral efficiency $R = 1$ bps/Hz is plotted in Figures 5 and 6. Figure 5 also compares direct transmission in terms of transmission capacity for different distance $d_{SR} = d_{RD} = 0.5$, 1.5, and 2. Likewise Figure 4, the transmission capacity of dual-hop relay performs better than that of direct transmission, especially in the relatively low outage probability regime and long distance. That is, using a dual-hop relay has more successful transmission nodes to satisfy outage probability and data rate. Conversely, the transmission capacity of dual-hop relaying is similar or lower than direct transmission in high-outage probability regime and short distance. But, since we utilize a relay to transmit over long distance and concern low-outage probability regime, the dual-hop relaying can be still efficient from transmission capacity point of view. Figures 4 and 5 reveal significant gain of dual-hop relaying compared to direct transmission in terms of density of attempted transmission and transmission capacity.

Finally, Figure 6 shows that transmission capacity of dual-hop relay at different transmission power $P_S = Pr = 20$ and 25 dB and distance $d_{SR} = d_{RD} = 1.5$ and 2. Figure 6 also shows that the existing region ($q^{-1}(\epsilon) > 0$) of transmission capacity of a dual-hop relay is growing with increasing transmitting power $P_S = Pr$ and decreasing distance $d_{SR} = d_{RD}$. This is due to the transmission capacity with thermal noise can be zero for small

Figure 4 The spatial intensity of attempted transmission associated with outage probability $q^{-1}(\epsilon)$ as a function of outage probability ϵ for the dual-hop relaying at the end-to-end spectral efficiency $R = 1$ bps/Hz in Rayleigh fading channel with path loss exponent $\alpha = 4$ and noise variance $\sigma_R^2 = \sigma_D^2 = 1$ and interference power $P_{iR} = 0.05 \times P_S (P_{iR} = P_{iD})$. Dual-hop relay is compared with direct transmission for different transmitting power $P_S = P_R = 15$ and 20 dB when distance between source and destination $d_{SD} = d_{SR} \times \sqrt{3}$.

Figure 5 Transmission capacity as a function of outage probability ϵ for the dual-hop relaying at the end-to-end spectral efficiency R = 1 bps/Hz in Rayleigh fading channel with path loss exponent α = 4, noise variance $\sigma^2_R = \sigma^2_D = 1$, transmitting power $P_S = P_R$ = 25 dB, and interference power $P_{iR} = 0.05 \times P_S(P_{iR} = P_{iD})$. Dual-hop relay is compared with direct transmission for different distance $d_{SR} = d_{RD}$ = 0.5,1.5 and 2 when distance between source and destination $d_{SD} = d_{SR} \times \sqrt{3}$.

transmission power and large distance. In common, the maximum transmission capacity is achieved at a specific outage probability in Figures 5 and 6. It is because that the transmission capacity consists of spatial density $q^{-1}(\epsilon)$ and success probability $(1 - \epsilon)$ from the definition C

Figure 6 Transmission capacity as a function of outage probability ϵ for the dual-hop relaying at the end-to-end spectral efficiency R = 1 bps/Hz in Rayleigh fading channel with path loss exponent α = 4 and noise variance $\sigma^2_R = \sigma^2_D = 1$ and interference power $P_{iR} = 0.05 \times P_S(P_{iR} = P_{iD})$ and $d_{SR} = d_{RD}$ = 1.5 and 2 when $P_S = P_R$ = 20 and 25 dB.

$(\epsilon) = Rq^{-1}(\epsilon)(1-\epsilon)$. In other words, as we have seen from Figure 4, $q^{-1}(\epsilon)$ increases consistently due to less outage constraint, but the success probability $(1 - \epsilon)$ goes to zero as the outage probability increases. Therefore, the transmission capacity decreases due to the success probability in high-outage probability regime and has maximum value at a specific outage probability.

6 Conclusion

This paper considers a dual-hop relaying in the presence of both noise and interference simultaneously, allowing a Poisson interference model. The outage probability of DF and AF strategies have been derived, especially we verified that the end-to-end *SINR* of AF relaying has Macdonald r.v's. The analytic and simulated results showed that outage probability of dual-hop relay had an error floor in high density of transmitting nodes and performance was greatly influenced by distance and interference power. Furthermore, we consider a metric of decentralized wireless network, called transmission capacity, for dual-hop relay with AF and DF strategies. We redefine the transmission capacity, because spatial intensity of attempted transmission associated with outage probability $q^{-1}(\epsilon)$ can be negative number due to thermal noise. Hence, we note that the existing region $(q^{-1}(\epsilon) > 0)$ of transmission capacity of dual-hop relay is growing with increasing transmission power. Our results reveal that transmission capacity of a dual-hop relay considering both noise and interference has better performance than direct transmission in a wireless ad hoc network.

Appendix
A Proof of Theorem 1
The *CDF* of end-to-end *SINR* at the relay X $\sim \gamma_R^{DF}$ is given by

$$
\begin{aligned}
F_X(x) &= \bullet\left\{ \frac{P}{Q + \sigma_R^2} < x \right\} \\
&= \int_0^\infty p_Q(q) \int_0^{x(q+\sigma_R^2)} p_P(p)\,\mathrm{d}p\,\mathrm{d}q \\
&= \int_0^\infty p_Q(q)\left[1 - \exp\left(-\frac{d_{SR}^\alpha}{P_S}x(q + \sigma_R^2)\right)\right]\mathrm{d}q \\
&= 1 - \int_0^\infty \exp\left(-\frac{d_{SR}^\alpha}{P_S}x(q + \sigma_R^2)\right) \frac{\mathrm{erf}(\sqrt{r_1})\lambda_1 P_{iR}^{\frac{1}{2}}}{4} \\
&\quad \times \left(\frac{\pi}{q}\right)^{3/2}\exp\left(-\frac{[\mathrm{erf}(\sqrt{r_1})]^2\pi^4\lambda_1^2 P_{iR}}{16q}\right)\mathrm{d}q \\
&= 1 - \frac{\mathrm{erf}(\sqrt{R_1})\lambda_1 P_{iR}^{\frac{1}{2}}\pi^{3/2}}{4}\exp\left(-\frac{d_{SR}^\alpha}{P_S}x\sigma_R^2\right) \\
&\quad \times \underbrace{\int_0^\infty \left(\frac{1}{q}\right)^{\frac{3}{2}}\exp\left[-\left(\frac{d_{SR}^\alpha}{P_S}xq + \frac{[\mathrm{erf}(\sqrt{r_1})]^2\pi^4\lambda_1^2 P_{iR}}{16q}\right)\right]\mathrm{d}q}_{A}
\end{aligned}
\tag{35}
$$

A can be changed to closed-form using following equation in Gradshteyn and Ryzhik [18]

$$\int_0^\infty x^{-n-\frac{1}{2}} e^{-(px+q/x)} dx = (-1)^n \sqrt{\frac{\pi}{p}} \frac{\partial^n}{\partial q^n} e^{-2\sqrt{pq}} \qquad (36)$$

In this case, we define $n = 1$, $p = \dfrac{d_{SR}^\alpha x}{P_S}, q = \dfrac{[\mathrm{erf}(\sqrt{R_1})]^2 \pi^4 \lambda_1^2 P_{iR}}{16}$. Then, the *CDF* of X can be written as

$$F_X(x) = 1 - \exp\left[-\left(\frac{d_{SR}^\alpha \sigma_R^2}{P_S} x + \frac{\mathrm{erf}(\sqrt{r_1}) \pi^2 \lambda_1}{2} \sqrt{\frac{d_{SR}^\alpha P_{iR}}{P_s} x} \right) \right], \quad x \geq 0 \quad (37)$$

Finally, we can obtain *CDF* and *PDF* of *X* in Theorem 1.

B Proof of Theorem 2

From the *PDF* and *CDF* of X and Y, we can compute outage probability of AF relaying as

$$
\begin{aligned}
P_{\mathrm{out}}^{\mathrm{AF}}(\beta) &= \bullet\left\{ \frac{XY}{X+Y+1} < \beta \right\} \\
&= F_Y(z) + \int_z^\infty F_X\left(\frac{z(y+1)}{y-z} \right) p_Y(y) dy \\
&= 1 - \int_z^\infty \exp\left[-\left(A\left(\frac{z(y+1)}{y-z} \right) + C\sqrt{\left(\frac{z(y+1)}{y-z} \right)} \right) \right] p_Y(y) dy \\
&= 1 - \int_z^\infty \left(B + \frac{D}{2\sqrt{y}} \right) \exp\left[-\left(A\left(\frac{z(y+1)}{y-z} \right) + By + C\sqrt{\frac{z(y+1)}{y-z}} + D\sqrt{y} \right) \right] dy \\
&= 1 - B \int_z^\infty \exp\left[-\left(A\left(\frac{z(y+1)}{y-z} \right) + By + C\sqrt{\frac{z(y+1)}{y-z}} + D\sqrt{y} \right) \right] dy \\
&\quad - \frac{D}{2} \int_z^\infty \frac{1}{\sqrt{y}} \exp\left[-\left(A\left(\frac{z(y+1)}{y-z} \right) + By + C\sqrt{\frac{z(y+1)}{y-z}} + D\sqrt{y} \right) \right] dy
\end{aligned}
\qquad (38)
$$

where

$$\beta = 2^{2R} - 1, \quad A = \frac{d_{SR}^\alpha \sigma_R^2}{P_S}, \quad B = \frac{d_{RD}^\alpha \sigma_D^2}{P_R} \qquad (39)$$

$$C = \frac{\mathrm{erf}(\sqrt{r_1})\pi^2 \lambda_1}{2} \sqrt{\frac{d_{SR}^\alpha P_{iR}}{P_s}}, \quad D = \frac{\mathrm{erf}(\sqrt{r_2})\pi^2 \lambda_2}{2} \sqrt{\frac{d_{RD}^\alpha P_{iD}}{P_R}}. \qquad (40)$$

Using *y - z = u*, we can obtain outage probability of AF relaying in Theorem 2.

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