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Foundations and Trends® in Networking

Published, sold and distributed by:
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
USA
Tel. +1-781-985-4510
www.nowpublishers.com
sales@nowpublishers.com

Outside North America:
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is S. Weber and J. G. Andrews, Transmission Capacity of Wireless Networks, Foundation and Trends® in Networking, vol 5, nos 2–3, pp 109–281, 2010

ISBN: 978-1-60198-518-7

Foundations and Trends® in Networking

Volume 5 Issues 2–3, 2010

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Foundations and Trends® in Networking, 2010, Volume 5, 4 issues. ISSN paper version 1554-057X. ISSN online version 1554-0588. Also available as a combined paper and online subscription.

Foundations and Trends® in
Networking
Vol. 5, Nos. 2–3 (2010) 109–281

DOI: 10.1561/1300000032

Transmission Capacity of Wireless Networks

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Abstract

Transmission capacity (TC) is a performance metric for wireless networks that measures the spatial intensity of successful transmissions per unit area, subject to a constraint on the permissible outage probability (where outage occurs when the signal to interference plus noise ratio (SINR) at a receiver is below a threshold). This volume gives a unified treatment of the TC framework that has been developed by the authors and their collaborators over the past decade. The mathematical framework underlying the analysis (reviewed in Section 2) is stochastic geometry: Poisson point processes model the locations of interferers, and (stable) shot noise processes represent the aggregate interference seen at a receiver. Section 3 presents TC results (exact, asymptotic, and bounds) on a simple model in order to illustrate a key strength of the framework: analytical tractability yields explicit performance dependence upon key model parameters. Section 4 presents enhancements to this basic model — channel fading, variable link distances (VLD), and multihop. Section 5 presents four network design

case studies well-suited to TC: (i) spectrum management, (ii) interference cancellation, (iii) signal threshold transmission scheduling, and (iv) power control. Section 6 studies the TC when nodes have multiple antennas, which provides a contrast vs. classical results that ignore interference.

Contents

1

Introduction and Preliminaries

Wireless networks are becoming ever more pervasive, and the correspondingly denser deployments make interference management and spatial reuse of spectrum defining aspects of wireless network design. Understanding the fundamentals of the performance and behavior of such networks is an important theoretical endeavor, but one with only limited success to date. Information theoretic approaches, well-summarized by [24], have been most successful when applied to small isolated networks, where background interference and spatial reuse are not considered. Large network approaches, typified by transport capacity scaling laws [82], have given considerable insight into scaling laws, but are generally unable to quantify the relative merits of candidate design choices or provide a tractable approach to analysis for spatial reuse or the SINR statistics. Our hope for the transmission capacity framework has been to develop a tractable approach to large network throughput analysis, that while falling short of information theory's ideals of inviolate upper bounds, nevertheless provides a rigorous and flexible approach to the same sort of questions, and ultimately provides the types of broad design insights that information theory has been able to achieve for small networks.

1.1 Motivation and Assumptions

This monograph presents a framework for computing the outage probability (OP) and transmission capacity (TC) [79, 80] in a wireless network. The OP is defined as the probability that a "typical" transmission attempt fails (is in outage) at the intended receiver, where outage occurs when the signal to interference plus noise ratio (SINR) at the receiver is below a threshold. Basing outage on the SINR, it is assumed that interference is treated as noise. The TC is defined as the maximum average number of concurrent successful transmissions per unit area taking place in the network, subject to a constraint on OP. The OP constraint may be thought of as a reliability and/or quality of service (QoS) parameter — strict requirements on the fraction of failed transmissions result in low spatial reuse, low area spectral efficiency (ASE, measured in bps/Hz per unit area), and thus lower TC, while relaxing the outage requirement improves, up to a point, the ASE and thus TC. Viewing the OP as a (strictly increasing) function of the intensity of attempted transmissions, the TC is computed by inverting this function for the transmission intensity at the target OP.

Note we use the word *capacity* in a distinctly different manner from its information–theoretic sense, i.e., Shannon capacity: the TC framework typically treats interference as noise[1] while Shannon theory does not, and TC measures capacity in a spatial sense, while Shannon theory does not. The *capacity* in TC is also distinct from the transport capacity of [34], defined as the maximum weighted sum rate of communication over all pairs of nodes, where each pair's communication rate is weighted by the distance separating them. The transport capacity optimizes over all scheduling and routing algorithms and the focus is on the asymptotic rate of growth of the sum rate in the number of nodes n, either keeping the network area fixed or letting the network area grow linearly with n. TC, on the other hand, is a medium access control (MAC) layer metric that neither precludes nor addresses routing.[2] Although transport capacity is more general in that it optimizes

[1] see §5.2 and the results of Section 6 as an exception: even here though the background (uncancelled) interference is then treated as noise.

[2] see §4.3 for an exception, where a simple multihop model is added.

scheduling and routing, the cost of this generality is that typically the transport capacity results are less specific than those obtainable under the TC framework. The results are less specific in the sense that results on the asymptotic rate of growth of the transport capacity as a function of n often do not specify the preconstant.

The advantages of using TC as a metric for wireless network performance are: (i) it can be exactly derived in some important cases, and tightly bounded in many others, (ii) performance dependencies upon fundamental network parameters are thereby illuminated, and (iii) design insights are obtainable from these performance expressions. More fundamentally, the TC captures in a natural way essential performance indicators like network efficiency (ASE), reliability (OP), and throughput (TP). In fact, TC is precisely maximization of TP under an OP constraint, as discussed in §3.4, and is proportional to ASE, as discussed in §5.1.

One limitation of the TC framework, at least as in this monograph, is the implicit assumption that the network employs the simplistic and suboptimal slotted Aloha protocol at the MAC layer. The TC can also be extended to model other contention-based MAC protocols at the cost of some tractability [25, 26], but we elect to stick to the simple slotted Aloha protocol, where each transmitter (Tx) independently elects whether or not to transmit to its receiver (Rx) in each time slot by flipping an independent biased coin [1]. If the point process describing the locations of contending transmitters at a snapshot in time form a Poisson point process (PPP), which we assume is the case, then under the Aloha protocol the locations of the active transmitters at some point in time also form a PPP, obtained by independent sampling of the node location PPP. The PPP model is necessary for preserving the highest level of analytical tractability of the TC framework, but of course it means that the computed TC is suboptimal. The difficulty in relaxing the Aloha assumption lies in the fact that any realistic and useful randomized MAC protocol involves coordination among competing transmitters, which necessarily spoils the crucial independence property of the PPP. One's valuation of the TC framework typically rests on weighing the advantage of having an explicit expression for an insightful network performance indicator with the disadvantage of that

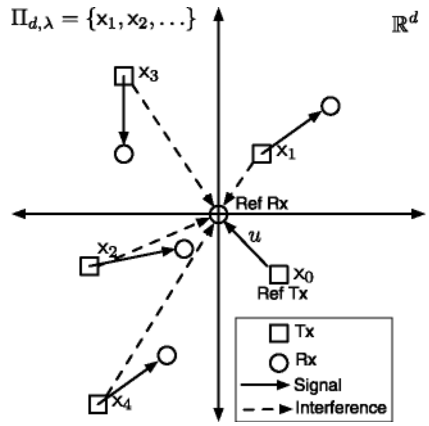

Fig. 1.1 A reference (typical) Rx located at the origin $o \in \mathbb{R}^d$ is paired with a reference Tx at distance u, and is subject to interference (dashed lines) from a PPP $\Pi_{d,\lambda}$ of interfering Tx's (each of which has a unique associated Rx).

performance corresponding to a suboptimal control law. Throughout this monograph, we generally adopt the following assumptions in order for the OP and TC to be computable. See Figure 1.1.

Assumption 1.1. The following assumptions are made:

(1) The network is viewed at a single snapshot in time for the purpose of characterizing its spatial statistics.
(2) Every potential Tx is matched with a prearranged intended Rx at a fixed distance u (meters) away[3]: these Tx–Rx pairs are in one to one correspondence.
(3) When mapping our results to a specific bit rate, we assume each Rx treats (uncancelled) interference as noise, and the rate from a particular Tx to its Rx at location o is given by the Shannon capacity $\mathsf{c}(o) \equiv \frac{1}{2}\log_2(1 + \mathsf{sinr}(o))$.
(4) The potential transmitters form a homogeneous PPP on the network arena, taken to be $\mathbb{R}^d$, for $d \in \{1,2,3\}$. This implies (i) the number of nodes in the network is countably infinite, and (ii) the number of potential transmitters in two disjoint

[3] The extension to random distances is straightforward and given in §4.2.

bounded sets of the plane are independent Poisson random variables (RV). See Figure 1.1.

(5) Every potential Tx decides independently whether or not to transmit with a common probability p_{tx}. It follows that the set of actual transmitters is also a (thinned) PPP.

A few remarks are in order:

(1) The TC computes the maximum spatial reuse which is computable by looking at the network at a single snapshot in time. This perspective neither addresses nor precludes multihop or routing considerations.
(2) Assumption (3) can be easily softened to account for any modulation and coding type that is characterized by an SINR "gap" from capacity. Typically, we directly utilize SINR for computing outage probability and TC and do not include the per-link rate in the results.
(3) Assumption (4) makes clear that our focus is on networks whose arena is the entire plane $\mathbb{R}^2$, and which have a countably infinite number of nodes. This along with Assumption (5) removes any concern about boundary effects and makes each node "typical" in a sense described below.

1.2 Key Definitions: PPP, OP, and TC

Assumption (1)–(3) allow us to formally define the OP.

Definition 1.1. Outage probability (OP). Define the constant R to be the spectral efficiency (in bits per channel use per Hz) of the channel code employed by each Tx–Rx pair in the network. Define the SINR threshold $\tau \equiv 2^{2R} - 1$ so that $R = \frac{1}{2}\log_2(1+\tau)$. For an arbitrary Tx–Rx pair with the Rx positioned at the origin $o \in \mathbb{R}^d$, let $\mathsf{c}(o) \equiv \frac{1}{2}\log_2(1+\mathsf{sinr}(o))$ be the *random* Shannon spectral efficiency of the channel connecting them when interference is treated as noise. The OP is the probability that the random spectral efficiency of the channel falls below the spectral efficiency of the code, or equivalently, the probability that the random SINR at the Rx is below the threshold τ:

$$\begin{aligned} q(o) &\equiv \mathbb{P}(\mathsf{c}(o) < R) \\ &= \mathbb{P}\left(\frac{1}{2}\log_2(1+\mathsf{sinr}(o)) < \frac{1}{2}\log_2(1+\tau)\right) \\ &= \mathbb{P}(\mathsf{sinr}(o) < \tau). \end{aligned} \tag{1.1}$$

It is worth emphasizing that the RV in $\mathbb{P}(\mathsf{c}(o) < R)$ is the capacity $\mathsf{c}(o)$ of the channel connecting the Tx–Rx pair, computed at the snapshot in time at which we observe the network, and not the rate R, which is assumed fixed. In particular, $\mathsf{c}(o)$ is a function of the RV $\mathsf{sinr}(o)$, which is quite sensitive to the distances between the Rx at o and the random set of interfering transmitters at the observation instant. The OP is the cumulative distribution function (CDF) of the RVs $\mathsf{c}(o)$ and $\mathsf{sinr}(o)$.

By the assumption that the transmitters (and receivers) form a PPP, it follows that all Tx–Rx pairs are typical, hence $q(o) = q$. More formally, we can condition on the presence of a test Tx–Rx pair where, without loss of generality, we assume the test Rx to be located at the origin o. The distribution of the PPP of potential transmitters is unaffected by the addition of this test pair[4]:

$$q(o) \equiv \mathbb{P}(\mathsf{c}(o) < R | \text{ Rx at } o) = \mathbb{P}(\mathsf{c}(o) < R). \tag{1.2}$$

Assumption (4)–(6) and the definition of OP allow us to formally define the TC. We first define a homogeneous PPP on $\mathbb{R}^d$. Figure 1.2 shows a portion of a sample PPP on $\mathbb{R}^2$ and illustrates the fact that the number of points in each compact set is a Poisson RV.

Definition 1.2. Homogeneous Poisson point process (PPP). A PPP with intensity $\lambda > 0$ in d-dimensions is a random countable

[4] This result is due to Slivnyak [67]. See, e.g., [8] Theorem 1.13 (p. 30), [36] Theorem A.5 (p. 113), [70] p. 41 and Example 4.3 (p. 121).

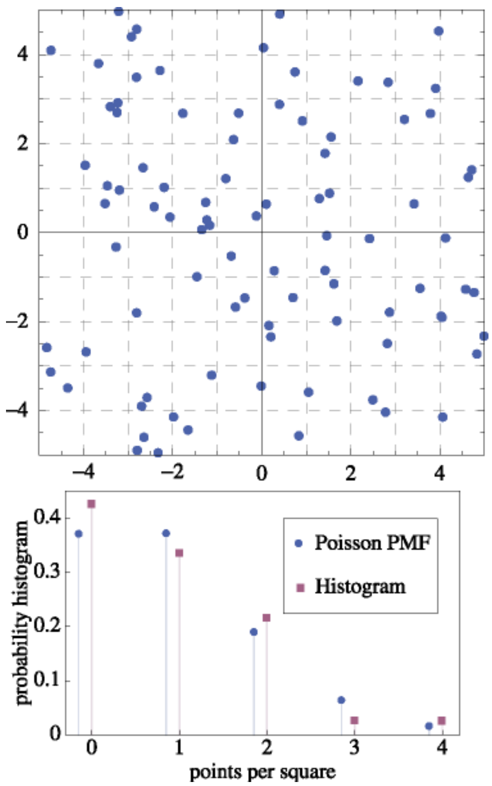

Fig. 1.2 **Top:** a realization of a homogeneous Poisson point process (PPP) of intensity $\lambda = 1$ on $\mathbb{R}^2$. **Bottom:** the histogram of the number of points in each of the 10×10 unit area squares and the corresponding Poisson PMF for $\lambda = 1$.

collection of points $\Pi_{d,\lambda} = \{\mathsf{x}_1, \mathsf{x}_2, \ldots\} \subset \mathbb{R}^d$ such that

- For two disjoint subsets $A, B \subset \mathbb{R}^d$ the number of points from Π in these sets are independent RVs: $\Pi(A) \perp \Pi(B)$, where $\Pi(A) \equiv |\Pi \cap A|$ is the number of points in Π in A.
- The number of points in any compact set $A \subset \mathbb{R}^d$ is a Poisson RV with parameter $\lambda|A|$, for $|A|$ the volume of A. That is:

$$\Pi(A) \sim \mathrm{Po}(\lambda|A|), \tag{1.3}$$

or equivalently,

$$\mathbb{P}(\Pi(A) = k) = \frac{1}{k!} \mathrm{e}^{-\lambda|A|} (\lambda|A|)^k, \quad k \in \mathbb{Z}_+. \tag{1.4}$$

Suppose $\Pi_{\text{pot}} \equiv \Pi_{d,\lambda_{\text{pot}}}$ is the PPP of potential transmitters with spatial intensity λ_{pot} discussed in Assumption 1.1 (4) and (5). Let $p_{\text{tx}} \in (0,1)$ be the common transmission probability employed by each node in Assumption 1.1 (5). It follows that the PPP of actual transmitters at the observation instant (denoted $\Pi_{d,\lambda}$) is a thinned version of Π_{pot} with corresponding thinned intensity $\lambda \equiv \lambda_{\text{pot}} p_{\text{tx}}$.

It is intuitive that the OP is increasing in λ: a higher spatial intensity of transmission attempts yields larger interference at each Rx, which decreases the SINR. We emphasize this dependence by writing the OP as $q(\lambda)$ and thereby view $q : \mathbb{R}_+ \to [0,1]$ as a map from the spatial intensity of transmission attempts to the corresponding OP.[5]

Fact 1.1. The OP $q(\lambda)$ is continuous, strictly increasing, and onto $[q(0),1)$, where $q(0)$ is the OP in the absence of interference.

Because of this fact, the inverse $q^{-1} : [q(0),1) \to \mathbb{R}_+$ is well-defined. For an outage constraint $q^* \in [q(0),1)$, the inverse OP $q^{-1}(q^*)$ is the (unique) intensity of transmission attempts associated with an outage probability of q^*. Each such transmission succeeds with probability $1 - q^*$, and as such $q^{-1}(q^*)(1 - q^*)$ is the spatial intensity of successful transmissions. This is what we call TC.

Definition 1.3. Transmission capacity (TC). Fix a maximum permissible OP $q^* \in [q(0),1)$. The TC is the maximum spatial intensity of successful transmissions subject to an OP of q^*:

$$\lambda(q^*) \equiv q^{-1}(q^*)(1 - q^*). \tag{1.5}$$

The intensity of failed transmissions is $q^{-1}(q^*)q^*$, and the summed intensity of successful and failed transmissions is naturally $q^{-1}(q^*)$. The TC (and all spatial intensities) is measured in units of (meters^{-d}), i.e., an "average" number of nodes per unit area.

[5] This redefines the OP $q(\lambda)$ as a function of the intensity $\lambda \in \mathbb{R}_+$ of the PPP — in Equations (1.1) and (1.2) $q(x)$ denoted the OP at location $x \in \mathbb{R}^d$.

Remark 1.1. TC and slotted Aloha. The TC $\lambda(q^*)$ has operational sigificance for a wireless network of potential transmitters positioned according to a PPP of intensity $\lambda_{\rm pot}$ and employing the slotted Aloha MAC protocol with transmission probability $p_{\rm tx}$. Namely, if $(q^*, \lambda_{\rm pot})$ are such that $\lambda(q^*) < \lambda_{\rm pot} \times (1 - q^*)$ then select

$$p_{\rm tx} = \frac{\lambda(q^*)}{\lambda_{\rm pot} \times (1 - q^*)}. \tag{1.6}$$

The resulting intensity of attempted transmissions $p_{\rm tx}\lambda_{\rm pot} = q^{-1}(q^*)$ will be such that the OP is $1 - q^*$. If $\lambda(q^*) \geq \lambda_{\rm pot} \times (1 - q^*)$ then the network does not need an Aloha MAC throttling transmission attempts to achieve an OP of q^*: setting $p_{\rm tx} = 1$ will result in an OP $q(\lambda_{\rm pot}) < q^*$.

1.3 Overview of the Results

The results presented in this volume are listed in Tables 3 (Section 1) through 6 (Section 6). We briefly discuss each section.

Section 1 (Table 3). The key concepts are in §1.2, specifically, Definition 1.1 of the outage probability (OP), Definition 1.2 of the (homogeneous) Poisson point process (PPP), and Definition 1.3 of the transmission capacity (TC).

Section 2 (Table 4). We first define the ball and annulus in $\mathbb{R}^d$. (Definition 2.1) and gives their volumes (Proposition 2.1). All results are given for arbitrary dimension d, where $\{1,2,3\}$ are the three relevant values.

Throughout the volume we denote RVs in sans-serif font (Remark 2.1 in §2.1), e.g., x. Note the acronyms and notation for standard probabilistic concepts in Definition 2.3. §2.2 gives a short but essential coverage of the void probability (Proposition 2.6), the mapping theorem (Theorem 2.7) and a derivative result on mapping distances (Proposition 2.8). The void probability underlies most performance bounds derived in this volume, and the mapping result allows translation from a PPP on $\mathbb{R}^d$ of intensity $\lambda \in \mathbb{R}_+$ ($\Pi_{d,\lambda}$) to an "equivalent" unit intensity PPP on $\mathbb{R}^1$ ($\Pi_{1,1}$).

We cover (spatial) shot noise (SN) processes in §2.3 (Definition 2.4), which are used to model the aggregate interference experienced by a reference Rx at the origin. We focus on power law SN (Definition 2.5) by assuming the impulse response function in the SN definition is taken to be the standard pathloss attenuation $|\mathsf{x}|^{-\alpha}$, where α is the pathloss exponent. We also introduce here the characteristic exponent $\delta = d/\alpha$, where to avoid trivialities we assume $\delta \in (0,1)$ (i.e., $\alpha > d$) throughout. The sum SN process (Σ) adds the interference contributions while the max SN (M) takes the largest contribution. The simple inequality $\mathsf{M} < \Sigma$ forms the basis of most of the bounds in this volume in that the distribution of M (Corollary 2.9) is Frechét (Definition 2.6), and also can be derived directly from the void probability in Proposition 2.6. The Campbell–Mecke result (Theorem 2.12) allows computation of moments (Proposition 2.13) of SN RVs. More important for us will be the series expansions of the SN distribution (Proposition 2.14) as these directly yield the asymptotic (tail) distributions (Corollary 2.15), which yield all the asymptotic performance results in this volume.

A critical observation is that the SN is a stable RV, this is the focus of §2.4. We define this class (Definition 2.7 and 2.8), and introduce the Lévy distribution (Definition 2.9) which is the only stable distribution of relevance to us with a closed form CDF, and corresponding to $\delta = \frac{1}{2}$. This allows the exact performance results in Section 3. We introduce the probability generating functional (PGFL) (Definition 2.10), and identify its connection with the Laplace transform (LT), the moment generating function (MGF), and the characteristic function (CF) of the SN RV.

The results in this section are tied together in §2.5 where we demonstrate the key property that the simple bound $\mathsf{M} < \Sigma$ is tight in the sense that the ratio of the complementary cumulative distribution functions (CCDFs) for these RVs approaches unity in the limit (Proposition 2.23). We derive a similar result using subexponential distributions (Definition 2.11) for a binomial point process (BPP).

Section 3 (Table 3). This section presents the main results on OP and TC in their barest, simplest form, so as to achieve maximum clarity. Exact OP and TC results are in §3.1. SINR is defined (Definition 3.1) and it is observed that the OP is the CCDF of the SN evaluated at a certain value. An explicit expression for the OP and TC (for $\delta = \frac{1}{2}$) is given (Corollary 3.2 and 3.4).

Asymptotic OP and TC results are in §3.2. The asymptotic CCDF of the SN (Corollary 2.15) yields the asymptotic OP (as $\lambda \to 0$) and TC (as $q^* \to 0$) in Proposition 3.5. The asymptotic TC is interpreted as sphere packing in $\mathbb{R}^d$, where the sphere radius depends upon the key model parameters δ, u, τ, d (Remark 3.1).

The $\mathsf{M} < \Sigma$ SN inequality forms the basis for the OP lower bound (LB) and TC upper bound (UB) in §3.3. We adopt the language of dominant interferers (Definition 3.3) to describe interferers capable by themselves of reducing the SINR seen at the origin below its threshold τ, but observe this concept is equivalent to taking the maximum interferer (Remark 3.2). The main result is the bound on OP and TC in Proposition 3.6.

In §3.4 we turn our attention to a third performance metric, the MAC layer throughput (TP), $\Lambda(\lambda)$, defined (Definition 3.4) as the spatial intensity of successful transmissions. A TP UB is obtained from the OP LB (Proposition 3.9). We make the key observation that "blind" maximization of TP leads to an associated OP of 67%. The natural design objective of maximizing TP subject to an OP constraint is shown to be precisely the TC, giving a more natural justification for this quantity as a meaningful performance measure (Proposition 3.10). In fact the TP and TC have the same unconstrained maximum and we relate their maximizers (Proposition 3.11).

Finally, §3.5 gives a UB on OP and an LB on TC. A useful expression for the OP in terms of its LB is derived (Proposition 3.13), which the OP LB and the three basic inequalities in §2.1 (Markov, Chebychev, and Chernoff) are combined to give three OP UBs. These are observed to vary both in terms of their tightness and their simplicity.

Section 4 (Table 4) extends the basic model in three ways: fading (§4.1), variable link distances (§4.2), and multihop (§4.3).

The bulk of this section is on fading (§4.1); the SINR under fading is defined (Definition 4.1). §4.1 is split into three subsections: exact results (§4.1.1), asymptotic results (§4.1.2), and bounds (§4.1.3). The main result in §4.1.1 is Proposition 4.2 which gives the exact OP and TC under the assumption that the signal fading is Rayleigh (exponential).

Note this exact result holds for all δ, while the only exact result available under the basic model in Section 3 is for $\delta = \frac{1}{2}$ (Corollary 3.2 and 3.4). For the asymptotic results in §4.1.2 we introduce the formalism of the marked PPP (MPPP) and exploit the important marking theorem (Theorem 4.6) which allows us to extend the distance and interference mapping results for PPPs from §2.2 to the MPPP case. The series expansions of the interference under fading (Proposition 4.9) are used to derive the asymptotic OP and TC (Proposition 4.11). An important observation is that fading in general degrades performance relative to the nonfading case (Corollary 4.12). In §4.1.3 the concept of dominant interferers used in Definition 3.3 is extended to incorporate fading (Definition 4.2), but under fading the strongest interferer need not be the nearest interferer to the origin. The main result is the OP LB (Proposition 4.13), where we observe the LB is in fact the MGF of a certain function of the signal fading RV.

§4.2 addresses variable link distances, i.e., the Tx–Rx distance is an RV. The SINR and OP for this model are defined in Definition 4.3 and 4.4, respectively, and we present asymptotic results (Proposition 4.14) and exact results (Corollary 4.17).

§4.3 extends the TC framework to a multihop scenario where sources send packets to destinations M hops away over a total distance R. Multihop TC is defined in Definition 4.5. Although some fairly strong assumptions must be made to preserve tractability, plausible insights can be drawn about the optimum hop count (given in Proposition 4.20) and end-to-end TC in terms of all the network parameters.

Section 5 (Table 5). The section on design techniques studies four natural approaches to improve the performance of a wireless network: §5.1 studies the performance when the spectrum is split into a number of channels, §5.2 considers performance when receivers are equipped with interference cancellation capabilities, §5.3 evaluates the performance when nodes only transmit when their signal fade is above a specified threshold, and §5.4 considers power control.

In §5.1 the design objective is to optimize the number of bands to form from the available spectrum, where each Tx selects a band uniformly at random. The intuitive tradeoff is that more bands give fewer interferers but this also means the bandwidth per band is smaller, and thus a higher SINR threshold is required to achieve a given data rate. We define the model in Definition 5.1 and 5.2. The spectral efficiency optimization problem is formalized in Proposition 5.1, and we characterize the solution in Proposition 5.4, and then specialize the result to both the high (Corollary 5.5) and low (Proposition 5.6) signal to noise ratio (SNR) regimes.

In §5.2 the usual limitations of interference cancellation (IC) are captured through the $(\kappa, K, P_{\min})$ Rx model (Definition 5.4) where κ is cancellation effectiveness, K is the maximum number of cancellable nodes, and $P_{\min}$ is the minimum received power. The SINR is defined in Definition 5.5, and the main result is the OP LB (Proposition 5.11).

In §5.3 the fading coefficient threshold (Definition 5.6) used to throttle transmission attempts naturally trades off between the quality and quantity of transmission attempts, and the TP metric Λ (Definition 5.7) illustrates this tradeoff. Asymptotic results are given in Proposition 5.13 and an LB on OP is given in Proposition 5.15.

In §5.4 the notion of fractional power control (FPC) is introduced, where the power control exponent sweeps between fixed power and channel inversion (Definition 5.8). The asymptotic results (Proposition 5.17) yield the optimal exponent is 1/2 (Proposition 5.18). The notion of dominant interferers is used once again (Definition 5.9) to compute the OP LB (Proposition 5.19).

Section 6 (Table 6). The final section introduces multiple antennas at both the Tx and Rx, resulting in some of the first analytical work on MIMO that properly accounts for background interference. The results are broken into two main categories, which are defined along with basics of the models in §6.2. §6.3 considers the case where despite the multiple antennas, only a single data stream is sent, with the balance of the antennas being used for diversity and/or interference cancellation. §6.4 considers the more general multistream case, where transmitters send more than one simultaneous stream to either a single receiver (spatial multiplexing (SM)) or to multiple users (space division multiple access (SDMA)). Finally, the practical implications and limitations of the results are discussed in §6.5.

In §6.3, the results are further categorized into diversity (§6.3.1) and interference cancellation (§6.3.2). For receive diversity, the OP of

MRC is given in Proposition 6.2 and the corresponding TC in 6.3. The result is equivalent for MRT (transmit MRC) and the generalization to $n_{\mathrm{T}} \times n_{\mathrm{R}}$ diversity beamforming (BF) is discussed in Remark 6.4. In §6.3.2, a TC lower bound is given on a suboptimal technique called partial ZF in Proposition 6.7 and a TC upper bound for MMSE in Proposition 6.8. These respectively bound the TC of MMSE and we see linear scaling can be achieved with the number of antennas.

In §6.4, we first consider a class of results for spatial multiplexing in §6.4.1, where multiple streams are transmitted from a single Tx to a single Rx. Proposition 6.10 and 6.11 give the optimal number of streams K^* and TC scaling in terms of $n_{\mathrm{T}} \leq n_{\mathrm{R}}$ for MRC and ZF receivers, respectively. This is extended to a Bell Labs space time (BLAST) receiver in Proposition 6.16. Then in §6.4.2, we turn our attention to streams being sent to multiple Rx's at the same time. The main result for MRC receivers is given in Proposition 6.17, with the appropriate scaling results given in Proposition 6.18.

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