

# Transmission Line Synthesis\*

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**Abstract**—RLC transmission line synthesis is a difficult problem because minimal net delay cannot be achieved or accurately predicted unless: 1) a termination scheme is chosen and properly implemented, and 2) output driver transition rates are constrained at or below the net's capability. This paper describes a method to concurrently find, for any RLC net, an optimal termination value, a maximum source transition rate, and an approximate net delay using the first few response moments. When optimal termination is accomplished and transition rates do not exceed the capabilities of the net, the resulting delay metrics are an interesting extension of the popular Elmore delay metric for RC interconnects. The task of physical RLC interconnect design is facilitated by the ease with which these first few moments are calculated for generalized RLC lines.

## I. INTRODUCTION

Because CMOS can easily provide balanced push-pull buffers and system power limits are being stretched, source termination has become the method of choice for transmission line control [7]. Sensing this trend, both the circuit and packaging communities are tailoring their respective products to provide variable source termination. MCM technologists propose integrating source terminating resistors in their MCM designs. Circuit designers routinely offer chips with programmable output buffers.

In this paper, a synthesis procedure for RLC nets is proposed that concurrently determines the optimal series termination resistance and the maximum source transition rate while estimating the subsequent net delay. Because this procedure "critically damps" the circuit response along the net, it thereby minimizes system power and maximizes signal noise immunity. And since optimal termination values, maximum transi-

tion rates, and approximate net delays can be concurrently computed with ease, this procedure is well suited for use in placement and routing or transmission line synthesis tools.

## II. THE SYNTHESIS DILEMMA

A transmission line synthesis tool might be used to iteratively place components and simultaneously route and size the RLC wiring. Viability of a placement and its interconnect routing would be based on how well thermal requirements and net delay objectives were met. The latter presents a dilemma.

Because excessive settling time increases delay in some sense, both under-damping and over-damping adversely impact delay. RLC interconnect delays cannot be accurately predicted unless a termination scheme is chosen and properly implemented. Furthermore, ideal termination values in RLC interconnects where topology, line parameters, load values, and rise times can all vary, are unique to every net. (Which is why MCM technologists have proposed imbedding series resistors in their packages.) A synthesis tool must, therefore, concurrently and efficiently determine optimal termination value, maximum source transition rate, and net delay.

## III. OPTIMAL TERMINATION USING CIRCUIT MOMENTS

Two methods for determining optimal termination using circuit moments have already been proposed for point-to-point RLC nets [6,8]. Both methods attempt to critically damp the circuit response at the end of the line and both use circuit moments to arrive at the solution. After a brief introduction of circuit moments, we will review [8].

The moments of a time-domain waveform,  $h(t)$ , are classically defined via the Laplace transform as follows:

$$H(s) = \int_0^{\infty} e^{-st} h(t) dt = m_0 + \frac{(-1)}{1!} m_1 s + \frac{(-1)^2}{2!} m_2 s^2 + \dots \quad (1)$$

where  $m_k$  are the Maclaurin series coefficients of  $H(s)$  which are related to the moments of  $h(t)$ :

$$m_k = \int_0^{\infty} t^k h(t) dt \quad (2)$$

While the concepts of moments stem from the fields of statistics and mechanics, they have been successfully applied to a number of problems. In one paper of particular interest, Elmore [3] computed the approximate delay and rise time of

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the step response using first and second moments of the impulse response.

### A. End of Line Waveform Matching

Waveform matching to find optimal termination was recently proposed in [8]. It can be best explained by first comparing the input and output response of a perfectly series terminated, unloaded, lossless, point to point net. Consider the multidrop daisy chain net shown in Fig.1, where the line is lossless, the source is located at  $C_0$ , the length of the line  $d = (d_1+d_2)$ ,  $R_s = Z_0$ , and  $C_0 = C_1 = C_2 = 0$ .

For this ideal case, the output response at the end of line identically matches the input response when the latter is shifted in time by the time of flight of the line. In the Laplace domain representation, this ideal matching can be stated as

$$e^{-sd\sqrt{LC}} - H(s) = 0. \quad (3)$$

where  $H(s)$  represents the transfer function at the end of the line,  $d$  is the length of the line, and  $L$  and  $C$  are the per unit length inductance and capacitance.

When applying this concept to lossy, loaded lines in [8], they attempt to match the transfer function  $H(s)$  to some unknown delay. The first term in (3) is replaced with  $e^{-sT_d}$ , where  $T_d$  is the unknown net delay to the end of the line. Expanding both  $e^{-sT_d}$  and  $H(s)$  in terms of Maclaurin series and collecting like coefficients of  $s$  yields

$$\left( (1-m_0) - (T_d-m_1)s + \left( T_d^2 - m_2 \right) \frac{s^2}{2!} - \dots \right) = 0. \quad (4)$$

Recognizing that the first term vanishes because  $m_0 = 1$  for lines with dc gain of 1, they then solve for  $R_s$  and  $T_d$  such that the next two terms also vanish. The first three terms of the far end response,  $H(s)$ , now match the first three terms of a pure delay shift,  $e^{-sT_d}$ . For an input ramp signal it is shown that the far end circuit response jumps onto the steady state asymptote as quickly as possible, while the transient response decays to zero as rapidly as possible (critically damping).

The work in [8] can also be interpreted “statistically”. Just like Elmore pointed out in [3], we can interpret the impulse response  $h(t)$ , like a probability density function. If  $H(s)$  were to match a pure delay,  $e^{-sT_d}$ , then  $h(t)$  would be given by  $\delta(t - T_d)$ . The mean of  $h(t)$  would be  $T_d$ , and the cumulative area and the *central moments* of  $h(t)$  would be as follows:

$$\int_0^{\infty} h(t) dt = 1 \quad (5)$$

$$\int_0^{\infty} (t - T_d)^k \cdot h(t) dt = 0 \quad \text{for all } k \geq 1 \quad (6)$$

End of line waveform matching accomplishes these goals in a limited way. The cumulative area of  $h(t)$  is one, and the 1st and 2nd central moments are shaped to zero. The first and second central moments are more commonly known as the mean,  $m_1$ , and the variance,  $\sigma^2$ , respectively, where

$$\sigma^2 = \left( m_2 - m_1^2 \right). \quad (7)$$

One interpretation of [8] is that  $h(t)$  is forced to be a response with zero variance and a mean equal to a pure delay.

### B. Shaping Central Moments about the Mean

The approach proposed here attempts to extend the idea in [8], and it’s link to Elmore’s work one step further. When low-loss, lightly loaded point-to-point and daisy-chain nets are perfectly series terminated, all points along the net monotonically rise to one half their final value as the incident wave passes. The reflected wave completes the switching by charging these points to their final value.

While the impulse responses at all points along the net are different, they all share one common characteristic. When a low-loss, lightly loaded transmission line is perfectly series terminated, all impulse responses are symmetric. Furthermore, all impulse responses are symmetric about the same mean. This is best illustrated with an example.

Consider the case of a three drop daisy chain net driven from one end of the net (Fig.1 with  $d=d_1+d_2$ , and the source  $R_s$  is located at  $C_0$ ). If this line were lossless, unloaded (i.e.  $C_0 = C_1 = C_2 = 0$ ), and perfectly series terminated (i.e.  $R_s = Z_0$ ), the middle and far end step responses would be like that shown in Fig.2. The far end response would identically match the input response when the latter were shifted in time by  $T_d = d\sqrt{LC}$ , the time of flight to the end of the line. The middle response would step up to its final value in two equal increments. The first step would occur at time  $T_d - dt$ , where  $dt$  is the time of flight distance between the middle and far ends. The second step would occur at time  $T_d + dt$ .

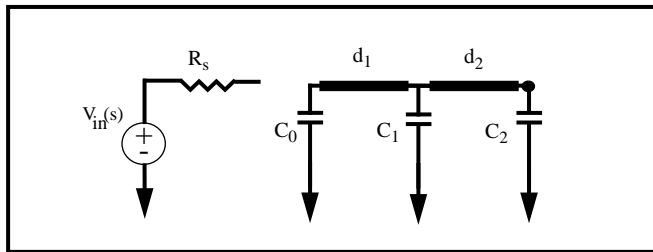


FIGURE 1: Series terminated three drop daisy chain net.

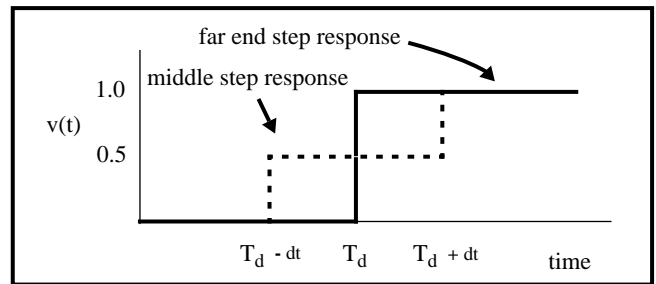


FIGURE 2: Ideal middle and far end step response.

The middle and far end impulse responses would be like that shown in Fig.3. If we treat these responses like probability density functions, the middle and far end responses share two characteristics. They're both symmetric and they're both centered about a mean of  $T_d$ .

The symmetry of a probability density function is measured by its skew [2], which is given by

$$\text{skew} = \frac{\mu_3}{\sigma^3} \quad (8)$$

where  $\mu_3$  is the 3rd central moment and  $\sigma$  is the standard deviation. The 3rd central moment of a probability density function is defined by (6) with  $k=3$  and  $T_d=m_1$ , and reduces to a simple algebraic expression involving  $m_1$ ,  $m_2$ , and  $m_3$ :

$$\mu_3 = m_3 - 3m_1m_2 + 2m_1^3 \quad (9)$$

Because a symmetric probability density function has zero skew, we will apply this symmetry measure to determine optimal series termination for point-to-point and daisy-chain nets. That is, we will solve for the parameters such that  $\text{skew} \approx 0$ . Although, this paper focuses entirely on series terminated nets where reflected wave switching is the objective, these techniques can be applied equally well to parallel terminated nets, where incident wave switching is the objective.

While setting the numerator  $\mu_3$  to zero is the primary goal of this technique, the role of denominator  $\sigma^3$  is also important. In statistics, probability density functions are strictly non-negative, and therefore, the 2nd central moment,  $\mu_2=\sigma^2$ , is always positive. When the step response of a network is monotonic, the network impulse response,  $h(t)$ , is a non-negative function, and so,  $\sigma \geq 0$ . Furthermore, as Elmore [3] observed, the step response rise time,  $\tau_r$ , is proportional to  $\sigma$ . Because of this relationship to rise time and line ringing, this technique will invoke the additional restriction that  $\sigma$  be strictly positive real at all points along the net. Although, input rise time will be used throughout this paper as the means of controlling  $\sigma$ , other techniques such as RC termination are equally effective.

Furthermore, as the next sections will show,  $\sigma$  is very useful in predicting RLC net delays and determining maximum transition rates. We will also show that  $\sigma^2$  can be used with  $\mu_3$  to determine a series termination design point with minimal net delay.

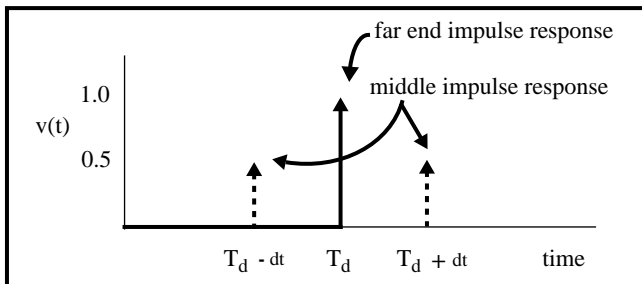


FIGURE 3: Ideal middle and far end impulse response.

In lossless LC trees, the following properties can be shown regarding this metric. First  $\mu_3$  is independent of position. The 3rd central moments at the ends of the net are identical to the 3rd central moments at all other points along the net. Second,  $\mu_3$  is of the form

$$\mu_3 = R_s(\alpha \cdot R_s^2 - \beta). \quad (10)$$

Hence, there's only one non-zero source resistance value for which  $\mu_3$  equals zero.

Clearly lossless source terminated LC trees are under-damped when the source resistance is small, and over-damped when the source resistance is large. The source resistance that critically damps the net is somewhere in between, and the 3rd central moment must change sign between these extremes.

Lossy RLC nets can be self terminated and various researchers have computed the optimal self termination for lossy point-to-point RLC nets [4,6]. Their results for optimal line resistance range from  $2 \cdot Z_0$  to  $\pi \cdot Z_0$  when the net is unloaded. Our metric (i.e. the line resistance at which the 3rd central moment at the end of the line vanishes), predicts an optimal line resistance of  $(15/2)^{1/2} \cdot Z_0 \approx 2.74 \cdot Z_0$ .

### III. DELAY ESTIMATION USING CIRCUIT MOMENTS

When low-loss, lightly loaded point to point or daisy chain nets have been optimally series terminated, we propose the following estimate for the RLC net delay:

$$\text{Net Delay} = m_1 + \sigma \quad (11)$$

We select this metric based on the knowledge that  $m_1 + \sigma$  bounds the median of any probability density function [2].

As an example, referring to the step response in Fig.2, it is obvious that the delay from the source end to the far end and middle are  $T_d$  and  $T_d + dt$  respectively. Using (2) to calculate  $m_1$  and  $m_2$  in Fig.3, the far end values are  $T_d$  and  $T_d^2$  while the middle point values are  $T_d$  and  $T_d^2 + dt^2$ . The variances, are therefore, zero and  $dt^2$  for far end and middle respectively. If we compute our net delay as prescribed in (11), this estimate yields net delays of  $T_d$  and  $T_d + dt$  for the far end and middle respectively. These values match the obvious interpretation of Fig.2, therefore the delay metric in (11) is exact for this ideal case.

While real point-to-point and daisy-chain nets will not exhibit perfectly symmetric impulse responses, the intent of reflected wave switching on these nets is to achieve a critically damped monotonic response at the far end while the points along the line step up to their final value in two equal increments. And when this condition is achieved, the 3rd central moments at all points along the line become negligibly small and  $m_1 + \sigma$  provides a good upper bound estimate of net delay, (provided  $\mu_2$  is greater than zero).

Care should be taken with these metrics. When  $\mu_3 \gg 0$ , the net is over-damped and the net delay is bounded by the Elmore delay [5]. When  $\mu_3 \ll 0$ , the net is under-damped and

the net delay is not predictable because  $\mu_2 < 0$ . However, when the net is critically damped,  $\mu_3 \approx 0$ ,  $\mu_2 > 0$  at all points on the net, and net delay is approximately given by  $m_1 + \sigma$ .

There are net topologies for which we cannot set  $\mu_3 \approx 0$  and  $\mu_2 > 0$  at all points using series termination alone. This metric is indicating that for nets with large discontinuities and stubs, critical damping requires additional parameters. Adding more termination (e.g. RC termination), or adjusting the net topology, or changing the driver rise time can be used to set  $\mu_3 \approx 0$  and  $\mu_2 > 0$ . Here we will consider rise time as a second independent parameter to adjust these metrics, and hence, terminate the net.

#### IV. RISE TIME EFFECTS ON PRECEDING METRICS

When the unit step input is replaced with a saturated unit ramp input, these “statistical” based metrics can be extended further. In [2], the distribution function,  $F(x)$ , of the sum of two independent variables is shown to be given by

$$F(x) = \int_{-\infty}^{\infty} F_1(x-z) dF_2(z) = \int_{-\infty}^{\infty} F_2(x-z) dF_1(z) \quad (12)$$

where  $F_1(x)$  and  $F_2(x)$  are the respective distribution functions of the two independent variables. Furthermore, the probability density function,  $f(x)$ , of the distribution function  $F(x)$  is given by the convolution of  $f_1(x)$  and  $f_2(x)$ , the probability density functions of the distribution functions  $F_1(x)$  and  $F_2(x)$ . Because the two variables are independent, the resultant mean, variance, and 3rd central moment of  $f(x)$  is given by the sums of the means, variances, and 3rd central moments of  $f_1(x)$  and  $f_2(x)$ .

Since the circuit response to a saturated unit ramp has the same form as (12),

$$V_{\text{ramp}}(t) = \int_{-\infty}^{\infty} r_{\text{sat}}(t-\tau) dV_{\text{step}}(\tau), \quad (13)$$

it can also be interpreted “statistically”. In this interpretation, the saturated ramp,  $r_{\text{sat}}(t)$ , is the distribution function of a uniform probability density function of height  $1/\tau_r$  and width  $\tau_r$ . The mean, variance, and 3rd central moments for a uniform probability density function of this height and width are  $(1/2)\tau_r$ ,  $(1/12)\tau_r^2$ , and 0 respectively. Therefore, the effect of a non-zero input rise time on the mean, variance, and 3rd central moment is as follows:

$$m_{1 \text{ ramp}} = m_{1 \text{ step}} + \frac{\tau_r}{2} \quad (14)$$

$$\sigma_{\text{ramp}}^2 = \sigma_{\text{step}}^2 + \frac{\tau_r^2}{12} \quad (15)$$

$$\mu_{3 \text{ ramp}} = \mu_{3 \text{ step}} \quad (16)$$

Although the 3rd central moment is independent of rise time, the skew, (8), depends on rise time since it is a measure of the symmetry of a probability density function. When an RLC net has minimal ringing, skew will be small at all points on the net and the minimum value of  $\sigma^2$  will be commensurate with the variance of the input rise time.

#### V. RLC SYNTHESIS

The approach proposed here is to minimize the skew for all nodes with the smallest possible rise time, thereby allowing the fastest possible signal. Since the variance is smallest at the extremities of the net, we first minimize  $\mu_3$  at the farthest point from the source. Therefore, step one is to compute the source resistance for which the 3rd central moment at the farthest point from the source is zero. If the line is lossless,  $\mu_3$ , hence skew, will be zero at all points along the line. Although this condition ensures the chosen resistance will damp out the aggregate line reflections as rapidly as possible, it does not guarantee monotonic response along the net.

Large discontinuities and excessive stub lengths inevitably produce line ringing that cannot be constrained with a single source termination value. Therefore, although the signal is reaching its final value as fast as possible, the signal can be “critically damped” only by adjusting either the input rise-time or the RLC net features to compensate for the discontinuities present in such line topologies. When the settling time is long,  $\sigma^2$  is small or even negative (since  $h(t)$  is not really a probability density function).

Therefore, step two in our procedure is to compute the maximum source transition rate for the unaltered net. We propose that the output response rise time must be commensurate with the input rise time. Using Elmore’s observation that rise time is proportional to the standard deviation, we force  $\sigma_{\text{ramp}}^2$  in (15) to satisfy the following relationship:

$$\sigma_{\text{ramp}}^2 = \left[ \sigma_{\text{step}}^2 + \frac{\tau_r^2}{12} \right] > \frac{(\alpha \cdot \tau_r)^2}{12} \quad (17)$$

where  $0 < \alpha < 1$  and  $\alpha$  is a function of the net topology. (For example, unbalanced RLC trees require a larger  $\alpha$  than simple daisy chain nets.) When the source transition rate has been degraded to this maximum rate, the waveform response will be nearly monotonic and net delay will be given by  $m_1 + \sigma$ .

#### VI. EXAMPLES

Six variations of the three drop net in Fig.1 are analyzed here. These six examples variously illustrate the effects of source location and capacitor placement and value on optimal termination and maximum transition rate for low loss multi-drop bidirectional nets.

In the first example the source was located at  $C_0$  and the circuit was analyzed for its optimal termination by setting the skew to zero at  $C_2$ . Referring to Fig.1, the transmission line

parameters and capacitor values were as follows:  $Z_0=50 \Omega$ ,  $R=0.1\Omega/\text{mm}$ ,  $L=0.335\text{nH}/\text{mm}$ ,  $C=0.134\text{pF}/\text{mm}$ ,  $d_1=50\text{mm}$ ,  $d_2=25\text{mm}$ , and  $C_0=C_1=C_2=3\text{pF}$ . The 2nd and 3rd central moments at  $C_0$ ,  $C_1$ , and  $C_2$  are shown as a function of  $R_s$  in Fig.4. Because the line is relatively low-loss, the 3rd central moments are very similar for all values of  $R_s$  and when  $R_s \approx 35\Omega$ , the 3rd central moment for the end is zero. The second central moments are all positive at this termination value, thereby satisfying (17) and indicating that a step input can be propagated without undue ringing. By using  $m_1$  and  $\sigma$  for this termination value in our delay equation,  $m_1 + \sigma$ , unit step response delays of 1.34, 1.01, and 0.75 ns were predicted from source to  $C_0$ , source to  $C_1$ , and source to  $C_2$ . Notice that, as expected, the maximum delay is at  $C_0$  due to the reflected wave switching.

The time domain waveforms for this example are shown in Fig.5. Analyzed using HSPICE\*, Fig.5 depicts the output response of the circuit in Fig.1 after a unit step input has been applied at  $C_0$  through a series resistance of  $35\Omega$ . The marks at 1.34, 1.01, and 0.75 ns on the time domain waveform indicate our unit step delay predictions. These waveforms show very little overshoot indicating the line was properly terminated. All waveforms reach approximately 50% of their final value at their predicted delays.

In the second example the transmission line parameters and capacitor values were held constant, and the source was

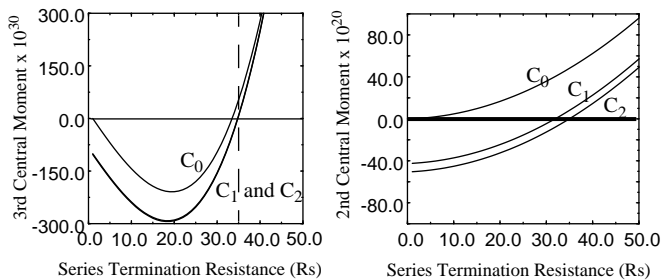


FIGURE 4: Third and second central moments vs. series termination resistance,  $R_s$ , for three drop net with source at  $C_0$ .

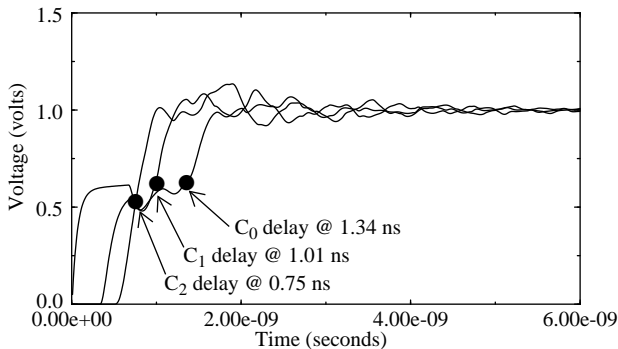


FIGURE 5: Time-domain waveforms for the example in Fig.4 along with delay estimates.

moved to  $C_1$ . The results of this analysis are shown in Fig.6 where the 2nd and 3rd central moments at  $C_0$ ,  $C_1$ , and  $C_2$  are plotted as a function of  $R_s$ . Again the 3rd central moments are very similar for all values of  $R_s$ . When  $R_s \approx 21\Omega$ , the 3rd central moment is zero at the end of the line (which is  $C_0$  in this example). The 2nd central moment however, is negative at  $C_0$  when  $R_s \approx 21\Omega$ . This indicates that the waveform response at  $C_0$  will exhibit considerable ringing when the net is driven with a unit step input. This can be expected for this example since we have a stub which is half the size of the main line. Using (17) with  $\alpha = 0.8$ , a rise time of 1.28 ns is required to damp the signal ringing due to this long stub. Using  $m_1$  and  $\sigma$  for this source resistance and rise time in our delay equation, unit ramp response delays of 1.34, 1.59, and 1.53 ns were predicted from source to  $C_0$ , source to  $C_1$  and source to  $C_2$  respectively.

Fig.7 depicts the output response of the circuit after a unit step input was applied at  $C_1$  through a series resistance of  $21 \Omega$ . Fig.8 depicts the output response when the unit step was replaced with a unit ramp with an 1.28 ns rise time. The waveforms in Fig.8 show little ringing indicating the line was properly terminated and the output response at  $C_0$  is approximately 80% of the input rise time. But are the waveforms in Fig.8 any faster than the waveforms Fig.7?

The issue here is settling time [1], and some heuristics are needed to answer this question. Settling time is most commonly equated with the last crossing of the 90% point. When waveform response is measured to this criteria, the 1.28 ns rise time improves net delay at both  $C_0$  and  $C_2$ . The marks at 1.34, 1.59, and 1.54 ns on the time domain waveform in Fig.8

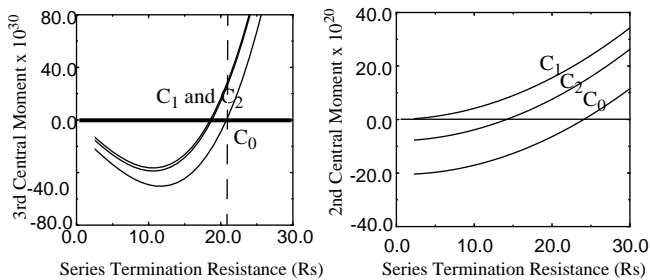


FIGURE 6: Third and second central moment vs. series termination resistance,  $R_s$ , for three drop net with source at  $C_1$ .

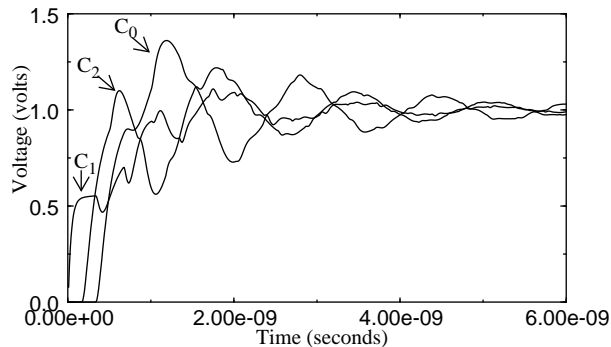


FIGURE 7: Time-domain waveforms for example in Fig.6 with unit step input.

\*HSPICE, Version H92 from Meta-Software, Inc.

indicate our delay predictions. All waveforms reach approximately 75% of their final value at their predicted delays.

In the final four examples, the source was again located at  $C_0$ , but the line was made lossless, total line length  $d$  was increased to 10 cm, and the values and locations of  $C_0$ ,  $C_1$  and  $C_2$  were varied. Optimal termination and minimum rise time was determined by setting the skew to zero and satisfying (17) at  $C_2$ . Because of the simple daisy chain net configuration, a smaller value of  $\alpha$  was used, (i.e.  $\alpha = 0.5$ ). To contrast these results against a more conventional metric, an effective impedance was also evaluated assuming the total lumped capacitance,  $C_0+C_1+C_2$ , was evenly distributed along the net.

$$Z_{\text{eff}} = \sqrt{L / \left[ C + \frac{(C_1 + C_2 + C_3)}{d} \right]} \quad (18)$$

The optimal source resistances and minimum acceptable rise times predicted by our metric are summarized in Table 1. The optimal source resistances predicted by the conventional metric, (i.e.  $R_s=Z_{\text{eff}}$ ), are also included.

In the first example, when the net was unloaded, both metrics predicted an optimal source resistance value equal to  $Z_0$ , the characteristic impedance of the line. Our metric, however, also predicted the net could be safely driven with a step input. In the second example, the capacitors  $C_0$ ,  $C_1$  and  $C_2$  were set to 5 pF, and  $C_1$  was located exactly in the center of the line. Both metrics predicted similar optimal source resistance values, and these values were, as expected, less than  $Z_0$ . Our metric also predicted that the line should be driven with a non-zero input rise-time to reduce undue line ringing.

In the third and fourth examples,  $C_1$  was first moved closer to the source, and then moved closer to the end of the line. The conventional metric, because it does not discriminate load placement, failed to predict a change in the optimal source resistance value. Our metric, however, predicted different resistance values and different minimum transition rates. When  $C_1$  was moved closer to the source, our metric predicted the expected result [7], (i.e. the optimal source resistance decreases while the minimum rise time increases). When  $C_1$  was moved to the opposite end of the net, our metric again

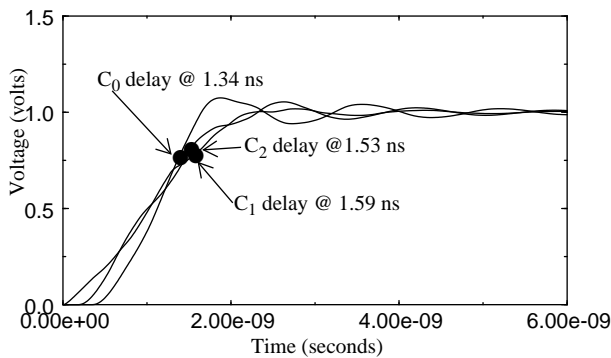


FIGURE 8: Time-domain waveforms for the example in Fig.6 with 1.28 ns ramp input along with delay estimates.

predicted the expected result, (i.e. the optimal source resistance increases while the minimum rise time decreases).

	$R_s$ @ skew=0	minimum rise-time	$R_s = Z_{\text{eff}}$ (dist. net)
$d_1=d_2=5\text{cm}$ & $C's=0\text{pF}$	50.0 $\Omega$	0.0 ns	50.0 $\Omega$
$d_1=d_2=5\text{cm}$ & $C's=5\text{pF}$	32.4 $\Omega$	1.31 ns	34.3 $\Omega$
$d_1=2\text{cm}, d_2=8\text{cm}$ & $C's=5\text{pF}$	29.0 $\Omega$	1.67 ns	34.3 $\Omega$
$d_1=8\text{cm}, d_2=2\text{cm}$ & $C's=5\text{pF}$	34.8 $\Omega$	1.10 ns	34.3 $\Omega$

TABLE 1. Optimal termination characteristics for lossless three drop net in Fig.1 when capacitor values and placement are varied.

## VII. CONCLUSIONS AND FUTURE WORK

RC interconnect design uses well-established metrics for estimating the signal delay. In contrast, RLC transmission lines synthesis for interconnect routing has no similar corresponding estimate for delay. This paper presents a technique for determining the conditions of critical damping for series terminated transmission line nets, and subsequently for estimating net-delays in terms of the circuit moments. This metric forms the basis for a delay-estimation tool to be used at the core of a transmission line interconnect router.

Future work will examine more complicated net topologies such as lossy multi-drop RLC nets, and other termination schemes such as parallel and RC termination will be considered.

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