Transmission loss allocation through a modified Ybus — Source link

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Transmission loss allocation through a modified Y_{bus}

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Abstract: A methodology to allocate the active power transmission loss among agents of a power pool is proposed. The approach is based on the inclusion of the admittances equivalent to bus power injections in the bus admittance matrix. For a given power-flow solution, the relationship between the branch currents and the load/generator current injections is determined using a modified bus admittance matrix, which allows the power loss of each transmission line to be expressed in terms of bus current injections. The proposed technique is simple to implement and flexible enough to allow the assignment of loss parcels to a preselected set of buses. An example, with a six-bus system illustrates the main steps of the proposed allocation strategy, and numerical results obtained with the IEEE 57-bus system are used to assess the quality of the loss allocation.

1 Introduction

Deregulation of the power market has created an ever-growing pressure for all components of the cost/price to be clearly identified and assigned equitably between all parties, taking care to avoid or minimise any temporal or spatial cross-subsidies [1]. One of the main concerns related to the decomposition of the cost/price of electric energy in deregulated power markets is the allocation of the active power transmission loss among the consumers. Although no power system variable is affected by this process, the revenue and payment reconciliation are dependent on the criterion adopted for this purpose.

Recently there have been a number of papers on the computation of loss factors. The calculation of the losses due to current flowing in each branch of the network (variable loss), and those due to iron loss and dielectric in transformers and cables (fixed loss), is well established. A number of methods have been proposed to apportion these losses (and hence costs) between customers. Some of the main methods found in the literature can be categorised into classes described as follows.

References [2–4] trace losses back from the network branch to the load. These strategies generally involve an algorithm to determine how the losses attributed to generators/loads accumulate as one traverses through the network. Either the algorithm allows loss attribution to be specified according to a user-defined formula, or a loss-sharing formula is implicitly included. Other approaches consider changes in total network loss caused by either infinitesimal or finite changes in the demand at a load or generator, which give rise to marginal or direct loss coefficients, as proposed in [1]. The reported apparent equivalence of these nodal loss distribution factors, which are relatively easy to compute, is interesting. References [5, 6] propose the decomposition of the Lagrange multipliers of the optimal power flow (OPF) solution into components corresponding to the load, transmission loss and congestion. While knowledge of marginal transmission loss coefficients would be very useful in the context of planning or managing the power network, it is not clear why their use should be preferred to direct attribution of losses in the context of charging for losses. These coefficients do not assure a fair division of the power loss, usually requiring adjusting factors for the summation of the loss parcels assigned to the buses to match the value of the power loss obtained through the solution of the power network equations [1, 7]. If the loss is divided proportionally to the bus power injection this requirement is satisfied. However, whether close to or far from the generating units, all load buses are similarly penalised, which makes this allocation strategy somewhat arbitrary.

Transmission loss nodal factors can also be obtained through suitable manipulation of current injections of a power-flow solution and the bus impedance matrix, as proposed in [8]. These factors are claimed to be a *natural* way of sharing the power losses among the power systems agents. Changes in the demand are entirely reflected in the loss factors attributed to buses with modified load (which reduces the level of cross subsidy), and the value of the power loss is accurately recovered (which is convenient for the reconciliation and payment processes). However, as with other approaches [1], nodal loss coefficients obtained through conventional Z_{bus} appear to treat each load or generator in the same way, although the validity of deriving loss coefficients for power sources and sinks simultaneously is questionable.

Recently the integration of incremental transmission loss coefficients [9] or alternatively marginal costs [10, 11] has been proposed. Although these allocation techniques have good properties with respect to the reconciliation process and cross-subsidy, usually a large number of conventional or optimal power-flow solutions are necessary to perform the desired integration, which makes them computationally burdensome.

Different results with respect to the power transmission loss are expected from the application of different power-loss allocation strategies, and the existence of an ideal scheme for sharing power loss is debatable. However, to reduce the degree of arbitrariness of any allocation technique, the following properties are required [1, 3, 9]:

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• loss parcels must reflect the true cost that the corresponding user imposes on the network;

• cross subsidies must be avoided (if possible) or at least minimised;

• loss parcels must be consistent with a power-flow solution;

• the strategy must be simple to understand and and easy to implement.

In this work a strategy to allocate the active power transmission loss based on a modification of the bus admittance matrix is proposed. This technique allows attribution of the power-loss parcels alternatively to generation or load buses. First, for a given power-flow solution, a set of buses (generation/load) is selected to share the power loss and the corresponding power injections of the remaining set of buses are converted to equivalent admittances and included in the bus admittance matrix. Next, the branch currents are expressed as functions of the current injections of the selected buses, allowing the amount of the power loss of each transmission line corresponding to each bus of the chosen set to be evaluated. A six-bus test system is employed to show the main steps of the modified $\boldsymbol{Y}_{bus}$-based technique. Numerical results obtained with the IEEE 57-bus system illustrate the quality of the loss allocation determined via the proposed methodology.

2 Theoretical basis

2.1 Prerequisites

The power system can be defined as a set of nodes and branches with current injection at some or all nodes. In this paper a bus is considered as a generation bus only if it delivers real power to the network. The corresponding node could then be classified as a source (or generator, with suffix g). If current injection occurs at a bus with zero or negative real power it would be classified as a sink (or load, including lumped shunt components, with suffix l). Buses with zero injection are eliminated.

Fixed power system losses occur whether a load/generator is connected or not, and charging for these should be done on the basis of annual maximum demand (or some other fixed rate) rather than actual usage. Loss allocation factors will be derived here for variable losses only (though these would include the effect of increased current in branches that supply fixed losses). We also assume that, since current flows from a set of sources to a set of sinks, either the set of sources or the set of sinks can be regarded as responsible for losses, but not both simultaneously. The variable loss in a branch can therefore be attributed in two different ways.

The steady-state power-flow equations relating the node current injections I_{bus} and complex bus voltages V_{bus} can be expressed by

$$\boldsymbol{I}_{bus} = \boldsymbol{Y}_{bus}\boldsymbol{V}_{bus}$$

where Y_{bus} is the bus admittance matrix; or in partitioned form

$$\begin{bmatrix} \boldsymbol{I}_g \\ \boldsymbol{I}_l \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{gg} & \boldsymbol{Y}_{gl} \\ \boldsymbol{Y}_{lg} & \boldsymbol{Y}_l \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_g \\ \boldsymbol{V}_l \end{bmatrix} \tag{1}$$

where subscript g and l refer to generators and loads. The dependency of loss on both source and sink can be clearly seen by noting that the real power loss in a transmission system is given by [12]

$$P_l = \boldsymbol{I}_{bus}^{t*}\boldsymbol{R}_{bus}\boldsymbol{I}_{bus} \tag{2}$$

where $\boldsymbol{Z}_{bus}$ is the $n \times n$ bus impedance matrix, with $\boldsymbol{Z}_{bus} = \boldsymbol{Y}_{bus}^{-1} = (\boldsymbol{G}_{bus} + \jmath\boldsymbol{B}_{bus})^{-1} = \boldsymbol{R}_{bus} + \jmath\boldsymbol{X}_{bus}$, and the superscript $t*$ denotes the complex transpose conjugate. (2) is the basis of the loss-allocation method proposed in [8]. The use of the conventional admittance matrix imposes that loss fractions be attributed to both load and generation buses simultaneously. Thus there is no flexibility to allocate losses for load buses only (or generation buses only).

2.2 Branch currents attributed to sinks

Any power injection into a sink can be explicitly represented by a current injection. Power injections due to generation buses however, need to be eliminated. The admittance component representation of the power injection of a generation bus is found by dividing the complex current injection by the corresponding complex busbar voltage, both of which are found by obtaining a load-flow solution for the network. The so calculated admittance would be not be physically realisable, since it would contain a negative conductance in this case.

If the nodal admittance matrix is set up to include component representations of sources at nodes, but the effects of loads are retained as current injections, (1) then becomes

$$\begin{bmatrix} 0 \\ \boldsymbol{I}_l \end{bmatrix} = \left\{ \begin{bmatrix} \boldsymbol{Y}_{gg} & \boldsymbol{Y}_{gl} \\ \boldsymbol{Y}_{lg} & \boldsymbol{Y}_{ll} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Y}_G & 0 \\ 0 & 0 \end{bmatrix} \right\} \begin{bmatrix} \boldsymbol{V}_g \\ \boldsymbol{V}_l \end{bmatrix} \tag{3}$$

where $\mathbf{Y}_G$ is a diagonal matrix, with dimension equal to the number of generation buses, whose ith diagonal term represents the admittance equivalent to the complex generation at bus i for the power flow solution; that is $Y_{G_{ii}} = I_{bus_i}/V_{bus_i}$. The generation bus voltages $\mathbf{V}_g$ can then be expressed in terms of $\mathbf{V}_l$ as

$$\boldsymbol{V}_g = -\left[\boldsymbol{Y}_{gg} + \boldsymbol{Y}_G\right]^{-1}\boldsymbol{Y}_{gl}\boldsymbol{V}_l \tag{4}$$

and eliminating $\boldsymbol{V}$g from (3) yields the expression

$$\boldsymbol{I}_l = \left[\boldsymbol{Y}_{ll} - \boldsymbol{Y}_{lg}\left(\boldsymbol{Y}_{gg} + \boldsymbol{Y}_G\right)^{-1}\boldsymbol{Y}_{gl}\right]\boldsymbol{V}_l \tag{5}$$

On the other hand, the branch currents $\boldsymbol{I}_{br}$ can be expressed in terms of the nodal voltages as

$$\boldsymbol{I}_{br} = \left(\boldsymbol{Y}_p\boldsymbol{C}_b\right)\begin{bmatrix} \boldsymbol{V}_g \\ \boldsymbol{V}_l \end{bmatrix} = \left[\boldsymbol{A}_{bg}|\boldsymbol{A}_{bl}\right]\begin{bmatrix} \boldsymbol{V}_g \\ \boldsymbol{V}_l \end{bmatrix} \tag{6}$$

where $\mathbf{Y}_p$ is a diagonal matrix whose terms are the series admittance of the transmission lines, $\boldsymbol{C}_b$ is a branch-node incidence matrix, $[\boldsymbol{A}_{bg}|\boldsymbol{A}_{bl}] = \boldsymbol{Y}_p\boldsymbol{C}_b$ and matrices $\boldsymbol{A}_{bg}$ and $\boldsymbol{A}_{bl}$ result from the partition of columns corresponding to generation and load buses, respectively. The substitution of (4) into (6) allows currents $\boldsymbol{I}_{br}$ to be expressed in terms of the injected source currents; that is

$$\boldsymbol{I}_{br} = \left[\boldsymbol{A}_{bg}|\boldsymbol{A}_{bl}\right]\begin{bmatrix} -\left[\boldsymbol{Y}_{gg} + \boldsymbol{Y}_G\right]^{-1}\boldsymbol{Y}_{gl} \\ \boldsymbol{U}_l \end{bmatrix}\boldsymbol{V}_l \tag{7}$$

where $\boldsymbol{U}_l$ is the identity matrix; and using $\boldsymbol{V}_l$ from (5)

$$\boldsymbol{I}_{br} = \boldsymbol{K}_{bl}\boldsymbol{I}_l \tag{8}$$

where

$$\boldsymbol{K}_{bl} = \left[\boldsymbol{A}_{bg}|\boldsymbol{A}_{bl}\right]\begin{bmatrix} -\left[\boldsymbol{Y}_{gg} + \boldsymbol{Y}_G\right]^{-1}\boldsymbol{Y}_{gl} \\ U_l \end{bmatrix} \left[\boldsymbol{Y}_{ll} - \boldsymbol{Y}_{lg}\left(\boldsymbol{Y}_{gg} + \boldsymbol{Y}_G\right)^{-1}\boldsymbol{Y}_{gl}\right]^{-1}$$

is a matrix which relates the branch currents to the load bus current injections. Note that, since the component representations of lumped shunts, loads and current injections exactly match the load-flow source currents, the branch currents are numerically equal to those obtained from the solution of the steady-state network equations.

2.3 Branch currents attributed to sources

The branch currents $\boldsymbol{I}_{br}$ can be expressed in terms of the injected source currents by adopting a procedure similar to that previously outlined for sinks. In this case, the admittance component representation of a load would be found by dividing the sink current injection by the sink busbar voltage, both of which are again obtained from a load-flow solution for the network. The calculated admittance would be physically realizable as a passive *RLC* network, in this case.

If the nodal admittance matrix is set up to include lumped shunt components and component representations of loads at nodes, but to retain sources as current injections, the procedure adopted previously gives

$$\begin{bmatrix} \boldsymbol{I}_g \\ 0 \end{bmatrix} = \left\{ \begin{bmatrix} \boldsymbol{Y}_{gg} & \boldsymbol{Y}_{gl} \\ \boldsymbol{Y}_{lg} & \boldsymbol{Y}_{ll} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & Y_L \end{bmatrix} \right\} \begin{bmatrix} \boldsymbol{V}_g \\ \boldsymbol{V}_l \end{bmatrix} \tag{9}$$

where $\boldsymbol{Y}_L$ is a diagonal matrix, with dimension equal to the number of load buses and whose *i*th term is $Y_{L_{ii}} = I_{bus_i}/V_{bus_i}$. Now the relationship between the branch currents and the current injection of generation buses is given by

$$\boldsymbol{I}_{br} = \boldsymbol{K}_{bg}\boldsymbol{I}_g \tag{10}$$

where

$$\boldsymbol{K}_{bg} = [\boldsymbol{A}_{bg}|\boldsymbol{A}_{bl}] \begin{bmatrix} \boldsymbol{U}_g \\ -[\boldsymbol{Y}_{ll} + \boldsymbol{Y}_L]^{-1}\boldsymbol{Y}_{lg} \end{bmatrix} \left[\boldsymbol{Y}_{gg} - \boldsymbol{Y}_{gl}(\boldsymbol{Y}_{ll} + \boldsymbol{Y}_L)^{-1}\boldsymbol{Y}_{lg}\right]^{-1}$$

is a matrix which relates the branch currents to the generation bus current injections. Note that in all cases source and sink representations and source and sink currents are identical in the load-flow and admittance-matrix solutions.

Equations (8) and (10) could also be used to decompose power flows from sources to sinks. This would give useful information such as which generators were supplying the fixed losses (modelled as loads) in the network, and how the network capacity is being used by the generators and loads.

2.4 Attributed losses

The total active power losses can be expressed in matrix form as

$$\begin{aligned} P_l &= \boldsymbol{I}_{br}^{t^*}\boldsymbol{R}_p\boldsymbol{I}_{br} \\ &= (\boldsymbol{K}_{bl}\boldsymbol{I}_l)^{t^*}\boldsymbol{R}_p(\boldsymbol{K}_{bl}\boldsymbol{I}_l) \\ &= \boldsymbol{I}_l^{t^*}(\boldsymbol{K}_{bl}^{t^*}\boldsymbol{R}_p\boldsymbol{K}_{bl})\boldsymbol{I}_l \end{aligned} \tag{11}$$

where $\boldsymbol{R}_p$ is a diagonal matrix, whose terms are the series resistance of the transmission lines and $(\boldsymbol{K}_{bl}^{t^*}\boldsymbol{R}_p\boldsymbol{K}_{bl}\boldsymbol{I}_l)$ is a symmetrical matrix. Matrices $\boldsymbol{K}_{bl}$ and $\boldsymbol{K}_{bg}$ assign coefficients relating branch currents to current injections of loads and generators, respectively, but we need to attribute branch losses to loads and/or generators. The active power transmission loss corresponding to the *j*th branch is given by $P_{l_j} = I_{br_j}^* r_j I_{br_j}$ where I_{br_j} and r_j are the current and the series resistance, respectively, of the *j*th transmission line. From (8) the current of the *j*th branch is expressed in terms of the load bus current injections as

$$I_{br_j} = \boldsymbol{K}_{bl_j}\boldsymbol{I}_l$$

where $\boldsymbol{K}_{bl_j}$ is the *j*th row of matrix $\boldsymbol{K}_{bl}$; and thus the power loss of branch *j* is

$$\begin{aligned} P_{l_j} &= (\boldsymbol{K}_{bl_j}\boldsymbol{I}_l)^{t^*}\boldsymbol{r}_j(\boldsymbol{K}_{bl_j}\boldsymbol{I}_l) \\ &= \boldsymbol{I}_l^{*^t}(\boldsymbol{K}_{bl_j}^{t^*}r_j\boldsymbol{K}_{bl_j})\boldsymbol{I}_l \end{aligned}$$

where the product $\boldsymbol{I}_l^{*^t}(\boldsymbol{K}_{bl_j}^{t^*}r_j\boldsymbol{K}_{bl_j})$ provides the participation factors of load buses in the active power loss corresponding to the *j*th branch. The loss fraction assigned to the *i*th load bus is given by the product of its nodal factor by the corresponding bus current injection. Summing over all the branches the parcels corresponding to *i*th load bus would give its total attributed loss.

Observe that, since the loss is expressed in terms of the complex current injections, the participation factors are also complex numbers. The transmission loss fraction attributed to bus *j* may be expressed more rigorously as $\mathrm{Re}(\boldsymbol{I}_l^{*^t}(\boldsymbol{K}_{bl_j}^{t^*}r_j\boldsymbol{K}_{bl_j})\boldsymbol{I}_l)$.

To attribute losses to generators a similar process is carried out for the branch currents attributed to sources. In this case, the power loss is expressed as

$$P_l = \boldsymbol{I}_g^{t^*}(\boldsymbol{K}_{bg}^{t^*}\boldsymbol{R}_p\boldsymbol{K}_{bg})\boldsymbol{I}_g \tag{12}$$

with the vector of participation factors of generators given by the complex vector $\boldsymbol{I}_g^{t^*}(\boldsymbol{K}_{bg}^{t^*}\boldsymbol{R}_p\boldsymbol{K}_{bg})$ and the parcel of generation buses in the active power loss of the *j*th transmission line calculated from $\mathrm{Re}(\boldsymbol{I}_g^{*^t}(\boldsymbol{K}_{bg_j}^{t^*}r_j\boldsymbol{K}_{bg_j})\boldsymbol{I}_g)$.

Whether losses are being attributed to the set of sources or the set of sinks, it would be possible for the total loss attributed to a member of that set to be negative. At first this seems counter-intuitive, as losses are always positive, but an example of this would occur if the components of a branch current attributed to a large inductive load and a small capacitive load were almost antiphase. Losses attributed to the last-mentioned would be negative but both the total loss and the loss attributed to the inductive load would be less than the corresponding values when the capacitive load was switched off.

No special treatment is needed for reactive power compensation sources, i.e. if power is transferred to the network the component is treated as a *source*; alternatively, if power is drawn from the network it is treated as a *sink*. If reactive power compensation sources behave as sinks and variable losses are to be attributed to the sinks, a choice can be made as to whether to include reactive power sources in the set of sinks to which the losses are attributed. Consider the example of the capacitance and conductance to earth of a line which can be modelled by a lumped lossy shunt capacitor at each end of the line. These *components* could either be treated explicitly as sinks at each node or their contribution implicitly included in the bus admittance matrix. If the former choice were made then the contributions to network variable losses of these notional components could be evaluated. The network line losses can therefore be expressed equally well as a function of external loads and assumed lossy capacitive components at the line ends. Different results would obviously be obtained for the explicit and implicit treatments, but each would be the unique solution to loss attribution to different sets of sinks.

In a similar manner the system admittance matrix can include discrete (power absorbing) reactive power compensation components either explicitly or implicitly. In the last case an extra row and column would be present corresponding to each component and this augmented system matrix would allow the losses to be expressed as a function of the augmented set of sinks. The fixed network losses, e.g. due to the transformer magnetising loss, can be modelled as sinks and as such would contribute to the variable loss. If these sinks were modelled explicitly, variable loss attributable to the network could be separated out if required.

3 Worked example

A simple example without fixed losses can be worked through to show the application of the proposed allocation method. The six-bus system, whose transmission line data are shown in Table 1, is used for this purpose. Two generators (located at buses 1 and 2) supply the power demand (located at buses 3, 5 and 6), while bus 4 is a zero injection (transfer) bus.

Table 1: Six-bus system: transmission line data

Line	From	To	R (%)	X (%)	B_{sh} (%)	Tap
1	1	4	8.00	37.00	3.00	—
2	1	6	12.30	51.80	4.20	—
3	2	3	72.30	105.00	0.00	—
4	2	5	28.20	64.00	0.00	—
5	3	4	0.00	13.30	0.00	1.041
6	4	6	9.70	40.70	3.00	—
7	5	6	0.00	30.00	0.00	1.049

Table 2 summarises the power flow solution by the Newton–Raphson method. Columns 2, 3 and 4 show, respectively, bus complex voltages, current injections and power injections. The admittance equivalent to the relationship between bus current injection and voltage (computed from the values in columns 2 and 3) is given in column 5.

3.1 Loss allocated to sources only

If the active power transmission loss is assigned to generation buses only, the complex power injection of buses 3, 4, 5 and 6 are converted into equivalent admittances; that is, the terms of the (4×4) diagonal matrix $\boldsymbol{Y}_L$ of (9) are $0.559\angle 166.70$, $0.000\angle 0$, $0.366\angle 149.04$ and $0.544\angle 174.29$ per unit (from the last column of Table 2). These terms are included in the diagonal of matrix $\boldsymbol{Y}_{bus}$, as required by (9).

The branch currents and the current injections at the generation buses are related by (10) which numerically (in per-unit values) is

$$\begin{bmatrix} 0.5960\angle -24.33 \\ 0.5179\angle -23.45 \\ 0.0974\angle -27.22 \\ 0.2230\angle -40.36 \\ 0.4861\angle -172.74 \\ 0.1199\angle -24.64 \\ 0.1192\angle -166.95 \end{bmatrix} = \begin{bmatrix} 0.5457\angle -1.45 & 0.0296\angle -98.76 \\ 0.4688\angle -1.51 & 0.0098\angle 29.22 \\ 0.0295\angle 170.3 & 0.4073\angle 8.05 \\ 0.0295\angle -9.60 & 0.5994\angle -5.46 \\ 0.4311\angle 155.20 & 0.1263\angle -66.90 \\ 0.1035\angle -7.26 & 0.0356\angle 66.23 \\ 0.1844\angle -177.3 & 0.3778\angle -15.14 \end{bmatrix} \begin{bmatrix} 1.098\angle 22.03 \\ 0.318\angle -36.38 \end{bmatrix} \tag{13}$$

and the participation factors of the generation buses 1 and 2 in the active power transmission loss, evaluated as $\boldsymbol{I}_g^{t^*}(\boldsymbol{K}_{bg}^{t^*}\boldsymbol{R}_p\boldsymbol{K}_{bg})$ are $0.0568\angle 22.71$ and $0.0667\angle 34.38$, respectively. The loss parcels corresponding to these buses given by Re $(\boldsymbol{I}_g^{*^t}(\boldsymbol{K}_{bg}^{t^*}\boldsymbol{R}_p\boldsymbol{K}_{bg})\boldsymbol{I}_g)$ are 0.0624 and 0.0213 p.u. (for a 100 MVA base).

3.2 Loss allocated to sinks only

In this case the admittances equivalent to the power injection of buses 1 and 2 are included in the matrix $\boldsymbol{Y}_{bus}$ according to (3). These values are $\boldsymbol{Y}_{G_{11}} = 0.998\angle -22.03$ and $\boldsymbol{Y}_{G_{22}} = 0.289\angle -26.47$ per unit, respectively (from the fourth column of Table 2). The vector of branch currents (in per unit values), which corresponds to right side of (13), is represented in terms of load bus current

Table 2: Six-bus system: power flow results

Bus	$\boldsymbol{V}_{bus}$ p.u.∠deg	$\boldsymbol{I}_{bus}$ p.u.∠deg	$\boldsymbol{S}_{bus}$ p.u.∠deg	$Y_{G(D)}$ to Y_{Gii} p.u.∠deg	Y_{Lii} p.u.∠deg
1	1.100∠0.000	1.098∠22.03	1.208∠−22.03	0.998∠−22.03	—
2	1.100∠−9.91	0.318∠−36.38	0.345∠−46.29	0.289∠−26.47	—
3	1.005∠−14.28	0.562∠152.42	0.565∠138.14	—	0.559∠166.70
4	0.983∠−10.63	—	—	—	—
5	0.978∠−15.26	0.357∠133.78	0.349∠118.52	—	0.366∠149.04
6	0.960∠−13.29	0.523∠161.00	0.502∠147.71	—	0.544∠174.29

injections as

$$
\left[\begin{array}{ll}
0.5717\angle-177.99 & 0.5636\angle-178.25 \\
0.2133\angle-166.63 & 0.1854\angle-169.64 \\
0.2066\angle-171.13 & 0.1502\angle-173.55 \\
0.0656\angle 121.00 & 0.0857\angle 162.98 \\
0.7961\angle 11.26 & 0.2551\angle 139.89 \\
0.2547\angle-7.67 & 0.2777\angle-3.36 \\
0.1816\angle 114.85 & 0.1784\angle 134.26
\end{array}\right.
$$

$$
\left.\begin{array}{ll}
0.3562\angle-168.38 & 0.2908\angle-174.09 \\
0.4628\angle-175.82 & 0.4682\angle-176.58 \\
0.1112\angle 43.19 & 0.0360\angle 169.13 \\
0.2943\angle-176.68 & 0.1862\angle 177.34 \\
0.2830\angle 89.99 & 0.2271\angle 118.85 \\
0.2745\angle 174.11 & 0.3335\angle-179.41 \\
0.7183\angle 6.71 & 0.2372\angle 153.55
\end{array}\right]
$$

$$
\begin{bmatrix}
0.5622\angle 152.42 \\
0.0000\angle 0.00 \\
0.3578\angle 133.78 \\
0.5232\angle 161.00
\end{bmatrix}
$$

The load bus participation factors in the active power transmission loss, calculated from (11), and the corresponding fractions of the active power transmission loss (obtained by multiplying the participation factor by current injection) are, respectively,

$$
\begin{array}{lcll}
\textit{Bus} & & & \\
3 \Rightarrow 0.0550\angle-151.05 & & 0.0309 & \\
4 \Rightarrow 0.0513\angle-150.56 & \text{and} & 0.0000 & \text{p.u.} \\
5 \Rightarrow 0.0615\angle-150.72 & & 0.0210 & \\
6 \Rightarrow 0.0616\angle-151.05 & & 0.0317 &
\end{array}
$$

Table 3 shows the generated and consumed powers and the allocation of the active power transmission loss according to two methodologies. The first is based on the bus impedance matrix as proposed in [8]. It provides the loss parcels shown in column 6, both generation and load buses sharing the transmission loss. The two columns show the loss fractions attributed to generation buses only (column 7) and to load buses only (column 8) resulting from the application of the proposed allocation technique. The analysis of loss parcels attributed to generation buses reveals the the greatest fraction of loss corresponds to bus 1, which has the largest magnitude of bus current injection (or complex power injection). In the case of load buses the largest loss fraction is allocated to bus 6, which has the second largest load, indicating that not only the bus current magnitude but also the localisation of the bus in the network is taken into account in the loss allocation.

4 Numerical results

To assess the quality of the loss allocation determined through the proposed methodology, loss parcels were attributed to load buses of the IEEE 57-bus system. The consistency of these parcels was analysed through their comparison with those obtained by cooperative-game theory and determined by scaled Lagrange multipliers. The former was chosen because, although it requires a considerable computational effort [13], it has very desirable properties for a cost allocation rule [7]. Lagrange multipliers were selected because they are a byproduct from the OPF solution, having been frequently applied for allocating transmission losses, and their scaling requires a negligible computational effort. Two cases are studied. In the first the loss factors are computed for a base load. In the second the demand of a preselected set of load buses is increased by 40% and new loss fractions are assigned to all load buses. These new fractions are compared with those from the base case to determine how the increase in the transmission loss (due to the load increase) is shared.

4.1 Base case

The OPF solution which minimises the active power transmission loss for the base case of the 57-bus network indicates that 1270.98 MW are necessary to supply a total demand of 1250.80 MW, and the active power transmission loss is 20.18 MW. Generation buses (1, 2, 3, 6, 8, 9 and 12) have a total load of 595.00 MW, and thus the active power effectively injected in the transmission system to supply load and loss is 675.98 MW (the active power transmission loss representing approximately 3% of this amount).

The generated active power of bus 12 (150 MW) is smaller than the active power load (377 MW). For this reason, in terms of active power injection, this bus can be seen as a sink (which absorbs 227 MW from the transmission system) and thus must share the active power transmission loss with load buses.

Table 4 presents the power demand of the buses for which loss parcels were attributed and summarises the results of the loss allocation according to previously mentioned procedures. The application of the proposed methodology (denoted $\boldsymbol{Y}_{bus}^{\text{mod}}$) provides the loss parcels presented in column 4 of Table 4. The last two columns of this Table show the contribution of each load bus to the active power transmission loss, evaluated according to

Table 3: Six-bus system: loss allocation

Bus	Generation		Load		Z_{bus} method	Proposed method	
	P_g (Mw)	Q_g (MVAR)	P_d (Mw)	Q_d (MVAR)	(Mw)	$\boldsymbol{Y}_{bus}^{G}$ (Mw)	$\boldsymbol{Y}_{bus}^{L}$ (Mw)
1	112.00	45.34	—	—	3.88	6.24	—
2	31.37	15.62	—	—	1.44	2.12	—
3	—	—	55.00	13.00	0.96	—	3.09
4	—	—	00.00	00.00	—	—	—
5	—	—	30.00	18.00	0.77	—	2.10
6	—	—	50.00	5.00	1.31	—	3.17
Total	143.37	60.96	135.00	36.00	8.36	8.36	8.36

co-operative game theory (denoted CGT) and by scaled Lagrange multipliers (denoted SLM). The scale factor used for this purpose is expressed as P_l'/P_l (where $P_l' = \sum \lambda_{p_i} P_{d_i} + \sum \lambda_{q_i} Q_{d_i}$ is the value of the active power transmission loss estimated via Lagrange multipliers and P_l is the value of the active power loss obtained with OPF). Buses 4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46 and 48 are transfer buses. These buses have null power injection and therefore no loss parcels are attributed to them.

Some interesting aspects can be noted by analysing Table 4. With respect to the revenue reconciliation, it can be seen that all allocation procedures provide loss parcels whose total summation matches the total active power loss with satisfactory accuracy. Note also that Lagrange multipliers were scaled for this purpose, otherwise the recovered value of the loss would be greater than that given by OPF. In the case of the allocation based on co-operative game theory, it is well known that the accuracy of the recovered value depends on the number of intervals used to integrate the Lagrange multipliers of the OPF [11, 13]. Here, 50 intervals of integration were used to obtain the loss parcels of the fifth column of Table 5, resulting in an error of 1.78% with respect to the value of the power loss calculated from OPF. The proposed technique is based on a re-arrangement of the power-flow equations and thus it does not need either scaling factors or repeated solutions to recover the power loss value with high accuracy.

Table 4: 57-bus system: loss allocation base case

Bus	Power demand		Loss allocation method		
			Y_{bus}^{mod}	CGT	SLM
	P_{d_i} (MW)	Q_{d_i} (MVAR)	P_{l_i} (Mw)	P_{l_i} (Mw)	P_{l_i} (Mw)
5	13.00	4.00	0.09	0.02	0.38
10	5.00	2.00	0.12	0.12	0.15
12	227.0	24.0	7.55	7.71	7.02
13	18.00	2.30	0.40	0.40	0.54
14	10.50	5.30	0.25	0.24	0.32
15	22.00	5.00	0.28	0.25	0.65
16	43.00	3.00	1.21	1.26	1.32
17	42.00	8.00	0.72	0.75	1.26
18	27.20	9.80	0.17	0.02	0.79
19	3.30	0.60	0.11	0.10	0.10
20	2.30	1.00	0.10	0.10	0.07
23	6.30	2.10	0.28	0.27	0.20
25	6.30	3.20	0.35	0.32	0.20
27	9.30	0.50	0.28	0.25	0.29
28	4.60	2.30	0.09	0.07	0.14
29	17.00	2.60	0.13	0.06	0.50
30	3.60	1.80	0.25	0.24	0.12
31	5.80	2.90	0.52	0.49	0.20
32	1.60	0.80	0.12	0.21	0.06
33	3.80	1.90	0.30	0.28	0.13
35	6.00	3.00	0.43	0.42	0.20
38	14.00	7.00	0.60	0.59	0.44
41	6.30	3.00	0.10	0.10	0.19
42	7.10	4.00	0.34	0.34	0.23
43	2.00	1.00	0.03	0.03	0.06
44	12.00	1.80	0.39	0.38	0.37
47	29.70	11.60	1.02	0.99	0.92
49	18.00	8.50	0.59	0.59	0.56
50	21.00	10.50	0.90	0.91	0.66
51	18.00	5.30	0.42	0.43	0.55
52	4.90	2.20	0.16	0.15	0.15
53	20.00	10.00	0.88	0.81	0.63
54	4.10	1.40	0.09	0.08	0.12
55	6.80	3.40	0.01	0.00	0.20
56	7.60	2.20	0.42	0.42	0.25
57	6.70	2.00	0.42	0.42	0.22
Total	655.80	160.0	20.12	19.82	20.19

The loss fractions obtained through the proposed allocation technique and co-operative game theory are very similar. These parcels are affected by the power load magnitude but this is not the predominant aspect of these allocation procedures. Note that although load of bus 5 (13.00 MW, 4.00 MVAR) is greater than load of bus 10 (5.00 MW, 2.00 MVAR), this is not reflected in the loss apportions of these buses, 0.09 (0.02) (bus 5) and 0.12 (0.12) (bus 10). On the other hand, Lagrange multipliers of the OPF solution for the base case are similar in magnitude, ranging from 1.01 to 1.17 (active power balance) and from 0.01 to 0.07 (reactive power balance). As a consequence the loss parcels obtained from the application of these scaled multipliers practically depend only on the magnitude of the power load.

4.2 Load increase at buses

Aiming at assessing the level of cross-subsidy of the loss allocation through the proposed procedure, the active and reactive power load of buses 50 to 57 was increased by 40%, maintaining the power factor of the base case. The OPF solution for this new load level provides a power loss of 23.39 (increase of 3.21 MW or 15.91% with respect to the loss of the base case).

In addition to reconciliation revenue, the aim in this case was to determine how the load increase affects the loss attribution. It was expected that the load change would result in new loss fractions assigned to all buses, but bearing in mind that the smaller the differences corresponding to the buses with unchanged load, the smaller the level of cross-subsidy.

Table 5 shows the nodal loss fractions (columns 4 to 6) of the load buses. Comparison of columns 4 and 5 of Tables 4 and 5 shows that the largest changes in the individual contribution of each bus correspond to buses 50 to 57. According to the proposed methodology, the power-loss apportionment relative to these buses increased 2.75 MW with respect to the previous case (3.29 MW from the base case to 6.04 MW after load increase). This means that 85.67% of the loss increase (3.21 MW) was attributed to buses whose demand has been modified. Similar figures are obtained if the loss parcels are estimated through the methodology based on co-operative game theory. In this case, the total loss fraction attributed to buses 50 to 57 is 2.65 MW (3.22 MW from the base case to 5.87 MW after load increase), which represents 82.55% of the loss increase. On the other hand, the allocation procedure based on scaled Lagrange multipliers assigns a total loss of 2.78 MW to buses 50 to 57 in the base case and 4.33 MW after the demand change, which represents an attribution of 48.29% of the loss increase to the mentioned buses. These percentages indicate that, in contrast to the technique based on scaled Lagrange multipliers, the procedure proposed here provides results consistent with those obtained via co-operative game theory, showing a low level of cross-subsidy.

Table 5: 57-bus system: loss allocation with load change at selected bus

Bus	Power demand		Loss Allocation Method		
			Y_{bus}^{mod}	CGT	SLM
	P_{d_i} (Mw)	Q_{d_i} (MVAR)	P_{l_i} (Mw)	P_{l_i} (Mw)	P_{l_i} (Mw)
5	13.00	4.00	0.11	0.03	0.42
10	5.00	2.00	0.13	0.14	0.17
12	227.0	24.0	7.75	7.91	7.69
13	18.00	2.30	0.42	0.41	0.60
14	10.50	5.30	0.26	0.25	0.35
15	22.00	5.00	0.27	0.26	0.71
16	43.00	3.00	1.21	1.29	1.44
17	42.00	8.00	0.65	0.77	1.37
18	27.20	9.80	0.17	0.01	0.86
19	3.30	0.60	0.11	0.11	0.11
20	2.30	1.00	0.11	0.10	0.08
23	6.30	2.10	0.29	0.29	0.22
25	6.30	3.20	0.37	0.36	0.22
27	9.30	0.50	0.30	0.27	0.31
28	4.60	2.30	0.10	0.08	0.15
29	17.00	2.60	0.18	0.07	0.55
30	3.60	1.80	0.26	0.26	0.13
31	5.80	2.90	0.55	0.55	0.22
32	1.60	0.80	0.13	0.13	0.06
33	3.80	1.90	0.32	0.33	0.14
35	6.00	3.00	0.46	0.46	0.22
38	14.00	7.00	0.63	0.62	0.48
41	6.30	3.00	0.10	0.10	0.21
42	7.10	4.00	0.37	0.38	0.25
43	2.00	1.00	0.03	0.03	0.07
44	12.00	1.80	0.40	0.40	0.41
47	29.70	11.60	1.05	1.04	1.01
49	18.00	8.50	0.64	0.64	0.61
50	29.40	14.70	1.53	1.55	1.03
51	25.20	7.42	0.69	0.68	0.84
52	6.86	3.08	0.35	0.32	0.24
53	28.00	14.00	1.88	1.75	1.01
54	5.74	1.96	0.20	0.18	0.19
55	9.52	4.76	0.00	0.00	0.30
56	10.64	3.08	0.68	0.69	0.38
57	9.38	2.80	0.69	0.70	0.34
Total	691.44	174.80	23.39	23.16	23.39

5 Conclusions

The strategy of loss allocation proposed, based on a modification of the bus admittance matrix, can be applied for any optimal or conventional power flow solution, requiring a moderate computational effort. Unlike the approaches based on the integration of Lagrange multipliers of OPF, it requires only a single steady-state power-flow solution. From analysis of the numerical results it is observed that the basic requirements of any allocation process are satisfied. The loss attribution of each power network user is accurately evaluated such that the revenue reconciliation is achieved naturally without need for scaling factors. If the load increases with respect to a base case, the loss variation is assigned predominantly to the buses with modified demand, indicating that the level of cross-subsidy is low. These features emphasise the potential of the proposed approach to solve problems of active power transmission loss allocation.

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