Transmission Loss Through Barriers Lined with Heterogeneous Porous Materials

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1. Introduction

Recently, several studies have revealed the benefit of performing air holes in porous material to improve their absorption efficiency. Olny [1] showed both theoretically and experimentally that the absorption coefficient of porous materials could be significantly increased in a large frequency band in the case of properly designed macroperforated highly resistive porous materials. Atalla et al [2] used a finite element based numerical formulation to model such configurations. Transmission loss through heterogeneous materials is also a major issue in all industries. Indeed, it is important to predict how the isolation performance of systems involving porous materials is affected by the presence of air cavities (leaks) such as holes carrying electric wires or plumbing pipes for instance. In addition, one may wonder if the transmission loss of double wall barriers lined with porous material can be improved or at least not deteriorated by keeping constant or decreasing the weight of the system, if heterogeneities such as air cavities or solid inclusions are added in the porous material. This paper investigates how the normal incidence transmission loss of heterogeneous porous materials inserted in an infinite rectangular wave-guide is affected by solid heterogeneities and acoustic cavities. The proposed model is based on a finite element Biot-Allard's formulation for the porous patches and classical finite element formulation for the solid inclusions. The coupling between the porous material and the wave-guide is accounted for explicitly using the modal behavior of the wave-guide. In this paper, numerical results are presented regarding the effect of geometric and physical parameters on the transmission loss performance of single highly resistive plastic foam. Results regarding the performance of multilayered systems involving heterogeneous porous materials will be shown during the oral presentation.

2. Theory

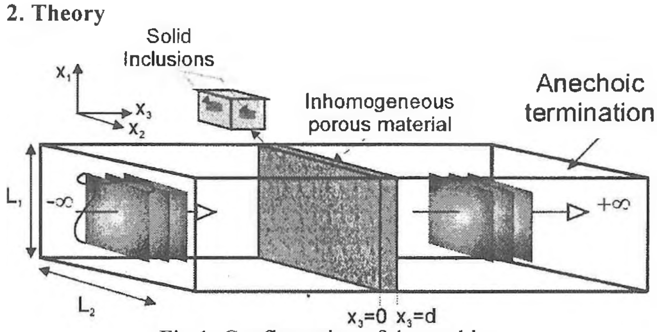

Fig.1: Configuration of the problem

The geometry of the problem is depicted in Fig.1. It consists of a three-dimensional patchwork inserted in a semi-infinite rectangular wave-guide. An incoming plane wave propagating in the wave-guide excites the system. Each 3-D patch is rectangular and either made from a homogeneous porous material modeled using Biot-Allard's theory or from a solid elastic material. The density and sound speed in the wave-guide are noted ρ_0 and c_0, respectively. In the following, a temporal dependency $e^{j\omega t}$ for all the fields is assumed. It is also supposed that the solid patches are located within the porous material, that is the patches on the front and rear faces of the material are all porous.

The classical weak integral form associated to the porous material has been given in Atalla [2]. Here, a modified form which has been presented in Sgard [3] is used:

$$\begin{aligned}&\int_{\Omega_p}\left[\underline{\underline{\tilde{\sigma}}}^s(\underline{u}):\underline{\underline{\varepsilon}}^s.(\underline{\delta u})-\omega^2\tilde{\rho}\underline{u}.\delta\underline{u}\right]d\Omega+\int_{\Omega_p}\left[\frac{\phi^2}{\omega^2\tilde{\rho}_{22}}\underline{\nabla}p.\underline{\nabla}\delta p-\frac{\phi^2}{\tilde{R}}p\delta p\right]d\Omega\\&-\int_{\Omega_p}\frac{\phi^2\rho_0}{\tilde{\rho}_{22}}\delta(\underline{\nabla}p.\underline{u})d\Omega-\int_{\Omega_p}\phi\left(1+\frac{\tilde{Q}}{\tilde{R}}\right)\delta(p\underline{\nabla}.\underline{u})d\Omega\\&-\int_{\partial\Omega_p}\phi\left[\underline{U}.\underline{n}-\underline{u}.\underline{n}\right]\delta p\,d\Gamma-\int_{\partial\Omega_p}\left[\underline{\underline{\sigma}}^t.\underline{n}\right]\delta\underline{u}\,d\Gamma=0\quad\forall\left(\delta\underline{u},\delta p\right)\end{aligned}\tag{1}$$

Ω_p and $\partial\Omega_p$ refer to the porous-elastic domain and its bounding surface. $\underline{u}$ and p are the solid phase displacement vector and the interstitial pressure in the porous-elastic medium, respectively. $\underline{U}$ is the fluid macroscopic displacement vector. $\delta\underline{u}$ and δp refer to their admissible variation, respectively. $\underline{n}$ denotes the unit normal vector external to the bounding surface $\partial\Omega_p$. $\underline{\underline{\tilde{\sigma}}}^s$ and $\underline{\underline{\varepsilon}}^s$ are the in-vacuo stress and strain tensors of the porous material. $\underline{\underline{\tilde{\sigma}}}^t$ is the total stress tensor of the material given by: $\underline{\underline{\tilde{\sigma}}}^s=\underline{\underline{\sigma}}^t+\phi\left[1+\frac{\tilde{Q}}{\tilde{R}}\right]p\underline{\underline{1}}$.

Note that $\underline{\underline{\tilde{\sigma}}}^s$ accounts for structural damping in the skeleton through a complex Young's modulus $E(1+j\eta_s)$. ϕ stands for the porosity, $\tilde{\rho}_{22}$ is the modified Biot's density of the fluid phase accounting for viscous dissipation, $\tilde{\rho}$ is a modified density given by $\tilde{\rho}=\tilde{\rho}_{11}-\frac{\tilde{\rho}_{12}}{\tilde{\rho}_{22}}$ where $\tilde{\rho}_{11}$ is the modified Biot's density of the solid phase accounting for viscous dissipation. $\tilde{\rho}_{12}$ is the modified Biot's density which accounts for the interaction between the inertia forces of the solid and fluid phase together with viscous dissipation. $\tilde{Q}$ is an elastic coupling coefficient between the two phases, $\tilde{R}$ may be interpreted as the bulk modulus of the air occupying a fraction ϕ of the unit volume aggregate.

This modified form is particularly suited for treating the coupling of porous-solid patches since it allows for the surface terms to vanish provided that one ensures the continuity of displacements. No calculation of coupling matrices is needed which saves considerable time during the assembly process.

For a porous material placed into a wave guide, the surface terms corresponding to the front ($\partial\Omega_p^1$) and rear ($\partial\Omega_p^2$) faces of the porous material lead to:

$$\int_{\partial\Omega_p^i} \delta(p\underline{u}.\underline{n})d\Gamma - \int_{\partial\Omega_p^i} \frac{1}{\omega^2\rho_0}\frac{\partial p}{\partial n}\delta p d\Gamma \qquad (2)$$

This equation is the result of applying continuity conditions at the air-porous interface. The first term amounts to the calculation of a classical coupling matrix whereas the second term can be rewritten using the orthogonal modes of the rectangular wave-guide, as [2]:

$$\begin{aligned}\int_{\partial\Omega_p^i} \frac{1}{\omega^2\rho_0}\frac{\partial p}{\partial n}\delta p d\Gamma = \frac{1}{j\omega}\int_{\partial\Omega_p^i}\int_{\partial\Omega_p^i} \underline{\underline{A}}^i(\underline{x},\underline{y})p(\underline{y})\delta p(\underline{x})d\Gamma_y d\Gamma_x \\ -\frac{\varepsilon}{j\omega}\int_{\partial\Omega_p^i}\int_{\partial\Omega_p^i} \underline{\underline{A}}^i(\underline{x},\underline{y})p_b(\underline{y})\delta p(\underline{x})d\Gamma_y d\Gamma_x \quad i=1,2\end{aligned} \qquad (3)$$

where ε=1 if i=1 and 0 if i=2; $\underline{x}=(x_1,x_2)$ and $\underline{\underline{A}}^i$ is an admittance operator given by

$$\underline{\underline{A}}^i(\underline{x},\underline{y}) = \sum_{(m,n)} \frac{k_{mn}^i}{\rho_0\omega N_{mn}}\varphi_{mn}(\underline{x})\varphi_{mn}(\underline{y}) \qquad (4)$$

with:

$$\varphi_{mn}(\underline{x}) = \cos\left(\frac{m\pi}{L_1}\right)\cos\left(\frac{n\pi}{L_2}\right), \left(k_{mn}^i\right)^2 = \left(k^i\right)^2 - \left(\frac{m\pi}{L_1}\right)^2 - \left(\frac{n\pi}{L_2}\right)^2 \text{ and}$$

$$N_{mn} = \int_{\partial\Omega_p^i} \left|\varphi_{mn}(\underline{x})\right|^2 d\Gamma_x$$

p_b is the blocked pressure loading at the porous material interface (x_3=0).

This form has the advantage of depicting the coupling with the wave guide in terms of radiation admittance in the emitter and receiver media together with a blocked-pressure loading. Note that at low frequencies (below the cut-off frequency of the wave-guide), higher modes lead to a purely imaginary admittance operator of an inertance type.

The transmission loss is defined as $TL = -10\log\tau$ where Π^t is the transmitted power given by

$$\Pi^t = -\frac{1}{2\omega}\Im\left[\int_{\partial\Omega_p^2} p^*(\underline{y})\underline{\underline{A}}^2(\underline{x},\underline{y})p(\underline{x})d\Gamma_x d\Gamma_y\right] \qquad (5)$$

and Π_{inc}, is the incident power which in the case of a plane wave excitation, of complex amplitude p_0, is given by $\Pi_{inc} = S|p_0|^2/(2\rho_0 c)$ where S is the cross-section of the wave guide.

3. Results

The normal incidence transmission loss through a 10cm wide and 3.75cm thick square heterogeneous plastic foam sample with characteristics given in Table 1 is considered. The influence on the transmission loss of the spatial distribution of very light elastic trapped heterogeneities (polystyrene ρ_s=2kg/m^3, E=3E^{+9}Pa) is studied. Figure 2 shows a cross section perpendicular to the x_3 axis inside the porous material together with the location of solid inclusions marked in light grey. Case (a) refers to the simple porosity (i.e without heterogeneities added); cases (b) to (d) represent respectively one single inclusion, 9 regularly spaced inclusions and 45 inclusions randomly distributed in 5 layers 0.53cm thick each, within the porous material (9 inclusion per layer). All inclusions are 2.68cm deep except for the random distribution. These configurations correspond to a percentage of 7.9% of the total volume of the porous material occupied by the trapped cavities. Figure 3 presents the normal incidence transmission loss for the different configurations interest. The transmission loss is not affected at low frequencies but small gains are found at high frequencies. The different distributions do not exhibit strong differences. Yet, case (b) is the most efficient at high frequencies while case (c) provides the best overall improvement. Case (d) yields an improvement around 1000Hz but performs less compared to the 2 other configurations .

ϕ	σ (kNm^{-4}s)	α_∞	Λ (μm)	Λ' (μm)	ρ_1 (kgm^{-3})	E (kPa)	ν	η
0.97	87	2.52	37	121	31	143	0.3	0.055

Table 1: Characteristics of the studied plastic foam

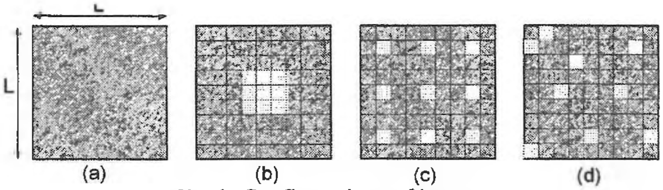

Fig. 2: Configurations of interest

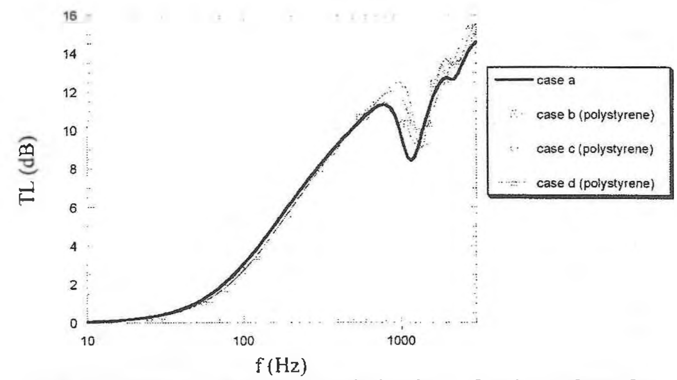

Fig. 3: Normal incidence transmission loss of a plastic foam for several distributions of solid trapped heterogeneities

4. Conclusion

The normal incidence transmission loss of porous media with added trapped solid heterogeneities has been predicted from a 3D numerical model wherein each porous patch is modeled using Biot-Allard poroelasticity equations. The coupling between the porous material and the wave-guide is accounted for explicitly using the modal behavior of the wave-guide. It has been shown that the normal incidence transmission loss of homogeneous materials could be slightly increased at sufficiently high frequencies by adding very light elastic solid inclusions like polystyrene ones. Further work involves studying the coupling of heterogeneous porous materials with elastic structures.

References

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