Geophys. J. Int. (1996) **125**, 559–566

Transmission properties of a plane fault

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Accepted 1995 December 22. Received 1995 December 18; in original form 1994 December 7

SUMMARY

Continuity conditions are derived for a fault modelled as a plane with isolated areas of slip. These slip areas are, for simplicity, taken to be such that their overall effect is that of a distribution of circular cracks; discontinuities in both normal and tangential components of displacement are allowed, depending on the internal conditions. Dry (gas-filled), partial or saturated liquid fill, or a fill of a weak visco-elastic solid are possible within the theory. The results are given in terms of the mean wave, which, at wavelengths long compared with the scale-lengths of the fault structure, is an accurate approximation to the displacement field. The continuity conditions that arise under this scheme are identical to those for a thin layer of visco-elastic material. However, unlike earlier, more empirical models of an 'averaged' fault, the parameters involved are directly related to the fault structure and include crack–crack interactions. It is clear from earlier work that a fault of this type is capable of supporting Stoneley waves.

Key words: effective medium theory, elastic-wave theory, fault models, seismic waves

1 INTRODUCTION

It is well known that a fault is not a simple air- or fluid-filled crack. The faces of a fault are pressed together by the lithostatic stress corresponding to the depth of burial. Fault movement commences when the shear tractions on the fault surface are sufficient to overcome friction and non-conformities in the two faces, and, in general, takes the form of slip. On the other hand, while inactive as a radiator of seismic waves, a fault acts as a passive reflector, refractor and scatterer of seismic radiation. This is because, although pressed together, the two faces do not exactly conform and so, while in some areas there is effectively welded contact, in others the faces are separated and the space between filled with gas, liquid, fault gouge or a combination of all three.

In order to calculate the effect of such a structure on seismic waves we take, as a model of an extended fault, an infinite plane containing a random distribution of co-planar cracks. Outside the area of the cracks, the two sides of the plane are in welded contact, while the cracks are, by definition, regions where the displacements on either side of the plane are allowed to be discontinuous (see Fig. 1). Interior conditions for the cracks are a matter of choice, but for practical purposes they are limited to 'dry' (gas-filled), fully or partially liquid-filled and 'cement-filled', where the cement is a weak visco-elastic solid. (If the cement is strong, the faces are in effectively welded contact.) The gas is usually air and the liquid usually water, possibly with dissolved gas and salts. In all cases, however, we assume that the relationship between imposed stress and displacement discontinuity on the crack is linear. The 'closed crack', for instance, which can transmit compression but cannot sustain tension (Dundurs & Comninou 1979) is not within the scope of this model.

Clearly this model of partial bonding is highly idealized. However, any study which aims, as this one does, to establish a direct relationship between overall properties and the microstructure (rather than introducing purely empirical parameters) must be specific about microstructure geometry. The representation of a fault as a plane distribution of cracks has been employed by, amongst others, Angel & Achenbach (1985), Schoenberg & Douma (1988) and Sotiropoulos & Achenbach (1988). [For a review of models of imperfect bonding, see Nagy (1992).] When two non-conforming surfaces are presented to each other, initial contact is made at isolated points and, as the pressure is increased, these contacts broaden out as a result of elastic and plastic deformation. Ultimately, contact will be achieved over most of the interface, leaving a relatively small number of open 'cracks'. This is the state that we wish to analyse here. There is a variety of literature dealing with the initial contact stage (kissing bond), mainly using Hertz contact theory at the points where the surfaces meet (Yoshioka 1994), but nothing that analyses the development of the later stage (partial bond) in which the interface is bonded over the major part of its area.

In this paper, we shall assume that all wavelengths are large compared with the scale-sizes of the structure of the fault (the

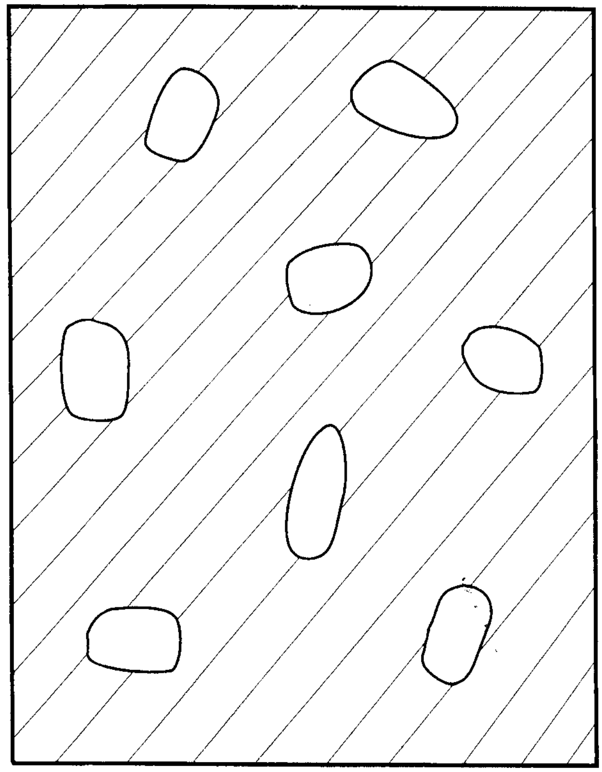

Figure 1. The fault model: a plane distribution of cracks in an otherwise welded (hatched region) interface.

diameter and spacing of the cracks) but are short compared with the overall length of the fault. It follows that an incoming wave will not 'see' the structure of the fault, which will behave as a uniform reflector. The effective boundary conditions at the fault form the main result. They contain parameters that depend directly on crack size and spacing length and on the interior condition on the cracks.

The method used is the method of smoothing (Keller 1964), so the results for reflection and refraction correspond to the 'mean wave'. At long wavelengths, the mean wave is a good approximation to the actual displacements. At the same time, scattering from the microstructure will be negligible. In other circumstances, the mean wave may be interpreted as an average over the radiation recorded at a number of different locations at a fixed distance from the fault.

The earliest attempt to model the structure of incomplete bonding appears to have been by Sezawa & Kanai (1940), who proposed a linear law of friction to represent the effect of a geological fault or block structure. In this, the traction and normal component of displacement are taken to be continuous across the bond, while the transverse component of displacement has a discontinuity whose rate of change is proportional to the transverse component of traction. Miller (1978) extended this model to allow for the non-linear dependence of the frictional force on the displacement discontinuity and its time derivative. This non-linear law is, however, approximated by a linear relation in order to obtain analytical results. The same interface condition of a discontinuity displacement proportional to transverse traction (where the constant of proportionality may depend on frequency) was employed by Newmark, Siess & Viest (1951) to represent incomplete bonding of composite beams. This was probably the first study in which the complex pattern of slip and weld was replaced by an averaged relation at the interface. Similar conditions were derived by Murty (1976) from consideration of a vanishingly thin layer of viscous fluid. Murty gives numerical results for the reflected and transmitted energy for an incident plane wave for either similar or different material properties either side of the fault. The transmission and reflection properties of a thin layer of fluid were also studied by Fehler (1982), specifically to interpret acoustic events from hydrofracture, and the effect of a thin, weak elastic layer was investigated by Jones & Whittier (1967), who considered the propagation of Stoneley waves along the interface.

Schoenberg (1980) extended Murty's (1976) model to include crack-opening displacements. The discontinuity in each component of displacement is taken to be proportional to the corresponding component of traction, and the constants of proportionality can all be different. Further generality was introduced by Tleukenov (1991), who considered thin layers filled with anisotropic elastic or linearly viscoelastic material, and also derived non-linear relations between displacement discontinuity and traction for thin layers of non-linearly viscoelastic and visco-plastic material.

Schoenberg & Douma (1988) also extended the empirical model to allow for a general linear relation between displacement discontinuity on the fault plane and the traction. This relation is, naturally, governed by a second-rank tensor. The main purpose of the paper by Schoenberg & Douma (1988) was to model the properties of a material with a volume distribution of aligned cracks or joints. The volume distribution was achieved by an extended sequence of parallel planes of cracks. By making a comparison with results for cracked or jointed material, the authors were, for the first time, able to relate the parameters of the linear slip interface to the microstructure of crack size and spacing. These relations were derived from results of Hudson (1981) and are correct to first order only where interactions between cracks can be neglected.

In this paper we derive the fault conditions in a more direct way and include interaction between cracks.

Schoenberg's (1980) continuity conditions were taken up by Pyrak-Nolte & Cook (1987), who showed that interface waves of Stoneley type may propagate along such a surface so long as the ratios of the properties of the materials on either side satisfy certain conditions. In particular, the Stoneley wave exists if the two materials are identical.

2 THE METHOD OF SMOOTHING

Following Keller (1964), we derive expressions for the mean displacement field $\langle \boldsymbol{u} \rangle$ in material that consists of a uniform solid matrix permeated by a random distribution of co-planar cracks. We define the scattering operator $\varepsilon \boldsymbol{S}^n$ for the nth crack: if $\boldsymbol{u}^n$ is the field incident on the crack, $\varepsilon \boldsymbol{S}^n \boldsymbol{u}^n$ is the scattered field. The parameter ε is in some sense small and the method consists of an expansion in ascending powers of ε. In fact, ε may be taken to be the crack number density ν^s multiplied by the square of the crack radius (Hudson 1980, 1981). This corresponds physically to an expansion where later terms refer to higher-order interactions between cracks, so that the zeroth term is the field in the absence of cracks, the first corresponds to scattering from individual cracks, the second to crack–crack interactions in pairs, and so on. This is expressed by the following equation (Hudson 1980):

$$\langle \boldsymbol{u} \rangle = \boldsymbol{u}^0 + \varepsilon \sum_{n_1} \langle S^{n_1} \rangle \boldsymbol{u}^0 + \varepsilon^2 \sum_{n_1} \sum_{n_2 \neq n_1} \langle S^{n_1} S^{n_2} \rangle \boldsymbol{u}^0 + O(\varepsilon^3), \tag{1}$$

where $\boldsymbol{u}^0$ is the field in the absence of cracks and $\langle\ \rangle$ implies the mean or expectation over a random ensemble.

On the assumption that all cracks are similar, so that all the S^n are the same except for the location of the crack, we obtain

$$\langle \boldsymbol{u} \rangle = \boldsymbol{u}^0 + \varepsilon N \langle S^1 \rangle \boldsymbol{u}^0 + \varepsilon^2 N(N-1)) \langle S^1 S^2 \rangle \boldsymbol{u}^0 + O(\varepsilon^3), \tag{2}$$

where N is the total number of cracks and S^1, S^2 correspond to two representative and non-identical cracks. This may be inverted by successive approximations to give $\boldsymbol{u}^0$ in terms of $\langle \boldsymbol{u} \rangle$:

$$\boldsymbol{u}^0 = \langle \boldsymbol{u} \rangle - \varepsilon N \langle S^1 \rangle \langle \boldsymbol{u} \rangle + \varepsilon^2 \{N^2 \langle S^1 \rangle^2 - N(N-1) \langle S^1 S^2 \rangle\} \langle \boldsymbol{u} \rangle + O(\varepsilon^3). \tag{3}$$

The cracks are assumed to be randomly distributed over a plane surface $\mathscr{S}$. If $\nu^s(\boldsymbol{x})$ is the number density of cracks per unit area, with $\boldsymbol{x}$ referring to the position of the centroid, the probability density for a single crack is ν^s/N. Then,

$$\langle S^1 \rangle = \frac{1}{N} \int_{\mathscr{S}} \bar{S}(\xi) \nu^s(\xi)\, dS_\xi, \tag{4}$$

where $\bar{S}(\boldsymbol{x})$ is the mean scattering operator for a crack with centroid at $\boldsymbol{x}$ and integration is over all points ξ of the fault plane $\mathscr{S}$. Similarly,

$$\langle S^1 S^2 \rangle = \frac{1}{N(N-1)} \int_{\mathscr{S}} \int \bar{S}(\xi^1) \bar{S}(\xi^2) \nu^s(\xi^2) \nu^s(\xi^1 | \xi^2)\, dS_1 dS_2, \tag{5}$$

where $\nu^s(\xi^1|\xi^2)$ is the number density of cracks centred at ξ^1 given that there is a crack centred at ξ^2.

The mean scattering operator $\bar{S}$ is assumed, for simplicity, to correspond to the scattering operator for a circular crack. It would be a simple matter to extend the analysis to a 'mean' elliptic crack. Other shapes would require the numerical solution for the displacement discontinuity on an isolated crack under arbitrary stress at infinity.

3 EVALUATION OF THE SCATTERING OPERATOR

We express the scattering operator $\bar{S}$ in terms of the Green's function $\boldsymbol{G}^k(\boldsymbol{x}, \boldsymbol{x}')$ for the solid matrix:

$$\varepsilon \bar{S}_i(\xi) \boldsymbol{v} = \int_\Sigma [V_k](\xi, \boldsymbol{x}') c^0_{kjpq} \frac{\partial G^p_i(\boldsymbol{x}, \boldsymbol{x}')}{\partial x'_q} n_j\, dS \tag{6}$$

(Hudson 1980), where $[\boldsymbol{V}](\xi, \boldsymbol{x}')$ is the displacement discontinuity at points $\boldsymbol{x}'$ on the crack face Σ due to the incident field $\boldsymbol{v}$; the crack is centred at ξ and is assumed to be plane with normal $\boldsymbol{n}$. The Green's function satisfies

$$\left(c^0_{ijkl} \frac{\partial^2}{\partial x_j \partial x_l} - \rho^0 \omega^2 \delta_{ik} \right) G^p_k(\boldsymbol{x}, \boldsymbol{x}') = -\delta(\boldsymbol{x} - \boldsymbol{x}') \delta_{ip}; \tag{7}$$

$\{c^0_{ijkl}\}$ are the elastic stiffnesses, ω is the angular frequency and ρ^0 is the density of the solid matrix.

Since the problem is linear, the discontinuity across the crack can be written as a linear functional U of the tractions $\boldsymbol{t}$

produced on the crack face by the incident field:

$$[V_k](\xi, \boldsymbol{x}') = \frac{a}{\mu} U_{ki}\{t_i(\boldsymbol{v}; \boldsymbol{x}'); \boldsymbol{X}\}, \tag{8}$$

where points on the crack face are designated by $\boldsymbol{x}' = \xi + \boldsymbol{X}$; a/μ is a scaling factor (Hudson 1980); a is the radius of a crack and μ the modulus of rigidity of the solid matrix. Eq. (6) now becomes

$$\varepsilon \bar{S}_i(\xi)\boldsymbol{v} = \frac{a}{\mu} c^0_{kjmq} n_j \int_\Sigma U_{k\ell}\{t_\ell(\boldsymbol{v}; \xi + \boldsymbol{X}); \boldsymbol{X}\} \frac{\partial G^m_i(\boldsymbol{x}, \xi + \boldsymbol{X})}{\partial \xi_q} n_j dS_X, \tag{9}$$

and, if the crack is sufficiently small (smaller than the wavelength of the incident radiation), we may regard $\boldsymbol{t}$ and $\boldsymbol{G}$ as constant on Σ, which leads to

$$\varepsilon \bar{S}_i(\xi)\boldsymbol{v} = \frac{a^3}{\mu} c^0_{kjmq} n_j c^0_{\ell rst} n_r \frac{\partial v_s(\xi)}{\partial \xi_t} \frac{\partial G^m_i(\boldsymbol{x}, \xi)}{\partial \xi_q} \bar{U}_{k\ell}, \tag{10}$$

where

$$\bar{U}_{k\ell} = \frac{1}{a^2} \int_\Sigma U_{k\ell}(1, \boldsymbol{X})\, dS_X. \tag{11}$$

4 THE OVERALL EFFECT OF A FAULT

It follows that the second term on the right-hand side of eq. (3) is

$$\varepsilon N \langle S^1_i \rangle \langle \boldsymbol{u} \rangle = \frac{a^3}{\mu} \int_{\mathscr{S}} v^s(\xi) \bar{U}_{k\ell} \left(c^0_{\ell rst} \frac{\partial \langle u_s \rangle(\xi)}{\partial \xi_t} n_r \right) \left(c^0_{kjmq} \frac{\partial G^m_i(\boldsymbol{x}, \xi)}{\partial \xi_q} n_j \right) dS_\xi. \tag{12}$$

This is the field due to displacement discontinuity

$$[u_k](\boldsymbol{x}) = \frac{a^3}{\mu} v^s(\boldsymbol{x}) \bar{U}_{k\ell} \sigma_{\ell r}(\langle \boldsymbol{u} \rangle, \boldsymbol{x}) n_r, \tag{13}$$

where $\boldsymbol{\sigma}(\langle \boldsymbol{u} \rangle, \boldsymbol{x})$ is the stress at the point $\boldsymbol{x}$ corresponding to displacements $\langle \boldsymbol{u} \rangle$.

Similarly, the third term on the right-hand side of (3) becomes

$$\begin{aligned}
\varepsilon^2 N(N-1)\langle S^1_i S^2 \rangle \langle \boldsymbol{u} \rangle &= A_{mqpt} A_{k\ell jr} \int\!\!\int_{\mathscr{S}} v^s(\xi^2) v^s(\xi^1 | \xi^2) \frac{\partial G^m_i(\boldsymbol{x}, \xi^1)}{\partial \xi^1_q} \frac{\partial^2 G^k_p(\xi^1, \xi^2)}{\partial \xi^1_t \partial \xi^2_\ell} \frac{\partial \langle u_j \rangle(\xi^2)}{\partial \xi^2_r} dS_2 dS_1 \\
&= \int_{\mathscr{S}} \left\{ \left(c^0_{kjmq} \frac{\partial G^m_i(\boldsymbol{x}, \xi^1)}{\partial \xi^1_q} n_j \right) \frac{a^3}{\mu} c^0_{\ell rpt} n_r \bar{U}_{k\ell} A_{uvnx} \int_{\mathscr{S}} v^s(\xi^2) v^s(\xi^1 | \xi^2) \frac{\partial^2 G^u_p(\xi^1, \xi^2)}{\partial \xi^1_t \partial \xi^2_v} \frac{\partial \langle u_n \rangle(\xi^2)}{\partial \xi^2_x} dS_2 \right\} dS_1,
\end{aligned} \tag{14}$$

where

$$A_{mqpt} = \frac{a^3}{\mu} c^0_{kjmq} c^0_{\ell rpt} n_j n_r \bar{U}_{k\ell}.$$

Once again this has the form of a field arising from a discontinuity in displacement:

$$[u]_k(\boldsymbol{x}) = \frac{a^3}{\mu} c^0_{\ell rpt} n_r \bar{U}_{k\ell} A_{uvnx} \int_{\mathscr{S}} v^s(\xi) v^s(\boldsymbol{x} | \xi) \frac{\partial^2 G^u_p(\boldsymbol{x}, \xi)}{\partial x_t \partial \xi_v} \frac{\partial \langle u_n \rangle(\xi)}{\partial \xi_x} dS_\xi. \tag{15}$$

The mean field $\langle \boldsymbol{u} \rangle$ can now be seen to be made up of an unperturbed field $\boldsymbol{u}^0$ together with displacement discontinuities on the surface $\mathscr{S}$. These are given by

$$[\langle u_k \rangle] = \frac{a^3}{\mu} v^s(\boldsymbol{x}) \bar{U}_{k\ell} \sigma_{\ell r}(\langle \boldsymbol{u} \rangle, \boldsymbol{x}) n_r - \frac{a^3}{\mu} \bar{U}_{k\ell} c^0_{\ell rpt} n_r \int_{\mathscr{S}} \{ v^s(\xi) v^s(\boldsymbol{x} | \xi) - v^s(\xi) v^s(\boldsymbol{x}) \} \frac{\partial^2 G^u_p(\boldsymbol{x}, \xi)}{\partial x_t \partial \xi_v} \frac{a^3}{\mu} c^0_{ijuv} \bar{U}_{iq} n_j c^0_{qmnx} \frac{\partial \langle u_n \rangle(\xi)}{\partial \xi_x} n_m dS_\xi + \cdots, \tag{16}$$

and may be regarded as a boundary condition on $\mathscr{S}$ between $\langle \boldsymbol{u} \rangle$ and the tractions $\langle \boldsymbol{t} \rangle$ on $\mathscr{S}$ associated with $\langle \boldsymbol{u} \rangle$:

$$\langle t_i \rangle = c^0_{ijpq} \frac{\partial \langle u_p \rangle}{\partial x_q} n_j. \tag{17}$$

With this substitution, we have

$$[\langle u_k \rangle] = \frac{a^3}{\mu} v^s(x) \bar{U}_{k\ell} \langle t_\ell \rangle(x) - \left(\frac{a^3}{\mu}\right)^2 \bar{U}_{k\ell} \bar{U}_{iq} c^0_{\ell rpt} n_r c^0_{ijuv} n_j \int_{\mathscr{S}} \{v^s(\xi) v^s(x|\xi) - v^s(\xi) v^s(x)\} \frac{\partial^2 G^u_p(x, \xi)}{\partial x_t \partial \xi_v} \langle t_q \rangle(\xi)\, dS_\xi + \cdots . \tag{18}$$

The term

$$v^s(\xi) v^s(x|\xi) - v^s(\xi) v^s(x)$$

is effectively zero for ξ outside a region of scale-size equal to the intercrack spacing and centred on x. If $\langle u \rangle$, and therefore $\langle t \rangle$, vary slowly on this scale, we may write the integral in eq. (18) approximately as

$$\int_{\mathscr{S}} v^s(\xi) \{v^s(x|\xi) - v^s(x)\} \frac{\partial^2 G^u_p(x, \xi)}{\partial x_t \partial \xi_v} dS_\xi \langle t_q \rangle(x). \tag{19}$$

If, in addition, v^s is constant within the crack–crack spacing distance, this becomes

$$-\frac{(v^s)^2}{\mu} K^s_{ptuv} \langle t_q \rangle ,$$

where, with the replacement $G(x, \xi) = \mathscr{G}(\xi - x)$,

$$K^s_{ptuv} = -\mu \int_{\mathscr{S}} [1 - n^s(x, X)] \frac{\partial^2 \mathscr{G}^u_p(X)}{\partial X_t \partial X_v} dS_X ,$$

and (20)

$$n^s(x, X) = \frac{v^s(x|x + X)}{v^s} .$$

The effect of the surface distribution of cracks is, therefore, a discontinuity in the mean field across $\mathscr{S}$ which is related to the tractions in the mean field by

$$[\langle u_k \rangle] = \left(\frac{v^s a^3}{\mu}\right) \bar{U}_{k\ell} \left\{\delta_{\ell q} + \left(\frac{v^s a^3}{\mu^2}\right) \bar{U}_{iq} c^0_{\ell rpt} c^0_{ijuv} n_r n_j K^s_{ptuv}\right\} \langle t_q \rangle \tag{21}$$

to second order. There is no discontinuity in traction on $\mathscr{S}$.

5 EVALUATION OF $\bar{U}$ AND K^s

The components $\bar{U}_{ij}$ are exactly those that appear in the expressions for a volume distribution of cracks, and expressions have been derived for them for the case of circular cracks under dry and fluid-filled conditions and for a weak viscoelastic solid filling (Hudson 1981), and also for partial saturation with liquid (Hudson 1988).

Referring to Hudson (1980), we can evaluate eq. (20) for K^s_{ptuv} in the long-wavelength limit:

$$K^s_{ptuv} = \frac{1}{32} \left(\int_0^\infty \frac{dn^s(X)}{dX} \frac{dX}{X} \right) \{\delta_{pu} \delta_{tv} (7 + \beta^2/\alpha^2) - (\delta_{pt} \delta_{uv} + \delta_{pv} \delta_{tu})(1 - \beta^2/\alpha^2)\} ; \qquad p, t, u, v \neq 3, \tag{22}$$

where α, β are the wave speeds in the solid, on the assumption that n^s depends on $X = |X|$ only and that $n = (0, 0, 1)$. There is no singularity at the lower limit of integration since n^s must be zero for X less than the diameter of a crack. Other components of K^s_{ptuv} are

$$K^s_{33uv} = \left(\int_0^\infty \frac{dn^s}{dX} \frac{dX}{X} \right) \frac{1}{8} \left(\frac{\beta^2}{\alpha^2} - 1 \right) \delta_{uv} = K^s_{uv33} = K^s_{3uv3} = K^s_{u33v} ; \qquad u, v \neq 3,$$

$$K^s_{3u3v} = \left(\int_0^\infty \frac{dn^s}{dX} \frac{dX}{X} \right) \frac{1}{8} (1 + \beta^2/\alpha^2) \delta_{uv} ; \qquad u, v \neq 3,$$

$$K^s_{u3v3} = \left(\int_0^\infty \frac{dn^s}{dX} \frac{dX}{X} \right) \frac{1}{8} (\beta^2/\alpha^2 - 5) \delta_{uv} ; \qquad u, v \neq 3, \tag{23}$$

$$K^s_{3333} = \left(\int_0^\infty \frac{dn^s}{dX} \frac{dX}{X} \right) \frac{1}{4} (1 - 3\beta^2/\alpha^2),$$

and otherwise zero.

In order to evaluate the integral in (22) and (23), we need to specify n^s. We have the constraints

$$n^s(X) \to 1 \quad \text{as } X \to \infty ,$$

$$v^s \int_{\mathscr{S}} (1 - n^s(X))\, dS = 1 , \tag{24}$$

as well as the above condition that n^s is zero for X less than a crack diameter. The precise choice of functional dependence of n^s on X will not affect the result except for a numerical factor close to unity. We choose

$$n^s = 1 - \exp[-(X-2a)^2/\ell^2], \qquad X \ge 2a,$$
$$= 0, \qquad 0 \le X < 2a,$$

where a is a crack radius and ℓ a parameter roughly equal to a crack spacing. The second of conditions (24) give

$$\frac{1}{v^s} = \pi \ell^2, \tag{25}$$

neglecting a term in a/ℓ.

Substitution for n^s in the integral gives

$$\int_0^\infty \frac{dn^s}{dX}\frac{dX}{X} = \frac{2}{\ell^2}\int_{2a}^\infty (X-2a)\exp[-(X-2a)^2/\ell^2]\frac{dX}{X}$$
$$= \sqrt{\frac{\pi}{\ell}}, \tag{26}$$

approximately, neglecting terms in a/ℓ once more. We have

$$K^s_{ptuv} = (v^s)^{1/2}\chi^s_{ptuv},$$

where

$$\chi^s_{ptuv} = \frac{\pi}{32}\{\delta_{pu}\delta_{tv}(7+\beta^2/\alpha^2) - (\delta_{pt}\delta_{uv} + \delta_{pv}\delta_{tu})(1-\beta^2/\alpha^2)\}; \qquad p, t, u, v \ne 3. \tag{27}$$

$$\chi^s_{33uv} = -\frac{\pi}{8}(1-\beta^2/\alpha^2)\delta_{uv} = \chi^s_{uv33} = \chi^s_{3uv3} = \chi^s_{u33v}; \qquad u, v \ne 3,$$

$$\chi^s_{3u3v} = \frac{\pi}{8}(1+\beta^2/\alpha^2)\delta_{uv}; \qquad u, v \ne 3,$$

$$\chi^s_{u3v3} = -\frac{\pi}{8}(5-\beta^2/\alpha^2)\delta_{uv}; \qquad u, v \ne 3,$$

$$\chi^s_{3333} = \frac{\pi}{4}(1-3\beta^2/\alpha^2).$$

All other terms are zero.

6 DISCUSSION

Representation of a fault by a surface distribution of plane cracks leads to a linear relation between the displacement discontinuity and the tractions on the fault. Thus it is similar in form to most previous continuity conditions proposed for a loosely bonded or non-rigid interface. The relation here is stated in terms of the mean wave. In the absence of scattering (that is, at long wavelengths) the mean wave may be equated to the measured displacements. At shorter wavelengths, when the variance about the mean wave (the scattering) is significant, the mean wave is equivalent to an average displacement taken over a suitable array of measurements.

A dry, fluid- or solid-filled circular crack gives rise to a diagonal form for $\{\bar{U}_{ij}\}$ with $\bar{U}_{11} = \bar{U}_{22}$ if the normal to the cracks is in the 3-direction and if the matrix and fill materials are isotropic. So, for a fault lying parallel to the 12-plane, this means that the continuity conditions (21) reduce to

$$[u_1] = \frac{\alpha}{\mu\omega}At_1, \qquad [u_2] = \frac{\alpha}{\mu\omega}At_2, \qquad [u_3] = \frac{\alpha}{\mu\omega}Bt_3, \tag{28}$$

where the non-dimensional parameters A and B are given by

$$\begin{aligned} A &= \left(\frac{\omega a}{\alpha}\right)(v^s a^2)\bar{U}_{11}\left\{1 + (v^s a^2)^{3/2}\bar{U}_{11}\frac{\pi}{4}(3-2\beta^2/\alpha^2)\right\}, \\ B &= \left(\frac{\omega a}{\alpha}\right)(v^s a^2)\bar{U}_{33}\{1 + (v^s a^2)^{3/2}\bar{U}_{33}\pi(1-\beta^2/\alpha^2)\}. \end{aligned} \tag{29}$$

The second term in the brackets in each case corresponds to interactions between cracks. The relationship expressed by (28) is exactly that proposed by Schoenberg (1980), but here the coefficients A and B are directly related to the microstructure.

Schoenberg & Douma (1988) show how their empirical parameters may be related to a specific model of microstructure by comparing their formulae for a volume distribution of cracks with those of Hudson (1981), which are based on a specific model of aligned circular cracks. These relations correspond to eqs (29) exactly to first order in the crack number density. Schoenberg & Douma (1988) go on to say that crack–crack interactions can be taken into account by using higher-order terms from Hudson (1980) which are proportional to the square of the number density. However, crack–crack interactions on a single fault generate a term in number density to the power (5/2). It is clear therefore that, at this level of accuracy, the response of a volume distribution of cracks cannot be constructed simply from a sequence of faults, although it *can* be done when working to first order only. This is not surprising: taking crack–crack interactions into account for a sequence of faults will involve interactions between the faults, unless they are widely spaced.

We can simplify this expression further by using expressions for $\bar{U}_{11}$ and $\bar{U}_{33}$ for a crack filled with material with bulk modulus κ' and rigidity $\mu' = \mu_f + i\omega\eta$, where μ_f is an elastic rigidity (which may be zero) and $\eta(\omega)$ a viscosity:

$$\bar{U}_{11} = \frac{16}{3}\left(\frac{\lambda+2\mu}{3\lambda+4\mu}\right)\Big/(1+M), \qquad \bar{U}_{33} = \frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\Big/(1+K), \tag{30}$$

where λ and μ are the Lamé parameters of the matrix solid, and (Hudson 1981)

$$M = \frac{4}{\pi}\left(\frac{a\mu'}{c\mu}\right)\left(\frac{\lambda+2\mu}{\lambda+\mu}\right), \qquad K = \frac{1}{\pi}\frac{\left(\kappa' + \frac{4}{3}\mu'\right)}{c\mu}\left(\frac{\lambda+2\mu}{3\lambda+4\mu}\right), \tag{31}$$

where c/a is the aspect ratio of the crack.

Writing

$$d = \frac{4c}{3} \tag{32}$$

for the mean thickness of the crack, and

$$r = \nu^s \pi a^2 \tag{33}$$

for the relative area of fracture on the fault, we get

$$A = \frac{r(\omega d/\alpha)\mu}{\left[\mu' + \frac{3\pi d\mu}{16a}(3-2\beta^2/\alpha^2)\left(1-\frac{4r^{3/2}}{3\sqrt{\pi}}\right)\right]}, \qquad B = \frac{r(\omega d/\alpha)\mu}{\left[\kappa' + \frac{4}{3}\mu' + \frac{3\pi d\mu}{4a}(1-\beta^2/\alpha^2)\left(1-\frac{4r^{3/2}}{3\sqrt{\pi}}\right)\right]}, \tag{34}$$

to the same order of accuracy as before.

The form of (34) is identical to that for a uniform thin layer, of thickness d, containing material with shear modulus

$$\mu' + \frac{3\pi d\mu}{16a}(3-2\beta^2/\alpha^2)\left(1-\frac{4r^{3/2}}{3\sqrt{\pi}}\right)$$

and bulk modulus (35)

$$\kappa' - \frac{\pi d\mu}{4a}\frac{\beta^2}{\alpha^2}\left(1-\frac{4r^{3/2}}{3\sqrt{\pi}}\right)$$

(Jones & Whittier 1967), except that the magnitudes of the displacement discontinuities are reduced by a factor r, the relative area of slip.

Thus the elastic rigidity of the crack infill is increased by the presence of the crack edges whereas its bulk modulus is decreased. However, the resistance of the layer to both tension and shear is increased.

These results are, of course, always subject to the constraint that the full scale of interaction between cracks is not taken into account and the formulae are not reliable anywhere near $r = 1$; that is, when slip occurs over almost the whole fault.

The reflection and refraction of plane waves at a model fault of this type depend on the values taken by A and B, which in our case depend on the parameters chosen for crack size, spacing and infill. This quantification is important as it may permit some information about cracks or fault parameters to be inferred from seismic observations. Earlier studies, in which A and B are purely empirical, show curves of reflection and refraction coefficients for a variety of values (Murty 1976; Schoenberg 1980). In addition, it has been shown (Jones & Whittier 1967; Pyrak-Nolte & Cook 1987) that it is possible for interface waves of Stoneley type to propagate along the interface. If A and B are real, such waves are non-attenuating.

A 2-D version of the fault model analysed here was studied by Sotiropoulos & Achenbach (1988), who gave expressions for the reflection of a plane wave at normal incidence. On the theory constructed here, the reflection coefficient for a plane P wave is

$$R=\frac{q}{1+q}, \tag{36}$$

where

$$q=\frac{iB(\lambda+2\mu)}{2\mu}$$

$$=i\frac{(\lambda+2\mu)}{2\mu}\frac{\omega v^s a^3}{\alpha}\bar{U}_{33}$$

to first order in the crack number density $v^s a^2$. If we write V as the integral of the displacement discontinuity over an individual crack as a result of an incident wave of unit displacement amplitude, we have

$$\bar{U}_{33}=\frac{\mu}{a}\frac{\alpha V}{i\omega(\lambda+2\mu)}$$

and so

$$q=\frac{v^s a^2}{2}V. \tag{37}$$

Eqs (36) and (37) are precisely the formulae given by Sotiropoulos & Achenbach (1988), although they take V to be the integrated crack-opening displacement for a representative crack, taking all the other cracks into account, whereas $\bar{U}_{33}$ is calculated for a single crack on its own. Clearly the two theories agree to first order. Sotiropoulos & Achenbach (1988) approximate the effect of surrounding cracks by introducing a dipole source on either side of the crack on which V is calculated. Here we give expressions that are exact to the next higher order in $(v^s a^2)$.

ACKNOWLEDGMENTS

We thank John Queen and Bill Rizer of Conoco Inc. for their advice on the construction of the theoretical model studied in this paper. The work was supported by Conoco (UK) Ltd through Contract No. 2.343 and the Natural Environment Research Council and is published with the approval of Conoco Inc. and the Director of the British Geological Survey.

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