Transmit Power Adaptation for Multiuser OFDM Systems

Jiho Jang, *Student Member, IEEE*, and Kwang Bok Lee, *Member, IEEE*

***Abstract*—In this paper, we develop a transmit power adaptation method that maximizes the total data rate of multiuser orthogonal frequency division multiplexing (OFDM) systems in a downlink transmission. We generally formulate the data rate maximization problem by allowing that a subcarrier could be shared by multiple users. The transmit power adaptation scheme is derived by solving the maximization problem via two steps: subcarrier assignment for users and power allocation for subcarriers. We have found that the data rate of a multiuser OFDM system is maximized when each subcarrier is assigned to only one user with the best channel gain for that subcarrier and the transmit power is distributed over the subcarriers by the water-filling policy. In order to reduce the computational complexity in calculating water-filling level in the proposed transmit power adaptation method, we also propose a simple method where users with the best channel gain for each subcarrier are selected and then the transmit power is equally distributed among the subcarriers. Results show that the total data rate for the proposed transmit power adaptation methods significantly increases with the number of users owing to the multiuser diversity effects and is greater than that for the conventional frequency-division multiple access (FDMA)-like transmit power adaptation schemes. Furthermore, we have found that the total data rate of the multiuser OFDM system with the proposed transmit power adaptation methods becomes even higher than the capacity of the AWGN channel when the number of users is large enough.**

***Index Terms*—Channel capacity, downlink, multicarrier, orthogonal frequency division multiplexing (OFDM), power control, water-filling.**

I. Introduction

THE GROWING demand for wireless multimedia services requires reliable and high-rate data communications over a wireless channel. However, high-rate data communications are significantly limited by intersymbol interference (ISI) because of the time dispersive nature of the wireless channel. Multicarrier systems have aroused great interest in recent years as a potential solution to the problem of transmitting data over wireless channels with large delay spread [1]–[3]. An orthogonal frequency division multiplexing (OFDM) system is one of the widely used multicarrier systems. The principle of the OFDM technique is to split a high-rate data stream into a number of lower rate streams, which are then simultaneously transmitted on a number of orthogonal subcarriers. As the symbol duration is increased for lower rate parallel streams, the relative amount of dispersion in time caused by multipath delay spread decreases. Moreover, the ISI can be almost completely eliminated by introducing a guard interval, which is a cyclic extension of the OFDM symbol.

In a single user OFDM system, when the channel state information (CSI) is available at the transmitter, the transmit power for each subcarrier can be adapted according to the CSI in order to increase the data rate [4]–[6]. The data rate of the single user OFDM system is shown to be maximized when the transmit power is adapted with the water-filling policy in frequency domain under the constraint of total transmit power [4], [5], or in frequency-time domain under the constraint of average transmit power [6]. The transmit power adaptation in the frequency domain is beneficial to increase the data rate in a channel whose transfer function is frequency dependent. In a time-varying wireless channel, the frequency-time domain transmit power adaptation yields greater data rate than the transmit power adaptation in the frequency domain only, because the time varying nature of the wireless channel can be exploited. The increase of data rate by using the transmit power adaptation in a single user OFDM system is owing to the spectral diversity effects (in frequency domain) and/or temporal diversity effects (in time domain).

In a multiuser OFDM system, each of the multiple users' signals may undergo independent fading because users may not be in the same locations. Therefore, the probability that all the users' signals on the same subcarrier are in deep fading is very low. Hence, for a specific subcarrier, if a user's signal is in deep fading, the others may not be in deep fading and the user in a good channel condition may be allowed to transmit data on that subcarrier yielding multiuser diversity effects [7]. Therefore, in a multiuser OFDM system, the multiuser diversity, as well as the spectral diversity may be exploited, if the transmit power for each user and for each subcarrier is appropriately adapted to the channel condition.

So far, several papers have dealt with the problem of transmit power adaptation for the multiuser OFDM system in a downlink transmission. In [8], the authors attempted to minimize the total transmit power under a fixed performance requirement and a given set of user data rates. They focused on the practical algorithms that can support real-time multimedia data whose data rates are generally fixed. However, in the formulation of transmit power minimization problem in [8], they did not allow more than one user to share a subcarrier without any mathematical reasoning. In [9], dynamic subchannel and power allocation was performed to maximize the minimum capacity of all the users under the total transmit power constraint and zero delay constraint. The authors of [9] also restricted their focus on exclusive assignment of each subcarrier to only one user in order to avoid the error propagation and the complexity of successive

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The authors are with the School of Electrical Engineering, Seoul National University, Seoul 151-742, Korea (e-mail: klee@snu.ac.kr).

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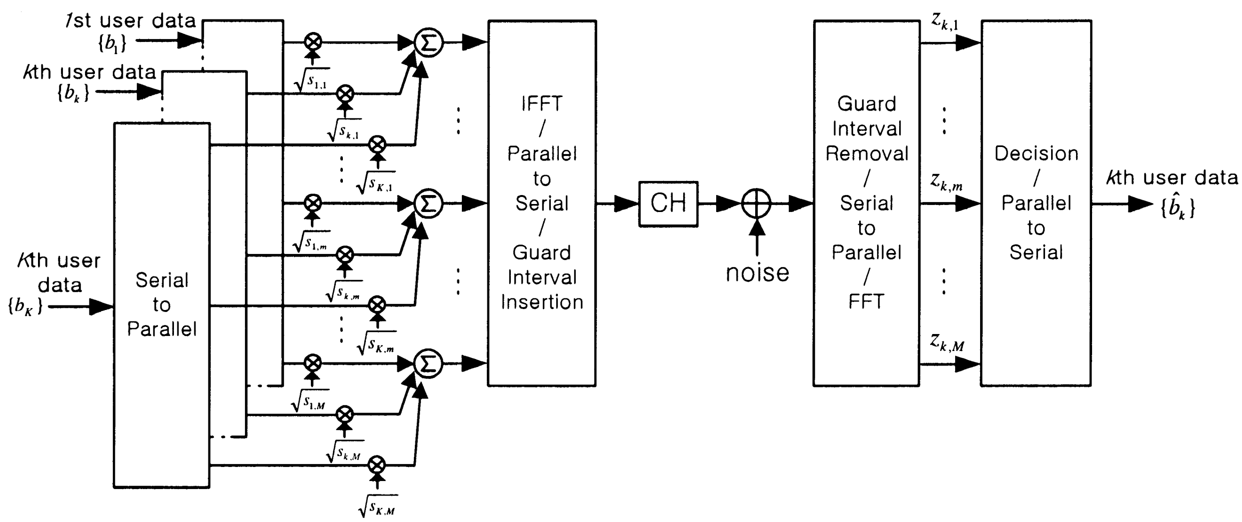

Fig. 1. Block diagram of a downlink multiuser OFDM system with transmit power adaptation.

decoding in the multiuser detection. The system model for the problems of transmit power minimization in [8] and minimum capacity maximization in [9] is viewed as a special case of the multiuser OFDM system. When the constraint that a subcarrier should be exclusively occupied by only one user is given, the multiuser OFDM system can be simplified as a frequency division multiple access (FDMA) system with dynamic subcarrier allocation. In this case, users transmit data through a number of orthogonal subcarriers assigned to their own independently and the interference from other users' signals does not exist.

In this paper, we focus on the development of practical transmit power adaptation method that maximizes the total data rate of the multiuser OFDM system in a downlink transmission under the constraint of total transmit power and bit-error rate (BER). We generally formulate the problem of data rate maximization by allowing that a subcarrier could be shared by multiple users. Without the constraint of exclusive assignment of each subcarrier for users, we should consider the interference from other users' signals on the same subcarrier. When multiple users are allowed to share a specific subcarrier, if a user's transmit power for the subcarrier is increased, the interference to other users using the same subcarrier may be increased also. Thus, the transmit power adaptation to maximize the total data rate becomes a complex problem to be solved analytically. Therefore, in this paper, the transmit power adaptation scheme is derived by solving the maximization problem via two steps: subcarrier assignment for users and power allocation for subcarriers. In the subcarrier assignment for users, we determine a set of users who should transmit data on a specific subcarrier to maximize the data rate for that subcarrier. In the step of power allocation for subcarriers, the amount of transmit power to be allocated for each subcarrier is determined to maximize the overall data rate.

The remainder of this paper is organized as follows. In Section II, a system model considered in this paper is given and the problem of data rate maximization in the multiuser OFDM system is formulated. In Section III, the transmit power adaptation scheme that maximizes the total data rate of the multiuser OFDM system is derived and a simple method to be implemented is also proposed. Numerical results for the proposed transmit power adaptation methods are shown in Section IV and conclusions are given in Section V.

II. System Model

In a downlink transmission of the multiuser OFDM system considered in this paper, the modulated signals on a number of subcarriers for multiple users are all summed together and are transmitted through the fading channel. A block diagram of the downlink multiuser OFDM system is depicted in Fig. 1. In the figure, $\{b_k\}$ denotes a set of data symbols for the kth user and $s_{k,m}$ represents the transmit power allocated to the kth user's mth subcarrier. The modulation and demodulation with a number of orthogonal subcarriers are performed by inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT) processes, respectively. Since the transmit power adaptation needs the knowledge of the CSI for all the users' subcarriers, we assume that the fading channel states are perfectly known by both the receiver and the transmitter.

When the CSI is available at the transmitter, the transmitter can adapt its transmit power for each user's subcarrier signal in a symbol-by-symbol manner to increase the data rate, assuming that the fading characteristics of the channel are constant over one OFDM symbol duration, but vary from symbol-to-symbol. We consider a wideband multipath fading channel in this paper. However, each subband is assumed to be narrow enough and the subband signals modulated on each subcarrier are assumed to undergo flat fading. Also, we assume that neither a channel coding scheme nor a multiuser detection with successive decoding is used.

Under the assumptions above, if the transmitted signal from the base station is detected by the kth user's receiver, the decision statistic $z_{k,m}$ for the kth user's mth subcarrier data symbol may be written as

$$z_{k,m} = b_{k,m}\sqrt{s_{k,m}}\alpha_{k,m} + \sum_{j=1, j\neq k}^{K} b_{j,m}\sqrt{s_{j,m}}\alpha_{k,m} + \eta_m \quad (1)$$

where $b_{k,m}$ is a data symbol on the kth user's mth subcarrier, K denotes the number of users and $\alpha_{k,m}$ is a random variable representing the fading for the mth subchannel between the base station and the kth user's receiver. Note that the fading effects of all the users' signals on the same subcarrier are identical at the desired user's receiver in downlink channels, although the fading for each user in a different location is independent. η_m denotes the additive white Gaussian noise (AWGN) with mean zero and variance σ^2. In OFDM systems, since the total bandwidth B is equally divided into M orthogonal subbands and the subband signals are transmitted in parallel, the bandwidth of the subcarrier signal becomes $B_m = B/M$ for all m and, therefore, the noise variance σ^2 can be rewritten as $\sigma^2 = N_0 B_m = N_0 B/M$, where N_0 is the noise power spectral density.

The first term in the right-hand side of (1) is a desired signal and the second term is the interference from other users' signals on the same subcarrier. Note that we have no restrictions on $s_{k,m}$ for all k and m and the interference term arises from sharing a subcarrier by multiple users. The interference term may be treated as Gaussian noise by the central limit theorem [10] under the assumption that the number of users, K is large enough. Consequently, the received signal-to-interference-plus noise ratio (SINR) for the kth user's mth subcarrier signal can be written as

$$\begin{aligned}\gamma_{k,m} =& \frac{E_\alpha\left[\left|b_{k,m}\sqrt{s_{k,m}}\alpha_{k,m}\right|^2\right]}{E_\alpha\left[\left|\sum_{j=1,j\neq k}^{K} b_{j,m}\sqrt{s_{j,m}}\alpha_{k,m}+\eta_m\right|^2\right]}\\ =& \frac{s_{k,m}\left|\alpha_{k,m}\right|^2}{\sum_{j=1,j\neq k}^{K} s_{j,m}\left|\alpha_{k,m}\right|^2+\frac{N_0 B}{M}}\end{aligned} \tag{2}$$

where $E_\alpha[\cdot]$ denotes the expectation operation conditioned on α and the data symbols, $b_{k,m}$ for all k and m are assumed to be independent random variables with zero mean and unit variance.

In order to formulate the data rate maximization problem, we first represent the data rate of the multiuser OFDM system using the SINR expression (2). Assuming that QAM modulation and ideal phase detection are used as in [11], the BER for the kth user's mth subcarrier signal is bounded by

$$\mathrm{BER} \le \frac{1}{5}\exp\left(\frac{-1.5\gamma_{k,m}}{(2^{q_{k,m}}-1)}\right) \tag{3}$$

where $q_{k,m}$ is the number of bits in each data symbol. Note that we replace the average signal-to-noise ratio (SNR) $\bar{\gamma}$ in the [11, eq. (17)] with the SINR (2) in this paper, because we treat the interference as Gaussian noise. Also note that the BER bound (3) is valid for $q_{k,m} \ge 2$ and $0 \le \gamma_{k,m} \le 30$ dB. For a given BER, rearranging (3) yields the maximum number of bits in a symbol to be transmitted for the kth user's mth subcarrier as

$$q_{k,m} = \log_2\left(1+\frac{\gamma_{k,m}}{\Gamma}\right) \tag{4}$$

where $\Gamma \triangleq -\ln(5\mathrm{BER})/1.5$. Note that Γ, which is a function of the required BER, has a positive value larger than 1 in the range of BER $< (1/5)\exp(-1.5) \approx 0.0446$. Since in the multiuser OFDM system, the total data rate is viewed as the sum of all the users' subcarriers' data rate, the total data rate of the multiuser OFDM system may be represented by

$$R = \sum_{k=1}^{K}\sum_{m=1}^{M}\frac{q_{k,m}}{T} = \frac{B}{M}\sum_{k=1}^{K}\sum_{m=1}^{M}\log_2\left(1+\frac{\gamma_{k,m}}{\Gamma}\right) \tag{5}$$

where T is the OFDM symbol duration which is given as $T = 1/B_m = M/B$. From the representation above, the total data rate of the multiuser OFDM system can be maximized, if the transmit power $s_{k,m}$ for all k and m, is appropriately adjusted by the transmit power adaptation method described in Section III.

III. Transmit Power Adaptation

In this section, we maximize the total data rate of the multiuser OFDM system by adapting the transmit power for each user and each subcarrier. Considering a downlink transmission, the constraint of total transmit power is written as

$$\sum_{k=1}^{K}\sum_{m=1}^{M} s_{k,m} = \bar{S} \tag{6}$$

where $\bar{S}$ denotes the total transmit power. Note that the maximization of the total data rate (5) under the constraint (6) is a complex problem because a subcarrier may be shared by multiple users in our formulation. When several users are allowed to share a subcarrier simultaneously, if the transmit power for a specific user's signal on that subcarrier is increased, the interference to other users' signals on the same subcarrier may be increased also. Thus, the SINRs of other users' signals on the same subcarrier is decreased as seen from (2) while SINR of the specific user's signal is increased. Therefore, in this paper, to make the maximization problem be tractable, we divide the problem into two steps: subcarrier assignment for users and power allocation for subcarriers. In the subcarrier assignment for users, we determine which users should transmit data on each subcarrier to maximize the data rate for that subcarrier. After a set of users to transmit data on each subcarrier is selected, the amount of transmit power to be allocated to each subcarrier is determined to maximize the overall data rate in the step of power allocation for subcarriers. By the two step approach, we may simplify the data rate maximization problem considered in this paper and may solve the problem analytically.

For the first step, we assign a subcarrier to a set of users to maximize the feasible data rate for that subcarrier and the subcarrier assignment strategy is found from the following theorem.

Theorem 1: The subcarrier assignment strategy for multiple users to maximize the data rate of a specific subcarrier in a downlink multiuser OFDM system is that the subcarrier should be assigned to only one user who has the best channel gain for that subcarrier. Therefore, a subcarrier should not be allowed to be shared by multiple users and only one user should transmit data on a subcarrier at a specific time.

Proof: See the Appendix.

Note that Theorem 1 provides a proof of the fundamental assumption in [8] and [9] that a subcarrier is exclusively assigned to only one user. Therefore, the derivation of Theorem 1 may be viewed as one of the main contributions of this paper. It should be pointed out that although the subcarrier assignment strategy for users has been found to be the same as the inherent assumption in [8] and [9], we have derived the results from the general formulation including interference from other users' signals on the same subcarrier by allowing multiple users to share a subcarrier. From Theorem 1, we may notice that since data symbols are transmitted through subcarriers with the best channel gains among multiple users', the data rate may be increased with the number of users owing to the multiuser diversity effects [7]. Unfortunately however, a user may not be assigned any subcarrier if the user has no best subcarrier, since we have no constraint on each user's data rate in this paper.

For the second step, we determine the amount of transmit power to be allocated to the subcarriers in order to maximize the overall data rate. When the subcarrier assignment for users is done by Theorem 1, the multiuser OFDM system can be viewed as a FDMA system with dynamic subcarrier allocation, where users transmit data through a number of subcarriers assigned to their own independently. Therefore, in the second step of power allocation, we may treat the multiuser OFDM system as a single user OFDM system virtually and need to consider only the transmit power allocation for subcarriers. Then, the total data rate (5) and the total transmit power constraint (6) may be rewritten as

$$R = \frac{B}{M}\sum_{m=1}^{M}\log_2\left(1 + s_{k_m^*}\left|\alpha_{k_m^*}\right|^2 \frac{M}{N_0 B\cdot\Gamma}\right) \tag{7}$$

where $k_m^* = \arg_k \max\{|\alpha_{1,m}|^2, |\alpha_{2,m}|^2, \ldots, |\alpha_{K,m}|^2\}$ for $m = 1, 2, \ldots, M$ and

$$\sum_{m=1}^{M} s_{k_m^*} = \bar{S}. \tag{8}$$

The method of transmit power allocation for subcarriers that maximizes the total data rate can be found by using the standard Lagrange multiplier technique. If we define the Lagrangian as

$$L = \frac{B}{M}\sum_{m=1}^{M}\log_2\left(1 + s_{k_m^*}\left|\alpha_{k_m^*}\right|^2 \frac{M}{N_0 B\cdot\Gamma}\right) + \lambda\left(\sum_{m=1}^{M} s_{k_m^*} - \bar{S}\right) \tag{9}$$

where λ is a Lagrange multiplier, then the solution for $s_{k_m^*}$ can be obtained by solving $\partial L/\partial s_{k_m^*} = 0$. Consequently, to maximize the total data rate of the multiuser OFDM system, the transmit power should be allocated as

$$\begin{cases} s_{k_m^*} = \frac{N_0 B\cdot\Gamma}{M}\left[\frac{1}{\lambda_0} - \frac{1}{\left|\alpha_{k_m^*}\right|^2}\right]^+, & \text{for } m = 1, 2, \ldots, M \\ s_{k,m} = 0, & \text{for } k \neq k_m^* \end{cases} \tag{10}$$

where $[x]^+ \triangleq \max\{x, 0\}$ and λ_0 is a threshold to be determined from the total transmit power constraint (8). Note that the transmit power adaptation method (10) is water-filling [12] over the subcarriers with the best channel gains among multiple users'. In other words, a user who has the best channel gain for a speccific subcarrier transmits data on that subcarrier with the amount of transmit power which is determined by the water-filling rule, i.e., more power when the channel gain is high and less power when the channel gain is low. Therefore, the transmit power adaptation scheme (10) may yield the spectral diversity effects, as well as the multiuser diversity effects.

It should be noted that the transmit power adaptation scheme (10) achieves the maximum data rate of the multiuser OFDM system provided that the (1), (2), and (3) are valid. Since Theorem 1 holds for arbitrary amount of transmit power, $\bar{s}_m$ allocated to the mth subcarrier, the data rate for each subcarrier is maximized by the exclusive assignment of the subcarrier to the only one user who has the best channel gain for that subcarrier. In addition, the overall data rate for entire bandwidth is maximized by the water-filling power allocation over the subcarriers. Therefore, the two step approach with the subcarrier assignment for the first step and the power allocation for the second step achieves the maximum total data rate of the multiuser OFDM system under the formulations and corresponding assumptions in this paper.

Unfortunately however, since there is no explicit method to calculate the water-filling level, λ_0, which should be determined for every symbol period to water-fill over the subcarriers, we have to resort to a numerical search method. It may be a computational burden to calculate λ_0 using a numerical search method for every symbol transmission. Therefore, to avoid the computational burden in the water-filling transmit power adaptation (10), we may adopt a simple equal power allocation method from the statements in [13] that the water-filling power allocation and the equal power allocation may yield marginal performance difference. In our equal power allocation, the total transmit power is equally distributed among the subcarriers after the subcarrier assignment for users in Theorem 1 is performed. The proposed equal power allocation strategy may be represented by

$$\begin{cases} s_{k_m^*} = \frac{\bar{S}}{M}, & \text{for } m = 1, 2, \ldots, M \\ s_{k,m} = 0, & \text{for } k \neq k_m^* \end{cases}. \tag{11}$$

Note that since only one user who has the best channel gain for each subcarrier transmits data by Theorem 1, the multiuser diversity may also be achieved in the proposed equal power allocation scheme (11) as in the water-filling transmit power adaptation method (10). However, since the transmit power is equally distributed over all the subcarriers regardless of the amount of channel gain, the spectral diversity effects in the equal power allocation may be smaller than in the water-filling-power allocation. The effects of multiuser diversity and spectral diversity on the total data rate of the multiuser OFDM system using the transmit power adaptation methods (10) and (11) will be shown in next section.

IV. Numerical Results

In this section, we evaluate the proposed transmit power adaptation methods (10) and (11) in terms of the average data rate normalized by the total bandwidth by computer simulations. In the computer simulations, we assume that each user's subcarrier signal undergoes identical Rayleigh fading independently and the average channel power gain, $E[|\alpha_{k,m}|^2]$ for all k and m, is assumed to be one. The average SNR is defined as $\bar{S}/(N_0 B)$ with the fixed total bandwidth B and the required BER is set to be BER $= 10^{-3}$. To obtain the average data rate, we have simulated 10 000 independent trials.

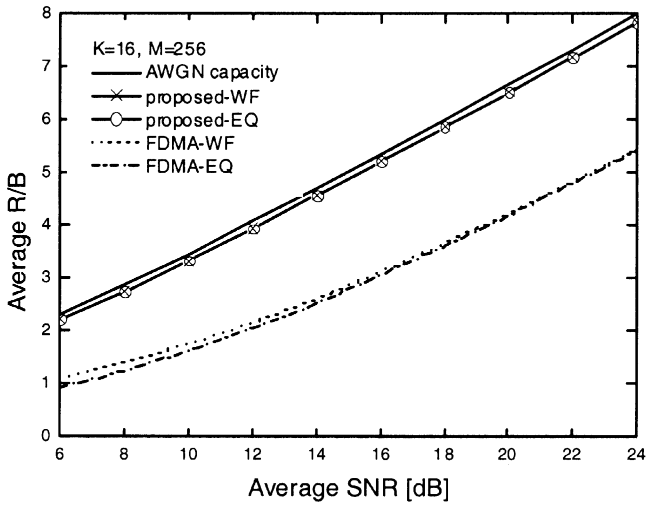

Fig. 2. Average data rate normalized by total bandwidth versus average SNR, when the number of users $K = 16$ and the number of subcarriers $M = 256$.

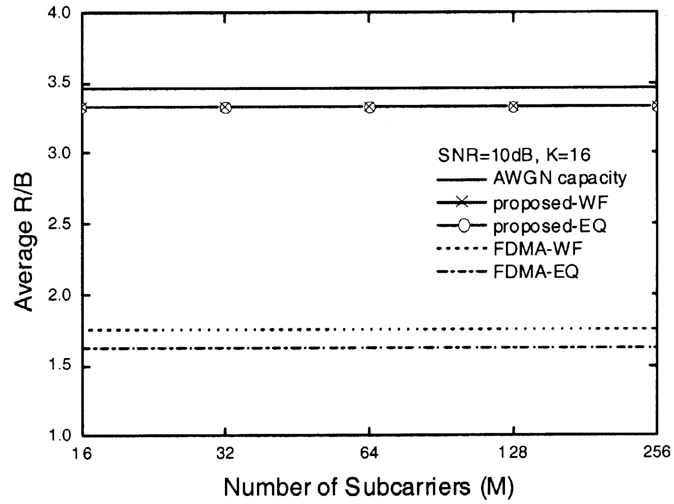

Fig. 3. Average data rate normalized by total bandwidth versus number of subcarriers M, when the number of users $K = 16$ and SNR = 10 dB.

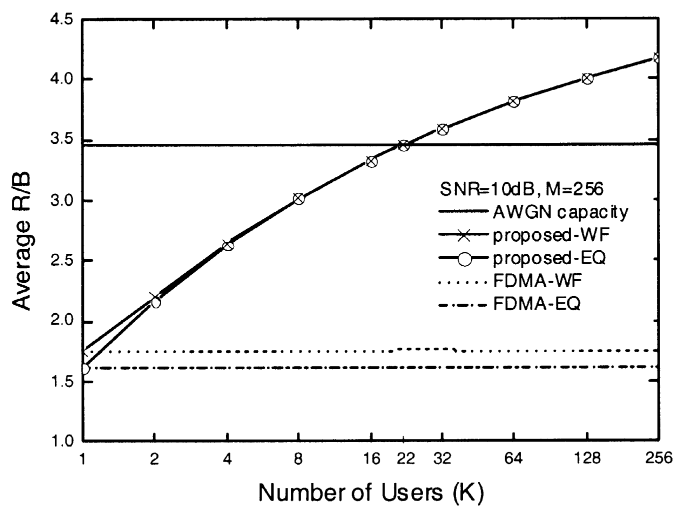

Fig. 4. Average data rate normalized by total bandwidth versus number of users K, when the number of subcarriers $M = 256$ and SNR = 10 dB.

In Figs. 2–4, the average data rate normalized by the total bandwidth is depicted for various scenarios. In the figures, the proposed transmit power adaptation methods (10) and (11) (marked *proposed-WF* and *proposed-EQ*, respectively) are compared with the conventional FDMA-like schemes as in [8]. In the FDMA-like schemes, the number of subcarriers assigned to each user is equal and the assignment of the subcarriers is fixed. We consider two FDMA-like schemes in this paper: one is a scheme where the total bandwidth is equally divided for all the users and the total transmit power is distributed over all the subcarriers with the water-filling policy (marked *FDMA-WF*, which is similar to OFDM-FDMA with Optimal Bit Allocation in [8]); the other is a scheme where the total bandwidth is equally divided for all the users and the total transmit power is equally distributed over all the subcarriers (marked *FDMA-EQ*, which is similar to OFDM-FDMA with equal bit allocation in [8]). For a benchmark, the capacity of the AWGN channel, which is given as $C = B\log_2(1 + \mathrm{SNR})$, is included also (marked *AWGN capacity*) in the figures.

Fig. 2 depicts the average data rate versus average SNR for various transmit power adaptation methods for the case of $K = 16$ and $M = 256$. This figure shows that the average data rate for all the methods increases with the average SNR and the two proposed methods outperform the FDMA-like schemes for all the values of the average SNR. The difference between the average data rate for the proposed methods and that for the FDMA-like schemes increases as the average SNR increases. Both of the proposed methods have almost the same performance although the transmit power allocation strategies are different for the two methods. This result shows that the effects of the spectral diversity achieved by the water-filling power allocation for subcarriers are not significant. Similar results have been shown in [13], where the channel capacity with CSI at the transmitter and receiver (i.e., water-filling power allocation) is just marginally larger than that with CSI only at the receiver (i.e., equal power allocation).

Fig. 3 depicts the average data rate versus the number of subcarriers, M, when the number of users, $K = 16$ and SNR = 10 dB. This figure shows that the average data rates for all the methods are constant regardless of the number of subcarriers. Although the data rate is a function of the number of subcarriers as seen from (5) and (7), the effects of the number of subcarriers on the data rate are negligible in the simulated environment.

In Fig. 4, the average data rate versus the number of users, K, is depicted for the case of $M = 256$ and SNR = 10 dB. This figure shows that the average data rate for the proposed methods increases significantly with the number of users, while the average data rate for the FDMA-like methods remains constant. Furthermore, the average data rate for the proposed methods becomes higher than the capacity of the AWGN channel, when the number of users, K is equal to or larger than 22. In the proposed schemes, since only one user who has the best channel gain transmits data on each subcarrier, the received average SNR for each subcarrier signal increases as the number of users increases and the total data rate increases as a result. The increase of the data rate with the number of users for the proposed schemes is mainly due to the effects of multiuser diversity. The multiuser diversity effects on the achievable data rate of the single-carrier multiuser system in a fading channel have been investigated in [7]. The authors of

[7] have shown similar results that the capacity of the multiuser system with the optimal power allocation among users increases as the number of users increases and the capacity of the multiuser system in a fading channel may be larger than the capacity of the AWGN channel when the number of users is large enough.

V. Conclusion

In this paper, we develop a transmit power adaptation method to maximize the data rate of multiuser OFDM systems in a downlink transmission. In a multiuser OFDM system, since multiple users' data symbols are transmitted in parallel through a number of orthogonal subcarriers simultaneously, both the multiuser diversity and the spectral diversity may be exploited using the transmit power adaptation scheme. In formulating the data rate maximization problem in the multiuser OFDM system, we allow that a subcarrier could be shared by multiple users simultaneously and the transmit power adaptation scheme is derived via two steps: subcarrier assignment for users and power allocation for subcarriers.

The transmit power adaptation method that maximizes the total data rate of the multiuser OFDM system is found that each subcarrier should be assigned to only one user who has the best channel gain for that subcarrier and the transmit power should be distributed over the subcarriers with the water-filling policy. To avoid the computational burden in calculating the water-filling level in the proposed transmit power adaptation method, we also propose an equal power allocation scheme. In our equal power allocation scheme, users with the best channel gain for each subcarrier are selected and then transmit power is equally distributed among the subcarriers. Results show that the total data rate for the two proposed transmit power adaptation schemes significantly increases with the number of users and is much greater than that for the conventional FDMA-like transmit power adaptation methods. Moreover, when the number of users is large enough, the total data rate of the multiuser OFDM system with the proposed transmit power adaptation methods is found to be even higher than the capacity of the AWGN channel.

Appendix

Proof of Theorem 1

Following from the discussions in Section III, in order to find out the transmit power adaptation rule that maximizes the total data rate of a multiuser OFDM system, we solve the subcarrier assignment problem first. Theorem 1 states that the data rate for a specific subcarrier can be maximized when the subcarrier is assigned to only one user who has the best channel gain for that subcarrier. Theorem 1 is proved by the Principle of Mathematical Induction.

Basis: When the number of users $K = 2$, the data rate for the mth subcarrier is written as

$$R_{m(K=2)} = \frac{B}{M}\log_2\left(1+\frac{s_{1,m}}{s_{2,m}+\frac{\sigma^2}{|\alpha_{1,m}|^2}}\frac{1}{\Gamma}\right) + \frac{B}{M}\log_2\left(1+\frac{s_{2,m}}{s_{1,m}+\frac{\sigma^2}{|\alpha_{2,m}|^2}}\frac{1}{\Gamma}\right) \quad \text{(A.1)}$$

where $\sigma^2 = N_0B/M$. Assuming that $\bar{s}_m$ is the arbitrary amount of transmit power allocated to the mth subcarrier, we may express the total transmit power constraint as $\sum_{m=1}^{M}\bar{s}_m = \bar{S}$ and $\sum_{j=1}^{2}s_{j,m} = \bar{s}_m$. Also, assuming that the channel power gains for the two users for the mth subcarrier have the relationship as $|\alpha_{1,m}|^2 \geq |\alpha_{2,m}|^2 \geq 0$, when the mth subcarrier is assigned to the 1st user only such that $s_{1,m} = \bar{s}_m$ and $s_{2,m} = 0$, the data rate for the mth subcarrier is rewritten as

$$R^*_{m(K=2)} = \frac{B}{M}\log_2\left(1+\bar{s}_m\frac{|\alpha_{1,m}|^2}{\sigma^2\Gamma}\right). \quad \text{(A.2)}$$

If we define

$$\Delta R_{m(K=2)} \triangleq R^*_{m(K=2)} - R_{m(K=2)} \quad \text{(A.3)}$$

then $\Delta R_{m(K=2)}$ can be represented by (A.4), shown at the bottom of the page. If we subtract the denominator from the numerator of the operand in the log function in (A.4) and set it as $\Omega_{m(K=2)}$, then $\Omega_{m(K=2)}$ becomes a quadratic function with respect to the variable $s_{2,m}$. Also, we may observe that the second derivative of $\Omega_{m(K=2)}$ with respect to $s_{2,m}$ is negative, i.e., $\partial^2\Omega_{m(K=2)}/\partial s_{2,m}^2 = -2(\bar{s}_m\Gamma|\alpha_{1,m}|^2/\sigma^2 + 2\Gamma - 1) \leq 0$, because $\Gamma > 1$ in the range of the required BER, BER $< (1/5)\exp(-1.5) \approx 0.0446$. In addition, the value of $\Omega_{m(K=2)}$ at $s_{2,m} = 0$ is zero and the value of $\Omega_{m(K=2)}$ at $s_{2,m} = \bar{s}_m$ is positive, i.e., $\Omega_{m(K=2)}|_{s_{2,m}=0} = 0$ and

$$\Omega_{m(K=2)}\big|_{s_{2,m}=\bar{s}_m} = \bar{s}_m\Gamma\left(\bar{s}_m+\frac{\sigma^2}{|\alpha_{1,m}|^2}\right)\left(\frac{|\alpha_{1,m}|^2}{|\alpha_{2,m}|^2}-1\right) \geq 0$$

because we assume that $|\alpha_{1,m}|^2 \geq |\alpha_{2,m}|^2 \geq 0$. Thus, $\Omega_{m(K=2)}$ always has positive value in the range of $0 \leq s_{2,m} \leq \bar{s}_m$ and, consequently, $\Delta R_{m(K=2)} \geq 0$, i.e., $R^*_{m(K=2)} \geq R_{m(K=2)}$ is held.

Induction Hypothesis: Suppose Theorem 1 is held when $K = k$, then

$$R^*_{m(K=k)} \geq R_{m(K=k)} \quad \text{(A.5)}$$

where

$$R^*_{m(K=k)} = \frac{B}{M}\log_2\left(1+\bar{s}_m\frac{|\alpha_{1,m}|^2}{\sigma^2\Gamma}\right) \quad \text{(A.6)}$$

$$\Delta R_{m(K=2)} = \frac{B}{M}\log_2\left(\frac{\Gamma\left(\Gamma+\bar{s}_m\frac{|\alpha_{1,m}|^2}{\sigma^2}\right)\left(s_{2,m}+\frac{\sigma^2}{|\alpha_{1,m}|^2}\right)\left(\bar{s}_m-s_{2,m}+\frac{\sigma^2}{|\alpha_{2,m}|^2}\right)}{\left(\bar{s}_m+s_{2,m}(\Gamma-1)+\frac{\sigma^2\Gamma}{|\alpha_{1,m}|^2}\right)\left(\bar{s}_m\Gamma+s_{2,m}(1-\Gamma)+\frac{\sigma^2\Gamma}{|\alpha_{2,m}|^2}\right)}\right) \quad \text{(A.4)}$$

and

$$R_{m(K=k)} = \frac{B}{M} \sum_{j=1}^{k} \log_2 \left(1 + \frac{s_{j,m}}{\sum\limits_{l=1, l \neq j}^{k} s_{l,m} + \frac{\sigma^2}{|\alpha_{j,m}|^2}} \frac{1}{\Gamma} \right) \tag{A.7}$$

under the condition of $\sum_{j=1}^{k} s_{j,m} = \bar{s}_m$ and $|\alpha_{1,m}|^2 \geq |\alpha_{2,m}|^2 \geq \cdots \geq |\alpha_{k,m}|^2 \geq 0$.

Induction Step: For the case of $K = k + 1$, we want to show that the relationship $R^*_{m(K=k+1)} \geq R_{m(K=k+1)}$ is held, where

$$R^*_{m(K=k+1)} = \frac{B}{M} \log_2 \left(1 + \bar{s}_m \frac{|\alpha_{1,m}|^2}{\sigma^2 \Gamma} \right) \tag{A.8}$$

and

$$R_{m(K=k+1)} = \frac{B}{M} \sum_{j=1}^{k+1} \log_2 \left(1 + \frac{x_{j,m}}{\sum\limits_{l=1, l \neq j}^{k+1} x_{l,m} + \frac{\sigma^2}{|\alpha_{j,m}|^2}} \frac{1}{\Gamma} \right) \tag{A.9}$$

under the condition of $\sum_{j=1}^{k+1} x_{j,m} = \bar{s}_m$ and $|\alpha_{1,m}|^2 \geq |\alpha_{2,m}|^2 \geq \cdots \geq |\alpha_{k,m}|^2 \geq |\alpha_{k+1,m}|^2 \geq 0$. Note that $x_{j,m}$ for $j = 1, 2, \ldots, k + 1$ denotes the transmit power allocated to the jth user's mth subcarrier in case of the number of users, $K = k + 1$. After rewriting (A.9) and using the relationship of $\sum_{l=1}^{k} x_{l,m} = \bar{s}_m - x_{k+1,m}$, we may find out that $R_{m(K=k+1)}$ in (A.9) is bounded by

$$\begin{aligned} R_{m(K=k+1)} = &\frac{B}{M} \sum_{j=1}^{k} \log_2 \\ &\cdot \left(1 + \frac{x_{j,m}}{\sum\limits_{l=1, l \neq j}^{k} x_{l,m} + x_{k+1,m} + \frac{\sigma^2}{|\alpha_{j,m}|^2}} \frac{1}{\Gamma} \right) \\ &+ \frac{B}{M} \log_2 \left(1 + \frac{x_{k+1,m}}{\sum\limits_{l=1}^{k} x_{l,m} + \frac{\sigma^2}{|\alpha_{k+1,m}|^2}} \frac{1}{\Gamma} \right) \\ \leq &\frac{B}{M} \sum_{j=1}^{k} \log_2 \left(1 + \frac{x_{j,m}}{\sum\limits_{l=1, l \neq j}^{k} x_{l,m} + \frac{\sigma^2}{|\alpha_{j,m}|^2}} \frac{1}{\Gamma} \right) \\ &+ \frac{B}{M} \log_2 \left(1 + \frac{x_{k+1,m}}{\bar{s}_m - x_{k+1,m} + \frac{\sigma^2}{|\alpha_{k+1,m}|^2}} \frac{1}{\Gamma} \right). \end{aligned} \tag{A.10}$$

Moreover, from (A.5) we may notice that the first term of the right-hand side of the inequality in (A.10) is also bounded by

$$\begin{aligned} &\frac{B}{M} \sum_{j=1}^{k} \log_2 \left(1 + \frac{x_{j,m}}{\sum\limits_{l=1, l \neq j}^{k} x_{l,m} + \frac{\sigma^2}{|\alpha_{j,m}|^2}} \frac{1}{\Gamma} \right) \\ &\qquad \leq \frac{B}{M} \log_2 \left(1 + (\bar{s}_m - x_{k+1,m}) \frac{|\alpha_{1,m}|^2}{\sigma^2 \Gamma} \right) \end{aligned} \tag{A.11}$$

and consequently we may rewrite (A.10) again as

$$\begin{aligned} R_{m(K=k+1)} \leq &\frac{B}{M} \log_2 \left(1 + (\bar{s}_m - x_{k+1,m}) \frac{|\alpha_{1,m}|^2}{\sigma^2 \Gamma} \right) \\ &+ \frac{B}{M} \log_2 \left(1 + \frac{x_{k+1,m}}{\bar{s}_m - x_{k+1,m} + \frac{\sigma^2}{|\alpha_{k+1,m}|^2}} \frac{1}{\Gamma} \right) \\ \triangleq &R^u_{m(K=k+1)}. \end{aligned} \tag{A.12}$$

If we define

$$\Delta R_{m(K=k+1)} \triangleq R^*_{m(K=k+1)} - R^u_{m(K=k+1)} \tag{A.13}$$

then, $\Delta R_{m(K=k+1)}$ can be represented by (A.14), shown at the bottom of the page. When we subtract the denominator from the numerator of the operand in the log function in (A.14) and set it as $\Omega_{m(K=k+1)}$, then $\Omega_{m(K=k+1)}$ becomes a quadratic function with respect to the variable $x_{k+1,m}$. Also, we may observe that the second derivative of $\Omega_{m(K=k+1)}$ with respect to $x_{k+1,m}$ is negative, i.e.,

$$\frac{\partial^2 \Omega_{m(K=k+1)}}{\partial x^2_{k+1,m}} = -2 \left(1 - \frac{1}{\Gamma} \right) \frac{|\alpha_{1,m}|^2}{\sigma^2} \leq 0$$

because $\Gamma > 1$ in the range of BER $< (1/5) \exp(-1.5) \approx 0.0446$. In addition, the value of $\Omega_{m(K=k+1)}$ at $x_{k+1,m} = 0$ is zero and the value of $\Omega_{m(K=k+1)}$ at $x_{k+1,m} = \bar{s}_m$ is positive, i.e., $\Omega_{m(K=k+1)}|_{x_{k+1,m}=0} = 0$ and

$$\Omega_{m(K=k+1)}\Big|_{x_{k+1,m}=\bar{s}_m} = \bar{s}_m \left(\frac{|\alpha_{1,m}|^2}{|\alpha_{k+1,m}|^2} - 1 \right) \geq 0$$

because we assume that $|\alpha_{1,m}|^2 \geq |\alpha_{k+1,m}|^2 \geq 0$. Therefore, $\Omega_{m(K=k+1)}$ always has positive value in the range of $0 \leq x_{k+1,m} \leq \bar{s}_m$ and, consequently, $\Delta R_{m(K=k+1)} \geq 0$, i.e., $R^*_{m(K=k+1)} \geq R^u_{m(K=k+1)} \geq R_{m(K=k+1)}$ is held.

Therefore, by the principle of mathematical induction, for all the values of $K \geq 2$, the relationship of $R^*_{m(K)} \geq R_{m(K)}$ is held and, thus, Theorem 1 is proved.

$$\Delta R_{m(K=k+1)} = \frac{B}{M} \log_2 \left(\frac{\left(\Gamma + \bar{s}_m \frac{|\alpha_{1,m}|^2}{\sigma^2} \right) \left(\bar{s}_m - x_{k+1,m} + \frac{\sigma^2}{|\alpha_{k+1,m}|^2} \right)}{\left(\Gamma + (\bar{s}_m - x_{k+1,m}) \frac{|\alpha_{1,m}|^2}{\sigma^2} \right) \left(\bar{s}_m - x_{k+1,m} + \frac{\sigma^2}{|\alpha_{k+1,m}|^2} + \frac{x_{k+1,m}}{\Gamma} \right)} \right) \tag{A.14}$$

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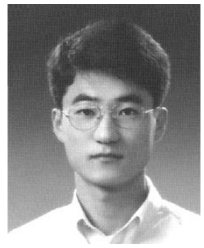
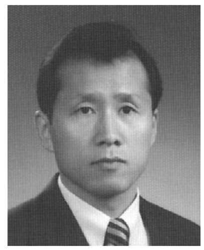


Jiho Jang (S'97) received the B.S. and M.S. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1996 and 1998, respectively, and is currently working toward the Ph.D. degree in electrical engineering at Seoul National University.

His current research interests include wireless communications, adaptive modulation, OFDM, and multicarrier systems.



Kwang Bok Lee (M'90) received the B.A.Sc. and M.Eng. degrees from the University of Toronto, Toronto, ON, Canada, in 1982 and 1986, respectively, and the Ph.D. degree from McMaster University, Canada, in 1990.

He was with Motorola Canada from 1982 to 1985 and Motorola USA, from 1990 to 1996, as a Senior Staff Engineer. At Motorola, he was involved in the research and development of wireless communication systems. He was with Bell-Northern Research, Canada, from 1989 to 1990. In March 1996, he joined the School of Electrical Engineering, Seoul National University, Seoul, Korea. Currently he is an Associate Professor in the School of Electrical Engineering. He was a Vice Chair of the School of Electrical Engineering from 2000 to 2002. He has been serving as a Consultant to a number of wireless industries. His research interests include mobile communications, communication theories, spread-spectrum, and signal processing. He holds ten U.S. patents and two Korean patents and has a number of patents pending.

Dr. Lee has been an Editor of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, Wireless Series in 2001 and is the Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He received the Best Paper Award from CDMA International Conference 2000 (CIC 2000).