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TRANSPORT IN DRIVEN PLASMAS

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ABSTRACT

A plasma in contact with an external source of power, especially a source that interacts specifically with high-velocity electrons, exhibits transport properties, such as conductivity, different from those of an isolated plasma near thermal equilibrium. This is true even when the bulk of the particles in the driven plasma are near thermal equilibrium. To describe the driven plasma we derive an adjoint equation to the inhomogeneous, linearized, dynamic Boltzmann equation. The Green's functions for a variety of plasma responses can then be generated. It is possible to modify the Chapman-Enskog expansion in order to incorporate the response functions derived here.

I. INTRODUCTION

The expansion developed by Chapman and Enskog[1] provides a generally useful description of transport in a plasma. This expansion, which applies when the mean-free path of particles is small compared to other lengths of interest, has been the starting point for describing plasmas near thermal equilibrium. At the heart of this method is a reduced description of each plasma species in terms of macroscopic fluid variables, namely, the density, temperature, and drift. It is these properties of the plasma which are transported in configuration space in order to describe the evolving plasma.

The transport numbers, such as the conductivity or the coefficients governing particle and heat diffusion, are calculated by considering small perturbations from thermal equilibrium. It is well-known, however, that certain phenomena are not described well by this method, namely, those processes involving the highly energetic particles, particularly electrons, which collide but infrequently. For example, the plasma conductivity, the so-called Spitzer-Härm conductivity,[2] is strictly derived only in the limit of the dc electric field, E, vanishing. For E finite, important corrections occur due to the high-velocity electrons, which tend to run away. If many of these electrons run away, the Spitzer-Härm conductivity may no longer provide a useful description. The regime in which only few electrons run away, where conductivity is a good description, is the small-mean-free-path limit, which arises for E small enough.

In the case of driven plasmas, however, the Spitzer-Härm conductivity may not be a useful approximation even for $E \to 0$. These plasmas are not near thermal equilibrium; they are actively heated by an external source of radiation or particles. One example of a driven plasma that is not described well by the Chapman-Enskog expansion, and hence not by Spitzer-Härm

conductivity, is a plasma in contact with external radiation that interacts specifically with the fast, collisionless electrons. This is a situation that commonly arises in rf-heated plasmas, particularly when the purpose is to generate toroidal current. The deviation from Spitzer-Härm conductivity is due to the larger number of fast electrons. These electrons have long mean-free paths, and even though small in number compared to the bulk electrons, they may play a large part in carrying current.[3]

In this report, we develop a description suitable for driven plasma. We consider those plasmas for which it is valid to apply the fundamental Chapman-Enskog approximation, that collisions are large enough to create a nearly Maxwellian distribution of particle velocities. We use this approximation to linearize the Boltzmann collision operator, and as a result we obtain the dynamic linearized, inhomogeneous Boltzmann equation. Plasma responses to arbitrary wave excitation affecting the fast electrons are then determined by solving an adjoint equation. The result is an improved characterization of the relatively collisionless phenomena associated with the external excitation of the plasma. A modification of the Chapman-Enskog expansion then allows a dual characterization of driven plasmas; a slowly evolving fluid description of the bulk properties and a dynamic response, linear in the excitation, for the driven relatively collisionless processes.

In order to put the present work in perspective, let us briefly review some of the approaches taken to this problem. Response functions for the current generated per power dissipated, J/P_d, were central to our early understanding of both beam-driven currents,[4] and wave-driven currents with low[5] and high[6] phase velocity waves. The early work, however, was characterized by a crude one-dimensional model of velocity space, and was unable to predict, for example, current drive by electron cyclotron (ECRH)

waves. Fisch and Boozer[7] used slowing-down equations to formulate the problem in two dimensions in velocity space. This gave the first precise response function for the current-drive efficiency and predicted the ECRH current-drive effect.

An improved derivation of the 2-D Fisch-Boozer response function was provided by Antonsen and Chu.[8] Drawing upon neoclassical techniques, these authors extended the Fisch-Boozer efficiency calculation to toroidal geometry by writing an adjoint equation to the steady-state linearized Boltzmann equation. Hirshman[9] and Taguchi[10,11] employed a similar technique to solve similar equations. Using the adjoint equation, Antonsen and Yoshioka[12] have calculated rf-induced particle transport across field lines. The Antonsen and Chu approach was also generalized to include a small dc electric field in order to evaluate the conductivity of a driven plasma.[3]

In order to interpret the recent high efficiency PLT ramp-up experiments,[13] response functions had to be identified and evaluated in the presence of a strong dc electric field. The most straightforward approach, adopted by Fisch and Karney,[14] generalized the Fisch-Boozer slowing-down equations to include the strong field. In addition to the current-drive efficiency, this work also identified the Green's function for runaway production. A comparison by Karney et al.[15] of this theory and the experiment showed very good agreement.

A better approach to the strong-field case is to build upon the Antonsen-Chu work. The present work seeks to do this. Here, an adjoint equation is written for the dynamic, linearized Boltzmann equation, allowing for the presence of a strong dc electric field. Also, in addition to a response function for the current, other response functions are identified and evaluated. It is interesting to note that were this adjoint equation solved

in the high-velocity limit by the method of characteristics, the characteristic equations would be the Fisch-Boozer slowing-down equations.

The adjoint formalism developed here also draws heavily on insights gathered in solutions to the homogeneous Boltzmann equation,[16-18] which exploit, too, the self-adjoint property of the collision integral. Here, however, the focus is on the effects associated with the wave-induced large deviations from Maxwellian velocity distributions.

The paper is organized as follows. In Sec. II we discuss the validity and the utility of the linearization of the Boltzmann equation. In Sec. III we derive the adjoint equation and we write the general Green's function response. This is our principal result. In Sec. IV we give several examples of the utility of this general equation. A modified Chapman-Enskog expansion, that includes the derived response functions in addition to the normal fluid terms, is given in Sec. V. In Sec. VI, we discuss ways of implementing numerical tools, based on these results, suitable for modeling a plasma experiment. A summary of our findings is given in Sec. VII.

II. LINEARIZED BOLTZMANN EQUATION

The Boltzmann equation provides an excellent description of the evolving plasma, but it is too complicated to apply directly to most problems of interest. Certain approximations, however, allow us to extract information from this equation reliably and efficiently. The most helpful simplifications that are unique to the wave-driven plasma rely on the localization in velocity space of the wave-driven fluxes and the fact that these fluxes often involve only very fast electrons.

To be specific, let us consider the electron distribution function $f(\mathbf{r},\vec{v},t)$ driven by some wave-induced flux $\vec{\Gamma}(\mathbf{r},\vec{v},t)$, evolving as described by the inhomogeneous Boltzman equation

$$\frac{df}{dt} = C(f,f) + C(f,f_i) - \frac{\partial}{\partial \vec{v}} \cdot \vec{F} \quad , \tag{1}$$

where $C(f,f)$ represents the self-collisions of electrons, $C(f,f_i)$ represents the scattering of electrons off ion distribution f_i, and the total time derivative, d/dt, is defined by

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} + \vec{F} \cdot \frac{\partial}{\partial \vec{v}} \tag{2}$$

where $\vec{F}$ is the acceleration due to the background dc electric field $\vec{E}$ and magnetic field B, i.e.,

$$F = \frac{e}{m} \left(\vec{E} + \vec{v} \times \vec{B} \right) \quad . \tag{3}$$

To be sure, we generally expect that the magnetic field will be so large that the conventional expansions leading, for example, to the drift kinetic equation are appropriate. To solve Eq. (1), we will make use of any of these expansions, which are useful whether or not the electrons are subject to wave-induced fluxes. However, here, we shall focus only on approximations that are specific to the problem at hand.

The main effect of the injection of intense rf-waves at high phase velocities parallel to the magnetic field $\vec{B}$ is to distort the electron distribution in the resonant region of velocity space. Waves at frequency ω are injected with a range of parallel wave numbers $k_\parallel$ such that $v_1 < \omega/k_\parallel < v_2$. The region of the electron velocity space resonant with the waves is $v_1 < v_\parallel < v_2$, where $v_\parallel$ is the electron velocity parallel to $\vec{B}$. In the resonant region the waves act to diffuse electrons, the so-called "quasilinear

diffusion," and if the injected power is intense enough, then a plateau of electrons forms. The plateau represents a distribution function $f(v_\parallel, v_\perp)$ of electrons such that $\partial f/\partial v_\parallel \simeq 0$ in the resonant region, with contours of f assuming levels in the parallel direction that are parameterized by $v_\perp$, the electron velocity perpendicular to the magnetic field.

For many problems of interest and, in particular, in the instance of the efficient generation of current, the wave-induced fluxes create a plateau of energetic (superthermal) electrons, while the bulk of the electron velocity distribution remains Maxwellian. This observation leads us to two helpful approximations. Firstly, most plateau electrons collide with bulk electrons, rather than with each other, since there are so many more bulk electrons. Hence, the collision integral may be linearized by letting $f = f_m + \tilde{f}$ and approximating

$$C(f,f) \simeq C(f_m,\tilde{f}) + C(\tilde{f},f_m) \quad , \tag{4}$$

where we used $C(f_m,f_m) = 0$, since a Maxwellian distribution has no further to relax by collisions.

The second helpful approximation arises from the inference that $\vec{\Gamma}$ is usually quite localized in velocity space, even though $\vec{\Gamma}$ itself may depend on f. More precisely, while $\vec{\Gamma}$ may, in general, depend on $v_\parallel$ and $v_\perp$ in both direction and magnitude, much information may be gleaned from the resonance condition $\omega - k_\parallel v_\parallel - n\Omega_e = 0$, where n is an integer and Ω_e is the electron cyclotron velocity. For $n = 0$, the Landau resonance, $\vec{\Gamma}$ must be in the parallel direction. This is a consequence of energy and momentum conservation between the wave and the particle. Similarly, for $n \neq 0$, we can infer the direction of $\vec{\Gamma}$, which for most applications, such as heating or current drive

with electron cyclotron waves, will be nearly in the perpendicular direction. If the spectrum of waves is narrow in $k_\parallel$-space, then only electrons within a narrow band in $v_\parallel$-space will be resonant, such as those electrons with $v_\parallel \approx v_{res}$. If this region is narrow and $v_{res} >> v_{Te}$, where v_{Te} is the electron thermal velocity, then even if the electron distribution is distorted by the waves, the wave-induced flux $\vec{\Gamma}$ is of known direction and finite only for $v_\parallel \approx v_{res}$, $v_\perp \approx v_{Te}$, or in other words, $\vec{v} \approx v_{res} \hat{i}_\parallel$. Only the magnitude of $\vec{\Gamma}$, which for Landau damping depends on $\partial f/\partial v_\parallel$ in the resonant region, remains unknown.

Knowledge of all but the magnitude of $\vec{\Gamma}$, together with the license to linearize Eq. (1) allows us to solve easily an important class of problems. Suppose that a certain power P_d is absorbed from the waves by the resonant electrons. As a result, there may appear in the plasma a wave-induced current J. The quantity J/P_d, which is a measure of the efficiency of the current drive, is independent of the magnitude of $\vec{\Gamma}$, since both J and P_d are proportional to it. The ratio, however, does depend upon both the direction and velocity locale of $\vec{\Gamma}$, which could be inferred approximately, as outlined above, without knowledge of the magnitude of $\vec{\Gamma}$. Other responses, such as the incremental bremmstrahlung radiation per power absorbed, can be computed in the same manner. The quantity J/P_d, and other quantities that are similarly inferred, describe the response of the plasma to the absorption of wave energy by resonant electrons, and provide insight as to how to inject these waves in order to maximize a desired response.

The magnitude of $\vec{\Gamma}$ is also an important quantity, since all plasma responses will be proportional to it. This quantity, however, may be calculated by more crude means. Further discussion of this point is deferred to Sec. VI.

III. ADJOINT EQUATION

The linearization of the Boltzmann equation enables us to find responses to arbitrary wave-induced fluxes. Instead of solving the Boltzmann equation, we solve an adjoint equation for the Green's function. In this section, we derive the adjoint equation. For simplicity, we shall restrict our attention to the homogeneous case, $\partial/\partial\vec{r} = 0$. However, we shall solve the problem in a finite velocity domain, which facilitates a numerical implementation.

There is a certain amount of arbitrariness in imposing a specific linearization. We shall adopt a linearization that will enable us to generalize easily the Chapman-Enskog expansion. We utilize the fact that the heating of the bulk of f occurs on a time scale long compared to the time scales for other processes of interest, while collisions, predominant in the bulk of f, assure for thermal velocities a nearly Maxwellian distribution. Thus, we linearize

$$f = f_m(1 + \phi) , \tag{5}$$

where

$$f_m \equiv n\left(\frac{2\pi T}{m}\right)^{-3/2} \exp\left(\frac{-\varepsilon}{T}\right) , \tag{6}$$

where density n and temperature T may be slow functions of time (and would be of space, too, in a more general consideration of the problem), and we defined the kinetic energy $\varepsilon \equiv mv^2/2$. For simplicity of presentation, we have made one further simplification in the Chapman-Enskog approach by linearizing about a stationary rather than the more general drifting Maxwellian distribution.

To find a unique solution with this procedure necessitates two compatibility conditions; we choose n and T such that f_m (integrated over velocity space) contains the same number of electrons and the same amount of energy as does f. Accordingly, $\phi(\vec{v}, \vec{r}, t)$ is orthogonal both to f_m and to εf_m, namely

$$\int_V f_m \phi \, d^3v = \int_V \varepsilon f_m \phi \, d^3v = 0 \ , \tag{7}$$

where V is the velocity space domain under consideration, which necessarily is finite in a numerical implementation of the method.

Substituting Eq. (5) into Eq. (1), and specializing to the homogeneous case, $\partial/\partial\vec{r} = 0$, we obtain

$$L\phi \equiv \frac{\partial}{\partial t} f_m \phi + \vec{F} \cdot \frac{\partial}{\partial \vec{v}} f_m \phi - C(\phi) = - \frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma} - \frac{\partial}{\partial t} f_m - \vec{F} \cdot \frac{\partial}{\partial \vec{v}} f_m \ , \tag{8}$$

where ϕ obeys a homogeneous initial condition and where we use

$$\frac{\partial f_m}{\partial t} = \frac{\partial n}{\partial t} \frac{\partial f_m}{\partial n} + \frac{\partial T}{\partial t} \frac{\partial f_m}{\partial T} \equiv \frac{\dot{n}}{n} f_m + \left(\frac{\varepsilon}{T} - \frac{3}{2}\right) \frac{\dot{T}}{T} f_m \tag{9}$$

although f_m is assumed to evolve on a time scale larger than the scale t on which ϕ evolves. In Eq. (9) we also adopted the convenient notation

$$C(\phi) \equiv C(f_m, f_m\phi) + C(f_m\phi, f_m) + C(f_m\phi, f_i) \ . \tag{10}$$

In order to assure the Eq. (7) continues to be obeyed as ϕ evolves in time, two solubility, or compatibility, conditions are applied to Eq. (8). To assure the orthogonality of ϕ to f_m, we integrate Eq. (8) over V to obtain

$$\frac{\partial}{\partial t}\int_V f_m \phi \, d^3v = -\int_\Sigma f_m \phi \, \vec{\Gamma}\cdot d\vec{a} + \int_V C(\phi) d^3v - \int_\Sigma \vec{\Gamma}\cdot d\vec{a} - \int_\Sigma f_m \vec{F}\cdot d\vec{a} - \dot{n}, \qquad (11)$$

where Σ is the surface in velocity space bounding V, and $\dot{n}$ is chosen so that the right-hand side of Eq. (11) vanishes. In practice, we wish to choose V large enough such that the boundary terms do not introduce severe particle losses, and such that particles which do leave through the boundary can be accounted for in some other simple way (such as free-streaming). The physical implication of decreasing n as a result of runaway production is that the bulk population is somewhat depleted of electrons and, hence, serves somewhat less effectively as a scatterer. Very fast electrons, i.e., those outside the domain V, are ineffective in contributing to the collisions of other electrons. It is often sufficient to account for these electrons merely by allowing them to stream freely from their point of exit on Σ. One fine point here is that we assume that the Maxwellian background is not confined to the bounded domain. This is consistent with the other approximations made so far, and it is a convenience.

The second compatibility condition gives us $\dot{T}$, the temperature evolution of the background electrons. Multiplying Eq. (8) by ε, integrating over the domain V, and applying Eq. (7), we get

$$\left(\frac{3}{2}\right)n\dot{T} = -\int_V \varepsilon\left[\frac{\partial}{\partial \vec{v}}\cdot\vec{\Gamma} + \vec{F}\cdot\frac{\partial}{\partial \vec{v}} f_m + \vec{F}\cdot\frac{\partial}{\partial \vec{v}} f_m\phi - C(\phi)\right]d^3v \, , \qquad (12)$$

where the first right-hand-side term above represents the energy input of the wave-driven flux, the second term vanishes, the third term contains the Joule heating effect of the rf-driven current, and the last term tends to zero as $V \to \infty$.

Equation (8), together with Eq. (9) and the expression for $\dot{n}$ and $\dot{T}$, comprise an inhomogeneous, linear, integro-differential equation for ϕ. This equation, difficult to treat analytically, may be solved numerically. The evolving solution, $\phi(t)$, automatically obeys the orthogonality conditions, so that the linearization, Eq. (5), continues to hold, which allows the collision integral to retain its form. Rather than solve Eq.(8) for each excitation $\vec{F}$, however, we consider an adjoint equation instead.

To find an operator adjoint to L, we first define a commutative operation on the two functions $\phi(\vec{v}, t')$ and $\psi(\vec{v},t')$ by

$$[\phi, \psi]_t \equiv \int_V d^3v \int_o^t \phi(\vec{v}, t-\tau)\, \psi(\vec{v}, \tau)\, d\tau \,, \tag{13}$$

where the operation [ ] is parameterized by t. Define a linear operator D_t operating on $\psi(\vec{v}, t')$ by

$$D_t\, \psi(\vec{v}, t') \equiv \left[f_m \frac{\partial}{\partial t'} \psi - f_m \vec{F}(\vec{v}, t-t') \cdot \frac{\partial}{\partial \vec{v}} \psi - C(\psi) \right] . \tag{14}$$

For functions ϕ and ψ with homogeneous initial and boundary conditions, it may be shown that the operator D_t is adjoint to the operator L with respect to the inner product $[\ ]_t$, i.e.,

$$[\phi, D_t\psi]_t = [\psi, L\phi]_t \,. \tag{15}$$

Suppose that $\phi(\vec{v},t')$ and $\psi(\vec{v}, t')$ are restricted to functions orthogonal both to f_m and to εf_m over the domain V. Suppose further that $\psi(\vec{v}, t')$ obeys the evolution equation

$$D_t\,\psi(\vec{v},\,t') = q_1 f_m + q_2\,\varepsilon f_m\,, \tag{16}$$

where the constants q_1 and q_2 are chosen to assure that the orthogonality conditions on ψ are obeyed subsequently, given that they are obeyed initially. The quantities q_1 and q_2 are independent of $\vec{v}$, but are linear functionals of ψ. We now supplement Eq. (16) with inhomogeneous initial and boundary conditions on ψ, namely

$$\psi(\vec{v},\,t' = 0) = \psi_o(\vec{v})\,, \tag{17a}$$

$$\vec{S}_\psi(\vec{v},\,t') = \vec{S}_B(\vec{v},\,t') \text{ on } \Sigma\,, \tag{17b}$$

where Σ is the boundary in velocity space to domain V, and $\vec{S}_\psi$ is the flux in velocity space associated with the operator D_t, i.e.,

$$\vec{S}_\psi(\psi) \equiv -f_m\,\vec{F}(\vec{v},\,t-\tau)\,\psi(\vec{v},\,\tau) + \vec{S}_c(\psi), \tag{18}$$

where $\vec{S}_c(\psi)$ is related to $C(\psi)$ by

$$C(\psi) = -\frac{\partial}{\partial \vec{v}} \cdot \vec{S}_c(\psi)\,. \tag{19}$$

For the collision operator C, we have the property

$$\int_V [\phi\,C(\psi) - \psi C(\phi)]\,d^3v = \int_\Sigma [\phi\,\vec{S}_c(\psi) - \psi\vec{S}_c(\phi)] \cdot d\vec{a}\,. \tag{20}$$

For $V \to \infty$, the surface terms vanish and we have the well-known self-adjoint property of the collision operator. For our present needs, however, we shall keep the boundary terms.

We take the inner product of ϕ and $D_t\psi$, where ϕ obeys Eq. (8) with homogeneous initial condition and ψ obeys Eqs. (16) and (17). We need to specify a boundary condition on Eq. (8) too; for our present purposes let us assume we know $S_c(\phi)$ on the boundary [but, we shall, in Eq. (23), give a more useful, less general, condition]. Exploiting the orthogonality properties of both ϕ and ψ, we obtain

$$\int_V d^3v\, f_m\, \phi(\vec{v},t)\, \psi_o(\vec{v}) + \int_o^t d\tau \int_\Sigma \phi(\vec{v},\tau)\, \vec{S}_B(\vec{v},t-\tau) \cdot d\vec{a}$$

$$= \int_V d^3v \int_o^t d\tau\, \vec{\Gamma}_*(\vec{v},t-\tau) \cdot \frac{\partial}{\partial \vec{v}}\, \psi(\vec{v},\tau)$$

$$+ \int_o^t d\tau \int_\Sigma \psi(\vec{v},t-\tau)\, [\vec{\Gamma}_* + \vec{S}_c(\phi)] \cdot d\vec{a}\ . \qquad (21)$$

The driven flux

$$\vec{\Gamma}_*(v,t) \equiv \vec{\Gamma}(v,t) + \vec{F}(v,t) f_m \qquad (22)$$

is the sum of the wave-induced flux $\vec{\Gamma}$ and the flux induced by the background fields that tend to accelerate the background Maxwellian distribution. The latter flux leads to the Ohmic current.

Equation (21) is a principal result of this work. Both ψ_o and $\vec{S}_B$ are arbitrary functions, and they may be used to express any moment of ϕ either in the interior of V or on its bounding surface Σ. Depending on the desired moment of ϕ, a different Green's function, ψ, is solved for by Eqs. (16) and (17). Once ψ is known, the right-hand side of Eq. (21) may be evaluated for arbitrary wave induced fluxes $\vec{\Gamma}$.

Note that in generalizing the steady-state adjoint equations to include time dependence, several significant steps were taken. The inner product, with respect to which the adjoint property is defined, was changed to include a convolution over time. Also note that the orthogonality conditions are imposed here to assure the continued validity of linearization in Eq. (5), and of the resulting simplification of the collision operator. These conditions reduce to the compatibility conditions of the steady-state theory in the limit $\partial/\partial t \to 0$. However, while the imposition of these conditions in the dynamic case results in considerable mathematical simplification, it is not necessary here for the existence of solutions.

IV. EXAMPLES

In this section, we give several examples showing how Eq. (21) may be used to find interesting plasma responses. For example, in addition to the current, the rf-induced runaway rate may be computed. A second example shows how contributions to the rf-induced plasma current (or for that matter radiation, etc.)[1] can be separated into runaway and nonrunaway contributions. A third example shows how analytic progress can be made, using Eq. (21), to find the rf-induced conductivity in certain limiting cases.

To calculate how rf waves affect the runaway production rate, we must first be precise in defining a runaway electron. If the parallel electric field points always in one direction (i.e., electrons are pushed unidirectionally), then electrons can be accelerated by this field to high enough energy that they overcome the dynamic frictional force of collisions. If the domain V is large, then an electron accelerated from the bulk of the distribution to the boundary Σ may be deemed a runaway, for frictional forces are extremely unlikely to return this electron to the domain V. It is

convenient to separate the boundary Σ into two parts; an electron appearing on Σ_{out} leaves the domain V, while an electron appearing on Σ_{in} is forced into the domain V, and Σ is the union of Σ_{out} and Σ_{in}. For large V, it is primarily the electric field that determines this dichotomy. A more precise separation of Σ takes into account the small, but finite, inwardly (i.e., into V) pointing collisional force in addition to the force of the electric field, so that Σ_{out} is slightly smaller than Σ_{in}. All runaway electrons appear at some time on Σ_{out}, and all electrons that appear on Σ_{out} are runaways, freely accelerating out of the domain V.

Equivalently, we can imagine solving a problem in which collisions operate within the domain V, but the collision operator vanishes outside this domain, allowing electrons to stream freely. Where the collision operator vanishes, the governing equation reverts from parabolic to hyperbolic. It is then correct to impose a boundary condition on just Σ_{in}, rather than on the full boundary Σ. The adjoint equation, on the other hand, is well-posed when a boundary condition is imposed only on Σ_{out}.

Define a flux $\vec{S}_\phi(\phi) \equiv \phi \vec{f}_m \vec{F} + \vec{S}_c(\phi)$. Then, on Σ_{in}, we specify $\vec{S}_\phi(\phi)$, the incoming flux into domain V. For the runaway problem, and most other problems of interest, this flux is zero. For the adjoint equation, we require that $\vec{S}_c(\psi) = 0$ on Σ_{in}. On Σ_{out}, the opposite holds. Here we specify $\vec{S}_c(\phi) = 0$, indicating that runaway electrons eventually stream freely. We are then left with the freedom to choose $\vec{S}_\psi(\psi)$ in order to pose the adjoint equation.

Recognizing the vanishing of $\vec{S}$ on Σ_{in} and the vanishing of $\vec{S}_c$ on Σ_{out}, we can manipulate Eq. (21) into the more useful form

$$\int_V d^3v\, f_m \phi(\vec{v},t)\psi_o(\vec{v}) + \int_o^t d\tau \int_{\Sigma_{out}} \phi(v,\tau)\vec{S}_\beta(\vec{v},\, t-\tau) \cdot d\vec{a}$$

$$= \int_V d^3v \int_o^t d\tau\, \vec{\Gamma}_*(\vec{v},\, t-\tau) \cdot \frac{\partial}{\partial \vec{v}} \psi(\vec{v},\tau) + \int_o^t d\tau \int_\Sigma \psi(v,t-\tau)\vec{\Gamma}_*(\vec{v},\tau)\cdot d\vec{a} \quad . \tag{23}$$

The above formulation also allows a slight refinement. The collisional slowing down may be distinguished from the collisional induced diffusion. Requiring that only the collisional diffusion vanish at the boundary gives a more accurate calculation of both the total flux at the boundary and the demarcation between Σ_{in} and Σ_{out}. For example, for the runaway problem, the flux at the boundary may be expressed as the sum $\hat{i}_\parallel eE_\parallel/m - \nu_s\vec{v}$, where $\nu_s\vec{v}$ represents the dynamical friction due to collisions. The boundary Σ_{out} is then defined to exist where this sum points outward from the domain V.

Let us find now the response function for the rf-induced runaway rate. The number of rf-induced runaway electrons, N_R, appearing on Σ_{out} between time $\tau = 0$ and time $\tau = t$ may be written as

$$N_R = \int_o^t d\tau \int_{\Sigma_{out}} \vec{S} \cdot d\vec{a} \quad , \tag{24}$$

where $\vec{S} = (\hat{i}_\parallel eE_\parallel/m - \nu_s\vec{v})\phi f_m$ on Σ_{out} is the more exact posing of the problem. The less exact posing takes $\nu_s \to 0$. In either event, the response function satisfies Eq. (16), $D_t\psi = Q_\phi$, with Q_ϕ assuring orthogonality, with initial condition

$$\psi(0) = 0 \quad , \tag{25a}$$

and boundary conditions

$$\vec{s}_B = f_m(\hat{i}_\parallel eE_\parallel/m - \nu_s \vec{v}), \quad \text{on} \quad \Sigma_{out} \tag{25b}$$

$$\vec{s}_B = 0 \quad , \quad \text{on } \Sigma_{in} \tag{25c}$$

Solving Eqs. (24) and (25) for ψ, and using Eq. (23), we get (neglecting non-rf-induced contributions),

$$N_R = \int_V d^3v \int_0^\tau d\tau\, \vec{\Gamma}(\vec{v}, t-\tau) \cdot \partial\psi(\vec{v}, \tau)/\partial\vec{v} \quad . \tag{26}$$

Note that $\psi(\vec{v},t)$ may be interpreted here as the probability with which a particle appears on Σ_{out} by time t given initial coordinate $\vec{v}$. For finite temperatures, eventually all particles would run away, since even those that slow down into the bulk distribution will ultimately scatter out of the bulk and run away. It is convenient, however, to label such electrons as stopped electrons, since they would run away on a much longer time scale. Suppose that the collision frequency were made artificially large as $v \to 0$ (or particles appearing near $v \to 0$ were frozen and not allowed to run away). Then particles running away without passing through the bulk would be termed runaways, and the others would be termed stopped. The probability of a particle running away given initial coordinate $\vec{v}$ may then be expressed as

$$R(\vec{v}) = \psi(\vec{v}, t \to \infty) \quad . \tag{27}$$

This runaway rate has been obtained previously through an equivalent formulation of the problem using Langevin equations.[14] The numerical

evaluation of the runaway rate was carried out by a Monte Carlo method. Solving Eqs. (24) and (25) directly, however, would be much easier.

It is often helpful to distinguish the contributions of distinct groups of electrons to a given effect. For example, both runaway and stopped electrons contribute to the current. By identifying the separate contributions, one can determine if there is anything to be gained by increasing or decreasing somehow the number of runaways, possibly by controlling their confinement time. In Ref. 15, distinguishing these contributions facilitated a comparison between theory and experiment.

In order to deduce the contribution to an effect, say the current, due solely to stopped electrons, we make use of the function $\vec{R}(\vec{v})$ defined in Eq. (27). We write the parallel current as $J = J_R + J_S$, where J_R is the runaway current and J_S is the current due to stopped electrons. We can write J_S as

$$J_S = \int_V d^3v \; f_m \; \phi(\vec{v},t)\{v_\parallel[1 - R(\vec{v})] + c_1 + \varepsilon c_2\} \quad , \tag{28}$$

where c_1 and c_2 are constants to be determined. Evidently, the appropriate Green's function, ψ_s, for J_S solves Eq. (16) with homogeneous boundary conditions and initial condition

$$\psi_s(\vec{v},\ t = 0) = v_\parallel[1 - R(\vec{v})] + c_1 + \varepsilon c_2 \quad , \tag{29}$$

with c_1 and c_2 determined by the orthogonality properties of ψ_s. Using Eq. (21), we then have $J_S = -[\psi_S,\ (\partial/\partial\vec{v})\cdot\vec{\Gamma}]$.

The last example in this section illustrates the ease with which analytic solutions can sometimes be found for the response functions. Analytic expressions are often available when the collision operator can be written in

the large-velocity limit. For example, the incremental conductivity due to the heating of superthermal electrons by rf waves can be calculated analytically. This is the so-called "hot conductivity." The simplification of the collision operator in the large-velocity limit is valid, because only the superthermal electron contribution to the current is important. In contrast, the Spitzer-Härm conductivity is not available analytically. Both slow and fast electrons contribute importantly to the Spitzer-Härm current and a drastic simplification of the collision operator is not possible.

In the high-velocity limit, the collision operator simplifies to

$$C(\phi) = -\frac{\nu_o f_m}{2u^3}\left[-u\frac{\partial}{\partial u}\phi + \left(\frac{1+Z}{2}\right)\frac{\partial}{\partial\mu}\left(1-\mu^2\right)\frac{\partial}{\partial\mu}\phi\right] \quad , \tag{30}$$

where $\nu_o = \omega_p^4 \ln\Lambda/2\pi n v_T^3$ is a collision frequency, $u \equiv v/v_T$, $\mu \equiv v_\parallel/v$, $v_T \equiv (T/m)^{1/2}$, and Z is the ion charge state. In order to find the hot conductivity, we need to solve Eq. (16) in the limit $\vec{F} = \hat{i}_\parallel eE_\parallel/m \to 0$. Also, since we wish to find the steady-state conductivity, rather than the transient effects, it is necessary only to calculate the time integral of ϕ rather than ψ itself.

Accordingly, define

$$\chi(\vec{v}) = \int_o^\infty d\tau\, \psi(\vec{v},\tau) \quad , \tag{31}$$

where $\psi(\vec{v},\tau)$ solves Eq. (16) with homogeneous boundary conditions and initial condition $\psi(\vec{v},\tau = 0) = ev_T u\mu$. Integrate Eq. (16) from $\tau = 0$ to $\tau = \infty$ to obtain

$$-f_m\left(\vec{F}\cdot\frac{\partial}{\partial\vec{v}}\right)\chi - C(\chi) = f_m e v_T u\mu \quad , \tag{32}$$

where we noted that $\psi(\vec{v},\tau) \to 0$ as $\tau \to 0$ (for finite conductivity), and that the constants q_1 and q_2 in Eq. (16) are, in this case, zero. Expanding χ in a power series in $E_\parallel$, i.e., $\chi = \chi_o + E_\parallel \chi_1 + \ldots$, allows us to solve Eq. (32) analytically. The first term gives the rf-driven current in the absence of a dc electric field. The second term gives the current that is bilinear in both the dc electric field and the rf power absorbed. The wave induced current is then given by

$$J = \int_V d^3v\, \vec{\Gamma} \cdot \frac{\partial}{\partial \vec{v}} \chi \quad . \tag{33}$$

From Eq. (33), we can find the hot conductivity, σ_H, which is identical to that given in Ref. 3.

V. MODIFIED CHAPMAN-ENSKOG EXPANSION

Although it cannot describe all the phenomena important to wave-driven plasmas, the expansion of Chapman and Enskog remains a simple and effective approach to describing transport important to the bulk of the electrons. The question is how we can modify this expansion so as to incorporate effects associated with the fast, collisionless electrons, especially those effects that can be described by Eq. (18) and its associated formalism. We seek an expansion that would reduce to the Chapman-Enskog expansion in the limit $\vec{\Gamma} \to 0$, and would, for finite $\vec{\Gamma}$, describe adequately the dynamics of the fast electrons. In this expansion, the heating of the bulk electrons by the fast electrons, as well as other effects that link the two distributions, should play a role in the evolution of the Chapman-Enskog variables (n, T, $\vec{c}$) for $\vec{\Gamma} \neq 0$. (Here we use $\vec{c}$ to describe the drift of a Maxwellian distribution, with the understanding that all these variables, n, T, $\vec{c}$, must be specified

separately for each species.) In this section, we describe an expansion that meets these criteria. We remark, however, that this expansion, like the Chapman-Enskog expansion itself, is not a unique choice, altnough it does possess some desirable features.

We begin with Eqs. (1) and (2), and we expand f using the formal expansion parameters θ and δ (assumed small but eventually set equal to one), i.e., we expand

$$f = f_o + \sum \theta^j f_j + \sum \delta^k h_k , \qquad j,k \geq 1 \tag{34}$$

where the f_j represent corrections primarily to the bulk electron distribution and the h_k represent the wave-driven corrections. Accordingly, the collisions among the f_j are assumed to take place on a much shorter time scale than collisions either between the f_j and the h_k, or among the h_k. We shall order

$$C(f_j,f_k) \sim \frac{1}{\theta} , \qquad j, k \geq 0 \tag{35a}$$

$$C(f_j,h_k) \sim 1 , \qquad j \geq 0, k \geq 1 \tag{35b}$$

$$C(h_j,h_k) \sim 1 , \qquad j, k \geq 1 . \tag{35c}$$

The $1/\theta$ Eq. (35a) ordering is inherited from the Chapman-Enskog (C-E) approach, whereas the other ordering, we shall see, allows us to describe better the dynamics of fast electrons.

In the C-E approach, the use of multiple time scales serves as a means of convenient bookkeeping. Here, in addition to the C-E time scales, we introduce a parallel scaling in δ, i.e.,

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau_o} + \theta \frac{\partial}{\partial \tau_1} + \theta^2 \frac{\partial}{\partial \tau_2} + \ldots + \delta \frac{\partial}{\partial t_1} + \delta^2 \frac{\partial}{\partial t_2} + \ldots, \tag{36}$$

which facilitates the formal expansion. We assume the functional dependencies

$$f_o = f_o \left(\tau_o, \tau_1, \tau_2, \ldots, t_1, t_2 \ldots, \vec{v}, \vec{r}\right) \tag{37a}$$

$$f_j = f_j \left(\tau_o, \tau_1, \tau_2, \ldots, \vec{v}, \vec{r}\right), \quad j > 1 \tag{37b}$$

$$h_j = h_j \left(\tau_o, t_1, t_2, \ldots, \vec{v}, \vec{r}\right), \quad j > 1. \tag{37c}$$

Consider terms in the expansion of order θ^k, which represent the Chapman-Enskog terms. To order $1/\theta$, we have

$$C\left(f_o, f_o\right) = 0 \quad , \tag{38}$$

which has the drifting Maxwellian solution

$$f_o = f_m \left(n, T, \vec{c}\right) \quad , \tag{39}$$

with n, T, and $\vec{c}$ functions of time and space. To higher order in θ, we have

$$\theta^o: \quad C\left(f_1\right) = \frac{d}{d\tau_o} f_o \tag{40a}$$

$$\theta^1: \quad C\left(f_2\right) = \frac{d}{d\tau_o} f_1 + \frac{\partial}{\partial \tau_1} f_o - C\left(f_1, f_1\right) \quad , \tag{40b}$$

where

$$\frac{d}{d\tau_o} \equiv \frac{\partial}{\partial \tau_o} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} + \vec{F} \cdot \frac{\partial}{\partial \vec{v}} \,. \tag{41}$$

Equations (40) represent the first terms of the Chapman-Enskog expansion. The f_j, $j \geq 1$ are chosen to contain no particles, current, or energy. Compatibility is assured, order by order, by using

$$\frac{\partial f_o}{\partial \tau_j} = \frac{\partial n}{\partial \tau_j} \frac{\partial f_m}{\partial n} + \frac{\partial T}{\partial \tau_j} \frac{\partial f_m}{\partial T} + \frac{\partial \vec{c}}{\partial \tau_j} \cdot \frac{\partial f_m}{\partial \vec{c}} \quad , \tag{42}$$

and solving for $\partial n/\partial \tau_j$, $\partial T/\partial \tau_j$, $\partial \vec{c}/\partial \tau_j$ to meet the requirements on the f_j. The C-E quantities n, T, $\vec{c}$ then evolve according to, e.g.,

$$\frac{dn}{dt} = \sum_{j=0} \theta_j \frac{\partial n}{\partial \tau_j} + \sum_{j=1} \delta^j \frac{\partial n}{\partial t_j} \,. \tag{43}$$

In the absence of contributions arising from $\partial/\partial t_j$, the terms f_o, f_1, f_2 are identical to the first three terms of the classic C-E expansion.

Matching terms of order δ^j allows us to describe additionally the effects of the wave-driven fluxes. For example, the first two equations in the expansion are

$$Lh_1 \equiv \frac{d}{d\tau_o} h_1 - C(h_1) = - \frac{\partial}{\partial t_1} f_o - \frac{\partial}{\partial \vec{v}} \cdot \vec{\Gamma} \tag{44a}$$

$$Lh_2 = - \frac{\partial}{\partial t_1} h_1 - \frac{\partial}{\partial t_2} f_o + C(h_1, h_1) \,. \tag{44b}$$

Note that Eq. (44a) is identical to Eq. (8) for $f_m \Phi$ except that f_o is slightly more general here, that h_1 can be spatially dependent, and that the inhomogeneous driving term associated with the Spitzer-Härm current has been accounted for in the C-E part of the expansion. The last difference means

that, consistent with our assumption of the low mean-free-path limit, we ignore any runaway electrons except those created by the waves. Those electrons could have been described had we ordered the term $\vec{F} \cdot \partial f_o/\partial \vec{v} \sim \delta$; however, that would take us further than we wish away from the C-E approach.

The differences between Eq. (44a) and Eq. (8) do not affect the transferability of the technique to solve these equations. Boundary and initial conditions are imposed similarly. The adjoint method used to solve Eq. (8) may be employed both in Eqs. (44a) and (44b) and higher order equations in the δ-expansion. The effect of the wave-driven fluxes on the bulk transport properties is then accounted for through the evolution equations for n, T, and $\vec{c}$, as in Eq. (43).

While the above procedure accurately treats to first order both the wave-driven fluxes and the bulk transport within the spirit of Chapman and Enskog, a question arises as to higher order terms in the expansion that are hybrid, i.e., bilinear in h_j and f_k. There is a certain amount of arbitrariness in treating these hybrid terms; their main effect would be to correct somewhat the properties of the bulk plasma, rather than to change the dynamics of the fast electrons. Accordingly, it is most expedient to account for these terms by allowing them to drive the f_K. To incorporate them, we assume $\delta \sim \theta$. So, for example, the lowest order hybrid terms, which are bilinear in f_1 and h_1 [which are computable from Eqs. (40a) and (40b)], drive the third order C-E term f_3, i.e.,

$$C(f_3) = \frac{d}{d\tau_o} f_2 + \frac{\partial}{\partial \tau_1} f_1 + \frac{\partial}{\partial \tau_2} f_o - C(f_2, f_1) - C(f_1, f_2)$$

$$- C(f_1, h_1) - C(h_1, f_1). \qquad (45)$$

In such a manner, we may solve order by order both for f_j and h_j, evolving n, T, $\vec{c}$ [e.g., through Eq. (43)] as we do so. Finally, we set the formal expansion parameters θ, $\delta = 1$. In practice, however, one may contemplate that only the first few terms of such an expansion are important.

IV. RESPONSE FUNCTIONS AND INFERENCE

In this section we address how the use of the response functions developed here may help us in understanding plasma experiments. There is an immediate and straightforward method of modeling the plasma. A second method uses the formalism developed here to develop important inferences.

The straightforward method is the traditional "transport code." Our best guesses as to transport coefficients, and other parameters thought to govern an evolving plasma, are collected to form a numerical description of the plasma. These best guesses could be arrived at either by theoretical insight into fundamental plasma processes, or by empirical evidence gathered in many experiments, or by a combination of the two approaches. The development in Sec. V indicates how the response functions here may easily be incorporated in such a scheme. The traditional transport description that relates to the bulk plasma is left untouched; however, effects associated with the rf-heating of fast electrons may be treated systematically.

Let us illustrate this procedure by means of an example: Suppose that rf waves are introduced into a tokamak plasma by means of waveguides or other injection devices situated on the periphery of the tokamak. The ray trajectories of the waves may be tracked numerically. The waves are followed until they are absorbed, giving up their energy and momentum to the absorbing plasma species at some point in the velocity space of that species and at some point in configuration space. The adjoint equations can be used to determine

the amount of bulk heating and current generation per wave power absorbed. To determine the amount of wave power absorbed, a number of approximations may be used. Antonsen and Hui[19] assume that the damping is linear. A more precise approach is to use the one-dimensional quasilinear equations, which take into account the flattening of the velocity distribution in response to the waves. Although neither of these approaches is exact in its determination of the power absorbed, the ratio of current generated to power absorbed, which is often more important, is an exact quantity.

Once the trajectory and the effects of the waves are determined, the bulk plasma properties may be evolved through transport equations. The result is a simulation of a plasma heating experiment. (see, e.g., Ref. 20).

In practice, simulating a plasma in this manner has not been particularly successful. The problem is that inherent in this approach is the multiplication of uncertainties. It is not absolutely certain what the wave phasing is at the plasma periphery; it is not certain whether these waves will be scattered by edge turbulence as they enter the tokamak; it is not certain that the plasma absorbs this radiation according to the approximate theory used; and it is not certain that other quantities governing the plasma heat, particle transport, or the background magnetic configuration are correctly given. The probability that the simulation will correctly predict an important experiment observable, however, is related to the product of the likelihoods that each process is correctly modeled. Nonetheless, the plasma transport code has undergone major development as a numerical tool for designing and interpreting experiments.

The second method of employing the response functions derived here to explain plasma experiments is through the identification of inferences. Instead of attempting a complete numerical description of the evolving plasma, this method seeks to isolate and understand specific mechanisms.

Consider, for example, the case of the rf-heated plasma. Suppose that in the interior of the plasmas a wave-induced flux $\vec{\Gamma}$ of electrons is produced in the resonant region of electron velocity space. The plasma responds in several ways. There may be, for example, an incremental production of current, runaway electrons, and plasma radiation at various angles and frequencies. Each of these quantities may be expressed as a function of $\vec{\Gamma}$ by use of the appropriate response function.

An understanding of the plasma can now be developed on the basis of conditional probabilities. Given that the wave power is absorbed by a certain group of electrons, a set of experimental observables may be recorded and related to each other. For example, an observed increase in the current density or poloidal field energy should be directly correlated with an increase in radiation at certain frequencies. The precise mechanism of wave propagation and damping need not be uncovered, then, in order to infer the location and velocity of the resonant electrons, as well as other local plasma parameters governing the radiation.

Similarly, if it is the theory of the radiation that is to be checked, then instead of plotting the observed radiation vs. input power, one might plot, e.g., the observed radiation vs. current-drive efficiency, or one might plot the observed radiation at one angle and frequency vs. that observed at a second angle and frequency. In these examples, one achieves an improved understanding of specific plasma processes, without the necessity of a complete picture of the evolving plasma.

The response functions derived here are especially suited for such an approach. As a numerical tool, the identification of such inferences and correlations can be of great help in understanding experiments. After all, such a tool mimics most closely the likely thought processes of the plasma

experimenter. Faced with an unusual experimental result, the experimenter is far more likely to attempt crude pattern matching and other correlations to reduce data, than to simulate conceptually the complete picture of the plasma evolution. A numerical tool that aids the experimenter in his natural approach to explaining an experiment may be of more direct use than the far more ambitious goal of providing a complete numerical simulation.

The above approach lends to measurements of plasma radiation far more importance than might otherwise be suspected. These measurements are of little use in the construction of a numerical simulation. On the other hand, they could provide a wealth of information about the absorption of waves in a driven plasma.

VII. SUMMARY AND CONCLUSIONS

This paper concerns itself with the description of effects associated with fast, relatively collisionless electrons, whose mean-free path may be long compared to other lengths of interest. It is possible that these few, fast electrons will contribute importantly to plasma transport, particularly in the case of an intensely rf-driven plasma. Intense rf waves interacting primarily with fast electrons produce particle fluxes in velocity space that can substantially alter the distribution of electrons. The result may be a large change in the plasma current, synchrotron radiation, bremsstrahlung, runaway production, or other quantities associated with the transport of fast electrons.

In order to calculate the plasma response to wave-induced fluxes in velocity space, we linearized the inhomogeneous dynamic Boltzmann equation. Although the linearized equation is not much more easily solved numerically than is the original equation, it is possible, because of the linearization, to find the Green's functions for quantities of interest. In Sec. III we

identified an adjoint operator, defined over a suitable inner product, that allowed us to generate adjoint equations for calculating these quantities. By imposing initial and boundary conditions on the adjoint equation, plasma responses may be calculated, via Eq. (23), for arbitrary wave-induced fluxes $\vec{\Gamma}$.

An important simplification in the practical application of Eq. (23) arises from our ability to infer information about $\vec{\Gamma}$ from the resonance condition, as discussed in Sec. II. This allows us to compare, for example, likely plasma responses per power dissipated as the phasing of the waves is varied.

We gave several examples in the use of these response functions in Sec. IV. These examples showed how useful adjoint equations may be defined, and how, in certain limits, analytic solutions are available.

The responses associated with these fast electrons are not fluid effects, and are not naturally found in small mean-free-path expansions of the Boltzmann equation, such as the expansion of Chapman and Enskog. In Sec. V, however, we showed how the C-E expansion may be modified to account for the wave-driven fluxes. An expansion, formally in two small parameters, was proposed: the frequently colliding bulk particles are treated fluidlike, exactly as in the C-E expansion, while the fast, long-mean-free-path electrons evolve subject to external forces and subject to collisions predominantly with bulk electrons and ions. The description of the fast electrons is facilitated by the use of adjoint equations, and the effect of these electrons on the bulk transport processes appears in the higher order terms describing the bulk particles.

Although the response functions derived here can be directly implemented in existing transport codes that simulate completely the evolving plasma, we speculate that a more advantageous use of this work is through the development of a set of correlations from which plasma behavior might be inferred.

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