TRANSPORT OF MHD COUPLE STRESS FLUID THROUGH PERISTALSIS IN A POROUS MEDIUM UNDER THE INFLUENCE OF HEAT TRANSFER AND SLIP EFFECTS

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An attempt has been made to examine the effects of magnetohydrodynamic couple stress fluid in peristaltic flow with porous medium under the impact of slip, heat transfer and wall properties. The expressions are obtained for temperature, coefficient of heat transfer and velocity. Influences of different parameters, the Hartmann number, Brinkman number and adaptability parameters on the temperature and warmth trade coefficient are discussed through outlines.

Key words: peristaltic flow, couple stress fluid, Darcy number, slip parameter, compliant walls.

1. Introduction

The automatic contraction and relaxation of certain process in the human body induces progressive waves through which food and waste fluid move and is termed on peristalsis, for example, movement of chime in the gastrointestinal tract, urine flow through the ureter, blood flow through blood vessels. Peristaltic principle is also used by mechanical devices like roller and finger pumps. Beaver and Joseph [1] gave the first boundary condition for a permeable wall, verifying it theoretically and justifying statistically. Saffmann [2] improved this and suggested a condition suitable for small permeability. Studies on of peristaltic transport through a channel with compliant wall have been carried out by some researchers [3-7].

With a number of prospective applications in biomedical science, the study on peristalsis with heat transfer and MHD has attracted many researchers. The heat transfer concept is applied in computing the blood stream rate with the underlying thermal conditions. Bio-heat transfer is widely used in the destruction of unwanted tissues, laser therapy, cryosurgery, perfusion of blood stream, skin smoldering, fever and hypothermia in vertebrates. Peristalsis involving MHD is studied on account of its connection with movement of conductive physiological fluids, viz., stream of blood in small vessels and the pumping machines. The effect of wall characteristics on peristalsis involving heat transfer has been studied by Radhakrishnamacharya and Srinivasalu [8]. Hayat *et al*. [9] analyzed the importance of a compliant wall and heat transfer on the peristaltic flow through a curved channel considering an incompressible viscous fluid. The influence of heat transfer and wall properties on the peristaltic movement of power law fluid was observed by Hayat *et al*. [10].

The investigation of peristalsis through a permeable medium has gained importance in investigating the reflux conditions which is useful for realizing the intricacy of many diseases such as bacterial stones, bladder stones, cystitis, bacterial affection of kidneys, etc. Kothandapani and Srinivas [11] concentrated on the MHD peristaltic motion under heat transfer to analyze the wall property effect on the flow of a couple stress fluid. The incompressible laminar flow in a porous conduit of a couple stress fluid was analyzed by

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Srinivasacharya *et al*. [12]. Considering the food bolus movement through the esophagus as a Jeffrey fluid in a conduit, Arun *et al*. [13] examined the wall property and heat transfer effects. Dheia *et al*. [14] considered the flow through a porous medium channel in continuation of the work of [12]. The peristaltic flow of a Williamson fluid in an asymmetric channel with a porous medium was analyzed for studying the heat transfer and magnetic field effects by Ramesh and Devakar [15].

Assuming the arterial wall of the porous medium channel to be realistic, we have to consider the channel wall with slip condition as no slip condition is not valid at the wall due to permeability. Thus applying the slip condition means that the viscous effect is no longer present in the fluid. The study of flow through the channels with the slip is applied in chemical engineering, e.g., two phase flows in bearings of porous slides, flow through pipes in which reactions occur at the wall, etc. Srinivas *et al*. [16] considered the MHD peristaltic motion to investigate the slip wall property and also the heat transfer effect. Das [17] considered the collective effect of the slip wall, MHD and transfer of heat on the Maxwell fluid flowing through a peristaltic porous medium channel. The flow of a Newtonian fluid with couple stress inside a permeable medium channel was analyzed by Hummady and Abdulhadi [18] to study the effect of transfer of heat with slip conditions. The movement of an MHD Newtonian fluid in a permeable medium having flexible walls was investigated by Reddy and Reddy [19] to study the consequences of velocity slip and Joule heating. Recently, Laxshiminarayana *et al*. [20] examined the heat transfer effect of a conducting Bingham fluid in a non-uniform conduit. The peristaltic motion was considered for the study in the slip and elastic wall properties. It was concluded that with an increase in the magnetic parameter and the Brinkman number, there is a decrease in the temperature field, whereas the observed gain in the Brinkman number resulted in the rise of the heat transfer coefficient.

In the present study, the impact of heat transfer and wall slip condition on the peristaltic motion of a magnetohydrodynamic couple stress fluid is analyzed. The fluid is considered to flow through a uniform permeable flexible channel having a compliant wall. The flow has been investigated under the assumption that the: Reynolds number is small and wavelength of the peristaltic wave is large.

2. Mathematical formulation

The peristaltic transfer in a uniform channel between two elastic walls is considered as shown in Fig.1. The flow considered is unsteady, two dimensional and incompressible. The fluid in the channel is a magnetohydrodynamic couple stress fluid. Here dimensions of the uniform channel flow in a porous medium are given as the width of the uniform channel *2d,* wave length λ and wave amplitude *a*. *x* is the axial coordinate and *y* is normal to it.
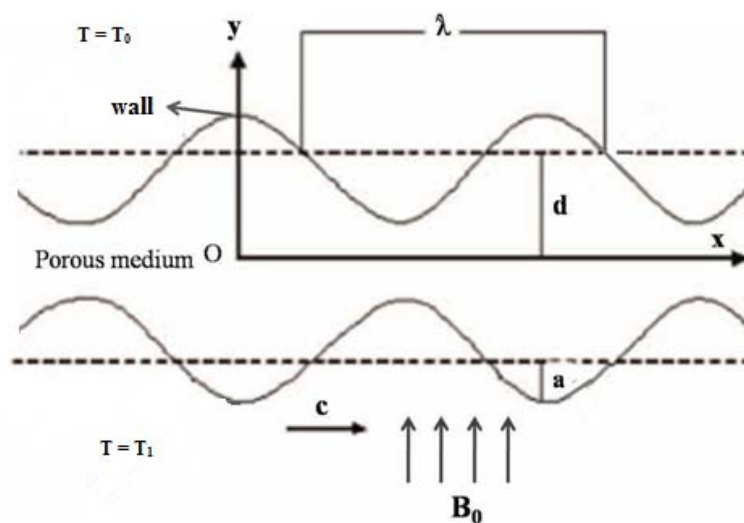

Fig.1. Physical model.

The peristaltic wave is moving with constant speed c along the channel wall and the geometry of the surface wall is

$$y = \eta(x,t) = d + a \sin \frac{2\pi}{\lambda}(x - ct).$$

(2.1)

The motion of the elastic wall is

$$L(\eta) = p - p_0$$

(2.2)

where L is an operator that represents the flexible wall motion with viscosity damping force given by

$$L = -\tau \frac{\partial^3 \eta}{\partial x^3} + m \frac{\partial^3 \eta}{\partial x \partial t^2} + C \frac{\partial^2 \eta}{\partial x \partial t}.$$

(2.3)

Here the elastic tension in the wall is given by τ, the mass per unit area is given by m and the coefficient of the viscous damping force is given by C.

Basic equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

(2.4)

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu \nabla^2 u - h \nabla^4 u - \mu \frac{u}{k} - \sigma B_0^2 u,$$

(2.5)

$$\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu \nabla^2 v - h \nabla^4 v - \mu \frac{v}{k},$$

(2.6)

$$G\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{K}{\rho}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + 9\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right\}.$$

(2.7)

In the equations above, the flow rates are u and v along x and y paths, respectively. As the plane is symmetrical, the normal velocity is zero. k, ρ, μ and h, respectively, represent the permeability of the porous medium, the fluid density, the coefficient of viscosity of the fluid and the constant associated with couple stress. Also, p, σ, K, T, G and 9 are the pressure, the electrical conductivity of the fluid, the thermal conductivity of the fluid, the temperature of the fluid, the specific heat at constant pressure and the kinematic coefficient of viscosity respectively. Further, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\nabla^4 = \nabla^2 \nabla^2$. Hear we assume $p_0 = 0$.

The corresponding boundary conditions are

$$\frac{\partial u}{\partial y} = 0, \qquad \frac{\partial^3 u}{\partial y^3} = 0, \quad \text{at} \quad y = 0,$$

(2.8)

$$u = -d \frac{\sqrt{Da}}{\beta} \frac{\partial u}{\partial y} \quad \text{at} \quad y = \pm \eta(x, t) \tag{2.9}$$

where Da denotes the Darcy number and β denotes the slip parameter.

After vanish the couple stresses

$$\frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = \pm \eta. \tag{2.10}$$

Referring to Mittra and Prasad [21] the dynamic boundary conditions are

$$\frac{\partial}{\partial x} L(\eta) = -\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \mu \nabla^2 u - h \nabla^4 u - \sigma B_0^2 u \quad \text{at} \quad y = \eta(x, t) \tag{2.11}$$

where

$$\frac{\partial}{\partial x} L(\eta) = \frac{\partial p}{\partial x} = -\tau \frac{\partial^3 \eta}{\partial x^3} + m \frac{\partial^3 \eta}{\partial x \partial t^2} + C \frac{\partial^2 \eta}{\partial x \partial t}. \tag{2.12}$$

The non- dimensional quantities used are

$$x' = \frac{x}{\lambda}, \quad y' = \frac{y}{d}, \quad t' = \frac{ct}{\lambda}, \quad u' = \frac{u}{c}, \quad v' = \frac{\lambda v}{cd}, \quad p' = \frac{d^2 p}{\mu \lambda c}, \quad \eta' = \frac{\eta}{d}, \quad \theta = \frac{T - T_0}{T_1 - T_0}. \tag{2.13}$$

The upper wall temperature is T_0 and at the lower wall the temperature is T_1.
Introducing non-dimensional variables in Eqs (2.4) - (2.11), after dropping the primes gives

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.14}$$

$$\begin{aligned} \mathrm{Re}\,\delta \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \left(\delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \\ -\frac{1}{\propto^2} \left(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{u}{k} - H_0^2 u, \end{aligned} \tag{2.15}$$

$$\begin{aligned} \mathrm{Re}\,\delta^3 \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \delta^2 \left(\delta^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \\ -\frac{\delta^2}{\propto^2} \left(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\delta^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \delta^2 \frac{v}{k}, \end{aligned} \tag{2.16}$$

$$\mathrm{Re}\delta\left(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)=\frac{1}{\mathrm{Pr}}\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+$$

$$+\mathrm{Ec}\left[2\left\{\delta^2\left(\frac{\partial u}{\partial x}\right)^2+\delta^2\left(\frac{\partial v}{\partial y}\right)^2\right\}+\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right].$$

(2.17)

The relevant conditions at the boundary are

$$\frac{\partial u}{\partial y}=0,\qquad\frac{\partial^3 u}{\partial y^3}=0\qquad\text{at}\qquad y=0\,,$$

(2.18)

and

$$u=-\frac{\sqrt{\mathrm{Da}}}{\beta}\frac{\partial u}{\partial y},\qquad\frac{\partial^2 u}{\partial y^2}=0,\qquad\text{at}\qquad y=\pm\eta(x,t)=\pm\left(1+\varepsilon\sin 2\pi(x-t)\right),$$

(2.19)

$$-\mathrm{Re}\delta\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{1}{\propto^2}\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-H_0{}^2 u=$$

$$=E_1\frac{\partial^3\eta}{\partial x^3}+E_2\frac{\partial^3\eta}{\partial x\partial t^2}+E_3\frac{\partial^2\eta}{\partial x\partial t}\qquad\text{at}\qquad y=\pm\eta(x,t).$$

(2.20)

Here, $\delta=\left(\dfrac{d}{\lambda}\right)$, $\varepsilon=\left(\dfrac{a}{d}\right)$, $\mathrm{Re}=\left(\dfrac{\rho c d}{\mu}\right)$, $\mathrm{Da}=\dfrac{k}{d^2}$, $\propto=\left(\sqrt{\dfrac{\mu}{h}}d\right)$, $H_0=B_0 d\sqrt{\dfrac{\sigma}{\mu}}$, $\mathrm{Pr}'\dfrac{\rho\vartheta G}{K}$, $\mathrm{Ec}=\dfrac{c^2}{G(T_1-T_0)}$, correspondingly stand for the wall slope parameter, the amplitude ratio, the Reynolds number, Darcy number, the couple stress parameter, the Hartmann number, the Prandtl number and the Eckert number.

The elasticity parameters are defined as $E_1=\left(\dfrac{-\tau d^3}{c\mu\lambda^3}\right)$, $E_2=\left(\dfrac{mcd^3}{\mu\lambda^3}\right)$, $E_3=\left(\dfrac{Cd^3}{\mu\lambda^2}\right)$. The parameter E_1 represents the rigidity, E_2 - the stiffness and E_3 - the viscous damping force in the wall.

3. Method of solving the problem

Considering the wavelength of the peristaltic wave to be large and Reynolds number to be small, Eqs (2.14) – (2.17) reduce to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,,$$

(3.1)

$$0=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}-\frac{1}{\alpha^2}\frac{\partial^4 u}{\partial y^4}-\frac{u}{k}-H_0{}^2 u\,,$$

(3.2)

$$0 = -\frac{\partial p}{\partial y},$$ (3.3)

$$Ec\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} = 0.$$ (3.4)

The boundary conditions (2.18) – (2.20) become

$$\frac{\partial u}{\partial y} = 0, \qquad \frac{\partial^3 u}{\partial y^3} = 0, \quad \text{at} \quad y = 0,$$ (3.5)

$$u = -\frac{\sqrt{Da}}{\beta}\frac{\partial u}{\partial y}, \quad \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = \pm\eta(x,t) = \pm\left(1 + \varepsilon\sin 2\pi(x-t)\right)$$ (3.6)

$$\frac{\partial^2 u}{\partial y^2} - \frac{1}{\alpha^2}\frac{\partial^4 u}{\partial y^4} - H_0^2 u = E_1\frac{\partial^3 \eta}{\partial x^3} + E_2\frac{\partial^3 \eta}{\partial x\partial t^2} + E_3\frac{\partial^2 \eta}{\partial x\partial t}$$

at $y = \pm\eta(x,t)$. (3.7)

Further

$$\theta = 0 \quad \text{on} \quad y = -\eta(x,t),$$ (3.8)

$$\theta = 1 \quad \text{on} \quad y = \eta(x,t).$$ (3.9)

The solution of Eqs (3.1) and (3.3), subject to the boundary conditions (3.5) – (3.7), is

$$u = \frac{E}{MH0^2}\left[\frac{m_1 m_2 - m2^2}{H0^2 \propto^2} + \frac{m_2}{m_1} - 1 - M - \frac{m_2\cosh\left(\sqrt{m_1}\,y\right)}{m_1 T_1} + \frac{\cosh\left(\sqrt{m_2}\,y\right)}{T_2}\right]$$ (3.10)

Solving Eq.(3.4), with boundary conditions (3.8) and (3.9), we get

$$\theta = P\left[\begin{array}{l} \dfrac{m_2^2}{4m_1 T_1^2}\left\{\dfrac{\cosh 2\sqrt{m_1}\,y}{2m_1} - \dfrac{M3}{2m_1} - y^2 + \eta^2\right\} + \\[2ex] + \dfrac{m_2}{4T_2^2}\left\{\dfrac{\cosh 2\sqrt{m_2}\,y}{2m_2} - \dfrac{M4}{2m_2} - y^2 + \eta^2\right\} + \\[2ex] - \dfrac{m_2^{\frac{3}{2}}}{T_1 T_2\sqrt{m_1}}\left\{\dfrac{\cosh M1 y}{M1^2} - \dfrac{\cosh M2 y}{M2^2} - \dfrac{M5}{M1^2} + \dfrac{M6}{M2^2}\right\} \end{array}\right] + \frac{y}{2\eta} + \frac{1}{2}.$$ (3.11)

Here

$$E = -\varepsilon\left[(2\pi)^3\cos 2\pi z(E_1 + E_2) - E_3(2\pi)^2\sin 2\pi z\right],$$

$$M = \frac{m_1 m_2 - m_2{}^2}{H0^2 \propto^2} + \frac{\sqrt{Da}}{\beta} m_2 \left(\frac{T_4}{\sqrt{m_2}} - \frac{T_3}{\sqrt{m_1}} \right),$$

$$H = \frac{1}{Da} + H_0{}^2, \qquad m_1 = \frac{\alpha^2 + \alpha\sqrt{\alpha^2 - 4H}}{2}, \qquad m_2 = \frac{\alpha^2 - \alpha\sqrt{\alpha^2 - 4H}}{2},$$

$$T_1 = \cosh\sqrt{m_1}\,\eta, \qquad T_2 = \cosh\sqrt{m_2}\,\eta, \qquad T_3 = Tanh\left(\sqrt{m_1}\eta\right),$$

$$T_4 = \tanh\left(\sqrt{m_2}\eta\right), \qquad M1 = \sqrt{m_1} + \sqrt{m_2}, \qquad M2 = \sqrt{m_1} - \sqrt{m_2},$$

$$M3 = \cosh 2\sqrt{m_1}\,\eta, \qquad M4 = \cosh 2\sqrt{m_2}\,\eta, \qquad M5 = \cosh M1\eta,$$

$$M6 = \cosh M2\eta, \qquad P = \frac{-Br\,E^2}{M^2 H0^4}, \quad \text{and} \quad Br = Ec\,p_r \quad \text{is the Brinkman number.}$$

The time average velocity $\bar{u}$ is

$$\bar{u} = \int\limits_0^1 u\,dt. \tag{3.12}$$

The heat transfer coefficient at the wall is

$$H = \eta_x \theta_y(\eta), \tag{3.13}$$

$$H = \left[2\pi\varepsilon\cos 2\pi(x - t) \right] \left[P \left\{ \frac{m_2{}^2}{4m_1 T_1{}^2} \left(\frac{\sinh 2\sqrt{m_1}\eta}{4m_1{}^{3/2}} - 2\eta \right) + \frac{m_2}{4T_2{}^2} \left(\frac{\sinh 2\sqrt{m_2}\eta}{4m_2{}^{3/2}} - 2\eta \right) + \right.$$

$$\left. - \frac{m_2{}^{\frac{3}{2}}}{T_1 T_2 \sqrt{m_1}} \left(\frac{\sinh M1\eta}{M1^3} - \frac{\sinh M2\eta}{M2^3} \right) \right\} + \frac{1}{2\eta} \right]. \tag{3.14}$$

4. Numerical results and discussion

The graphical results obtained from Eq.(3.11) are depicted in Figs 2-7 to show the effects of different parameters, i.e., the couple stress parameter α, slip parameter β, Hartmann number H_0, Darcy number Da, Brinkman number Br and the effect of elastic parameters on temperature θ and heat transfer H. From Fig.2, we observe that the temperature decreases as the couple stress parameter increases. Figure 3 reveals that with the gain in the slip parameter β there is a rise in the temperature. Looking at the effect of the Hartmann number H_0, from Fig.4, we a observe decrease in the temperature with an increase in H_0. Figure 5 shows an increase in the temperature with an increase in Da. With a rise in Br, the temperature also increases as shown in Fig.6. This result is in accordance with those obtained by Lakshminarayana *et al*. [20]. Figure 7 shows that by raising the elastic parameters, the temperature rises.
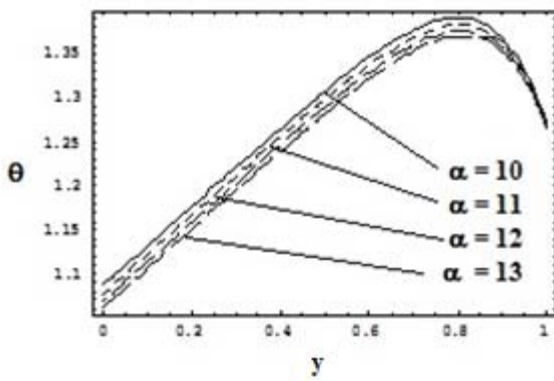
Fig..2. Plot of temperature.
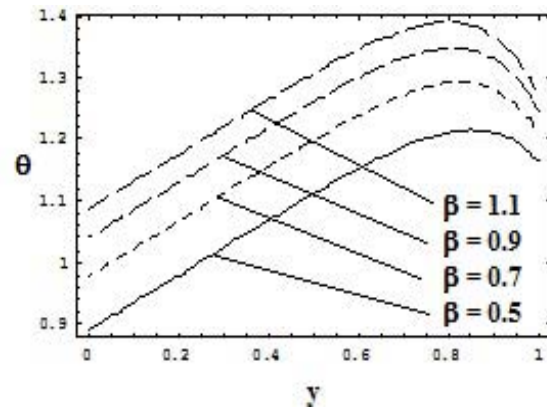$\varepsilon = 0.2$, E_1=0.1, E_2=0.5, E_3=0.3, $\beta = 1.1$, Da=0.1, H_0=2, Br=1, x=0.5, t=0.1

Fig.3. Plot of temperature.
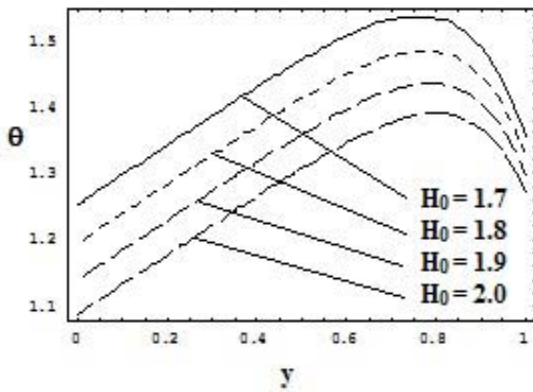$\varepsilon = 0.2$, E_1=0.1, E_2=0.5, E_3=0.3, $\alpha = 10$, Da=0.1, H_0=2, Br=1, x=0.5, t=0.1

Fig.4. Plot of temperature.
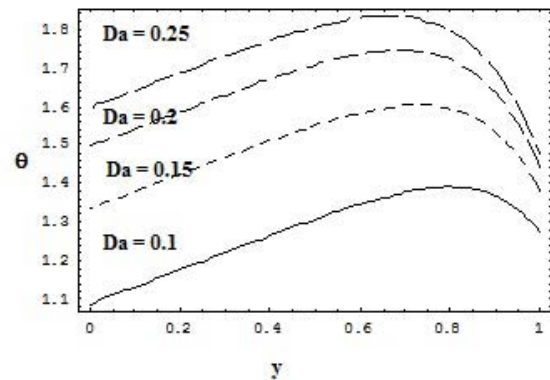$\varepsilon = 0.2$, E_1=0.1, E_2=0.5, E_3=0.3, $\alpha = 10$, $\beta = 1.1$, Da=0.1, Br=1, x=0.5, t=0.1

Fig.5. Plot of temperature.
$\varepsilon = 0.2$, E_1=0.1, E_2=0.5, E_3=0.3, $\alpha = 10$, H_0=2, $\beta = 1.1$, Br=1, x=0.5, t=0.1
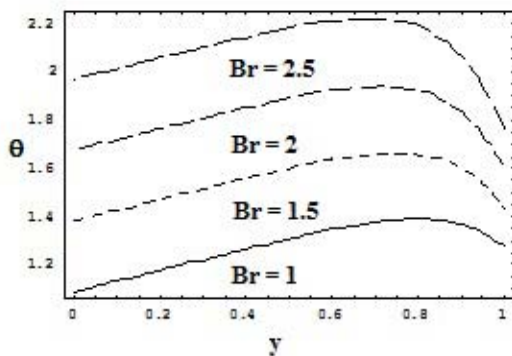
Fig.6. Plot of temperature.
$\varepsilon = 0.2$, E_1=0.1, E_2=0.5, E_3=0.3, $\alpha = 10$, $\beta = 1.1$, H_0=2, Da=0.1, x=0.5, t=0.1
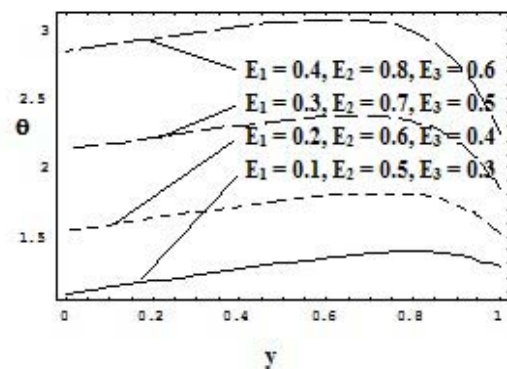

Fig.7. Plot of temperature.
$\varepsilon = 0.2$, $\alpha = 10$, $\beta = 1.1$, Da=0.1, H_0=2, x=0.5, t=0.1

A deviation of the heat transfer coefficient H at the channel wall is shown in Figs 8-13 for different values of α, β, H_0, Da, Br, E_1, E_2 and E_3 with preset values of the remaining parameters. These graphs

present the absolute value of the coefficient of heat transfer. The absolute value of the coefficient of heat transfer H reduces by enhancing H_0 and Br as depicted in Figs 8 and 9, whereas the magnitude of H is increased by increasing the Darcy number Da as shown in Fig.10. The magnitude of the coefficient of heat transfer H decreases by increasing elastic parameters E_1, E_2 and E_3 as shown in Fig.11. The effect of change in α and β on H is presented in Figs 12 and 13, but there is no appreciable difference.
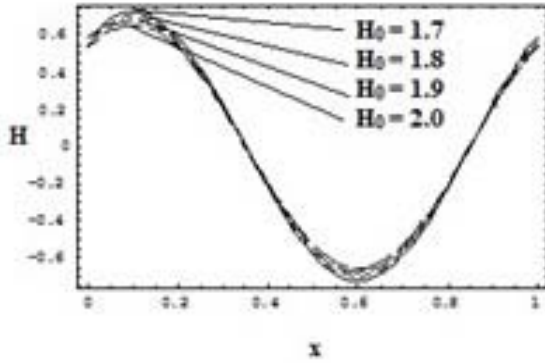

Fig.8. Plot of heat transfer coefficient.
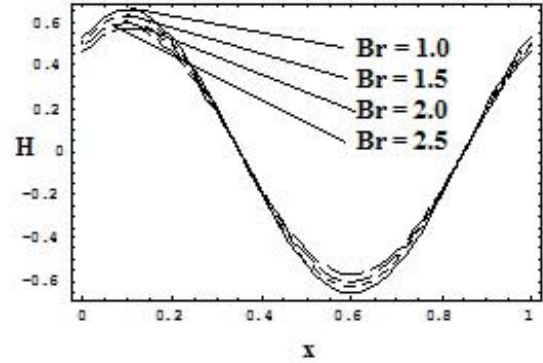$\varepsilon = 0.2$, $E_1=0.1$, $E_2=0.5$, $E_3=0.3$, $\alpha = 10$, $\beta = 1.1$, Da=0.1, Br=1, x=0.5, t=0.1

Fig.9. Plot of heat transfer coefficient.
$\varepsilon = 0.2$, $E_1=0.1$, $E_2=0.5$, $E_3=0.3$, $\alpha = 10$, $\beta = 1.1$, $H_0=2$, Da=0.1, x=0.5, t=0.1
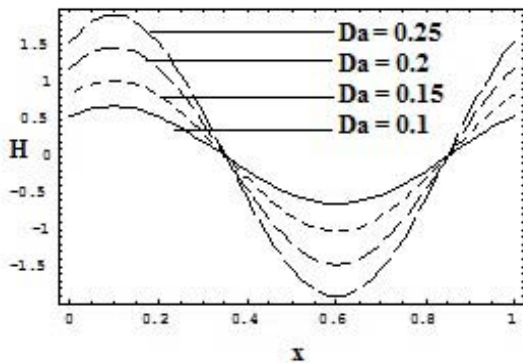

Fig.10. Plot of heat transfer coefficient.
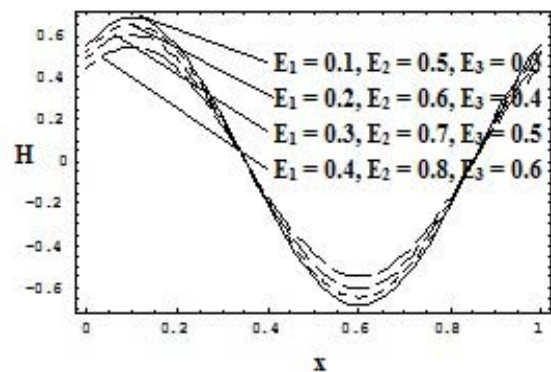$\varepsilon = 0.2$, $E_1=0.1$, $E_2=0.5$, $E_3=0.3$, $\alpha = 10$, $H_0=2$, $\beta = 1.1$, Br=1, x=0.5, t=0.1

Fig.11. Plot of heat transfer coefficient.
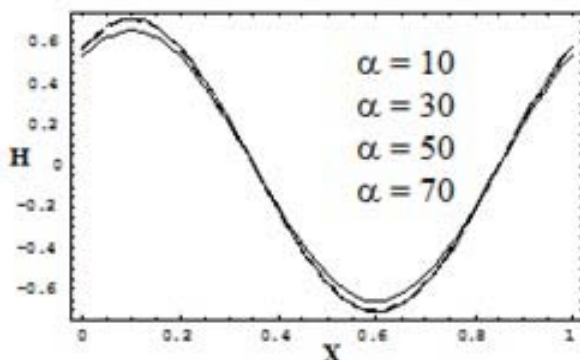$\varepsilon = 0.2$, $\alpha = 10$, $H_0=2$, $\beta = 1.1$, Da=0.1, x=0.5, t=0.1

Fig.12. Plot of heat transfer coefficient.
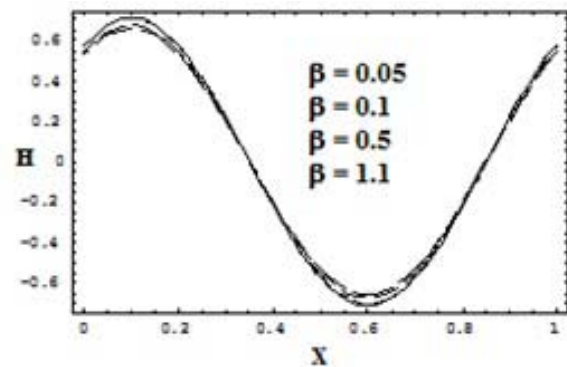$\varepsilon = 0.2$, $E_1=0.1$, $E_2=0.5$, $E_3=0.3$, $\beta = 1.1$, Da=0.1, $H_0=2$, Br=1, x=0.5, t=0.1

Fig.13. Plot of heat transfer coefficient.
$\varepsilon = 0.2$, $E_1=0.1$, $E_2=0.5$, $E_3=0.3$, $\alpha = 10$, Da=0.1, $H_0=2$, Br=1, x=0.5, t=0.1

Conclusion

The motion of a magnetohydrodynamic couple stress fluid in a uniform channel with a porous medium under peristalsis is examined to study the heat transfer and slip effects. A theoretical solution is obtained using long wavelength and small Reynolds number assumptions. The consequences are analyzed for a variety of values of the parameters, namely; the couple stress parameter, Hartmann number, slip parameter, Brinkman number Darcy number and elastic parameters. The following interesting points have been noticed.

- Temperature increases with the increasing slip parameter, Darcy number, Brinkman number and also the elastic parameters.
- Temperature decreases with the increasing couple stress parameter and Hartmann number.
- The choice of the couple stress parameter depends on the Hartmann number while modelling peristaltic transport of physiological fluids together with the application of a magnetic field.
- In the upper part of the absolute value, the heat transfer coefficient increases with an increase in the Darcy number.

Our study on heat transfer is carried out having in view various applications, such as: destruction of cancer tissues, accessing skin burns, dilution method in investigating blood flow, food processing, etc.

Nomenclature

a – amplitude of the wave
Br – Brinkman number
B_0 – magnetic field
C – coefficient of the viscous damping force
c – wave speed
Da – Darcy number
d – half width of the channel
Ec – Eckert number
E_1 – rigidity in the wall
E_2 – stiffness in the wall
E_3 – viscous damping force in the wall
G – specific heat at constant pressure
H_0 – magnetic parameter (Hartmann number)
h – constant associated with couple stress
K – thermal conductivity of the fluid
k – permeability of the porous medium
L – expression defined in Eq.(2.3)
m – mass per unit area
Pr – Prandtl number
p – pressure
p_0 – pressure applied on the outside surface of the wall because of tension in the muscles
Re – Reynolds number
T – temperature of the fluid
t – time
u – axial velocity in the wave frame
$\bar{u}$ – time dependent mean velocity
x – axial coordinate in the wave frame
y – transverse coordinate in the wave frame
α – couple stress parameter
β – slip parameter
δ – wall slope parameter
$\in$ – amplitude ratio

η   – geometry of the wall defined in (2.1)
λ   – wavelength
μ   – coefficient of viscosity of the fluid
ν – transverse velocity in the wave frame
ρ   – density
σ   – electrical conductivity of the fluid
τ   – elastic tension in the wall
ϑ   – kinematic coefficient

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