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Transport processes and the stripping of cluster galaxies

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Summary. The effects of viscosity, thermal conduction and turbulence on the flow of hot gas past a galaxy are considered. Transport processes are found to cause stripping of gas from galaxies at a rate which can often exceed that due to ram pressure alone. The results are applied to M86 in the Virgo cluster and UGC 6697 in A1367. The X-ray image of M86 suggests that ion mean free paths in the Virgo intergalactic medium are much smaller than their magnetic field free values, while turbulent viscous stripping provides a natural explanation for the appearance of UGC 6697.

1 Introduction

One consequence of the presence of hot gas in rich clusters (Mushotzky *et al.* 1978; Mitchell *et al.* 1979; Jones *et al.* 1979) is in stripping the interstellar medium from the constituent galaxies. Ram pressure of the hot gas can be sufficient to push interstellar medium bodily from a galaxy, and Gunn & Gott (1972) proposed that this may convert spiral galaxies into S0s by removing all of their gas (but see Gisler 1980). Gisler (1976) and Lea & De Young (1976) have also invoked ram pressure stripping to explain the absence of interstellar medium from many elliptical galaxies.

Ram pressure is not, however, the only means by which the intergalactic gas can strip galaxies. Cowie & Songaila (1977) have pointed out that, because of the long Coulomb mean free paths in the intracluster medium, thermal evaporation of the interstellar medium could be the dominant cause of mass loss. Livio, Regev & Shaviv (1980) have considered mass loss caused by Kelvin–Helmholtz instability on the boundary between the intracluster and interstellar gas. Although their reasoning is flawed, Kelvin–Helmholtz instability can lead to substantial stripping rates under some circumstances. In the present paper we will consider mass loss from cluster galaxies under a wide variety of circumstances caused by transport phenomena. We will find that the rate of stripping of a galaxy moving through a hot intracluster medium is substantial and often exceeds that due to ram pressure alone.

The nature of the gas flow in the vicinity of a galaxy is determined by the effective mean free paths in the intergalactic gas. We will divide our considerations roughly accordingly. In Section 2 we consider a laminar viscous flow and estimate the mass loss caused by the viscosity. In Section 3 we consider the effect of thermal conduction and show that evaporation will be the dominant mass loss mechanism when mean free paths are long. The effect of Kelvin–Helmholtz instability is considered in Section 4 and shown to be most important when mean free paths are small. Finally, in Section 5, we apply our results to M86 and UGC 6697. Details of the Kelvin–Helmholtz instability are discussed in the Appendix.

2 Laminar viscous stripping

Throughout this paper we will denote quantities measured in the hot intergalactic gas by a subscript 'h' and the same quantities measured in the interstellar medium with a subscript 'c'. We wish to consider the flow of intergalactic gas of density ρ_h and temperature T_h past the interstellar medium, of radius r, within a galaxy moving at velocity v. The temperature of the hot gas can be related to the velocity dispersion of the cluster containing it using the parameter τ by

$$kT_h = \mu m_H \sigma^2/\tau, \tag{1}$$

where μm_H is the mean mass per particle and σ is the line of sight velocity dispersion of the cluster. Since τ is of order unity (Cavaliere & Fusco-Femiano 1976; Nulsen & Fabian 1980) equation (1) shows that the motion of a galaxy through the hot gas is typically transonic.

The temperature, T_c, of the gas trapped within a galaxy can be related to the galaxy's velocity dispersion in a manner similar to equation (1). In a disc galaxy, however, rotationally supported gas can have a temperature much less than given by the analogue of (1) with $\tau \cong 1$. Since cluster velocity dispersions are typically 3–10 times those of galaxies (Faber & Jackson 1976; Dressler 1978) it follows that

$$T_h \gtrsim 10\text{–}100\, T_c. \tag{2}$$

This has two immediate consequences. Firstly, the gravitational potential of a galaxy will normally have a negligible effect on the hot gas. Secondly, continuity of pressure between the hot and cold gas means that the interstellar gas will be considerably denser than the intergalactic gas. Thus, to a first approximation, the flow of the hot gas is like that around a rigid body. Treating the interstellar medium as spherical, the viscous drag on it is then roughly (Batchelor 1967)

$$F_D = 6\pi\eta_h r v, \tag{3}$$

where η_h is the viscosity. Recalling that (Spitzer 1956)

$$\eta_h \cong 0.7\, \rho_h c_h \lambda_h,$$

where c_h is the isothermal sound speed,

$$c_h^2 = kT_h/\mu m_H, \tag{4}$$

and λ_h is the effective mean free path of the ions in the hot gas, the drag can be expressed

$$F_D \cong \pi r^2 \rho_h v^2\ 12/\mathrm{Re}, \tag{5}$$

where the Reynolds number is (Batchelor 1967)

$$\mathrm{Re} = 2.8\, (r/\lambda_h)\, (v/c_h). \tag{6}$$

The structure of the flow in the interface between the hot and cold gas is determined by the viscous diffusion of momentum. Continuity of the stress where the two fluids are in contact requires that u, the velocity there, satisfy

$$\rho_h \eta_h (v-u)^3 = \rho_c \eta_c u^3.$$

Applying continuity of the pressure and assuming the mean free paths are determined by Coulomb collisions, this gives

$$(v-u)/u=\sqrt{T_c/T_h}. \qquad (7)$$

Since $T_c \ll T_h$, gas will be stripped from the galaxy with velocities of order v. The drag (5) represents the momentum input to the surface layers of the interstellar medium so that gas is stripped at a rate

$$\dot{M}_{vis} \cong F_D/v=\pi r^2 \rho_h v(12/\mathrm{Re}). \qquad (8)$$

In the presence of effects which reduce the effective mean free path, equation (7) is replaced by

$$(v-u)/u=(T_h/T_c)^{1/2}(\lambda_c/\lambda_h)^{1/3}$$

so that u may be much less than v and the stripping rate may exceed that given in (8). In that case, however, the mean free path will usually be so small that the flow is no longer laminar (see below). The galaxy's gravity will only affect the stripping flow significantly if the restoring force on the gas leaving the galaxy is comparable to or greater than F_D, i.e. if

$$F_D \lesssim GM/r^2 \cdot (\dot{M}r/v)$$

or

$$GM/r \gtrsim v^2, \qquad (9)$$

where M is the mass of the galaxy inside r. Since $v^2 \sim \sigma^2$, condition (9) will only be satisfied for slowly moving galaxies. Even when (9) is satisfied we would expect some mass loss from the surface layers of the interstellar medium, the rate depending in detail on the viscosity and form of the galaxy's potential.

The reasoning giving the mass loss rate (8) requires: (a) that the flow be laminar; and (b) that the mean free path is sufficiently small to use the classical treatment of viscosity. Assumption (a) breaks down at high Reynolds number, i.e. $\mathrm{Re} \gtrsim 30$ (Batchelor 1967), while assumption (b) is clearly violated when $\lambda_h \gtrsim r$. Since $v \sim c_h$, we see from the definition (6) of the Reynolds number that the domain in which the laminar viscous stripping rate (8) applies is small. We now consider what happens for the remaining cases.

3 Thermal conduction

Cowie & Songaila (1977) have shown the importance of thermal evaporation in stripping cluster galaxies. Their result for the rate of evaporation from a spherical galaxy is (see Cowie & McKee 1977)

$$\dot{M}_{ev}=\pi r^2 \rho_h c_h \, 4\phi_s F(\sigma_0), \qquad (10)$$

where $\phi_s \cong 1$,

$$\sigma_0=1.84\,\lambda_h/r\phi_s, \qquad (11)$$

and the function $F(\sigma_0)=2\sigma_0$ for $\sigma_0 \lesssim 1$, increasing more slowly for $\sigma_0 \gtrsim 1$. The saturation parameter, σ_0, determines whether thermal conduction can be treated classically or is saturated. In the classical domain, $\sigma_0 \lesssim 1$ or $r \gtrsim \lambda_h$, the ratio of the evaporation rate to the viscous stripping rate (equation 8) is

$$\dot{M}_{ev}/\dot{M}_{vis} \cong 3.5, \qquad (12)$$

so that evaporation always causes the greater mass loss. It is clear that, while viscous stripping is less important than evaporation, the two processes are inseparable in a complete treatment of stripping. When $\sigma_0 \gtrsim 1$ both transport processes will saturate and the viscous mass loss rate is no longer given by equation (8). It is likely that thermal evaporation will remain dominant, so that the mass loss rate is given well enough by equation (10) for saturated conduction.

From equations (8) and (12) we see that in the classical domain the total rate of mass loss will certainly exceed

$$\dot{M}_{\rm Fid} = \pi r^2 \rho_{\rm h} v \tag{13}$$

for Re $\lesssim 30$. Similarly, in the saturated domain $\dot{M}_{\rm ev}$ exceeds $\dot{M}_{\rm Fid}$ unless $v \gg c_{\rm h} \cdot (\dot{M}_{\rm ev} \cong 8 \pi r^2 \rho_{\rm h} c_{\rm h}$ at $\sigma_0 = 1$ and F is an increasing function of σ_0). However, when $\sigma_0 > 1$ particles from the hot gas penetrate the interstellar medium and deposit considerable energy there. Thus, if $v \gg c_{\rm h}$, the kinetic energy flux in the hot gas is a more effective heat source than the thermal flux. This situation is unlikely to occur and we will not pursue it further, except to note that the evaporation rate can then exceed that given by (10).

The galaxy's gravity only affects the evaporation rate (10) significantly when the specific binding energy of the cold gas is comparable to or exceeds the thermal energy of the hot gas, i.e.

$$c_{\rm h}^2 \lesssim GM/r$$

(cf. equation 9). In that case the heat flow into the interstellar medium will not be much affected, but will be consumed mainly in unbinding the cold gas from the galaxy, rather than in heating it. $\dot{M}_{\rm ev}$ will thus be reduced by a factor of $\sim c_{\rm h}^2/(GM/r)$.

In the absence of magnetic fields the ion mean free path in the intracluster medium is generally large (Spitzer 1956):

$$\lambda_{\rm h} \simeq 11 T_8^2 n_{-3}^{-1} \,{\rm kpc}, \tag{14}$$

where $T_{\rm h} = 10^8 T_8$K and $\rho_{\rm h} = 10^{-3} m_{\rm H} n_{-3} \,{\rm cm}^{-3}$. However, the effective mean free path can be reduced dramatically by the presence of even very weak tangled magnetic fields. The Reynolds number (6) could then be very large indeed and we next pursue that case.

4 The domain Re $\gtrsim 30$, turbulent viscous stripping

When $\lambda_{\rm h} \lesssim r$, if the flow velocity exceeds the adiabatic sound speed in the hot gas a shock will form ahead of the interstellar medium. Since $v \sim \sigma$, the shock will generally be weak, but in any case the post-shock flow will always be subsonic in the vicinity of the interstellar medium. In the current section we will thus assume

$$v < s_{\rm h} = \sqrt{\gamma} c_{\rm h}, \tag{15}$$

where $\gamma = 5/3$ is the adiabatic index, so that the condition Re $\gtrsim 30$ means $\lambda_{\rm h} \ll r$ (equation 6).

Livio *et al.* (1980) attempted to calculate the rate of mass loss due to Kelvin–Helmholtz instability. However, they overestimated the true rate by neglecting the effect of the mass loss on the flow. (I note that equation (12) of Livio *et al.* 1980 also appears to be algebraically in error.)

Since, as was shown in Section 2, the galaxies generally move at velocities which are transonic in the hot intergalactic medium and supersonic relative to the cooler interstellar medium, we cannot ignore the compressibility of the gas in deriving growth rates for the Kelvin–Helmholtz instability. There is a brief discussion of Kelvin–Helmholtz instability in the Appendix.

Taking the fluid interface to lie in the x–y-plane and assuming that one fluid moves over the other at a velocity v in the x-direction, unstable oscillation of wavenumber k_x, k_y in the interface grow exponentially on a time-scale (see Appendix)

$$\tau_{\mathrm{KH}} \cong \frac{1}{ks_{\mathrm{c}}} \sqrt{\frac{1-(v^2/s_{\mathrm{h}}^2)\cos^2\theta}{(v^2/s_{\mathrm{h}}^2)\cos^2\theta}}, \tag{17}$$

where $k^2 = k_x^2 + k_y^2$, $k\cos\theta = k_x$ and $s_{\mathrm{h}} \gg s_{\mathrm{c}}$ is assumed. At the stage when the instability becomes non-linear, the velocity of the boundary between the fluids is

$$\delta v \cong \sqrt{1-(v^2/s_{\mathrm{h}}^2)\cos^2\theta}\; k^{-1}\, \tau_{\mathrm{KH}}^{-1} = s_{\mathrm{c}}\,(v/s_{\mathrm{h}})\cos\theta. \tag{18}$$

If the Kelvin–Helmholtz instability were to proceed unchecked, we would thus expect a mass loss from the interstellar medium of

$$\dot{M} \cong \pi r^2 \rho_{\mathrm{c}} \cdot s_{\mathrm{c}} \cdot (v/s_{\mathrm{h}}). \tag{19}$$

Recalling that the pressure is continuous between the hot and cold fluids equation (19) is what would be obtained from the reasoning of Livio *et al.* (1980), apart from an efficiency factor ~ 0.1 (see Arons & Lea 1980 for details). The reasoning giving equation (19) fails, however, because we cannot ignore the effect of the stripped gas on the flow past the galaxy. The total momentum flux past the galaxy is

$$F_{\mathrm{D}}' \cong \pi r^2 \rho_{\mathrm{h}} v^2, \tag{20}$$

while the momentum flux in the stripped gas corresponding to (19) is

$$\dot{M}v \cong r^2 \rho_{\mathrm{c}} v^2 (s_{\mathrm{c}}/s_{\mathrm{h}}),$$

so that their ratio is

$$F_{\mathrm{D}}'/\dot{M}v \cong \sqrt{T_{\mathrm{c}}/T_{\mathrm{h}}} \tag{21}$$

(where continuity of the pressure has been used). It is clear that, since T_{c} is usually considerably less than T_{h}, mass loss at a rate (19) would completely disrupt the flow near the galaxy, and the momentum flux (20) is usually inadequate to support such a high mass loss.

The above argument shows that Kelvin–Helmholtz instability is capable of introducing more than enough galactic gas into the boundary layer to smooth the velocity profile over $\sim r$. This it will do, quenching itself in the process. Stripping will proceed from the front of the galaxy smoothing the velocity gradient in the interface between the interstellar and intergalactic gas. This will suppress unstable modes with wavelengths smaller than about the thickness of the interface. Longer wavelength modes will continue to be able to grow causing further stripping and thickening of the interface. The process will terminate when the interface thickens to $\sim r$ and no unstable modes remain. The mass loss required to thicken the interface to this size is

$$\dot{M}_{\mathrm{tur}} \cong F_{\mathrm{D}}'/v = \pi r^2 \rho_{\mathrm{h}} v, \tag{22}$$

which is generally less than given by (19). It is apparent that stripping will occur preferentially from near the front of the galaxy, ahead of the place where the interface thickens to $\sim r$.

The above discussion ignored the effect of viscosity, which is to stabilize modes of wavelengths $\lesssim r/\mathrm{Re}$ (see, e.g. Betchov & Criminale 1967). The galaxy's gravity has also been neglected. Its influence is measured by the Richardson number (Tritton 1977).

$$\mathrm{Ri} = \frac{\text{buoyancy forces}}{\text{inertia forces}} \simeq \frac{2GM}{kv^2r^2} \tag{23}$$

for a mode of wavenumber k (assuming $\delta\rho/\rho \cong 1$). Provided that $\mathrm{Ri} \ll 1$, gravity can be ignored, so that long wavelength modes are most affected by gravity. We can estimate the effect of gravity upon the stripping rate by noting that, when

$$GM/r \gtrsim v^2,$$

the Richardson number (equation 23) $\cong 1$ for a mode of wavelength

$$l \cong rv^2/(GM/r).$$

Longer wavelength modes will be stable, so that the stripping process will saturate when the interface thickens to $\sim l$. The mass loss is then reduced by a factor $\sim v^2/(GM/r)$.

The stripping process outlined in this section is driven by the convection of momentum from the hot gas flow into the surface layers of the interstellar medium. This process is most effective at high Reynolds number, when the viscosity is negligible and the interface between the hot gas and the interstellar medium will become turbulent. We will thus refer to it as turbulent viscous stripping.

5 Discussion

We have considered several (effective) transport processes which cause stripping of galaxies moving through a hot intergalactic medium. These processes are supplementary to ram pressure stripping (Gunn & Gott 1972) and more or less independent of it. Recalling that $v \sim \sigma$, regardless of the particle mean free paths, in a cluster with velocity dispersion σ transport processes typically cause a galaxy to lose mass at a rate

$$\dot{M}_{\mathrm{typ}} = \pi r^2 \rho_{\mathrm{h}} \sigma, \tag{24}$$

reduced by a factor $\sim \sigma^2 r/GM$, if this quantity is less than ~ 1. The domains in which the various transport stripping processes apply are summarized in Fig. 1.

It was shown above that, for all but the most massive and/or slow moving galaxies, the stripping rate exceeds

$$\dot{M}_{\mathrm{Fid}} = \pi r^2 \rho_{\mathrm{h}} v,$$

so that the integrated mass loss will usually exceed the total mass of hot gas that a galaxy has passed through. By comparison, ram pressure will only strip galaxies when it can overcome gravity and push the interstellar medium bodily from them. In most rich clusters ram pressure stripping is very effective near the core, but causes negligible mass loss in the lower density regions making up the bulk of the cluster (Gisler 1976; Lea & De Young 1976). Ram pressure stripping of a spiral galaxy is also sensitive to its orientation. In the most extreme case, of a spiral galaxy moving edge on through the intracluster medium, ram pressure stripping is negligible, while any of the stripping processes discussed above will proceed at very nearly the rates given (with r being the radius of the gas disc).

We now consider the edge-on spiral galaxy UGC 6697 in A1367. Sullivan *et al.* (1980) have drawn attention to UGC 6697 as a possible example of a galaxy in the process of being stripped. UGC 6697 lies close to the edge of the X-ray emitting regions of A1367 (Jones *et al.* 1979) about 3 arcmin NW of NGC 3842. The galaxy is asymmetric, showing a lack of H II regions and dust in the SE of the disc with no corresponding lack in the NW. On the red Palomar print the disc of the galaxy appears to extend in the SE direction ~ 0.6 arcmin beyond the region containing bright H II regions. The integrated H I profile of UGC 6697 also lacks the 'horn' which would normally correspond to the SE side of the disc (Sullivan

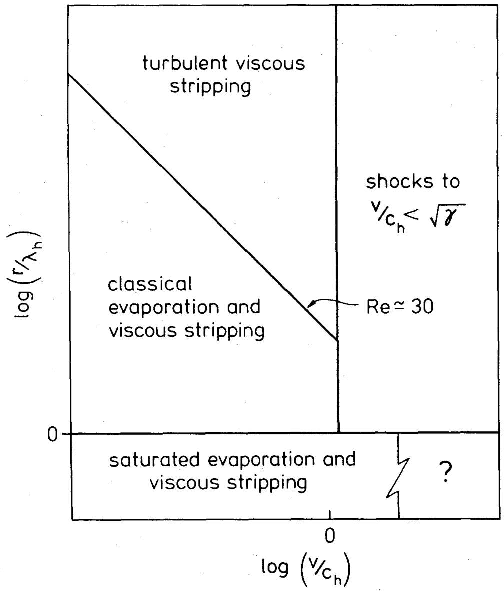

Figure 1. Transport processes causing stripping. The nature of the flow of a hot gas past the interstellar medium, of radius r, within a galaxy is determined by the mach number v/c_h in the hot gas and the ratio of r to the effective mean free path in that gas. The most important transport processes causing stripping are indicated for the cases in which they apply. Turbulent viscous stripping is discussed in Section 4, classical and saturated evaporation in Section 3 and classical viscous stripping in Section 2. Provided that $r \gtrsim \lambda_h$ when the hot gas flow is supersonic a shock will form ahead of the interstellar medium preventing v exceeding $\sqrt{\gamma}\, c_h$ near to it. When $r \lesssim \lambda_h$ transport processes saturate and can no longer be treated in the classical way. Cowie & McKee (1977) have considered saturated thermal conduction, which will be the dominant cause of stripping in subsonic saturated flows. We have not attempted to calculate what happens in supersonic saturated flows.

et al. 1980). Using the total H I mass of Sullivan *et al.* (1980), this suggests that $\sim 10^9 M_\odot$ of gas have been stripped from UGC 6697. The rate of turbulent viscous stripping is

$$\dot{M}_{tur} \cong 6 r_{30}^2 n_{-4} v_3 M_\odot \,\mathrm{yr}^{-1}, \tag{25}$$

where $\rho_h = 10^{-4} n_{-4} m_H \,\mathrm{g\,cm^{-3}}$, $r = 30\, r_{30}$ kpc and $v = 1000\, v_3 \,\mathrm{km\,s^{-1}}$, so that near the centre of A1367 $\sim 10^9 M_\odot$ of gas could easily be stripped from UGC 6697 in $< 2 \times 10^8$ yr. In this time the galaxy would have moved ~ 200 kpc and undergone $\sim 1/3$ of a rotation. Turbulent viscous stripping removes gas from the front edge of the galaxy, so that we would expect to see UGC 6697 as it now appears if it is moving nearly edge-on towards the SE through A1367. Neither evaporation nor ram pressure stripping easily produce such a large asymmetry in the rate of stripping of the disc.

It is interesting too to apply the discussion of Kelvin–Helmholtz instability to M86 in the Virgo cluster. Fabian, Schwarz & Forman (1980) have suggested that the X-ray emission from around M86 is due to weakly bound gas that has been swept from the galaxy by ram pressure. Although it is an example of ram pressure stripping, Kelvin–Helmholtz instability can explain the X-ray morphology of this source. Since the gas was weakly bound to M86, we treat it as a uniform sphere of density ρ_c and temperature T_c. M86 is moving through the Virgo cluster at a velocity of at least $\sim 1500 \,\mathrm{km\,s^{-1}}$ and since this is twice the cluster velocity dispersion, the ram pressure of the Virgo intergalactic medium on M86 will be considerably greater than its thermal pressure (see equation 1). (This also means that Fabian *et al.* have overestimated the density of the intergalactic medium required to explain the X-ray source.) In that case

$$\rho_c k T_c / \mu m_H \cong \rho_h v^2 \tag{26}$$

and the ram pressure of the intergalactic medium accelerates the gas cloud away from M86, displacing it by

$$\Delta x \cong \frac{1}{2}\frac{\pi r^2 \rho_h v^2}{(4/3)\pi r^3 \rho_c} t^2 = (3/8)\, c_c^2\, t^2/r \tag{27}$$

in a time t. The time taken to displace the cloud a distance of $\sim r$ from M86 is thus

$$t_{sep} \cong \sqrt{8/3}\, r/c_c, \tag{28}$$

which is the same as the growth time for the longest wavelength ($\sim r$) Kelvin–Helmholtz unstable modes (equation 17). Such modes can completely disrupt the cloud of cool gas, resulting in a fragmented cloud lying close behind the galaxy. This is consistent with the X-ray morphology seen by Forman *et al.* (1979). Because of the large density contrast between the M86 cloud and its surroundings, very little gas will have been stripped from the cloud at this stage and it will still be moving at very nearly the velocity of M86. I note that the disrupted appearance of the M86 cloud lends weight to the suggestion that the effective ion mean free path is considerably less than given in equation (14). If the mean free path were that large the gas flow near M86 should be laminar (equation 6) and we would expect to see a smooth gas distribution.

Finally, I note that the stripping processes discussed here also tend to disrupt free clouds moving through a hot and tenuous confining medium. Thus transport processes other than conduction could have a significant effect on the development of a multiphase interstellar medium.

6 Conclusions

Transport process in the hot intracluster medium cause substantial stripping of gas from cluster galaxies. The associated stripping rates often exceed those due to ram pressure alone. An interesting feature of the transport processes is that typical stripping rates are insensitive to inaccessible details of effective particle mean free paths.

Application of the results discussed here suggest that, at least in some cases, ion mean free paths are much less than their magnetic field free values. In that case, the process of turbulent viscous stripping (section 4) will be an important mechanism for gas loss from cluster galaxies.

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Appendix: the Kelvin–Helmholtz instability

The review by Gerwin (1968) contains a general discussion of the Kelvin–Helmholtz instability. The central results used here can be found in Miles (1958) and Fejer & Miles (1963).

We consider displacements of the form

$$h(x, y, t) = \xi \exp[-i(\omega t - k_x x - k_y y)], \tag{A1}$$

k_x and k_y real, in the boundary between a uniform 'hot' fluid (subscript h) moving with velocity v_0 in the x-direction over a uniform 'cold' fluid (subscript c) at rest in $z < 0$. The dispersion relation is

$$\frac{\sqrt{(\omega - k_x v_0)^2/k^2 s_\mathrm{h}^2 - 1}}{(\omega - k_x v_0)^2/k^2 s_\mathrm{h}^2} = -\frac{\sqrt{\omega^2/k^2 s_\mathrm{c}^2 - 1}}{\omega^2/k^2 s_\mathrm{c}^2}, \tag{A2}$$

where $k^2 = k_x^2 + k_y^2$, s is the adiabatic sound speed and $\sqrt{\ }$ is defined to have non-negative imaginary part. Equation (A2) has the formal solution

$$(\omega - k_x v_0)^2/k^2 s_\mathrm{h}^2 = \omega^2/(\omega^2 - k^2 s_\mathrm{c}^2), \tag{A3}$$

provided that $\omega^2/k^2 s_c^2$ is not real and $\geqslant 1$. This is a quartic for ω which always has two real solutions. The two remaining solutions are a complex conjugate pair if

$$0 < |k_x v_0/k| < (s_\mathrm{h}^{2/3} + s_\mathrm{c}^{2/3})^{3/2}, \tag{A4}$$

in which case one will correspond to a growing mode.

When $s_\mathrm{c} \ll s_\mathrm{h}$, provided that

$$(1 - k_x v_0/k s_\mathrm{h}) \gg (s_\mathrm{c}/s_\mathrm{h})^{2/3},$$

the dispersion relation has a fairly simple approximate solution. In that case the growth time of the instability is

$$\tau_\mathrm{KH} = [\mathrm{Im}(\omega)]^{-1}$$

$$\cong \frac{s_\mathrm{h}}{v_0} \frac{1}{k_x s_\mathrm{c}} \sqrt{1 - \left(\frac{k_x v_0}{k s_\mathrm{h}}\right)^2} \tag{A5}$$

and the phase speed of the wave is

$$\mathrm{Re}(\omega)/k \simeq v_0(k_x/k)\,(s_c/s_h)^2\,[1-(k_x v_0/k s_h)^2]^{-2}. \tag{A6}$$

These modes are essentially stationary in the cool gas as they grow ($\mathrm{Im}(\omega) \gg \mathrm{Re}(\omega)$), so that the finite size of the gas cloud only affects modes with wavelengths comparable to the cloud size or greater. When a growing mode becomes non-linear the velocity of the boundary is roughly

$$\delta v \simeq s_c(k_x v_0/k s_h). \tag{A7}$$