# Transportation Modeling: An Artificial Life Approach 

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#### Abstract

Artificial Life (ALife) uses biological knowledge and techniques to help solve different engineering, management, control and computational problems. Natural systems teach us that very simple individual organisms can form systems capable of performing highly complex tasks by dynamically interacting with each other. The main goal of this paper is to show how we can use ALife concepts (inspired by some principles of natural swarm intelligence) when solving complex problems in traffic and transportation. The Bee System that represents the new approach in the field of Swarm Intelligence is described. It is also shown in the paper that ALife approach can be successful to "attack" transportation problems characterized by uncertainty. The Fuzzy Ant System (FAS) described in the paper represents an attempt to handle the uncertainty that sometimes exists in some complex transportation problems. The potential applications of the Bee System and the Fuzzy Ant System in the field of Traffic and Transportation Engineering are discussed.


## 1. Introduction

Researchers in many scientific disciplines have started to develop various Artificial Life (ALife) models during the last decade. On one hand, ALife uses computational techniques to help study various biological phenomena. On the other hand, ALife uses biological knowledge and techniques to help solve different engineering, management, control and computational problems. Within the ALife scientific arena, researchers are developing artificial system that posses some of the basic characteristics of life. The entities in some artificial systems are virtual creatures that breed, learn, think, fight, collaborate, age, and die. Often, when making different ALife models, researchers try to explain complex system behavior that is derived from relatively simple rules.

The first ideas of making the analogy with Darwin's theory of evolution and the basic principle of the "survival of the fittest" when creating artificial systems were introduced during the last four decades
(evolutionary programming, evolution strategies, genetic algorithms). Scientists in areas like engineering, management, or control are copying or adapting systems from biology when facing complex problems. The development of Artificial Systems does not entail the complete imitation of natural systems, but explores them in the search for ideas and models.

The main goal of the paper is to show how we can use ALife concepts (inspired by some principles of natural swarm intelligence) when solving complex problems in traffic and transportation. The organization of the rest of the paper is described below. Solving complex transportation engineering problems by ALife techniques is discussed in Section 2. The Bee System as the new computational paradigm is introduced in Section 3. Section 4 is devoted to the Fuzzy Ant System that has been proposed to "attack" some transportation problems characterized by uncertainty. Conclusions and recommendations for further research are given in section 5.

## 2. Solving complex transportation engineering problems by artificial life techniques

Many practical real-world problems were formulated and solved using mathematical programming techniques during the last four decades. It is important to note, however, that the majority of real-world problems solved by some of the optimization techniques were of small dimensionality. Many traffic and transportation engineering problems are combinatorial by nature. Most of the combinatorial optimization problems are difficult to solve either because of the large dimensionality or because it is very difficult to decompose them into smaller sub-problems. These are most commonly found to be NP-complete problems that cannot be solved exactly in polynomial time. Typical representatives of this type of problems are the vehicle fleet planning and static and dynamic routing and scheduling of vehicles and crews for airlines, railroads, truck operations and public transportation services, designing transportation networks
and optimizing alignments for highways and public transportation routes through complex geographic spaces, different locations problems, etc.

There are few models in the literature based on ALife concepts that try to solve certain transportation problems. The ALife models that have been developed for solving complex transportation problems are inspired by social insects behavior. Interaction between individual insects in a colony of social insects has been well documented. The examples of such interactive behavior are bee dancing during the food procurement, ants' pheromone secretion, and performance of specific acts which signal the other insects to start performing the same actions. These communication systems between individual insects contribute to the formation of the "collective intelligence" of the social insect colonies. Recently, the term "Swarm Intelligence", denoting this "collective intelligence" has come into use [3, 4, 5, 6].

The self-organization of the ants is based on relatively simple rules of individual insect's behavior [13, 20, 2, 28, 36]. The ants successful at finding food leave behind them a pheromone trail that other ants follow in order to reach the food. The appearance of the new ants at the pheromone trail reinforces the pheromone signal. This comprises typical autocatalytic behavior, i.e., the process that reinforces itself and thus converges fast. The "explosion" in such processes is regulated by a certain limitation mechanism. In the ant case, the pheromone trail evaporates with time. In this behavioral pattern, the decision of an ant to follow a certain path to the food depends on the behavior of his nestmates. At the same time, the ant in question will also increase the chance that the nestmates leaving the nest after him follow the same path. In other words, one ant's movement is highly determined by the movement of previous ants.

An important result of the Artificial System development that was based on the Swarm Intelligence is the creation of the Ant System and the Ant Colony System. Ant Colony System [15, 16] is a relatively new metaheuristic for hard combinatorial optimization problems. Dorigo et al. [17] applied Ant System to the classical Traveling Salesman Problem. They also tested the approach proposed on asymmetric Traveling Salesman Problem, the quadratic assignment and the jobshop scheduling problem. Bullnheimer et al $[7,8]$ used the Ant System to solve the Vehicle Routing Problem in its basic form (homogenous fleet, capacity restriction, distance restriction, one central depot) and obtained very good results.

## 3. Computing with artificial bees: case study of traveling salesman problems

Self-organization of bees is based on a few relatively simple rules of individual insect's behavior [21, 22, 1, 18,
$23,29,30,19,24,10,11,12,37,38]$. In spite of the existence of a large number of different social insect species, and variation in their behavioral patterns, it is possible to describe individual insects' behavior as follows [9]. Each bee decides to reach the nectar source by following a nestmate who has already discovered a patch of flowers. Each hive has a so-called dance floor area in which the bees that have discovered nectar sources dance, in that way trying to convince their nestmates to follow them. If a bee decides to leave the hive to get nectar, she follows one of the bee dancers to one of the nectar areas. Upon arrival, the foraging bee takes a load of nectar and returns to the hive relinquishing the nectar to a food storer bee. After she relinquishes the food, the bee can (a) abandon the food source and become again an uncommitted follower, (b) continue to forage at the food source without recruiting nestmates, or (c) dance and thus recruit nestmates before returning to the food source. The bee opts for one of the above alternatives with a certain probability. Within the dance area the bee dancers "advertise" different food areas. The mechanisms by which the bee decides to follow a specific dancer are not well understood, but it is considered that the recruitment among bees is always a function of the quality of the food source. It is also noted that not all bees start foraging simultaneously. The experiments confirmed that new bees begin foraging at a rate proportional to the difference between the eventual total and the number presently foraging.

Lučić and Teodorović [26, 27], and Lučić [25] recently developed an initial version of the ALife model inspired by bees' behavior in nature. They use bees' behavior in nature as a source of ideas for development of an artificial system called the Bee System. The Bee System is composed of agents, or virtual creatures called artificial bees. The basic assumption is that artificial bees are capable of discovering "good" solutions for difficult combinatorial optimization problems.

The primary goal of this paper is to show the possible applications of the artificial systems inspired by collective social insects intelligence in solving complex traffic and transportation engineering problems. The development of the new heuristic algorithm for the Traveling Salesman Problem using the Bee System will serve as an illustrative example for such applications and will show the characteristics of the proposed concept. It is important to say that the Traveling Salesman Problem is closely related to the broad class of transportation problems whose typical representatives are the vehicle fleet planning, and static and dynamic routing and scheduling of vehicles and crews for airlines, railroads, truck operations and public transportation services.

## 3.1 solving the traveling salesman problem by the bee system

Let us denote by $\mathrm{G}=(\mathrm{N}, \mathrm{A})$ the network in which the bees are collecting nectar (the graph in which the traveling salesman route should be discovered). By $\mathrm{N}=$ $\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \ldots \mathrm{v}_{\mathrm{n}}\right\}$ we denote the set of nodes to be visited, and by $A=\left\{\left(\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{j}}\right): \mathrm{i} \neq \mathrm{j}\right\}$ the set of links connecting these nodes. Let us also randomly locate the hive in one of the nodes. When foraging, the artificial bees are trying to collect as much nectar as possible. Let us also assume that the nectar quantity that is possible to collect flying along a certain link is inversely proportional to the link length. In other words, the shorter the link, the higher the nectar quantity collected along that link. This means that the greatest possible nectar quantity could be collected when flying along the shortest traveling salesman route. Our artificial bees will collect the nectar during the certain prescribed time interval. After that, we will randomly change the hive position. The artificial bees will start to collect the nectar from the new location. We will then again randomly change the hive location, etc. The iteration in this searching process represents one change of the hive position. Our artificial bees live in an environment characterized by discrete time. Each iteration is composed of a certain number of stages. The stage is an elementary time unit in the bees' environment. During one stage the artificial bee will visit $s$ nodes, create partial traveling salesman tour, and after that return to the hive (the number of nodes $s$ to be visited within one stage is prescribed by the analyst at the beginning of the search process). In the hive the bee will participate in a decision making process. The artificial bee will decide whether to abandon the food source and become again an uncommitted follower, continue to forage at the food source without recruiting nestmates, or dance and thus recruit nestmates before returning to the food source.

During any stage, bees are choosing nodes to be visited in a random manner. The Logit model is one of the most successful and widely accepted discrete choice models. Inspired by the Logit model, we have assumed that the probability of choosing node $j$ by the $k$-th bee, located in node $i$ (during stage $u+1$ and iteration $z$ ) equals:

where:
$i, j$ - node indexes $(i, j=1,2, \ldots,|N|)$,
$d_{i j}-$ length of link $(i, j)$,
$k$ - bee index $(k=1,2, \ldots, B)$,
$B$ - the total number of bees in the hive,
z - iteration index ( $z=1,2, \ldots, M$ ),
$M$ - maximum number of iteration,
$u$ - stage index $(u=1,2, \ldots,\lceil(|N|-1) / s\rceil)$,
s - number of nodes visited by every artificial bee during one stage,
$\mathrm{n}_{i l}(r)$ - total number of bees that visited link $(i, l)$ in $r$-th iteration,
$b$ - "memory length"",
$\mathrm{g}_{k}(u, z)$ - last node that bee $k$ visits at the end of stage $u$ in iteration $z$,
$\mathrm{N}_{k}(u, z)$ - set of unvisited nodes for bee $k$ at stage $u$ in iteration $z$ (in one stage bee will visits $s$ nodes; we have $\left.\left|N_{\mathrm{k}}(u, z)\right|=|N|-u s\right)$, $a$ - input parameter given by analyst.

Let us discuss relation (1) in more details. The greater the distance between node $i$ and node $j$, the lower the probability that the $k$-th bee located in the node $i$ will choose node $j$ during stage $u$ and iteration $z$. The greater the number of iterations $z$, the higher the influence of the distance. In other words, at the beginning of the search process, artificial bees have "more freedom of flight". They have more freedom to search the solution space. The more iterations we make, the bees have less freedom to explore the solution space. The more we are approaching the end of the search process, the more focused the bees are on the flowers (nodes) in the neighborhood. Our artificial bees have memory and they can remember how many bees visited a certain link during the last $b$ iterations. The greater the total number of bees that visited a certain link in the past, the higher the probability of choosing that link in the future. This represents the interaction between individual bees in the colony.

For every bee we now know the nectar quantity collected by the bee (the length of the partial traveling salesman tour). After returning to the hive bees relinquish the nectar to a food storer bee. After relinquishing the food, the bee is making a decision about abandoning the food source or continuing the foraging at the food source. We assume that every bee can obtain the information about nectar quantity collected by every other bee. The probability that, at the beginning of stage $u+1$, bee $k$ will use the same partial tour that is defined in stage $u$ in iteration $z$ equals:

$$
\begin{equation*}
p_{k}(u+1, z)=e^{-\frac{L_{k}(u, z)-\min _{r e s z z}\left(L_{r}(u, z)\right)}{u z}} \tag{2}
\end{equation*}
$$

where $\mathrm{L}_{k}(u, z)$ is the length of partial route that is discovered by bee $k$ in stage $u$ in iteration $z$.

[^0]We can see from relation (2) that if a bee has discovered the shortest partial traveling salesman tour in stage $u$ in iteration $z$, the bee will fly along the same partial tour with the probability equal to one. The longer the tour that the bee has discovered, the smaller is the probability that the bee will fly again along the same tour. When a bee decides not to abandon the food source she can: (a) continue foraging at the food source without recruiting nestmates; (b) fly to the dance floor area and start dancing, thus recruiting nestmates before returning to the food source. The bee opts for one of the above alternatives with a certain probability. Within the dance area the bee dancers "advertise" different food areas. Because the real bees are, above all, social insects (the interaction between individual bees in the colony has been well documented), it is assumed in this paper that the probability $p^{*}$ of the event that the artificial bee will continue foraging at the food source without recruiting nestmates is very low:

$$
\begin{equation*}
p^{*} \ll 1 \tag{3}
\end{equation*}
$$

After relinquishing the food, and after making the decision to continue foraging at the food source, the bee flies to the dance floor and starts dancing with probability equal to $\left(1-p^{*}\right)$. Bee dancing represents the interaction between individual bees in the colony.

In the case when at the beginning of stage $u+1$, the bee does not use the same partial traveling salesman tour, the bee will go to the dancing area and will follow another bee(s). Every partial traveling salesman tour $\xi$ that is being advertised in the dance area has two main attributes: (a) the total length, and (b) the number of bees that are advertising the partial route. We introduce the normalized value of the total length of the partial traveling salesman tour and the normalized value of the number of bees advertising the partial tour. Both normalized values are defined in the following way: (a) Both normalized values can take any value between 0 and 1; (b) The smaller the total length normalized value, the better the partial tour; (c) The bigger the number of bees normalized value, the better the partial tour.

We have assumed in this paper that the probability that the partial route $\xi$ will be chosen by any bee that decided to choose the new route equals:

$$
\begin{equation*}
p_{\xi}(u, z)=\frac{e^{\rho \beta_{\xi}(u, z)-\theta \alpha_{\xi}(u, z)}}{\sum_{\tau \in Y(u, z)}^{\rho e_{\tau}(u, z)-\theta \alpha_{\tau}(u, z)}} \quad \xi \in Y(u, z), \forall u, z \tag{4}
\end{equation*}
$$

where:
$\rho, \theta-\quad$ parameters given by the analyst,
$\alpha_{\xi}(u, z)$ - the normalized value of the partial route length,
$\beta_{\xi}(u, z)-$ the normalized value of the number of bees advertising the partial tour,
$Y(u, z)$ - the set of partial tours that were visited by at least one bee.

The proposed Bee System was tested on a large number of numerical examples. Before relocating the hive to the next location we tried to improve the solution obtained by the bees in current iteration by applying the well-known 2 -opt or 3 -opt heuristic algorithms. The benchmark problems were taken from the following Internet address: http://www.iwr.uniheidelberg.de/iwr/comopt/software/TSPLIB95/tsp/.
The following 8 problem instances were considered: Eil51.tsp, Berlin52.tsp, St70.tsp, Pr76.tsp, Kroa100.tsp, Eil101.tsp, Tsp225.tsp and A280.tsp. All tests were run on an IBM compatible PC with PIII processor ( 533 MHz ). Table 1 presents the results obtained by the Bee System when the search is limited to 100 cycles.
Table 1. The results obtained by the Bee System enriched with 3-opt heuristic

| Problem | $\|$Optimal <br> Value <br> (O) | The best value obtained by the Bee System (B) | $\begin{gathered} \frac{(B-O)}{O} \\ (\%) \end{gathered}$ | Time <br> required <br> to find <br> the best <br> solution <br> (seconds) | Average <br> value <br> obtained by <br> the Bee <br> System <br> over 20 <br> runs (A) | $\begin{aligned} & \text { St. } \\ & \mathrm{Dev} . \\ & \text { (SD) } \end{aligned}$ | $\begin{gathered} \frac{(A-O)}{O} \\ (\%) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Eil51 | 428.87 | 428.87 | 0 | 37 | 428.87 | 0 |  |
| Berlin52 | 7544.37 | 7544.37 | 0 | 1 | 7544.37 | 0 |  |
| St70 | 677.11 | 677.11 | 0 | 22 | 677.11 | 0 |  |
| Pr76 | 108159 | 108159 | 0 | 11 | 108159 | 0 |  |
| Kroa100 | 21285.4 | 21285.4 | 0 | 10 | 21285.4 | 0 |  |
| Eil101 | 640.21 | 640.21 | 0 | 1741 | 643.05 | 1.7 | 0.44 |
| Tsp225 | 3859 | 3876.05 | 0.44 | 5153 | 3905.32 | 18.9 | 1.2 |
| A280 | 2586.77 | 2600.34 | 0.53 | 13465 | 2627.45 | 12.31 | 1.5 |

In all instances with less than 100 nodes, the Bee System produced the optimal solution. The times required to find the best solutions by the Bee System are very low. In other words, it was able to produce "very good" solutions in a "reasonable amount" of computer time. The best solution of one studied benchmark problem (A280.tsp) discovered by the Bee System is presented in the Figure 1.


Figure 1. Bee System solution to the TSP problem instance A280.tsp (limited to 100 cycles)

## 4. Transportation problems characterized by uncertainty: artificial life approach

A wide range of traffic and transportation engineering parameters are characterized by uncertainty, subjectivity,
imprecision, and ambiguity [31, 32, 35]. Human operators, dispatchers, drivers, and passengers use subjective knowledge or linguistic information on a daily basis when making decisions. Drivers, passengers, or dispatchers make decisions about route choice, mode of transportation, most suitable departure time. In each case the decision maker is a human. The environment in which a human expert (human controller) makes decisions is often complex, making it difficult to formulate a suitable mathematical model. Thus, the development of fuzzy logic systems seems justified in such situations. Developing models for solving difficult combinatorial optimization problems characterized by uncertainty is a very important and challenging research task.

ALife approach can be successfully used to "attack" transportation engineering problems characterized by uncertainty. One can assume that virtual creatures (artificial ants, or artificial bees) use approximate reasoning in decision-making process. The control strategies of agents can be easily formulated in terms of numerous descriptive rules. The Fuzzy Ant System (FAS) described in this paper represents an attempt to handle the uncertainty that sometimes exists in some complex transportation problems. Before explaining FAS, let us briefly describe "classical" Ant algorithms.

### 4.1 Basic characteristics of the Ant algorithms

It is known that real ants in nature are capable of discovering the shortest path from a food source to the nest. It is also well known that real ants deposit pheromone on the ground while walking. At the same time, real ants follow pheromone that was deposited previously by other ants. Going form the nest towards a food source, real ants choose a particular path with a certain probability. The probability that a certain path will be chosen significantly depends on the amount of pheromone that has been deposit on that path. The higher the amount of pheromone, the higher is the probability that a path is chosen. Dorigo et al. [17] solved the Traveling Salesman Problem using the Ant System. Dorigo et al. [17] proposed that artificial ants search the solution space. In this way, artificial ants simulate real ants that search their environment looking for a shortest path between food source and nest. The searching process is characterized by the existence of adaptive memory. The adaptive memory corresponds to the pheromone trails. In the environment of the artificial ants the time is discrete. At the beginning of the search process (time $t=0$ ), the ants are located in different towns. Dorigo et al denoted by $\tau_{\mathrm{ij}}(\mathrm{t})$ the intensity of trail on edge $(i, j)$ at time $t$. At time point $t=0$ the value of $\tau_{\mathrm{ij}}(0)$ is equal to a small positive constant $c$. At time $t$ each ant is moving from the current town to the next town. Reaching the next town at time $(t+1)$, each ant is making the next move towards
next, yet unvisited town. Being located in town $i$, ant $k$ is choosing next town $j$ to be visited at time $t$ with the transition probability $p_{\mathrm{ij}}{ }^{\mathrm{k}}(t)$ defined by the following relation:

$$
p_{i j}^{k}(t)=\left\{\begin{array}{lr}
\frac{\left[\tau_{i j}(t)\right]^{k}\left[\eta_{i j}\right]^{\beta}}{\sum_{n \in \Omega^{k}(t)}\left[\tau_{i k}[(t)]^{j}\left[\eta_{i k}\right]^{\beta}\right.}, & \text { if } j \in \Omega_{i}^{k}(t)  \tag{5}\\
0, & \text { otherwise }
\end{array}\right.
$$

where:
$\Omega_{\mathrm{i}}^{\mathrm{k}}(t)$ - set of feasible nodes to be visited by ant $k$ (the set of feasible nodes is updated for each ant after every move),
$\eta_{\mathrm{ij}}=1 / d_{\mathrm{ij}}$ - "visibility",
$\mathrm{d}_{\mathrm{ij}}-$ Euclidean distance between node $i$ and node $j$,
$\alpha, \beta$ - parameters representing relative importance of the trail intensity and the visibility.

The greater the importance the analyst gives to the visibility, the greater the probability that the closest towns will be selected. The greater the importance given to the trail intensity on the link means that that link is highly desirable because many ants already passed through that link.

Dorigo et al. [17] assumed $m$ moves performed by $m$ ants in the time interval $(\mathrm{t}, \mathrm{t}+1)$. Every ant will complete a traveling salesman tour after n iterations. Dorigo et al [17] named $n$ iterations of the algorithm the cycle. They proposed to update the trail intensity after each cycle in the following way:

$$
\begin{equation*}
\tau_{i j}(t+n)=\rho \tau_{i j}(t)+\Delta \tau_{i j}(t, t+n) \tag{6}
\end{equation*}
$$

where:
$\rho$ is the coefficient $(\rho<1)$ such that $(1-\rho)$ represents evaporation of the trail between time $t$ and $t+n$.

The total increase in trail intensity along link (i, j ) after one completed cycle is equal to:

$$
\begin{equation*}
\Delta \tau_{i j}(t, t+n)=\sum_{k=1}^{m} \Delta_{i j}^{k}(t, t+n) \tag{7}
\end{equation*}
$$

where $\Delta_{\mathrm{ij}}{ }^{\mathrm{k}}(t, t+n)$ is the quantity of pheromone laid on link $(i, j)$ by the $k$-th ant during time interval $(t, t+n)$.

The quantity $\Delta_{\mathrm{ij}}{ }^{\mathrm{k}}(t, t+n)$ is given by:
$\Delta_{i j}^{k}(t, t+n)= \begin{cases}\frac{Q}{L_{k}(t, t+n)}, & \begin{array}{l}\text { if the } \mathrm{k}-\text { th ant walks along the ink }(\mathrm{i}, \mathrm{j}) \\ \text { in its tour during time interval }(\mathrm{t}, \mathrm{t}+\mathrm{n})\end{array} \\ 0, & \text { otherwise }\end{cases}$
where:
$Q$ - the constant,
$L_{\mathrm{k}}(t, t+n)$ - the tour length of the $k$-th ant within cycle $(t$, $t+n$ ).

As we can see, in order to discover a good solution, artificial ants cooperate through the deposition of the
pheromone on graph edges. In other words, pheromone deposition represents exchange of information between ants. In order to increase the efficiency of the Ant System, Dorigo and Gambardella [15, 16], and Dorigo et al. [14] developed the Ant Colony System.

### 4.2 FAS: Fuzzy Ant System

We propose $a$ Fuzzy Ant System (FAS) that represents a combination of the Ant Colony System and Fuzzy Logic. In this way, the Ant System is combined with a non-quantitative approach. The basic modification is in the way in which transition probabilities are calculated. Fuzzy logic could be used to calculate an ant's utility to visit the next node.

When deciding about the next node to be visited (in the case of the Traveling Salesman problem), the ant takes into account "visibility", as well as pheromone trail intensity. We assume that the ant can perceive the particular distance between nodes as "small", "medium" or "large", and the trail intensity as "weak", "moderate" or "strong", etc. Possible membership functions of these fuzzy sets are shown in Figure 2.

Depending on the distance to the next node, as well as the trail intensity, the ant will have a stronger or weaker utility to choose the considered link. These utilities can be described by appropriate fuzzy sets. Approximate reasoning algorithm for calculating the utility of choosing the next link (in the case of Traveling Salesman Problem) could be composed, for example, of the rules of the following type:
If distance is SHORT and trail intensity is STRONG
Then utility is VERY HIGH
The approximate reasoning algorithm could replace relation (5) for calculating transition probabilities. In this way, it is possible to calculate an ant's utilities even in the cases where some of the input data are only approximately known.

Teodorović and Lučić [33] developed the model for schedule synchronization in public transit based on Fuzzy Ant System. Trips between nodes in public transit networks may be made with or without making transfers. Transfers usually represent inconvenience to the passengers. Because badly coordinated transfers can significantly increase waiting times, it is especially important to carefully synchronize schedules in the cases of larger headways. At the same time, badly coordinated transfers can decrease the total number of passengers in public transit and their switching to competitive modes. While making schedule synchronization, it is necessary to try to minimize the total waiting times of all passengers at transfer nodes in transit network. Very often only approximate numbers of transfer passengers are known.

The model proposed by Teodorović and Lučić [33] for schedule synchronization when the number of transfer passengers is only approximately known, is based on Fuzzy Ant System.


Figure 2. Membership functions of the fuzzy sets describing distance to the next node and trail intensity

Let us note the network shown in Figure 3. The network is composed of a few layers. The total number of layers in the network, $m$, is equal to the total number of transit lines. The number of nodes in every layer equals the number of possible transit line departure times. There is a full connectivity between the two neighboring layers. Let us assume that all ants are located in origin O and that all of them travel to destination D. Every node in the network is described by the appropriate coordinates. For example, the coordinates $(i, j)$ describe $j$-th node located in the $i$-th layer. This node represents $j$-th possible departure moment of the $i$-th transit line.

Ants located in origin O go towards destination D . The ants have few options when choosing the first node in the first layer (departure time of the first transit line). The number of choices is equal to the number of possible departure times. Pheromone will be deposited at every node that was visited by at least one ant. At the beginning of the search process we will assume that the pheromone trail is very low in every node and that it is equal to some small positive constant. An ant starts his trip from the origin, chooses one node in the first layer, then moves to the second layer, chooses one node from the second layer, etc. An ant's utility for choosing the next node is calculated using the approximate reasoning algorithm.

The typical rule in this approximate reasoning algorithm reads:
If waiting time is SHORT and trail intensity is WEAK Then utility is HIGH


Figure 3. Transit lines and possible departure times
The nodes with better utility values are more likely to be selected by an ant. The performed numerical tests showed that the Fuzzy Ant System could produce very good results when solving schedule synchronization problem in public transit.

Recently, Lučić and Teodorović [34] also successfully combined the Bee System with Fuzzy logic in order to solve vehicle routing problem with uncertain demands at nodes (the real value of demand at a node is only known when the vehicle reaches the node). The obtained results are very promising.

## 5. Conclusion

The Bee System has been successfully applied to the classical Traveling Salesman Problem. The results obtained are considered to be very good. A Fuzzy Ant System (FAS) representing a combination of the Ant Colony System and Fuzzy Logic is sussesfully applied for
solving schedule synchronization in public transit. The Fuzzy Ant System shows that it is possible to successfully combine quantitative and a non-quantitative approach when developing ALife models.

It seems that there are a lot of potential applications for the ALife techniques in the field of Traffic and Transportation Engineering. Some typical ones, for example, are designing an airline or utility network (designing the best hub and spoke architecture), selecting the "best" path to connect two points on a map (alignment problem for public transportation routes), vehicle and crew routing and scheduling problems, or the Gate Assignment Problem (assignment of arriving aircraft to available gates). Natural systems teach us that very simple individual organisms can form systems capable of performing highly complex tasks by dynamically interacting with each other. Preliminary results with ALife based models are very promising. These results show that the development of new models based on ALife principles could significantly contribute to the solution of complex transportation engineering problems.

## Acknowledgments

We wish to thank the National Science Foundation for this research was supported by a grant \#CMS-0085430.

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[^0]:    *While foraging in stage $u$, every artificial bee has the ability to notice the total number of bees in every link. The maximum number of stages that bee can recall represents memory length.

