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REVIEW ARTICLE

Transverse dynamics in cavity nonlinear optics (2000–2003)

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Abstract

We review publications on transverse dynamics in cavity nonlinear optics during the years 2000–2003. Topics covered are transverse pattern dynamics, localized structures, scaling of turn-on transients, properties of nonlinear PDEs, VCSELs, quadratic media, light valves with optical feedback, quantum properties of light, and quantum images.

Keywords: cavity nonlinear optics, transverse pattern dynamics, localized structures, solitons, turn-on transients, VCSELs, quadratic media, light-valve with optical feedback, quantum properties of light, quantum images

1. Introduction

This review was initially intended to report progress on transverse effects in nonlinear optics during the four years 2000–2003. It soon appeared that the number of topics and papers to be covered under this umbrella is gigantic. Therefore, we have limited the scope of this paper to transverse problems in passive and active *cavities* during those years. Bose–Einstein condensates are not retained on this basis, although the case against this choice could easily be argued. Conference proceedings have not been retained in the bibliography because too often they simply duplicate material published in regular scientific journals.

This review does not fall into any of the usual categories of scientific papers. It was commissioned by the journal and we were given complete freedom to define its *genre*. With a few preconceived ideas, we started to work on the paper and its *genre* was progressively defined as the work progressed. It is not a pedagogical introduction to the subjects covered in this paper. That would be far too much for a single paper. For the same reason, it is not a review either. The best description we can give is that we eventually aimed at a commented comprehensive list of references. Thus, this paper is meant for the active research physicist and engineer working either in one of the topics covered here or in an adjacent topic and wishing to have a quick overview of what is going on in one of the topics covered by this paper. Therefore, when there are conflicting results, we have been careful to expose the various facets of the problem without taking sides. In a few cases, we have also included a historical background when we felt that recent research papers tend to forget some seminal papers.

The field of transverse effects in nonlinear optics has witnessed a sudden acceleration at the beginning of the third millennium with the explosive growth of the soliton theme, as witnessed by recent review papers [1–12]. An excellent review of the soliton subtopic can be found in the book of Kivshar and Agrawal [13]. Let us also cite three topical issues that have been published on solitons. The first deals mostly with mathematical aspects of optical solitons [14–28] while the other two special issues [29–35] and [36–44] are more oriented towards applications in optics.

A topical issue directly related to the subject of this review was published in this journal [45]. It contains a collection of conference papers devoted to dynamics and pattern formation in nonlinear optical systems. Other books and review papers dealing with transverse effects in nonlinear optics during this period are [46–74].

Another source of excitement has been the experimental realization of a read/write system based on cavity solitons in semiconductor cavities [75, 76], opening hopes to achieve an integrated all-optical information processor.

Another contender for the title of heavyweight topic is the physics of photorefractive materials. A book thoroughly covering transverse patterns in photorefractive materials has just appeared [77].

A number of papers stress the mathematical properties of partial differential equations more than the nonlinear optical side. For these papers, we have retained studies of the Ginzburg–Landau and Swift–Hohenberg equations only if they have a clear relation to optics while disregarding studies of the nonlinear Schrödinger equation which is mostly associated with propagation problems.

2. Transverse pattern dynamics

The formation of dissipative structures far from equilibrium has been widely studied since the early 1970s. Turing was the first to propose, in the context of morphogenesis, a mechanism explaining the spontaneous emergence of patterns out of a homogeneous state [78]. This bifurcation, known as the Turing bifurcation, arises from a competition between two processes: diffusion (and/or diffraction in optics), which tends to restore spatial uniformity, and nonlinearity, which is typically generated by light–matter interactions and which is responsible for the amplification of spatial inhomogeneities. The balance between these two effects can stabilize spatial structures [79]. Self-organization and pattern formation in extended nonlinear optical systems has been presented in a number of excellent reviews [45–74]. In most cases, the intrinsic wavelength of the emerging dissipative patterns is solely determined by dynamical parameters and not by geometrical constraints imposed by the boundary conditions. Practically, only systems of large size (large aspect ratios or large Fresnel numbers) compared with the scale of the selected pattern have been studied. In this limit, the presence of boundaries does not affect the dynamics of the systems. In this case, the distance between the largest eigenvalues of the linear problem is very small, leading to a quasicontinuous spectrum of the Laplace operator. So far, many problems related to the interaction of instabilities, secondary instabilities affecting the Turing branches of solutions, the role of the walk-off, and the chromatic dispersion remain open questions.

The interaction between a saddle node bifurcation and a Turing mode in the dynamics of the degenerate optical parametric oscillator (DOPO) induces the formation of a new type of stable phase-locked mixed-mode hexagonal structure below the lasing threshold. Without this interaction, hexagonal structures do not exist [80]. The role of the quasineutral mode in the interaction between two Turing instabilities having the same critical wavelength has been studied for a passive system in the nascent bistability limit [40]. If the two critical wavelengths are different, a normal form in the limit where the two instabilities are close to each other has been derived. This situation arises for instance in a coherently driven semiconductor cavity. In that system, an analytical study shows that there exist an infinity of branches of periodic solutions. In the bifurcation diagram, the field envelope can either smoothly join the two instability points or form an isolated branch of solutions [81]. In the type II DOPO, standing wave intensity patterns for the two polarization components of the field arise from the nonlinear competition between two concentric rings of unstable modes. Close to threshold a linear stability predicts standing waves with the same wavelength for the two polarization components [82]. In this device, waves are the predominant solutions and the defects are vectorial dislocations that appear at the boundaries of domains where travelling waves of different phases or wavevector orientations are formed [83]. The walk-off effect on pattern selection in the DOPO was also studied [84]. Thermal effects can lead to periodic mode hopping in a multilongitudinal mode cw OPO if it is either triply resonant or doubly resonant with a weakly resonant pump. Such oscillations have been observed experimentally in a type II OPO in both configurations [85]. Self-focusing as well as spatially ordered structures is observed in singly resonant intracavity second-harmonic generation [86]. A weakly nonlinear multiple-scale analysis has been performed for this system. The amplitude equations explain the absence of roll patterns, in agreement with numerical results [87].

Different scenarios have been established where either the up- or down-conversion processes dominate the spatiotemporal behaviour, such as doubly resonant intracavity second-harmonic generation (SHG) in the presence of competing nondegenerate parametric downconversion. It has been shown that for positive cavity detuning of the fundamental frequency the threshold for parametric oscillation is lower than that of transverse, pattern forming instabilities [88]. The possibility of obtaining exact solutions above threshold for the parametric oscillation process allows detailed analytical investigations of the parametric instability, that are supplemented by numerical analysis [89]. The influence of a secondary instability affecting the Turing branches has been assessed: solutions leading to a sequence of bifurcations showing spatial-period multiplying and quasiperiodicity, and optical turbulence has been reported in Kerr cavities [90]. The influence of noise in the formation of hexagonal patterns was also studied for that system [91]. The experimental observation of noisy pattern precursors has been reported for a Kerr slice medium with optical feedback [92]. The interaction between scalar and vectorial instabilities in two-photon resonant Kerr cavities gives rise to an elliptically polarized stripe pattern [93]. Synchronization in a noise-sustained structure resulting from the interplay between walk-off and noise fluctuations has been theoretically reported in a vectorial intracavity down-conversion system [94]. The influence of spatial nonuniformities leading to the selection of a wavenumber outside the Eckhaus-stable band has been investigated experimentally and numerically in a modulated multimode laser [95]. Transverse patterns in a spherical cavity close to confocality with Gaussian pump profile are reported in [96]. Pattern selection in nonlinear cavities with plane and curved mirrors has been discussed [97].

In another line of research, theoretical studies have shown that the chromatic dispersion, due to the intracavity medium, plays an important role in the space–time dynamics of nonlinear optical systems. It was proved that when chromatic dispersion operates together with diffraction the type II SHG exhibits a three-dimensional (3D) Turing instability leading to the formation of 3D periodic structures. These structures consist of regular 3D lattices of bright or dark light drops travelling at the group velocity of light within the cavity [98]. This behaviour was first predicted for a coherently driven passive ring cavity [99]. In that purely analytical study, it was proved that body-centred cubic structures dominate the nonlinear dynamics of the dispersive and diffractive Kerr nonlinearity near the Turing instability. In an independent work, 3D structures such as lamellae and tetrahedral patterns have been reported for the DOPO [100]. It was also proved analytically and numerically that different 3D structures exist (lamellae, 3D hexagons, and body centred cubic patterns) and have overlapping domains of stability in type II SHG [101]. By using the normal form analysis, a general condition under which tetrahedral structures are stable was derived [102]. Finally, the interaction between saddle node bifurcation and 3D Turing instability has been investigated for the DOPO. Such an interaction leads to the stabilization of body-centred cubic and hexagonally packed cylinders of dissipative optical crystals [103].

3. Localized structures

3.1. Background

Localized structures (LSs) belong to the class of dissipative structures found far from equilibrium. They correspond to stationary or time-dependent pulses in the transverse directions of a resonant cavity. Structures in the form of solitary waves [104], or in the form of diffractive autosolitons [105], have been predicted in an optical bistable ring cavity. These localized solutions correspond to two locked switching waves or fronts between the two stable homogeneous steady states forming the bistability loop. Later, another type of localized structures (also called cavity solitons) in one and two transverse dimensions has been predicted in the weak dispersive limit where their formation does not require steady state bistability [106]. They arise in the regime where a monostable homogeneous steady state exhibits a subcritical Turing instability. A subcritical bifurcation induces mulitistability between the homogeneous steady state and branches of self-organized patterns selected by the system's dynamics. Soon afterwards, the same phenomenon was found in the Lugiato–Lefever model describing the dynamics of a coherently driven passive Kerr cavity [107]. They are homoclinic or multisolitonic solutions connecting the homogeneous and the periodic solutions in the domain where they coexist as stable solutions. The homoclinic nature of these solutions implies that for a given set of control parameters, the number and the space distribution of LSs immersed in the bulk of the basic homogeneous steady state are determined by the initial condition. A large variety of cavity solitons that consist of bright and/or dark pulses are found in the weak dispersion limit [108]. The subject has attracted growing interest because of its potential application in information technology [109, 110]. The first experimental evidence of LS in optical systems has been found in lasers with saturable absorbers [111, 112], in liquid crystals with light valves [113, 114], and in a single-mirror feedback system using sodium vapour as a nonlinear medium [115].

3.2. Passive and active media

The stability of LSs has been analysed and the interaction potential between two LSs has been studied in the limit of absorptive bistability [116] and in the weakly dispersive limit [40]. In these two limits the interaction potential is constructed in terms of Bessel functions that describe the asymptotic behaviour of LSs. It has been shown that LSs can exhibit a curvature instability affecting their circular shapes. This instability consists of an elliptical deformation of the LS, followed by the splitting process leading to the formation of hexagonal structures [117].

It is well known that Kerr cavities support stable two-dimensional LSs in a wide parameter range, and can be destabilized via a self-pulsing instability affecting the LS branch of solutions [107]. This instability has also been analysed in two dimensions [118].

For the vectorial Kerr medium, it was shown that cavities with different losses for the two polarization components support both dark and bright localized structures [95, 119].

In coherently driven passive cavities with a threshold nonlinearity, i.e., a dielectric constant which is piecewise constant, an analytical form is obtained for the field distribution of localized dissipative structures [120, 121]. In lasers with saturable absorbers, the selection of transverse patterns has been shown to be tuned by the absorber. It leads to the formation of transverse patterns in regimes where the free-running laser exhibits only a stable homogeneous state, via a frequency-pulling phenomenon. If the absorber inhibits structure formation, transverse propagating pulses may appear [122]. It was shown that stable LSs arise in this system [123]. The effect of detuning and relaxation processes on their stability and the bifurcations in (1 + 1)D was also assessed [124, 125]. A bifurcation from a stationary LS to a slowly moving LS has been described analytically [124]. Conditions for the existence of completely localized 3D dissipative laser solitons have been found [126] and nonparaxial corrections have been derived [127, 128]. The dynamics of a 3D laser bullet was analysed [129]. The collisions of pairs of 3D solitons were also considered, leading to the characterization of a new collision regime resulting in the formation and the propagation of a switching wave [130]. The formation of pulsating solitons has been reported [131]. Multivortex asymmetric rotating solitons were revealed numerically [132].

By investigating solitons in lasers with a saturable absorber, it is shown that stochastic (Anderson) localization can stabilize dissipative spatial solitons: spatial solitons in resonators with randomly distorted mirrors are more stable than those in perfect mirror resonators [133]. The interaction of two weakly overlapping localized structures has been analysed [134].

The presence of the saturable absorber is not a necessary condition to generate stable LSs in one or two dimensions. It has been proved that broad-area lasers with three-level media operating in the cascade configuration support the formation of localized structures. These solutions consist of spots of two-photon emission embedded in a background of single-photon emission [135]. The Fourier filtering technique can be used to prevent off-axis emission. This procedure allows for the stabilization of LSs in the two-level dense amplifying media [136].

A Fourier-transform-based, computer-assisted, technique to find the stationary solutions of a model describing a saturable absorber in a driven optical cavity has been proposed [137, 138]. The transformation of a soliton propagating in a bulk into solitons propagating in a coherently pumped cavity can be achieved but is followed by the emergence of the negative-energy Vakhitov–Kolokolov modes directly associated with the locking of the soliton phase to the phase of the coherent pump. Mutual annihilation of the positive- and the negative-energy modes leads to a Hopf instability of the cavity solitons [139].

Most of the work on photorefractive media is realized without resonant cavities. See [77] for an up-to-date review of this topic. A remarkable exception is the study of pattern formation in cavities driven by a field which lacks spatial and/or temporal coherence. Pattern formation was first observed experimentally and analysed theoretically with a cavity containing the photorefractive crystal SBN:60 ($Sr_{0.6}Ba_{0.4}Nb_2O_6$) driven by a field which is temporally incoherent but spatially coherent [140]. The same research group has predicted cavity pattern formation with spatially and temporally incoherent light [141]. The experimental confirmation was reported at the CLEO 2003 conference.

Another result based on photorefractive materials in cavities is an experimental study of the detuning-induced transverse mode selection and of the corresponding variation of the oscillation threshold in an optical oscillator with gain provided by a photorefractive crystal has been reported. The issues of the selection of the transverse mode close to threshold and the value of the pump parameter at threshold are addressed. Competition between right and left travelling waves, resulting in a winner-takes-all dynamics, is also reported [142].

The periodic spatial modulation of refractive index allows one to modify, and even to invert, the sign of the diffraction coefficient. This phenomenon has been recently studied in conjunction with so-called bandgap solitons in nonlinear optics [60], in atomic Bose–Einstein condensates [143, 144], and in dissipative nonlinear photonic crystal resonators [145].

The existence of stable dissipative spatial solitons at low intensities in patterned electrode semiconductor optical amplifiers has been predicted theoretically [146] and the observation reported soon afterwards [147].

4. Scaling of turn-on transients

In many instances of cavity nonlinear optics, the steady state properties are governed by a closed equation for the field intensity $I = |E|^2$. Therefore, if E is a solution, $-E = e^{i\pi} E$ is also a solution. Below, we refer to these states as asymptotic states. Domain walls, which belong to the family of localized structures, result from this phase indetermination: they are solutions connecting through a thin and steep but continuous layer the two asymptotic states. In the DOPO, they lead to the formation of either localized single stripe [148], or circular domains [149]. In optical bistability, single stripes and circular domains have been shown to exist and to coexist [150].

In this connection, the kinetics of the evolution from a random initial distribution of the two asymptotic states to the final localized structure has raised a still open problem. This kinetics is analysed by determining $R(t)$ which is the average size of connected domains containing only one type of asymptotic solution. In the short time limit, this size grows exponentially (linearized theory). However, in the long time limit, a variety of power laws have been found. This question was studied first for a Turing instability occurring close to the saddle-node bifurcation in the DOPO and for a Swift–Hohenberg model [151]. It was shown that the long time kinetics becomes a nonlinear process obeying a power law $R(t) \propto t^{\alpha}$ with $\alpha = 1/3$. Comparison between the propagation and the mean-field models of the DOPO yields the same power law [152]. The same behaviour was also found in type I SHG [153]. Beyond the mean-field approximation, the DOPO supports large amplitude localized structures, and their kinetics verify the scaling law with either $\alpha = 1/3$ or $1/2$, depending on the sign of the detuning parameters [154]. In the same reference, the power law with $\alpha = 1/5$ was obtained for the growth of the labyrinth patterns. In the mean-field approximation of a DOPO, numerical simulations indicate, that close to resonance or for positive detuning, domain walls are ruled by curvature effects with an $\alpha = 1/2$ growth law [155].

A single power law is not the rule: in optical bistability, the time evolution of $R(t)$ displays multiple power laws with $\alpha = 2/3$, $1/3$, 1, $1/2$, and $1/5$ [150].

For vectorial Kerr resonators, the growth law associated with the circular localized structures is determined by curvature effects and leads to $\alpha = 1/2$ [156]. The dynamics of the curvature close to the bifurcation point was studied and leads to the growth law $\alpha = 1/4$ [157]. The influence of the drift on the curvature-driven dynamical process has been investigated, motivated by an experimental observation in a liquid crystal light valve system [158]. The transition from rolls to labyrinths and their stability has been reported for both variational and nonvariational models [154].

5. Properties of nonlinear PDEs

As already explained in the introduction, this section focuses on two PDEs only: the Ginzburg–Landau (GL) and the Swift–Hohenberg (SH) equations [159]. Only two books, in the optics community, attempt to explain the intricacies of the derivation of these equations from the Maxwell–Schrödinger equations and therefore their limits of validity [160, 161]. Let us also mention a good tutorial presentation of pattern dynamics [162].

Few papers deal with the SH equation. The existence, stability, and bifurcation structure of static localized solutions has been assessed in one transverse dimension, near the robust existence of stable fronts between homogeneous solutions and periodic patterns for the cubic real SH equation [163]. A family of solutions is formed by the stationary pulses. They are solutions of the form $\psi(t, z) = f(t)\exp(-iz\Omega)$ where Ω is a free parameter. In the case of a complex quintic SH equation, the equation for real f has been solved analytically, leading to cnoidal solutions (of which the bright soliton is a limit case) and snoidal solutions (of which the dark soliton is a limit case). These solutions were further generalized to complex f, describing chirped solitons [164]. Generalizing their method, the same authors extended their analysis to the complex cubic SH equation [165]. The solutions again include particular types of solitary wave solutions, bright and dark soliton solutions and periodic solutions in terms of either elliptic Jacobi functions or the Weierstrass function. Although these solutions represent only a small subset of the large variety of possible solutions of the complex cubic and quintic SH equations, they are the first examples of exact analytic solutions found thus far. Using a Swift–Hohenberg equation with a stochastic source, it is shown that

(i) the temporal noise spectra are of the form $1/f^{\alpha}$ where $\alpha = 1 + (3 - D)/4$ with D the spatial dimension of the system and
(ii) the spatial stochastic fluctuations of the stripe position are subdiffusive [166].

The GL equation has attracted much more attention. Papers published on this topic can be classified according to three main criteria:

(i) equations can have real or complex coefficients. A complex diffusion coefficient means that there is both diffusion and diffraction.
(ii) The nonlinearity can be a polynomial of degree three or five.
(iii) The transverse dynamics can be driven by a Laplace operator in one or two dimensions.

One transverse dimension. For the complex cubic GL equation, the influence of walls and corners (with Dirichlet and Neumann boundary conditions) was assessed [167]; the limit of small nongradient corrections was shown to be singular [168]; an analytical approximation for a solitary pulse was obtained [169]. For the complex quintic GL equation, a systematic analysis of two-dimensional axisymmetric doughnut-shaped localized pulses with the inner phase field in the form of a rotating spiral was achieved with an indication that they can be stable, in contrast to their NLS counterpart [170]. In coupled real cubic GL equations, the generation of a two-dimensional quasiperiodic pattern was studied [171], while in coupled complex cubic GL equations, the presence of uniformly propagating localized objects behaving as coherent structures was found [172]. More complicated solutions for the complex cubic and quintic GL equations have been reported numerically [173–175]. All these numerical studies are performed either with periodic boundaries or for an infinite system. The influence of walls may qualitatively change these results [176]. A technique to forecast spatiotemporal time series, based on a proper orthogonal or Karhunen–Loève decomposition to encode large spatiotemporal data sets, has been applied to the complex 1D GL equation in a finite domain [177].

Two transverse dimensions. Models were studied where the complex cubic GL equation has a real coefficient in one direction and an imaginary coefficient in the other direction, that is, each transverse space direction supports either diffusion or diffraction. Numerically stable completely localized pulses are found, corresponding to spatiotemporal solitons (‘light bullets’) in the optical cavities [178]. For a laser cavity, this model requires periodic boundary conditions. This analysis reveals stationary single-humped and multi-humped solutions [179]. For the complex quintic GL equation, three novel varieties of spiralling and nonspiralling axisymmetric solitons have been found. These are irregularly ‘erupting’ pulses and two different types of very broad stationary ones found near a border between ordinary pulses and expanding fronts [180]. A complex quintic GL equation has been shown to lead to stable fully localized 2D pulses. The model also generates 1D patterns in the form of simple localized stripes, which may be stable, or may exhibit an instability transforming them into oblique stripes with zigzags. The straight and oblique stripes may stably coexist with the $(2 + 1)$D pulse, but not with each other [181]. A qualitatively distinct class of spiral waves has been reported for a complex quintic GL equation. These are stable clusters of localized states rotating around a central vortex core emerging due to interference of the tails of the individual states involved [182]. Since light is in general elliptically polarized, it is quite natural to consider the 2D vectorial extension of the complex cubic equation. Dynamical properties of localized structures of topological character have been analysed in this frame [183, 184]. A complex cubic GL driven by a periodic non-autonomous forcing has been shown to display all the complexity of spatially extended systems, such as phase domains, labyrinths, and phase spatial solitons [185].

6. VCSELs

Vertical-cavity surface-emitting lasers (VCSELs) have a definite advantage over edge-emitting semiconductor lasers: they operate in an essentially single-mode regime. Also, their size and characteristics lead to dense-packing capability, low threshold current, high modulation bandwidth, narrow circular beam profile, and simple but still efficient coupling to optical fibres. Such a dream device is, however, close to a utopia and nature is more subtle. First, if it is true that VCSELs essentially operate on a single mode, that mode is vectorial and corresponds to two scalar modes which differ by their electric field polarization. The resulting polarization dynamics was addressed in a seminal paper [186] which has induced much research on this topic. This subfield is not covered in the present review since it deals essentially with longitudinal modes. Second, even if only one longitudinal mode is excited, it appears that many transverse modes belonging to that longitudinal mode lase simultaneously, unless special precautions are taken. VCSELs have, ideally, a cylindrical shape and therefore have the same empty cavity mode structure as optical fibres. Using the known structure of optical fibre transverse modes [187], a model adapted to semiconductor physics and VCSEL properties, including several quantum wells in the active region, was proposed [188, 189]. This model describes the VCSEL in terms of rate equations, coupling the modal field amplitudes to the free-carrier density which obeys a diffusion equation. Spatial gratings have been shown in these early papers to play an essential role in the mode–mode competition.

Research dealing with cavity solitons in VCSELs culminated with the experimental demonstration of solitons (existence, stability, and independence) [75, 76]. A first principle theory of cavity solitons in VCSELs made of an MQW GaAs/AlGaAs structure has been proposed [190, 191], following the approach of Haug and Koch for semiconductor gain media [192]. Other papers analyse the pattern formation in connection with the modulational instability [39, 43, 193, 194], propose a versatile numerical technique to obtain cavity solitons in bulk or MQW GaAs/AlGaAs microresonators [138, 195], predict the temperature-induced motion of cavity solitons [196], and exploit the intrinsic mobility properties of cavity solitons to realize periodic motion along the maximum intensity of a Gauss–Laguerre doughnut mode (TEM^*_{10} or TEM^*_{01} for instance), suitable in principle to provide soliton-based, all-optical clocking or synchronization [197]. The combination/competition of the different timescales of the dynamical variables together with diffraction and carrier/thermal diffusions have been shown to be responsible for a Hopf instability giving rise to regenerative oscillations, travelling patterns, and cavity solitons [198].

Many results have been obtained on the transverse dynamics in VCSELs expressed in terms of a small number of either modes or patterns. Optical patterns such as rolls, rhombs, or hexagons have been observed in optically driven passive GaAs/GaAlAs vertical-cavity microresonators, depending on the wavelength detuning between input and cavity fields. Cavity thickness fluctuations have been shown to contribute to the pattern selection [199]. The basic modelling of VCSELs is still a hot topic and here again many models (many of them claiming to be comprehensive!) have been proposed. Topics covered include spatio-temporal modelling with polarization dynamics [200], modal expansion versus spatio-temporal description [201], effect of carrier transport on transverse mode selection [202], multimode antiphase dynamics with optical feedback [203], analytic solutions [204] of a comprehensive circuit-model [205], and spatio-spectral interaction mechanisms of carriers and photons are observed and explained in [206]. The spatial mode structure of an MQW device has been revisited in [207, 208]. Among the competing models, we have an effective index model that has been used to predict modal frequencies [209], a vectorial model used to explain different intensity patterns observed in the orthogonal polarizations [210], a rigorous and efficient vector model [211], a model, claiming to be comprehensive, including the interdependent processes of carrier transport and heat generation and dissipation with an effective index adopted for the evaluation of the optical fields in the complex layer structure [212], a model that includes thermal effects [213, 214], a system-oriented model, also claiming to be comprehensive [215], and a coupled mode model [216].

The transverse multimodal structure has been described [217]. The turn-on/off dynamics of single- and bi-transverse modes has been analysed in data transmission experiments [218]. Competition and bistability between orthogonally polarized transverse modes has been studied [219]. Since VCSELs are multimode, attention has been paid to mechanisms or techniques that may lead to single-mode operation. These include transverse mode control by frequency-selective feedback [220], the necessity to use a heat sink [221, 222], a selective surface coating technique [223], an injected signal [224], and the thickness of the concave micromirror layer [225, 226] or of a convex mirror [227]. Predictably, the response to a periodic modulation of the driving current [228–234] or to an injected signal (pulsed or harmonically modulated) [235–237] has been actively studied.

Additional papers deal with relative noise intensity [238], transverse standing wave patterns [239], transverse mode locking [240], application to quantum billiards [241], amplitude-squeezed emission from transverse modes [242], and influence of anisotropies [243].

A special mention goes to a group of papers dealing with ‘passive VCSELs’, i.e., material processed and prepared to become a VCSEL except that the electrodes are not added and the cavity is driven by an external high intensity pulsed laser beam instead of an electric current [244]. In this device, experiments have led to the observation of patterns, indications of localization of structures [245–248] and bistability [249]. Bright and dark spatial solitons are observed [250]. The bright solitons have been proved to be bistable. They can be written and erased by incoherent optical injection [251–253] or by injection of light coherent with the background illumination [254]. A hexagonal pattern has also been reported in this configuration [255].

7. Quadratic media

As far as quadratic media *in cavities* are concerned, there is no coherent stream of research but only scattered results, mostly for parametric amplifiers.

Soliton excitation in an OPO naturally selects a strictly defined frequency difference between the signal and idler fields. The frequency selection is closely linked to the relative energy balance between the idler and signal fields [256]. The stability range of solitons in a DOPO is enhanced by increasing the diffraction of the pump wave [257]. In the limit of large pump detuning, the fundamental cavity soliton of an OPO is shown analytically to be the usual sech function. Its stability is determined and a Hopf bifurcation resulting in a periodically pulsing localized structure is obtained in closed form [258]. For a singly resonant OPO, a numerical model which includes diffraction in two transverse dimensions and idler absorption has been developed [259]. For a cw DOPO, the complex spatio-temporal dynamics in two transverse dimensions has been shown to occur closer to the signal generation threshold in the case of a higher finesse for the pump field than for the signal field [260]. As could be expected, patterns in a DOPO are shown to emerge through the interplay between diffractions of the coupled field as in the Turing instability [261]. The formation of cavity solitons in a high finesse, doubly resonant DOPO in two transverse dimensions has been studied analytically and numerically, leading to bright, dark, and oscillating solitons [262].

The coupling between diffraction and walk-off (i.e. divergence of the ordinary and extraordinary beams due to the birefringence of the $\chi^{(2)}$ medium) has been analysed. This includes

(i) transverse nonlinear front (or domain wall) propagation in DOPO with positive detuning [263],
(ii) shrinkage of the region of convective instabilities due to negative detuning [264], and
(iii) Eckhaus and zigzag phase instabilities for waves propagating in the walk-off direction [265].

Walking solitons are slowly varying field envelopes having the property $E_i = A(z - vt)$. A family of walking solitons has been obtained for the DOPO below threshold [266]. The influence of spatial inhomogeneities on solitons in a cavity filled with a quadratically nonlinear material has been studied. It was shown quasi-analytically that perturbation may trap the solitons or set them in motion [267].

Nonlinear frequency conversion taking place within a soliton-induced waveguide has been demonstrated in an OPO with a reduction 33% of the OPO threshold. It is predicted that this threshold may be reduced to less than 4% of its original value [268].

It has been shown theoretically and experimentally that basic geometrical effects can prevent transverse wavevector matching for TEM $_{00}$ modes and thus dramatically change the beam structures when an OPO is pumped by a resonant (or double-pass) beam [269].

The interaction and locking at discrete distances of domain walls with oscillatory tails lead to asymptotically stable spatial disorder. Noise can either suppress it by privileging highly correlated dynamical states consisting of arrays of spatial solitons or it can induce packed arrays of cavity solitons in the limit of large pump finesse relative to the signal finesse [270, 271].

Near-resonant conditions lead to a strong interaction between saddle node and modulational instabilities in diffractive OPOs. The study of this interaction reveals a new type of stable phase-locked mixed-mode hexagonal structure below threshold. Without this interaction, hexagonal structures do not exist [80]. These results were obtained for an OPO with intracavity saturable absorber, but remain exact if the saturable absorber is removed.

Stochastic resonance has been studied in models of OPOs in the presence of a spatially uniform time-periodic driving and in a regime where two equivalent states with equal intensity but opposite phase exist. Stochastic resonance is inhibited at low driving amplitudes by the presence of localized states which prevent the front motion but enhanced for larger driving amplitudes, in the regime where localized states cease to be stable [272].

Transverse patterns in a triply resonant OPO have been studied for a spherical cavity close to confocality with Gaussian pump profile, but the signal and idler intensities may be made of many rings, either stationary or time dependent [96].

In a plane wave $3\omega \rightarrow 2\omega, \omega$ OPO with intracavity SHG, self-phase locking of the pump and subharmonic waves, freezing the phase diffusion noise is demonstrated [273].

For an intracavity type II SHG in a planar waveguide resonator, where two orthogonally polarized pump photons at frequency ω generate one signal photon at frequency 2ω, a transition of the Ising–Bloch type has been found, manifesting itself in a transition from static to moving polarization fronts [274].

Bloch domain walls have been found in OPO with cavity birefringence and/or dichroism taken into account. The associated dynamical behaviour is caused by the fact that walls of opposite chirality move spontaneously with opposite velocity [275].

The possibilities offered by intracavity type II SHG for all-optical parallel processing of images have been investigated by injecting an image in a linearly polarized pump beam and a homogeneous field with orthogonal polarization. Depending on the relative field amplitudes, either frequency and polarization transfer or contrast enhancement and contour recognition is favoured [276].

Most models have been derived in the mean-field limit. Beyond the mean-field approximation, models (often called propagation models) have been derived to study the formation of localized structures in a DOPO [154, 277]. A quantitative comparison between the mean-field and the propagation models for different values of mistunings in terms of stability domains and intensities amplitude has been published in [278].

8. Light valve with optical feedback

8.1. Liquid crystal cells

Following the pioneering work of the Akhmanov group [279, 280], the most popular type of light valve with optical feedback uses a liquid crystal as nonlinear medium. The liquid crystal layer is placed in a resonant cavity and is in contact with one of the mirrors which also supports a photoconductive layer, leading to a hybrid electro-optical photoconductor–liquid-crystal structure whose transparency can be controlled by the intensity-dependent nonlinear refractive index of the liquid crystal. The feedback loop begins with a beam splitter extracting a fraction of the field from the cavity. That field is sent back to the cavity via the photoconductive layer. The main feature is that in the feedback loop a device is inserted that modifies the transverse properties of the field. This has been called 2D feedback. The most common property that is modified in this way is the light polarization but any other transverse property of the beam (such as its centre or diameter) could serve as a variable.

A good review of the subject, starting from the basic principles of light valves with optical feedback, was published at the beginning of the period analysed here [281]. This paper reviews several physical configurations and the mechanisms by which they lead to certain rules of pattern selection.

From the liquid-crystal viewpoint, feedback, which introduces a dependence of the electric field on the liquid-crystal director, renders the Fredericksz transition first order [282].

The transverse spatial structures can be studied, e.g., with the feedback as control parameter which can be gradually tuned from purely diffractive to mixed interferential and diffractive [114]. Self-organized spot patterns have been analysed numerically from the point of view of their control. It has been shown how nonlinearity, diffraction, and diffusion can be designed for stable spots and stable spot motion [283]. The primary bifurcation corresponds to the destabilization of the homogeneous state to periodic patterns such as rolls. A secondary instability leads to roll dislocations [284]. It has been shown experimentally that using a Fourier filtered control signal can stabilize selectively unstable periodic patterns and can eliminate spatially chaotic regimes [285]. Defects can be swept out of a spontaneously formed hexagonal intensity pattern containing several dislocation-type defects using Fourier filtering [286].

Like most other nonlinear systems, light valves with optical feedback display localized structures (LSs). By discussing how a single LS depends on the system spatial frequency bandwidth, it was shown that a modification of the LS tail leads to the possibility of tuning the interactions between LS pairs, and thus the equilibrium distances at which

LS bound states form [287]. Another source of LSs appears to be the simultaneous presence of bistability and diffraction. In this regime, new features appear, such as the LS emergence on successive and concentric rings and their motion along the rings. The radial and azimuthal dynamics of the LSs in this regime has been described [288].

By modulating the input field with appropriate amplitude and frequency, chaotic domains are formed at different locations of the transverse space. The detection of a variety of different unstable periodic orbits and their presence for different detector sizes are strong indications that the dimensionality of the spatio-temporal chaos can be drastically reduced [289]. If the output of the system is a set of chaotically spaced spots, useful geometrical methods which apply to systems with a finite number of objects in a spatially restricted region can be used. A deeper insight is gained from the Voronoi construction [290] due to the diagnostic capability of the Voronoi cells for the local and global spatial spot arrangements [291].

Finally, geometrical constraints (cavity length and structure) may compete with physical constraints [292, 293]. For instance, for small diffusion and close to threshold, the system is forced to fulfil the geometrical constraints giving rise to a phase dynamics of quasi-crystals. For larger diffusion, the system fragments into spatial domains giving rise to a competition between different patterns.

8.2. *Na cells*

Another group of papers deals with a Na cell placed in a slightly different configuration, namely a single-mirror (or ‘open cavity’) feedback scheme [294]. Here also, LSs have been found experimentally. Experiments confirm the theoretical prediction that LSs have an oscillatory decaying tail originating from diffraction. Bound states of two or more constituents have been observed. These clusters contain several preferred mutual distances. Numerical simulations show that the LS interactions are mediated by the oscillatory tails [115]. In the vicinity of a parameter region with bistability between two homogeneous states, large amplitude peaks as well as dark holes exist as stable localized states on a hexagonal background. Moreover, resonant interaction between oscillatory and stationary inhomogeneous modes produces a nonstationary background which may force the localized states to drift [295].

A rich transition scenario between patterns of different symmetries (hexagons and squares) close to and beyond threshold, dependent on the magnitude of an external magnetic field, has been observed [296]. Secondary bifurcations from hexagons to squares occur via a time-dependent state whose origin has been traced back to noise-driven hexagon–square competition in a region of bistability between these two states [297]. The selection between hexagonal and square polarization patterns as a function of an external magnetic field yields a transition sequence that goes from squares via negative hexagons, squares, and positive hexagons again to squares, close to the instability onset. Well above threshold the hexagons give way to squares via a secondary bifurcation [298].

Rhombic and triangular patterns have been observed after a spontaneous symmetry-breaking bifurcation [299], while target and spiral patterns appear spontaneously from a Hopf bifurcation at a finite wavenumber [300]. Polarization degrees of freedom can be used to tailor and/or to manipulate optical nonlinearities in order to optimize the conditions for the pattern forming process [301].

Systematic deviations between the experiment and the linear stability analysis of the infinitely extended system have been shown to result from the finite diameter of the input beam [302]. Critical and noncritical slowing down have been demonstrated for the switching dynamics of bistable localized states [303]. Finally, quasi-patterns with an eightfold rotational symmetry and irregular two- and three-mode patterns have also been reported [304] and a secondary bifurcation from hexagons has been shown to yield patterns formed by 12 wavevectors [305].

9. Quantum properties of light

Quantum effects in pattern formation for a DOPO with walk-off have been carefully analysed using a method in which the pump field is treated as a c-number variable but is driven by the c-number representation of the quantum subharmonic signal field. This allows one to include the effects of the fluctuations in the signal on the pump, which in turn act back on the signal. Nonclassical effects, in the form of squeezing, survive just above the threshold of the convective regime. Above threshold, the macroscopic quantum noise suppresses these effects [306]. In the case of spatially tilted macroscopic signal beams, the transverse pattern formed in a DOPO above threshold still displays quantum correlations between the two beams, so that their intensity difference exhibits sub-Poissonian statistics [307]. While strong correlations between the fluctuations of the signal modes emitted at the critical wavenumber and with opposite wavevector are present both below and above threshold of the DOPO close to an instability for the formation of a square pattern, no feature signalling the square character of the pattern forming above threshold has been identified below threshold in the spatio-temporal second-order coherence [308]. A Q-representation has been used to show that non-classical correlations in an OPO are present just above the threshold, in the regime in which stripe patterns are formed, but that they also persist further above threshold in the presence of spatially disordered structures [309].

The spatial spectrum shows maximum squeezing at $\mathbf{k} = 0$ in singly resonant SHG below threshold for spatial modulational instability for parameters for which the intracavity fields are modulationally stable. In contrast, under conditions of modulational instability maximum squeezing occurs at the finite wavenumber $|\mathbf{k}| = k_c$, where k_c is the classical critical wavenumber [310].

An experimental demonstration of coincidence imaging using a classical source proves that coincidence imaging does not require entanglement. It is further found that any kind of coincidence imaging technique which uses a ‘bucket’ detector in the test arm is incapable of imaging phase-only objects, whether a classical or quantum source is employed [311].

10. Quantum images

Nonclassical spatial correlations arise in the cross section of transverse multimode optical beams as a consequence of nonlinear wave mixing phenomena. These correlations are a macroscopic manifestation of quantum entanglement. Quantum images is the term coined to describe inhomogeneous field distributions generated by quantum fluctuations. A review has covered most of the past millennium research on this topic [312]. In a more recent review, an up-to-date discussion of the basic concepts underlying the field can be found [313]. There have been developments mainly in two directions: entangled imaging and sub-shot noise displacements. Both rely on the properties of entangled photons and are mostly based on the properties of quadratic media, though it was recently realized that entangled imaging can be realized by means of incoherent sources, using a beam splitter to induce the necessary entanglement [311, 314]. Most papers on entangled imaging deal with laser beams used to pump a cavityless $\chi^{(2)}$ medium. In addition, the detection/diagnostic set-up, which is an essential part of the physical process, has nothing to do with cavity nonlinear optics. Therefore, they fall outside the focus of this review. The same applies to other related applications, such as quantum lithography [315–317], the quantum laser pointer [318–320], and quantum teleportation [321] which have also been developed using entangled states in cavityless set-ups.

A few results have been obtained in cavities. Quantum images have been shown to persist when passing from the degenerate to the non-degenerate case of OPO [322]. For type I LBO, a theoretical, numerical, and experimental study of the spatio-temporal properties of the spontaneous parametric emission around degeneracy has been performed [323, 324]. Spatial quantum noise properties of the one-dimensional transverse pattern formation instability in intracavity SHG have been investigated numerically. Close to the threshold for pattern formation, beams with opposite directions of the off-axis critical wavenumbers are shown to be highly correlated. This is observed for the fundamental field, for the second-harmonic field, and also for the cross-correlation between the two fields. Nonlinear correlations involving the homogeneous transverse wavenumber are also described [325]. The spatial distribution of quantum noise in the twin beams produced by a type-II OPO operating in a confocal cavity above threshold has been studied experimentally [326].

Finally, the quantum fluctuations for a Kerr medium in a planar resonator, taking into account the vectorial character of the radiation field, have been studied using a Langevin treatment based on the Wigner representation [327].

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