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TRANSVERSE VIBRATION OF TWO AXIALLY MOVING BEAMS CONNECTED BY AN ELASTIC FOUNDATION

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ABSTRACT

Transverse vibration of two axially moving beams connected by a Winkler elastic foundation is analyzed analytically. The system is a model of paper and paper-cloth (wire-screen) used in paper making. The two beams are tensioned, translating axially with a common constant velocity, simply supported at their ends, and of different materials and geometry. Due to the effect of translation, the dynamics of the system displays gyroscopic motion. The Euler-Bernoulli beam theory is used to model the deflections, and the governing equations are expressed in the canonical state form. The natural frequencies and associated mode shapes are obtained. It is found that the natural frequencies of the system are composed of two infinite sets describing in-phase and out-of-phase vibrations. In case the beams are identical, these modes become synchronous and asynchronous, respectively. Divergence instability occurs at the critical velocity; and, the frequency-velocity relationship is similar to that of a single traveling beam. The effects of the mass, flexural rigidity, and axial tension ratios of the two beams, as well as the effects of the elastic foundation stiffness are investigated.

1. INTRODUCTION

Axially moving materials have many engineering applications like magnetic tape systems, fiber winders, power transmission belts, textile and paper web handling machinery [1]. Axially moving materials typically are modeled as a string or as an Euler-Bernoulli beam [2, 3].

Web is a generic name used for thin, flexible continuous materials such as magnetic tapes and papers. Paper making is one of the oldest of the industries involved with web handling, with more than a century of history. In the papermaking process, paper fibers are mixed with water, and this pulp slurry is sprayed onto a large, flat, fast-moving wire-screen, sometimes called the paper-cloth. As the wire-screen translates along the paper machine, the water drains out, and the fibers bond together. The paper web is pressed between rolls in order to squeeze out more water and it is further dried by heated rollers. The stiffness of paper increases as it is dried along the path of the machine. The paper is eventually rolled and removed from the machine. Vibration problems can arise during transport of the paper-wire system, where excessive vibration could cause the paper to separate from the wire-screen prematurely. In this work the translating wire/paper system is modeled as two translating beams, connected by an elastic foundation. The elastic foundation is used, without much justification, to represent the bonding between the wire and the paper.

To the best of the authors' knowledge the vibration of such a system has not been considered in the literature. On the other hand, vibration of translating single beams/strings and vibration of non-translating, double beam/string systems have been studied extensively.

The vibration of a translating string supported by an elastic foundation was studied by Perkins [5], Wickert [4] and Parker [6]. Perkins studied the axially moving, string supported by a distributed elastic foundation and obtained the natural frequencies and corresponding mode shapes and examined the subcritical frequencies [5]. Parker found that the elastic foundation does not alter the lowest critical speed, but the supercritical stability is changed by the elastic foundation [6]. Oz *et al.* investigated the transition from string to beam for an axially moving material and obtained the natural frequencies expressions [7].

Tabarrok *et al.* derived the governing equation of motion of an axially moving beam including the effect of flexural rigidity of the beam [8]. Barakat investigated the transverse vibrations of moving thin rod, and obtained the natural frequencies and corresponding mode shapes for fixed and simply supports at ends [9]. Wickert and Mote presented a closed form solution for axially moving continua, subjected to arbitrary excitations and initial conditions, using complex modal analysis and Green's function method [4,10]. Wickert investigated the nonlinear vibration of an axially moving, tensioned, Euler-Bernoulli beam [11]. Ulsoy investigated the transverse vibration with coupling between spans of axially moving beam and the effect of tension variation [12]. Chakraborty *et al.* analyzed the free and forced vibration of a traveling beam having an intermediate guide, including non-linear effects, using the complex normal mode method [13]. They modeled the guide as a purely elastic constraint with no inertia, and found that the choice of a suitable guide location plays an important role in controlling the vibrations. Riedel and Tan investigated the free response of an elastically constrained, axially moving string and beam using transform function method, and studied the effects of speed and tension on the natural frequencies [14].

The use of two (or more) non-translating beams, connected by elastic foundation(s) is common in engineering, and a variety of problems adopt it as a model. The basic model uses a Winkler foundation, in which the beams are connected through closely spaced, but non-interconnected linear springs, which is defined by the foundation modulus *k*.

A considerable number of theoretical and experimental work on the transverse vibrations of such systems has been performed. Seelig *et al.* solved the system of double beams connected by elastic foundation [15]. They obtained the natural frequencies and associated mode shapes for various supports at ends limited to equal masses and flexural rigidities. Kessl determined the resonances of a simply supported, elastically connected double beam system that is subjected to a cyclic load [16].

Although most of the studies on transverse vibrations of double beam systems consider numerical approximations, many researchers tried to analyze the vibrations using analytical solution, He *et al.* presented an analytical solution for the coupled transverse and longitudinal vibration of multi-span beam systems with arbitrary boundary conditions [17]. They used the energy method and Hamilton's principal to derive the governing equations of motion and the corresponding essential and natural boundary conditions for each beam. Using Green function method, Kukla obtained analytical solution for systems of axially loaded beams with several boundary conditions [18]. He considered system of double beams connected by translational springs. Chen *et al.* presented the exact solutions for the natural frequencies and mode shapes of non-uniform beams with multiple spring-mass systems using Numerical Assembly Method (NAM) [19].

Vu *et al.* developed a closed form solution for the vibration of a double beam system subject to harmonic motion by decoupling the governing equations [20]. Their method is valid only for identical beams and identical boundary conditions for each side. Oniszczuk presented exact analytical solutions and performed full theoretical vibration analysis of a discrete two degree of freedom system with arbitrary damping [21]. His analysis is valid only for identical masses, viscous damping, and spring constants as his method is based on decoupling the differential equations of motion through a specific transform of spatial displacement. Cha proposed an alternative formulation for determining the eigenvalues of a linear elastic structure carrying any number of concentrated masses, springs, and viscous dampers [22]. Oniszczuk applied the theory proposed in reference [21] on a damped vibration analysis of an elastically connected complex double string system and obtained analytical solutions for a special case of identical masses, springs, and dampers [23].

Oniszczuk also analyzed undamped free transverse vibrations of an elastically connected complex double beam system [24]. He applied the classical modal expansion method for the case of simply supported beams. He obtained an analytical solution using pre-assumed mode shape function and determined the natural frequencies and the complete dynamic response.

2. PROBLEM STATEMENT

The system shown in Fig. 1 consists of two parallel, slender, prismatic and homogeneous beams, joined by a Winkler foundation of stiffness *k*. Both beams have the same length between the two supports, simply supported at ends, axially translating, and axially tensioned, to p_1 and p_2 as shown.

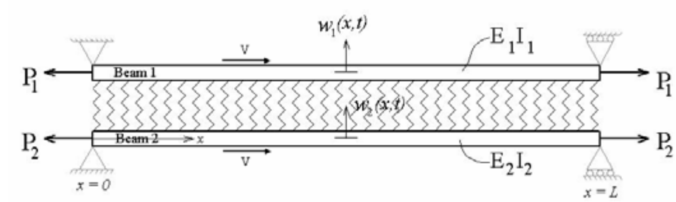

Figure 1 Double beams connected by elastic foundation

The coupled governing equations of the free transverse vibrations for the system are derived using Bernoulli-Euler beam theory and can be written as:

$$m_1\left(\frac{\partial^2 w_1}{\partial t^2}+2V\frac{\partial^2 w_1}{\partial x\partial t}+V^2\frac{\partial^2 w_1}{\partial x^2}\right)+E_1I_1\frac{\partial^4 w_1}{\partial x^4}-p_1\frac{\partial^2 w_1}{\partial x^2}+k\left(w_1-w_2\right)=0 \qquad (1)$$

$$m_2\left(\frac{\partial^2 w_2}{\partial t^2}+2V\frac{\partial^2 w_2}{\partial x\partial t}+V^2\frac{\partial^2 w_2}{\partial x^2}\right)+E_2I_2\frac{\partial^4 w_2}{\partial x^4}-p_2\frac{\partial^2 w_2}{\partial x^2}+k\left(w_2-w_1\right)=0 \qquad (2)$$

where $w_i = w_i(x,t)$ are the transverse deflections of the two beams (i=1, 2), x is the spatial coordinate, t is the time, m_i the mass per unit length, E_i is the Young's modulus, I_i is the second moment of area of the beam cross section, k is the stiffness of the Winkler foundation, V is the axial translation speed of the beams. Both beams are assumed to be simply supported at their ends x=0 and x=L. The simply support boundary conditions are:

$$w_1(0,t)=\frac{\partial^2 w_1(0,t)}{\partial x^2}=w_1(L,t)=\frac{\partial^2 w_1(L,t)}{\partial x^2}=0 \qquad (3)$$

$$w_2(0,t)=\frac{\partial^2 w_2(0,t)}{\partial x^2}=w_2(L,t)=\frac{\partial^2 w_2(L,t)}{\partial x^2}=0 \qquad (4)$$

The two governing equations can be written in the following non-dimensional form:

$$\frac{\partial^4 W_1}{\partial X^4}-(\mu^2-\nu^2)\frac{\partial^2 W_1}{\partial X^2}+2\nu\frac{\partial^2 W_1}{\partial X\partial T}+\frac{\partial^2 W_1}{\partial T^2}+K(W_1-W_2)=0 \qquad (5)$$

$$\frac{\partial^4 W_2}{\partial X^4}-((\frac{R_s}{R_p})\mu^2-(\frac{R_s}{R_m})\nu^2)\frac{\partial^2 W_2}{\partial X^2}+2\nu(\frac{R_s}{R_m})\frac{\partial^2 W_2}{\partial T\partial X}+(\frac{R_s}{R_m})\frac{\partial^2 W_2}{\partial T^2}+R_sK(W_2-W_1)=0 \quad (6)$$

where

$$X=\frac{x}{L};\ W_i=\frac{w_i}{L};\quad T=t(\frac{E_1I_1}{m_1L^4})^{1/2};\quad \mu^2=\frac{p_1L^2}{E_1I_1};$$
$$K=\frac{kL^4}{E_1I_1};\ R_m=\frac{m_1}{m_2};\ R_s=\frac{E_1I_1}{E_2I_2};\ R_p=\frac{p_1}{p_2} \quad (7)$$
$$\bar{\lambda}=\frac{\lambda}{(\frac{E_1I_1}{m_1L^4})^{1/2}};\ \nu=V(\frac{m_1L^2}{E_1I_1})^{1/2}$$

where T is the non-dimensional time, μ^2 is the non-dimensional tension parameter, K is the non-dimensional elastic foundation stiffness, R_m is the mass ratio of the beams, R_s is the flexural stiffness ratio of the beams, R_p is the axial load ratio, $\bar{\lambda}$ is the non-dimensional complex natural frequency, and ν is the non-dimensional axial translation speed. The non-dimensional forms of the boundary conditions become:

$$W_1(0,T)=\frac{\partial^2 W_1(0,T)}{\partial X^2}=W_1(1,T)=\frac{\partial^2 W_1(1,T)}{\partial X^2}=0 \quad (8)$$
$$W_2(0,T)=\frac{\partial^2 W_2(0,T)}{\partial X^2}=W_2(1,T)=\frac{\partial^2 W_2(1,T)}{\partial X^2}=0 \quad (9)$$

3. SOLUTION METHOD

The response of the lower beam can be written in terms of the response of the upper beam, by using Eq. (5) as follows:

$$W_2=\frac{1}{K}[\frac{\partial^4 W_1}{\partial X^4}-(\mu^2-\nu^2)\frac{\partial^2 W_1}{\partial X^2}+2\nu\frac{\partial^2 W_1}{\partial X\partial T}+\frac{\partial^2 W_1}{\partial T^2}+KW_1] \quad (10)$$

Equations (5) and (6) can then be combined into a single eighth-order partial differential equation:

$$\frac{\partial^8 W_1}{\partial X^8}+A_1\frac{\partial^6 W_1}{\partial X^6}+A_2\frac{\partial^6 W_1}{\partial X^5\partial T}+A_3\frac{\partial^6 W_1}{\partial X^4\partial T^2}+A_4\frac{\partial^4 W_1}{\partial X^4}+A_5\frac{\partial^4 W_1}{\partial X^3\partial T}+A_6\frac{\partial^4 W_1}{\partial X^2\partial T^2}+A_7\frac{\partial^4 W_1}{\partial X\partial T^3}+A_8\frac{\partial^4 W_1}{\partial T^4}+A_9\frac{\partial^2 W_1}{\partial X^2}+A_{10}\frac{\partial^2 W_1}{\partial T^2}+A_{11}\frac{\partial W_1}{\partial T}=0 \quad (11)$$

where the coefficients are given as,

$$A_1=(1+\frac{R_s}{R_m})\nu^2-(1+\frac{R_s}{R_p})\mu^2 \quad (a)$$
$$A_2=2\nu(1+\frac{R_s}{R_m}) \quad (b)$$
$$A_3=(1+\frac{R_s}{R_m}) \quad (c)$$
$$A_4=K(1+R_s)+(\frac{R_s}{R_m}\nu^2-\frac{R_s}{R_p}\mu^2)(\nu^2-\mu^2) \quad (d)$$
$$A_5=2\nu(\frac{R_s}{R_m}\nu^2-\frac{R_s}{R_p}\mu^2)+2\nu(\frac{R_s}{R_m})(\nu^2-\mu^2) \quad (e)$$
$$A_6=(\frac{R_s}{R_m}\nu^2-\frac{R_s}{R_p}\mu^2)+(\frac{R_s}{R_m})(\nu^2-\mu^2)+(2\nu)^2(\frac{R_s}{R_m}) \quad (f) \quad (12)$$
$$A_7=4\nu(\frac{R_s}{R_m}) \quad (g)$$
$$A_8=(\frac{R_s}{R_m}) \quad (h)$$
$$A_9=K(\frac{R_s}{R_m}\nu^2-\frac{R_s}{R_p}\mu^2)+R_sK(\nu^2-\mu^2) \quad (i)$$
$$A_{10}=R_sK(1+\frac{1}{R_m}) \quad (j)$$
$$A_{11}=2\nu R_sK(1+\frac{1}{R_m}) \quad (k)$$

The system of equations given by (5) and (6) can be written in the form of a system of second order differential equations as:

$$\boldsymbol{M}\boldsymbol{W}_{,TT}+\boldsymbol{G}\boldsymbol{W}_{,T}+\boldsymbol{K}^*\boldsymbol{W}=\boldsymbol{f} \quad (13)$$

Where a subscripted comma $_{,T}$ indicates partial differentiation, and

$$\boldsymbol{W}=\begin{Bmatrix}W_1\\W_2\end{Bmatrix},\quad \boldsymbol{M}=\begin{bmatrix}1&0\\0&(\frac{R_s}{R_m})^2\end{bmatrix},\quad \boldsymbol{G}=\begin{bmatrix}2\nu&0\\0&2\nu(\frac{R_s}{R_m})\end{bmatrix},\quad \boldsymbol{f}=\begin{Bmatrix}f_1\\f_2\end{Bmatrix} \quad (14)$$

$$\boldsymbol{K}^*=\begin{bmatrix}(\nu^2-\mu^2)&0\\0&(\frac{R_s}{R_m}\nu^2-\frac{R_s}{R_p}\mu^2)\end{bmatrix}\frac{\partial^2}{\partial X^2}+\begin{bmatrix}1&0\\0&1\end{bmatrix}\frac{\partial^4}{\partial X^4}+\begin{bmatrix}1&-1\\-R_s&R_s\end{bmatrix}K \quad (15)$$

$\boldsymbol{M}$, $\boldsymbol{G}$, $\boldsymbol{K}^*$ are the mass, gyroscopic and stiffness operators, respectively, and $\boldsymbol{f}$ is the vector external forces. In general, f_1 and f_2 are functions of X and T. As the system response depends on the applied load and initial conditions, the system can be described using state variables to fully describe any arbitrary initial conditions. On the other hand, the orthogonality of eigenfunctions with respect to each operator is of fundamental importance in vibrations. It plays an indispensable role in the solution of the differential equations of motion associated with the vibration of linear systems [10] and it is well confirmed [25-28]. Thus, the equations of motion can be expressed in the form of state space representation in the first-order form [27]:

$$\boldsymbol{A}\boldsymbol{U}_{,T}+\boldsymbol{B}\boldsymbol{U}=\boldsymbol{q} \quad (16)$$

where the state and excitation vectors are:

$$\boldsymbol{U}=\{W_{1,T}\ W_{2,T}\ W_1\ W_2\}^T\ ,\quad \boldsymbol{q}=\{f_1\ f_2\ 0\ 0\}^T \quad (17)$$

and the matrix differential operators are:

$$\boldsymbol{A}=\begin{bmatrix}\boldsymbol{M}&0\\0&\boldsymbol{K}^*\end{bmatrix}\ ;\quad \boldsymbol{B}=\begin{bmatrix}\boldsymbol{G}&\boldsymbol{K}^*\\-\boldsymbol{K}^*&0\end{bmatrix} \quad (18)$$

Equation (16) is the canonical form of the equation of motion (13), where **A** is a symmetric and **B** is a skew symmetric matrix.

The inner product of two vectors U_1 and U_2 is defined as

$$\langle U_1, U_2 \rangle = \int_0^1 U_1^T \overline{U}_2 dX \quad (19)$$

where the over bar denotes complex conjugation.

4. NATURAL FREQUENCIES ANALYSIS

The general solution Eq. (16) can be written in the form:

$$U_i(X,T) = \text{Re}\left\{\hat{\phi}_i(X) e^{\overline{\lambda} T}\right\} \quad (20)$$

where the eigenvalues $\overline{\lambda}$ and the eigenfunctions $\hat{\phi}_i$ are complex, and $i = 1, 2$. By assuming the eigenfunction for beam-1 in the general form as follows:

$$\hat{\phi}_1(X) = \sum_{k=1}^{8} c_k e^{i\gamma_k X} \quad (21)$$

and substituting Eqs. (20) and (21) in Eq. (10), the eigenfunction for beam-2 is found to be:

$$\hat{\phi}_2(X) = \sum_{k=1}^{8} B_k c_k e^{i\gamma_k X} \quad (22)$$

with,

$$B_k = \frac{1}{K}\left[\gamma_k^4 + (\nu^2 - \mu^2)\gamma_k^2 + 2\nu i \gamma_k \overline{\lambda} + \overline{\lambda}^2 + K\right]. \quad (23)$$

In order to obtain the eigenvalues for the double-beam system, boundary conditions, in Eqs. (8-9) are evaluated using Eqs. (21-22). This results in eight homogeneous algebraic equations, which is represented in matrix form as:

$$\mathbf{A}\,\mathbf{c} = 0 \quad (24)$$

where $\mathbf{c} = \{c_1\ c_2\ c_3 \ldots c_8\}^T$ is the coefficient vector, and $\mathbf{A}$ is the matrix of coefficients. In order to have a nontrivial solution, the determinant of matrix $\mathbf{A}$ must be zero. This gives the characteristic equation of the system. The natural frequencies are determined from the solution of the characteristic equation. A computer program, using the symbolic mathematics language Mathematica™, is developed to determine these complex natural frequencies.

5. RESULTS AND DISCUSSION

In this section the free transverse vibrations of two axially translating simply supported, tensioned beams connected by an elastic foundation are investigated. First the mode shapes and the natural frequencies are analyzed. Then, the effects of foundation stiffness and axial tension are investigated for different system parameters.

5.1 Mode Shapes and Natural Frequencies

This work showed that the natural frequencies of the translating double beam system are divided into two fundamental odd and even sets ω_{1n} and ω_{2n}, where the subscript n = 1, 2, … The distinction becomes clear in Fig. 2 where the first four mode shapes, corresponding to the first four natural frequencies are plotted, for the parameters K = 100, ν = 5, μ = 10, $R_m = R_p = R_s = 1$. As expected the mode shapes have real and imaginary parts. However, the mode shapes for ω_1 and ω_3 show that the two beams experience synchronous deflection. On the other hand, the mode shapes for ω_2 and ω_4 show asynchronous deflections. It is also seen that the mode shapes are not symmetrical with respect to the mid-span of the beams, due to the effects of translation, as was also observed by Wickert and Mote [10]. The separation of mode shapes into asynchronous and synchronous behaviors is the result of the coupling of the two beams by the Winkler foundation, and it is observed for non-translating beam systems (e.g., [24]).

In case the beams are not identical, the vibration of the two beams still show in-phase and out-of-phase characteristics, for odd- and even-modes, respectively, as shown in Figure 3, for a case where the mass ratio R_m = 0.6. Close inspection of the figure shows that the modes are not parallel to each other. This effect is observed for other R_m values as well as R_p and R_s values, and parallelism of the modes deteriorate further with decreasing values of R_m, R_p, and R_s. To the best of our knowledge this is the first observation of this behavior for a translating beam system.

The natural frequencies of this system as a function of the non-dimensional transport speed is shown in Figure 4. Figure 4a shows that similar to a single, simply supported, axially moving beam analyzed in reference [10], the first natural frequency vanishes at the critical speed $\nu_c = \left(\mu^2 + \pi^2\right)^{1/2}$. This result is expected, as the odd numbered natural frequencies are not affected by the presence of the Winkler foundation. Hence, the onset of divergence instability for the double beam system analyzed here is identical to the case of the single beam, and the elastic stiffness does not alter the divergence instability. The even numbered frequencies behave in a similar way to the odd frequencies, except their divergence occurs at larger translation speeds.

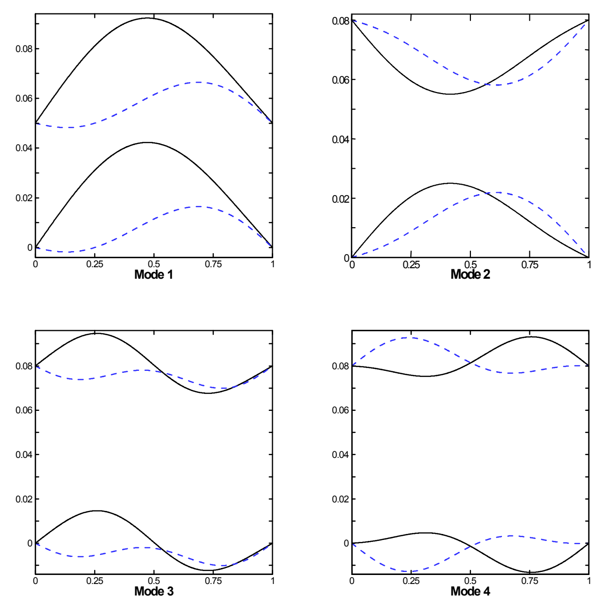

Figure 2 The first four complex mode shapes. The real (solid) and imaginary parts (dashed) for K = 100, ν = 5, μ = 10, R_m = R_p = R_s = 1.

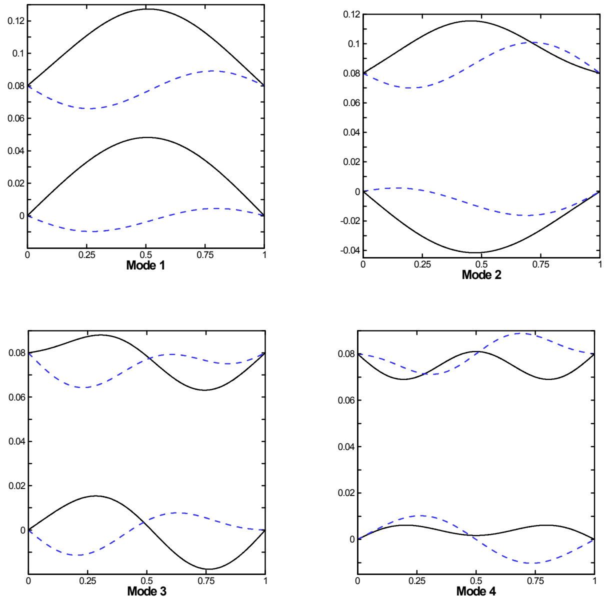

Figure 3 The first four complex mode shapes. The real (solid) and imaginary parts (dashed) for K = 100, ν = 5, μ = 10, R_p = R_s = 1, R_m =0.60.

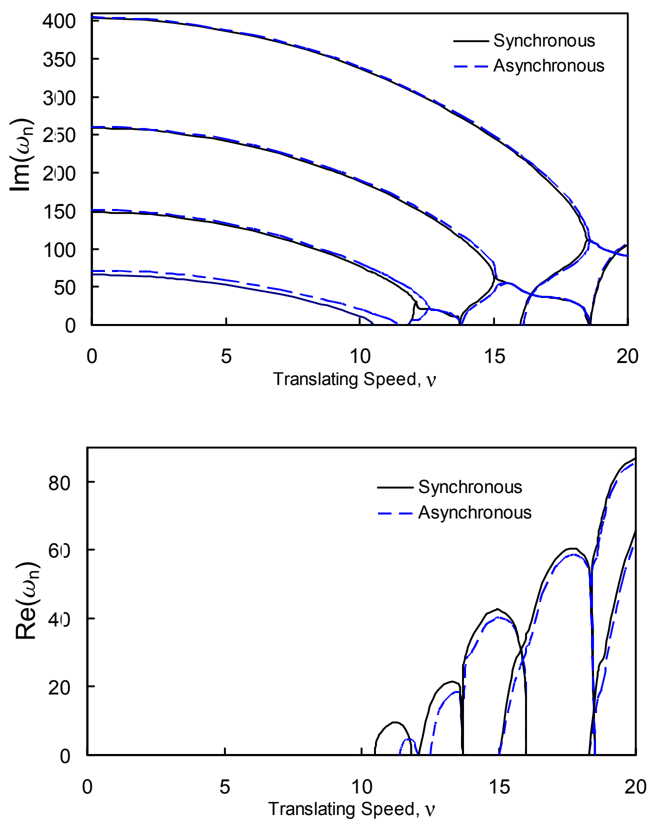

Figure 4 The imaginary and real parts of the frequency spectrum for translating double beams system; the solid curves indicate the synchronous frequencies and dashed curves indicate the asynchronous frequencies. (K = 100, μ = 10, R_m = R_s = R_p =1).

The real part of the natural frequencies plotted in Figure 4b, shows that flutter instabilities occur along with divergence for translation speeds greater than ν_c. This behavior is not altered as compared to the single beam case. Moreover, the asynchronous frequency spectrum behaves qualitatively the same way as the synchronous part.

5.2 Effect of Foundation Stiffness on Natural Frequencies

In order to demonstrate the effect the elastic foundation stiffness K on the vibration of the system, the first six natural frequencies of the system are investigated, for different values of the non-dimensional tension ratio R_p, mass ratio R_m, bending rigidity ratio R_s, in Figure 5. The imaginary part of the natural frequencies are given for $0 \le K \le 500$, while the base values are chosen as $R_m = R_s = R_p = 1$, μ = 10, and ν = 5.

Figure 5a shows the effect of elastic foundation stiffness for the base values. This figure shows that the synchronous mode frequencies, $\omega_1, \omega_3, \omega_5$, are not affected by the stiffness of the foundation, K, and remain 53, 131, 242, respectively, as expected. On the other hand, the asynchronous frequencies, $\omega_2, \omega_4, \omega_6$, increase slightly with increasing K. This increase is due to the general stiffening of the system with increasing K values.

Figure 5b shows the effect of mass ratio parameter R_m = 0.1. Note that if it is assumed that R_m = 0.1 corresponds to a ten fold increase in the mass of beam-2, all of the other non-dimensional parameters affected by mass given in Equation (7) remain constant. This figure shows that decreasing the mass ratio to R_m = 0.1 causes the natural frequencies to increase slightly with respect to Figure 5a. But, otherwise the natural frequencies remain nearly constant with increasing K.

The effect of bending rigidity ratio R_s = 0.1 is given in Figure 5c. This R_s value should be considered to correspond to a 10 fold increase of the bending rigidity of beam-2. Using R_s = 0.1 causes the even mode frequencies to move higher, but the odd mode frequencies remain nearly at their previous (R_s = 1) values, except for ω_5. It is clear that increasing the elastic foundation stiffness K does not significantly affect the natural frequencies.

Figure 5d shows the effect of axial tension ratio for R_p = 0.3. It is seen that the even mode frequencies increase, while the odd mode frequencies remain nearly constant. As in the other cases the elastic foundation stiffness has a small effect on the frequencies.

In summary, rendering R_m, R_s, and R_p values less than one does not affect the odd frequencies but increases the even frequencies. This effect is expected as mass, bending-rigidity and tension of beam-2 increase with decreasing R_m, R_s, and R_p values. The foundation stiffness has a very small effect on frequencies over a wide range (0 – 500).

5.3 Effect of Axial Tension on Natural Frequencies

The effects the non-dimensional tension parameter μ on the natural frequencies of the system are shown in Figure 6, where the imaginary parts of the natural frequencies are presented for

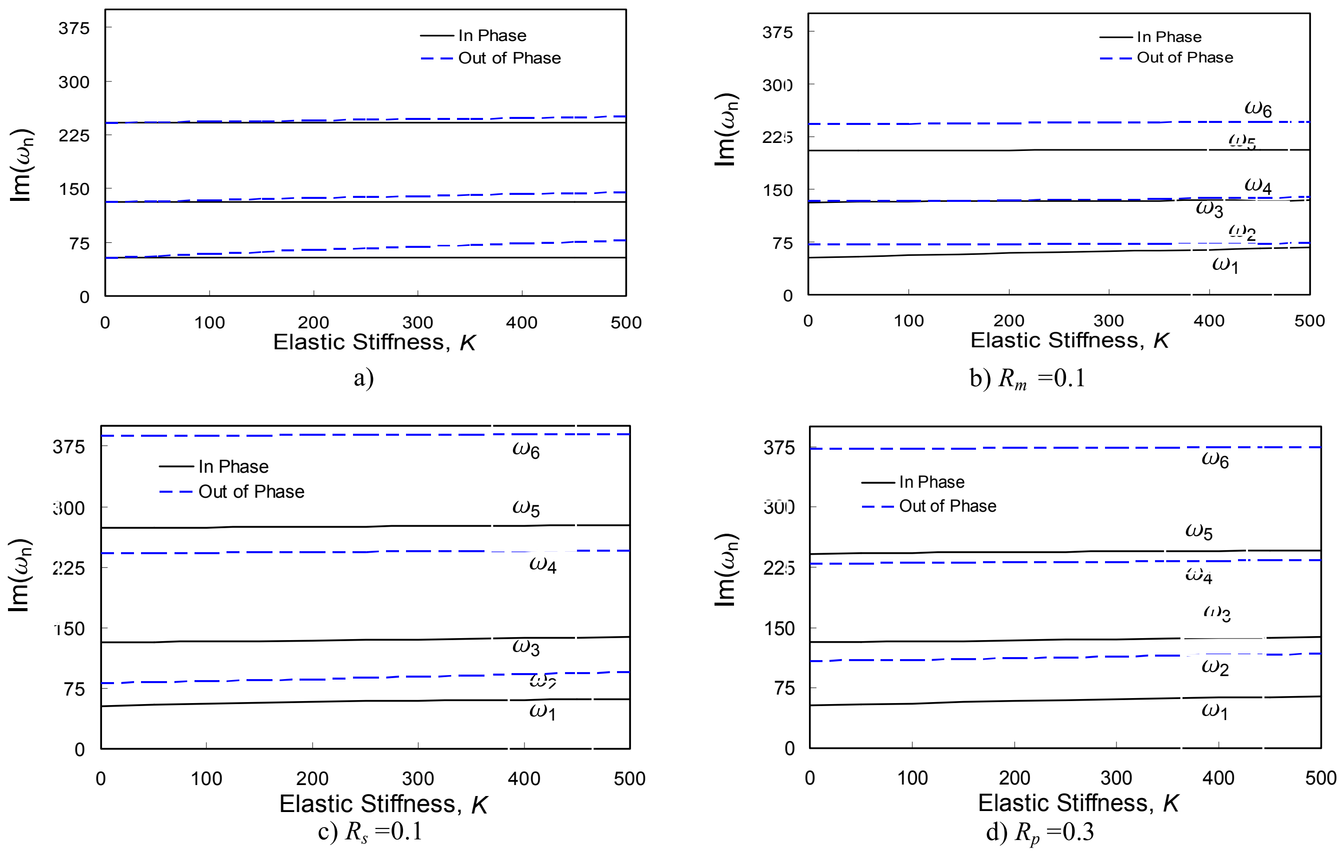

Figure 5 The natural frequencies versus non-dimensional elastic foundation stiffness K for $\mu = 10$, $\nu = 5$, a) $R_m = R_s = R_p = 1$; b) $R_m = 0.1$ and $R_s = R_p = 1$; c) $R_m = 1$, $R_s = 0.1$ and $R_p = 1$; and d) $R_m = R_s = 1$ and $R_p = 0.3$.

different values of R_m, R_s, and R_p. As before, the base values are chosen as $R_m = R_s = R_p = 1$, $K = 100$ and $\nu = 5$, unless otherwise noted.

Figure 6a shows the effect of axial tension for base parameters. This figure shows that frequencies are increasing with increasing μ, while the values of the odd-mode and even-mode natural frequencies remain relatively close. Note that for these parameters, the critical value of the tension parameter is approximately $\mu_{cr} = 3.9$.

Figure 6b shows the effect of mass ratio parameter $R_m = 0.1$. The odd-mode frequencies are not significantly affected, while the even-mode frequencies move higher with increasing μ. When the mass ratio is reduced to 0.1 it is found that the critical value of the tension parameter is approximately $\mu_{cr} = 3.2$.

The effect of bending rigidity ratio, $R_s = 0.1$, is shown in Figure 6c. This figure shows that the first two odd-mode frequencies are not affected significantly by the drop of R_s, however the third odd-mode frequency and the even-mode frequencies increase with respect to $R_s = 1$. The critical value of the tension parameter is approximately $\mu_{cr} = 2$.

Figure 6d shows the effect of the axial load ratio. Even-mode frequencies increase significantly for larger values of the tension parameter, by the reduction of R_p to 0.3, as compared to $R_p = 1$. The critical value of the tension parameter is approximately $\mu_{cr} = 2.8$.

In summary, with increasing tension parameter the stiffness of the system increases, causing the natural frequencies of the odd- and even-modes to increase. Increasing the mass, bending-rigidity and tension of beam-2, as described in Section 5.2, affects the even-modes; and causes their natural frequencies to increase, as expected.

6. SUMMARY AND CONCLUSIONS

The free transverse vibration of an elastically connected axially loaded, simply supported, axially translating double beam system is considered. The two beams have the same length, translation speed, and boundary conditions. The system of governing partial differential equations is cast in the first order canonical form as state space form. The natural frequencies and mode shapes are obtained. Divergence instability occurs at the critical speed, and flutter and divergence instabilities coexist in post critical speeds. It is found that, in the case of identical beams the presence of the elastic foundation does not affect the critical speed.

In general, the natural frequencies of the system are composed of two infinite sets, ω_{1n} and ω_{2n}. When the two beams are identical, the free vibrations are described by synchronous and asynchronous vibrations, with ω_{1n} and ω_{2n}, respectively. The vibrations still show in-phase and out-of-phase characteristics, as the parameters of the beams change.

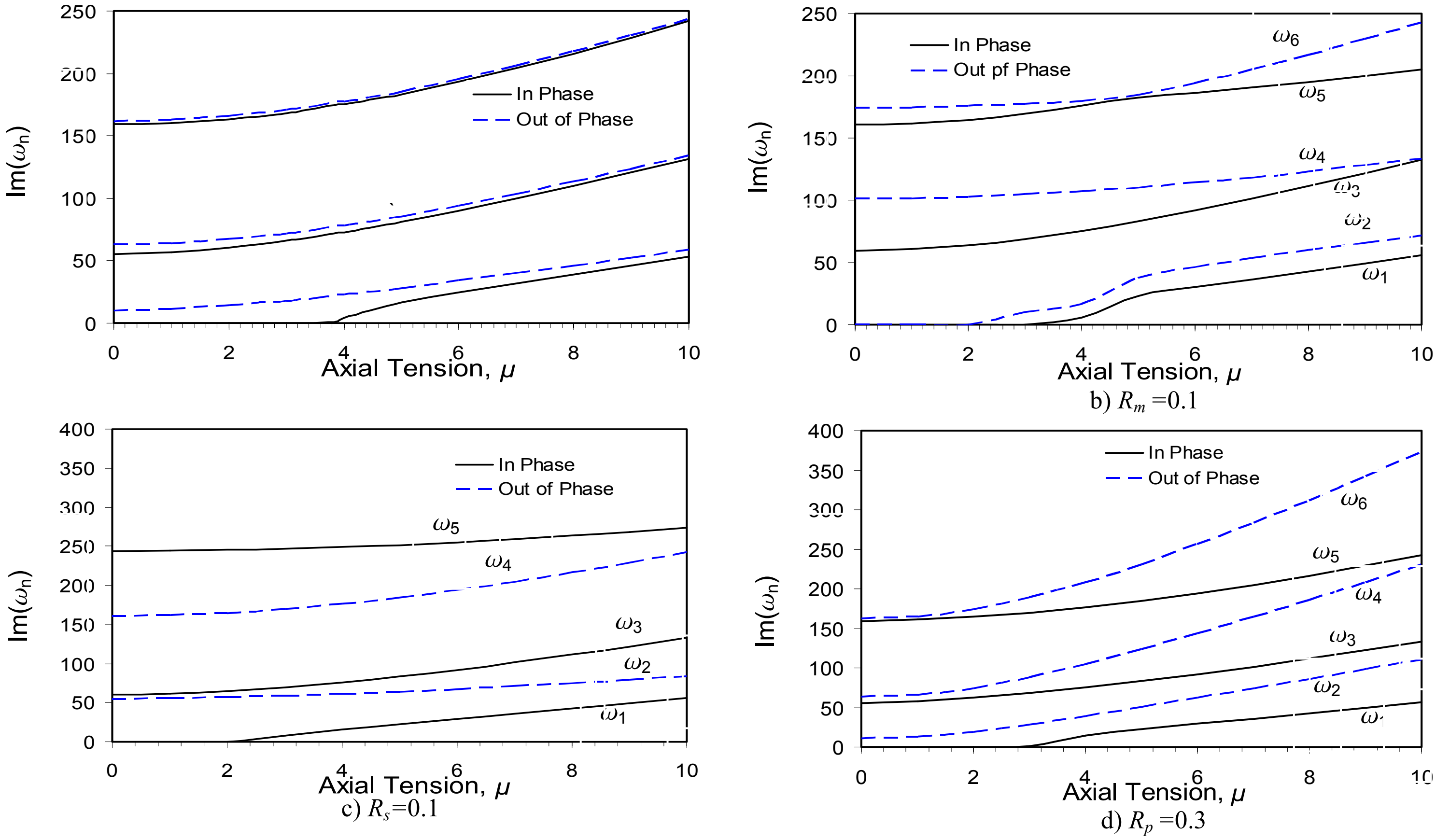

Figure 6 The natural frequencies versus non-dimensional axial tension parameter, μ, for K = 100, ν = 5, a) $R_m = R_s = R_p = 1$; b) $R_m = 0.1$ and $R_s = R_p = 1$; c) $R_m = 1$, $R_s = 0.1$ and $R_p = 1$; and d) $R_m = R_s = 1$ and $R_p = 0.3$.

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