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Trapped-Ion Quantum Simulator

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Abstract

Coherent manipulations involving the quantized motional and internal states of a single trapped ion can be used to simulate the dynamics of other systems. We consider some examples, including the action of a Mach Zehnder interferometer which uses entangled input states. Coherent manipulations can also be used to create entangled states of multiple trapped ions; such states can be used to demonstrate fundamental quantum correlations.

1. Introduction

Stimulated, in part, by the interest in quantum computation and quantum communication [1], a number of papers have investigated the possibility of synthesizing or "engineering" arbitrary quantum states of trapped ions (for recent reviews, see Refs. [2] and [3]). To the extent that this can be accomplished for a large number of trapped ions, such a system would allow general quantum computations, including the factorization of large numbers [1, 4]. By anybody's reckoning, factorizing large numbers is a daunting task. Therefore, it is desirable that a quantum computer, or a system which can generate arbitrary entangled states, have wider applicability. Various possibilities have been explored in the recent literature [5–11]. Some of these proposals extend the ideas of Feynman who considered whether or not one quantum system could be used to simulate the behavior of another quantum system [12]. In this spirit, we discuss some simple examples of how a single trapped ion might be used to simulate the behavior of other quantum systems, such as entangled particles acted on by a Mach Zehnder interferometer. We also briefly discuss how the states of multiple trapped ions can be entangled; these states can be employed in fundamental demonstrations of quantum measurements.

2. Coupling of a two-level trapped ion to its motion

We first consider a single ion trapped in a 3-D harmonic well with oscillation frequencies ω_x, ω_y and ω_z along three cartesian axes. This situation is closely approximated by a single ion confined in a Paul (rf) trap. We will be interested in two internal states of the ion which we label as $|\uparrow\rangle$ and $|\downarrow\rangle$, and which are separated in energy by $\hbar\omega_0$. We apply a (classical) radiation field or fields (typically laser fields) of the form

$$\boldsymbol{E}(\boldsymbol{x}, t) = \boldsymbol{E}_0 \cos(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t + \phi). \tag{2.1}$$

The Hamiltonian which describes the (resonant) coupling between the ion's internal states and its motion (provided by $\boldsymbol{E}(\boldsymbol{x}, t)$) can be written in the rotating-wave approximation as

$$H_{\mathrm{I}} = \hbar\Omega(S_+)^{\varepsilon}\, \mathrm{e}^{\mathrm{i}[\boldsymbol{k}\cdot\boldsymbol{x} - \delta t + \phi]} + h.c., \tag{2.2}$$

where Ω is the coupling strength (Rabi frequency), S_+ is the raising operator for the internal states ($S_+|\downarrow\rangle = |\uparrow\rangle$), $\boldsymbol{x}$ is the ion's position relative to its equilibrium position, $\delta \equiv \omega - \omega_0$, and ϕ is a phase factor of the field [3]. In eqs (2.1) and (2.2), $\boldsymbol{k}$ is the wavevector of the field for single photon transitions or $\boldsymbol{k}$ is the difference between the two wavevectors when two-photon stimulated-Raman transitions are used [13]. Similarly, ω is the frequency of the applied field for single-photon transitions, or is the difference in frequencies of the two applied fields when stimulated-Raman transitions are used. The exponent ε is equal to 1 when internal state transitions are involved and $\varepsilon = 0$ when the internal state is unchanged (stimulated-Raman transitions).

In an interaction picture of the ion's motion, this Hamiltonian becomes [3]

$$H_{\mathrm{I}} = \hbar\Omega(S_+)^{\varepsilon}\, \mathrm{e}^{-\mathrm{i}(\delta t - \phi)} \prod_{j=x,y,z} \exp\left(\mathrm{i}[\eta_j(a_j\,\mathrm{e}^{-\mathrm{i}\omega_j t} + a_j^{\dagger}\,\mathrm{e}^{\mathrm{i}\omega_j t})]\right) + h.c., \tag{2.3}$$

where a_j and $a_j^{\dagger}$ are the lowering and raising operators for harmonic motion in the jth direction, and $\eta_x \equiv \boldsymbol{k}\cdot\hat{\mathbf{x}}x_0$ is the Lamb-Dicke parameter in the x direction, where $x_0 \equiv \sqrt{\hbar/(2m\omega_x)}$ (m is the ion mass), and similarly for η_y and η_z. Now, assume that Ω is small enough, and that ω_x, ω_y and ω_z are incommensurate so that we can (resonantly) excite only one spectral component of the possible transitions induced by this interaction. For a particular resonance condition $\delta = -l_x\omega_x - l_y\omega_y - l_z\omega_z$ (l_j integers), and in the Lamb-Dicke limit, we find

$$H_{\mathrm{I}} \simeq \hbar\Omega\, \mathrm{e}^{\mathrm{i}\phi}(S_+)^{\varepsilon} \prod_{j=x,y,z}\left[\delta_{l_j,|l_j|}\frac{(\mathrm{i}\eta_j a_j)^{|l_j|}}{|l_j|!} + (1-\delta_{l_j,|l_j|})\frac{(\mathrm{i}\eta_j a_j^{\dagger})^{|l_j|}}{|l_j|!}\right] + h.c. \tag{2.4}$$

The two mode case where $\varepsilon = l_z = 0$, $l_x, l_y \neq 0$ is considered by Drobný and Hladký [14], and in a different excitation scheme by Steinbach, *et al.* [15]. If the Lamb-Dicke limit is not rigorously satisfied, we must consider higher-order terms in the expansion of the exponentials of eq. (2.3) [16]; specific examples are discussed by Wallentowitz and Vogel [17], and Steinbach, *et al.* [15]. These nonlinear terms appear as corrections to the Rabi frequencies for Fock states and have been observed in the experiments of Ref. [18].

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3. Simulations of processes in optics

Referring to eq. (2.4), the carrier and first red and blue (z-motion) sidebands on internal state transitions (e.g., $\varepsilon = 1$, $l_x = l_y = 0$, $l_z = 0$, ± 1) are used in experiments to cool the ion to the ground state of motion [13, 19], for quantum logic [20], and to generate nonclassical motional states [2, 18]. The upper and lower sidebands ($l_z = \pm 1$) correspond to emission and absorption of single vibrational quanta or "phonons" associated with internal state changes; this is directly analogous to the emission and absorption of single photons into a cavity by an atom inside. An interesting system which could be simulated with these couplings is a "phonon maser" which provides vibrational amplification by stimulated emission [21]. The case $\varepsilon = 0$, $l_x = l_y = 0$, $|l_z| = 1$ has been used to create coherent [18] and Schrödinger cat [22] states of motion. Coherent states of ion motion correspond to coherent states in optics. The case $\varepsilon = 0$, $l_x = l_y = 0$, $|l_z| = 2$ has been used to create squeezed states of ion motion [18]. A realization of the Hamiltonian $H_\mathrm{I} \propto S_+(a^\dagger)^2 + h.c.$ ($\varepsilon = 1$, $l_x = l_y = 0$, $l_z = -2$) for ions has been reported by Leibfried, *et al.* [23]. This is similar to the case of two-photon excitation in cavity QED analyzed by Buck and Sukumar [24] and Knight [25]. An example of an interesting new case would perhaps be the realization of three-phonon downconversion (e.g., $\varepsilon = 0$, $l_x = 3$, $l_y = -1$, $l_z = 0$); this is accomplished by driving a two-mode resonance using stimulated-Raman transitions where the difference in frequencies of the two laser beams is equal to $\omega_y - 3\omega_x$. This case corresponds to three-photon downconversion in quantum optics (see Refs. [15], [26], and references therein). A suggestion to realize a Hamiltonian proportional to $a_x^2 a_y^\dagger + h.c.$, ($\varepsilon = 0$, $l_z = 0$, $l_x = 2$, $l_y = -1$) is discussed by Agarwal and Banerji [27].

Clearly, a very large number of possibilities could, in principle, be realized just for a single ion; moreover, the number of possibilities increases dramatically if we consider all modes of motion for multiple trapped ions. The only limitation on how high $|l_j|$ in eq. (2.4) can be is that Ω be chosen sufficiently small that couplings to other (unwanted) resonances are avoided. This will require that decoherence be small enough to see the desired dynamical behavior before coherence is lost. Finally, the analogy to optics discussed here should not be surprising since a single ion's motion (for one mode) and a single mode of the radiation field are both described by quantized harmonic oscillators.

4. Mach Zehnder interferometer with entangled states

Realization of the various Hamiltonians indicated in eq. (2.4) can lead to simulation of various devices of practical interest. As an example, we can simulate the action of a Mach Zehnder interferometer for various input states. We consider H_I to act on two modes of ion motion; to be specific, we will assume these are the x and y modes. The analogy with a Mach-Zehnder interferometer for bosons is that the two input modes to the boson interferometer are replaced by the x and y modes of ion oscillation. The (50/50) beamsplitters in the boson interferometer are replaced by an operator [28–30]

$$B_\pm = \exp\left[\pm i\pi(a_x^\dagger a_y + a_x a_y^\dagger)/4\right] \tag{4.1}$$

This operator can be realized by applying the interaction in eq. (2.4) with $\varepsilon = l_z = 0$, and $l_x = -l_y = 1$ for a time given by $\Omega\eta_x\eta_y t = \pi/4$. A differential phase shift between the two arms of the interferometer can be simulated by shifting the relative phases of the fields in eq. (2.4) between successive applications of $B_\pm$. In a particle (e.g., boson) interferometer, one typically measures the number of particles in either one or both output modes. For single ions, the experiments so far have only one convenient observable, the internal state of the ion (either $|\downarrow\rangle$ or $|\uparrow\rangle$). Nevertheless, we can fully characterize the action of the phonon interferometer by repeating the experiment many times and measuring the density matrix of the output state [23, 31].

It will be interesting to characterize the action of the interferometer for various nonclassical input states. One interesting input state is the two-mode Fock state $|n_x\rangle_x|n_y\rangle_y$ [32]. This state could be prepared by applying the Fock state creation techniques described in Ref. [18] sequentially to the ion's x and y modes. This state is interesting because it has been shown that one could approach the Heisenberg uncertainty limit in a Mach Zehnder interferometer by measuring the distribution of bosons in the output modes [32–34]. The observable is the variance of the number of particles detected in one of the output ports when the arms of the interferometer are of approximately equal length. As the difference in length of the arms deviates from equality, the variance increases sharply. An alternative technique for studying the action of a beamsplitter on the two-mode Fock states has been suggested by Gou and Knight [35] when $\omega_x = \omega_y$. Here, a beamsplitter could be simulated by first preparing $|n_x\rangle_x|n_y\rangle_y$ along two orthogonal axes and then probing along two other axes (x' and y') which are rotated (in the xy plane) with respect to the first. This technique could also be used to analyze, for example, the $(|0\rangle_{x'}|2\rangle_{y'} + |2\rangle_{x'}|0\rangle_{y'})/\sqrt{2}$ state from an initially prepared $|1\rangle_x|1\rangle_y$ state [35].

Another interesting input state to consider is the state $B_\pm^\dagger(|N\rangle_x|0\rangle_y + |0\rangle_x|N\rangle_y)/\sqrt{2}$. (Equivalently, the state after the first beam splitter is $(|N\rangle_x|0\rangle_y + |0\rangle_x|N\rangle_y)/\sqrt{2}$.) This state has been shown to yield exactly the Heinsenberg uncertainty limit for an interferometer for any value of N and any difference of the lengths of the arms [8]. The observable is the parity of the number of particles measured in one of the output ports. For example, we could measure the number of particles $N(x)$ in the x output port. The result of this measurement is assigned the value $(-1)^{N(x)}$.

For a single ion, the state after the first beamsplitter could be prepared from the $|\downarrow\rangle|0\rangle_x|0\rangle_y$ state by the following two steps:

(1) Apply a $\pi/2$ pulse on the Nth blue sideband of mode x ($\varepsilon = 1$, $l_x = -N$, $l_y = l_z = 0$); this creates the state $(|\downarrow\rangle|0\rangle_x + |\uparrow\rangle|N\rangle_x)|0\rangle_y/\sqrt{2}$.

(2) Apply a π pulse on the Nth blue sideband of mode y ($\varepsilon = 1$, $l_x = l_z = 0$, $l_y = -N$); this creates the state $|\uparrow\rangle(|N\rangle_x|0\rangle_y + |0\rangle_x|N\rangle_y)/\sqrt{2}$.

After the second beamsplitter, we have a state which can be written as

$$\Psi_\mathrm{final} = |\uparrow\rangle \sum_{n_x=0}^{N} C_{n_x}|n_x\rangle_x|N - n_x\rangle_y. \tag{4.2}$$

We now want to measure n_x, record the value $N(x)$, and assign the value $(-1)^{N(x)}$ to the overall measurement. Effectively, this assignment can be accomplished if we can find interaction M which provides the mapping

$$M\Psi_{\text{final}} = |\uparrow\rangle \sum_{n_x \text{ even}}^{N} C_{n_x} e^{i\phi(n_x)} |n_x\rangle_x |N - n_x\rangle_y + |\downarrow\rangle \sum_{n_x \text{ odd}}^{N} C_{n_x} e^{i\phi(n_x)} |n_x\rangle_x |N - n_x\rangle_y. \quad (4.3)$$

After this mapping, we need only measure the internal state; if the ion is found in the $|\uparrow\rangle$ we assign the value $+1$ to the measurement; if the ion is found in the $|\downarrow\rangle$ state, we assign the value -1. The mapping M can be achieved by applying radiation with $\boldsymbol{k}\|\hat{\mathbf{x}}$ at the carrier frequency ($\varepsilon = 1$, $l_x = l_y = l_z = 0$) and insuring $\Omega_{n_x, n_x} t = 2\pi m \pm n_x \pi$ where m is an integer. Here Ω_{n_x, n_x} is the Rabi frequency for $|\downarrow\rangle \leftrightarrow |\uparrow\rangle$ which depends on n_x due to terms in the expansion of $\exp(i\boldsymbol{k} \cdot \boldsymbol{x})$ which are nonlinear in $\boldsymbol{x}$. We find [3]

$$\Omega_{n,n} t \simeq \Omega t\, e^{-\eta_x^2/2}\left[1 - n\eta_x^2\left(1 + \frac{\eta_x^2}{4} - n\frac{\eta_x^2}{4}\right)\right]. \quad (4.4)$$

Therefore, if we satisfy $\Omega \exp(-\eta_x^2/2)t = 2\pi m$ and $\eta_x^2(1 + \eta_x^2/4) = (2m)^{-1}$, we achieve the desired mapping as long as the contribution to the phase from the term proportional to n_x^2 in this equation is small compared to π. Therefore we require $m \gg N^2/8$ or, equivalently, $\eta_x \ll 2/N$.

If N is large and/or the Lamb-Dicke parameter is very small, creating the state $|\uparrow\rangle(|N\rangle_x|0\rangle_y + |0\rangle_x|N\rangle_y)/\sqrt{2}$ by steps (1) and (2) above may be very slow. If we use an auxiliary internal state, we can speed up this process by employing first-order sidebands. To be specific, we will assume we can realize a coupling of the form of eq. (2.4), between the $|\uparrow\rangle$ state and auxiliary state which we label $|A\rangle$. We assume state $|A\rangle$ is lower in energy than state $|\uparrow\rangle$ so that $S_{A+}|A\rangle = |\uparrow\rangle$. A particular realization of states $|\downarrow\rangle$, $|\uparrow\rangle$, and $|A\rangle$ is described in Ref. [20]. As an example, starting with the state $|\downarrow\rangle|0\rangle_x|0\rangle_y$, we can create the state $(|\uparrow\rangle(|3\rangle_x|0\rangle_y + |0\rangle_x|3\rangle_y)/\sqrt{2}$ with the following steps (and appropriate choices of ϕ):

$$|\downarrow\rangle|0\rangle_x|0\rangle_y - (S_+(\pi/2) + h.c.) \rightarrow$$

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)|0\rangle_x|0\rangle_y - (S_{A+}(\pi) + h.c.) \rightarrow$$

$$\frac{1}{\sqrt{2}}(|A\rangle + |\downarrow\rangle)|0\rangle_x|0\rangle_y - (S_{A+}\, a_x^\dagger(\pi) + h.c.) \rightarrow$$

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle|1\rangle_x + |\downarrow\rangle|0\rangle_x)|0\rangle_y - (S_{A+}\, a_x(\pi) + h.c.) \rightarrow$$

$$\frac{1}{\sqrt{2}}(|A\rangle|2\rangle_x + |\downarrow\rangle|0\rangle_x)|0\rangle_y - (S_+\, a_y^\dagger(\pi) + h.c.) \rightarrow$$

$$\frac{1}{\sqrt{2}}(|A\rangle|2\rangle_x|0\rangle_y + |\uparrow\rangle|0\rangle_x|1\rangle_y) - (S_+\, a_y(\pi) + h.c.) \rightarrow$$

$$\frac{1}{\sqrt{2}}(|A\rangle|2\rangle_x|0\rangle_y + |\downarrow\rangle|0\rangle_x|2\rangle_y) - (S_{A+}\, a_x^\dagger(\pi) + h.c.) \rightarrow$$

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle|3\rangle_x|0\rangle_y + |\downarrow\rangle|0\rangle_x|2\rangle_y) - (S_+\, a_y^\dagger(\pi) + h.c.) \rightarrow$$

$$\frac{1}{\sqrt{2}}|\uparrow\rangle(|3\rangle_x|0\rangle_y + |0\rangle_x|3\rangle_y). \quad (4.5)$$

In this expression, the notation $S_{A+}\, a_x^\dagger(\pi) + h.c.$ means the operator $S_{A+}\, a_x^\dagger + h.c.$ is applied for a time sufficient to drive a π pulse, etc. From this, it is straightforward to see how to generate the state $|\uparrow\rangle(|N\rangle_x|0\rangle_y + |0\rangle_x|N\rangle_y)/\sqrt{2}$ for N odd. For N even, we can, for example, substitute two carrier transitions for two sideband transitions in the above steps.

One final example of a two-mode interferometer which directly yields Heisenberg $1/N$ phase sensitivity is a "beam-splitter" which creates that state $(|\downarrow\rangle|N\rangle_x|0\rangle_y + |\uparrow\rangle|0\rangle_x|N\rangle_y)/\sqrt{2})$. This state (for N even) could be created as in eq. (4.5) except the last two operations are replaced by the operation $S_{A+}(\pi) + h.c.$ For example, to create the state $(|\uparrow\rangle|2\rangle_x|0\rangle_y + |\downarrow\rangle|0\rangle_x|2\rangle_y)/\sqrt{2})$, we replace the last two steps of eq. (4.5) by

$$\frac{1}{\sqrt{2}}(|A\rangle|2\rangle_x|0\rangle_y + |\downarrow\rangle|0\rangle_x|2\rangle_y) - (S_{A+}(\pi) + h.c.) \rightarrow$$

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle|2\rangle_x|0\rangle_y + |\downarrow\rangle|0\rangle_x|2\rangle_y). \quad (4.6)$$

For N odd, we can, for example, substitute two carrier transitions for two sideband transitions in the above steps. If an auxiliary state is not available, this state can be created by first making the initial dual Fock state $(|\uparrow\rangle + |\downarrow\rangle)|N/2\rangle_x|N/2\rangle_y/\sqrt{2}$ with the methods described in Ref. [18]. (In this example, we assume N is even.) Next, we apply $N/2$ π-pulses alternating between the two interaction Hamiltonians $H_1 = \Omega\eta_x\eta_y S_+ a_x^\dagger a_y + h.c.$ and $H_2 = \Omega\eta_x\eta_y S_+ a_x a_y^\dagger + h.c.$. In this way, the ion is stepped through the sequence

$$|\downarrow\rangle|N/2\rangle_x|N/2\rangle_y - (S_+(\pi/2) + h.c.) \rightarrow$$

$$\frac{1}{\sqrt{2}}(|\downarrow\rangle|N/2\rangle_x|N/2\rangle_y + |\uparrow\rangle|N/2\rangle_x|N/2\rangle_y) - (H_1) \rightarrow$$

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle|N/2+1\rangle_x|N/2-1\rangle_y + |\downarrow\rangle|N/2-1\rangle_x|N/2+1\rangle_y) - (H_2) \rightarrow$$

$$\frac{1}{\sqrt{2}}(|\downarrow\rangle|N/2+2\rangle_x|N/2-2\rangle_y + |\uparrow\rangle|N/2-2\rangle_x|N/2+2\rangle_y) - (H_1) \rightarrow$$

$$\cdots - (H_2) \rightarrow \frac{1}{\sqrt{2}}(|\downarrow\rangle|N\rangle_x|0\rangle_y + |\uparrow\rangle|0\rangle_x|N\rangle_y). \quad (4.7)$$

The interactions H_1 and H_2 follow from eq. (2.4) with $\varepsilon = 1$, $l_x = -1$, $l_y = 1$, $l_z = 0$, and $\varepsilon = 1$, $l_x = 1$, $l_y = -1$, $l_z = 0$ respectively. The kth pulse has Rabi frequency $\Omega\eta_x\eta_y\sqrt{(N/2+k)(N/2-k+1)}$ in the Lamb-Dicke regime. After a relative phase is accumulated in the two "paths" of the interferometer (simulated by adjusting the phase of the laser pulses as discussed above), then the steps in eqs (4.7) or (4.5) and (4.6) are reversed. Upon measuring the probability of occupation in state $|\downarrow\rangle$ or $|\uparrow\rangle$, the interference fringes exhibit $1/N$ phase sensitivity.

If the Lamb-Dicke criterion is not satisfied, the two components of the wavefunction superposition may experience different Rabi frequencies during each pulse, leading to undesired evolution. However, as long as $\eta_x = \eta_y$ it can be shown that the system will evolve as in eq. (4.7), even when the Lamb-Dicke criterion is not satisfied [3].

5. Quantum correlations

The coherent manipulations on single ions discussed above can be extended to multiple ions [4]. As one step towards this goal, a controlled-not quantum logic has been demonstrated [20] between qubits formed with the ground and first excited state for one mode of motion ($|n_x = 0\rangle_x \equiv |0\rangle_m$, $|n_x = 1\rangle_x \equiv |1\rangle_m$) and the ion's internal states ($|\downarrow\rangle \equiv |0\rangle$, $|\uparrow\rangle \equiv |1\rangle$). The controlled-not gate exhibited the logic

$$|\varepsilon_1\rangle_m|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle_m|\varepsilon_1 \oplus \varepsilon_2\rangle \tag{5.1}$$

where $\oplus$ signifies addition mod 2. For a collection of ions in a trap, we select a particular mode (say the center-of-mass mode along the axis of a linear Paul trap) to comprise the motional qubit. By first mapping the internal state of ion i onto the motional qubit (which is shared by all ions), performing the logic in eq. (5.1) between the motional qubit and ion j, followed by reversing the first mapping step, we can realize a controlled-not logic operation between ion i and ion j [4]

$$|\varepsilon_1\rangle_i|\varepsilon_2\rangle_j \rightarrow |\varepsilon_1\rangle_i|\varepsilon_1 \oplus \varepsilon_2\rangle_j. \tag{5.2}$$

Application of these gates to small numbers of trapped ions can lead to interesting experiments which may shed light on the viability of local hidden-variables theories. For example, for two ions, starting with the state $|0\rangle_1|0\rangle_2$ we can apply a $\pi/2$ pulse to the internal states of ion 1 followed by a controlled-not between ions 1 and 2

$$\begin{aligned}|0\rangle_1|0\rangle_2 &\rightarrow \frac{1}{\sqrt{2}}(|0\rangle_1 + |1\rangle_1)|0\rangle_2 \\ &\rightarrow \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 + |1\rangle_1|\rangle_2).\end{aligned} \tag{5.3}$$

If the states of the resulting entangled particles are detected outside of each other's light cones, then, for particular sets of measurements, we may derive Bell's inequalities [36] which local hidden-variables theories must obey, but which quantum mechanics violates. The experiments performed by Aspect and co-workers [37] (and more recent versions – see Ref. [38] and A. Zeilinger and P. Kwiat, these proceedings) provide strong evidence against local hidden-variables theories. The Aspect *et al.* experiments used polarization measurements on entangled pairs of photons. The detection of the photons' polarization states occurred outside each others' light cones. Thus, the measurement on one photon could not have affected the other measurement, which closed possible "loopholes" in the proof of quantum mechanics over other explanations.

However, some loopholes still remain open. Since the photon detection in the Aspect *et al.* experiments was not 100% efficient, the group had to make assumptions that the photons they measured were a "fair" sample of the whole population of events. Thus, their experiments do not rule out the (seemingly implausible) possibility of local hidden-variables theories in which the hidden variables cause some sub-ensemble of the photon pairs to preferentially interact with the measurement apparatus.

In the system of two ions, we may detect the state of either ion with nearly 100% efficiency through the use of "electron-shelving" (for a discussion, see Ref. [3]). On the other hand, it may be difficult to perform measurements on two ions outside each other's light cone. Such a measurement would require separating the ions by a distance larger than the speed of light times the measurement time. In principle of course, the ions could be first entangled and then placed in different traps which could be separated by large distances before measurements were performed. Alternatively, it may be possible to entangle distant pairs of ions using optical fibers [11]. Nonetheless, an experiment with two entangled ions confined in the same trap could be viewed as complementary to those of Aspect and others: the photon experiments definitively close loopholes of causality, and the ion experiments could close loopholes due to detection inefficiency. Such experiments have the additional appeal of studying EPR on massive particles (E. Fry, these proceedings). EPR states of atoms have recently been created in an atomic beam using the methods of cavity QED (S. Haroche, these proceedings); if detection efficiency can be improved, these experiments could also close loopholes due to detection inefficiency. Finally, even though measurements of quantum correlations between entangled ions cannot be easily performed outside each other's light cone, one can argue strongly that the ions cannot transfer information by any known mechanism. Therefore, if the observed correlations violate Bell's inequalities, the correlations are established by some new force of nature or are, in fact, inherent in the structure of quantum mechanics.

An intriguing possibility for ions is the possibility of making "GHZ states" [39, 40]. For three ions, the GHZ state has the form

$$\Psi = \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2|0\rangle_3 + e^{i\phi}|1\rangle_1|1\rangle_2|1\rangle_3). \tag{5.4}$$

This state can be made starting with the state $|0\rangle_1|0\rangle_2|0\rangle_3$, applying the first two steps shown in eq. (5.3), and following with a controlled-not gate between ions 1 and 3 [4]. For such a state, a single measurement can distinguish between the predictions of quantum mechanics and those of any local hidden-variables theory [39, 40].

Aside from these possibilities, Bell states, GHZ states, and Schrödinger-cat states are highly entangled, and are thus of inherent interest for the study of uniquely quantum behavior. As the experiments improve, it will be interesting to push the size of entangled states to be as large as possible. The question is not whether we can make states which have the attributes of Schrödinger cats, but how big can we make the cats? Certain theories which address the measurement problem will be amenable to experimental tests, for example, quantitative limits on spontaneous wavefunction collapse theories [41, 42] can be established. The isolation from the environment exhibited by trapped ions, coupled with the control possible over their quantum state and high detection efficiency make them an interesting laboratory for the study of fundamental issues in quantum mechanics.

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