World Applied Sciences Journal 21 (12): 1857-1869, 2013
ISSN 1818-4952

DOI: 10.5829/idosi.wasj.2013.21.12.2696

Trigonometric Curve Fitting Based on Multi-step Differential Transformation Method and the Application of Non-linear Oscillatory Systems

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Abstract: In this article, trigonometric curve fitting on the multi-step differential transformation method (MsDTM) is implemented to give approximate analytical solutions of nonlinear ordinary differential equation such as non-linear oscillation systems. In addition, the solution obtained from MsDTM applied trigonometric curve fitting. Our proposed approach reveals that results to approximate analytical solutions of non-linear oscillation systems. Some plots are presented to show the reliability and simplicity of the methods.

Key words: Curve fitting . the multi-step differential transformation method . non-linear oscillatory systems

INTRODUCTION

This study will consider the following general non-linear oscillations [1-4]:

$$y'' + w_0^2 y + \lambda g(y) = f(t) \tag{1}$$

with initial conditions

$$y(0) = A,\ y'(0) = 0 \tag{2}$$

Here g is a nonlinear function depending on y and its derivatives.

A review of the recently developed analytical methods is given in review article and the comprehensive book by He [5, 6]. In the last decade, several computational methods have been applied to solve nonlinear oscillation systems. Some of these well-known methods are such as: variational iteration method [7], homotopy perturbation method [8], homotopy analysis method [9], Exp-function method [10, 11], linearized perturbation method [12], modified Lindstedt-Poincare methods [13], iteration perturbation method [14], bookkeeping parameter perturbation method [15], energy balance method [16], amplitude frequency formulation [17], max-min approach [18, 19], rational harmonic balance method [20], Mickens iteration procedure [21].

The differential transform method (DTM) is a numerical as well as analytical method for solving integral equations, ordinary, partial differential equations and differential equation systems. The concept of the differential transform was first proposed by Zhou [22] and its main application concern with both linear and nonlinear initial value problems in electrical circuit analysis. The DTM gives exact values of the nth derivative of an analytic function at a point in terms of known and unknown boundary conditions in a fast manner. This method constructs, for differential equations, an analytical solution in the form of a polynomial. It is different from the traditional high order Taylor series method, which requires symbolic computations of the necessary derivatives of the data functions. The Taylor series method is computationally taken long time for large orders. The DTM is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. Different applications of DTM can be found in [23-26].

The aim of this Letter is to apply DTM and multi-stage DTM [27] to solve the non-linear oscillator equation. The numerical solutions are compared with the multi-stage DTM and by classical RK4.

This paper is organized as follows:

In section 2, 3 and 4, we describe DTM and multi-stage DTM briefly. To show in efficiency of this method, we give some examples and numerical results in Section 5. The conclusions are then given in the final section 6.

SOLUTION APPROACHES

Consider a general equation of n-th order ODE

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$$f\left(t, y, y', \ldots, y^{(n)}\right) = 0 \quad (3)$$

subject to the initial conditions

$$y^{(k)}(0) = d_k, k = 0 \ldots n-1 \quad (4)$$

system in (3), we will next present the solution approaches of (3) base on standard DTM and MsDTM separately.

SOLUTION OF BY DTM

To illustrate the differential transformation method (DTM) for solving differential equations systems, the basic definitions of differential transformation are introduced as follows. Let y(t) be analytic in a domain D and let t = t_i represent any point in D. The function y(t) is then represented by one power series whose center is located at t_i. The differential transformation of the kth derivative of a function y(t) is defined as follows:

$$Y(k) = \frac{1}{k!}\left[\frac{d^k y(t)}{dt^k}\right]_{t=t_i}, \forall t \in D \quad (5)$$

In (5), y(t) is the original function and Y(K) is the transformed function. As in [22-28] the differential inverse transformation of Y(K) is defined as follows:

$$y(t) = \sum_{k=0}^{\infty} Y(k)\left(t - t_i\right)^k, \forall t \in D \quad (6)$$

In fact, from (5) and (6), we obtain

$$y(t) = \sum_{k=0}^{\infty} \frac{\left(t - t_i\right)^k}{k!}\left[\frac{d^k y(t)}{dt^k}\right]_{t=t_i}, \forall t \in D \quad (7)$$

Eq. (7) implies that the concept of differential transformation is derived from the Taylor series expansion. From the definitions of (5) and (6), it is easy to prove that the transformed functions comply with the following basic mathematics operations (Table 1)

In real applications, the function y(t) is expressed by a finite series and (5) can be written as

$$y(t) = \sum_{k=0}^{N} Y(k)\left(t - t_i\right)^k, \forall t \in D \quad (8)$$

Equation (8) implies that $\sum_{k=N+1}^{\infty} Y(k)\left(t - t_i\right)^k$ is negligibly small.

Table 1: Operations of the one dimensional differential transform

Original function	Transformed function
$y(t) = f(t) \mp g(t)$	$Y(k) = F(k) \mp G(k)$
$y(t) = \xi f(t)$	$Y(k) = \xi F(k)$
$y(t) = \frac{df(t)}{dt}$	$Y(k) = (k+1) F(k+1)$
$y(t) = \frac{d^n f(t)}{dt^n}$	$Y(k) = \frac{(k+n)!}{k!} F(k+n)$
$y(t) = e^{\lambda t}$	$Y(k) = \frac{\lambda^k}{k!}$
$y(t) = \sin(wt + \varphi)$	$Y(k) = \frac{w^k}{k!}\sin\left(\frac{k\pi}{2} + \varphi\right)$
$y(t) = \cos(wt + \varphi)$	$Y(k) = \frac{w^k}{k!}\cos\left(\frac{k\pi}{2} + \varphi\right)$
$y(t) = \int_0^t f(t)dt$	$Y(k) = \frac{F(k-1)}{k}$
$y(t) = t^r$	$Y(k) = \delta(k-r) = \begin{cases} 1, & k = r, \\ 0, & \text{otherwise} \end{cases}$
$y(t) = f(t)g(t)$	$Y(k) = \sum_{k_1=0}^{k} F(k_1) G(k - k_1)$
$y(t) = f(t)g(t)h(t)$	$Y(k) = \sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2} F(k_1) G(k_2 - k_1) H(k - k_2)$

SOLUTIONS BY MSDTM

Let [0,T] be the interval over which we want to find the solution of the initial value problem (3). In actual applications of the DTM, the approximate solution of the initial value problem (3)-(4) can be expressed by the finite series,

$$y(t) = \sum_{i=0}^{N} b_i t^i \quad t \in [0,T] \quad (9)$$

Assume that the interval [0,T] is divided into N subintervals [t_{n-1}, t_n], n = 1,2,…,N of equal step size h = T/N by using the nodes t_n = nh. The main ideas of the multi-step DTM are as follows [43]. First, we apply the DTM to Eq. (3) over the interval [0,t_1], we will obtain the following approximate solution,

$$y_1(t) = \sum_{i=0}^{N} b_{1i} t^i, \; t \in [0, t_1] \quad (10)$$

using the initial conditions $y_1^{(k)}(0) = d_k$. For n≥2 and at each subinterval [t_{n-1}, t_n], we will use the initial conditions $y_n^{(k)}(t_{n-1}) = y_{n-1}^{(k)}(t_{n-1})$ and apply the DTM to Eq. (3) over the interval [t_{n-1}, t_n], where t_i in Eq. (3) is replaced by t_{n-1}. The process is repeated and generates a sequence of approximate solutions $y_n(t)$, n = 1,2,…,N for the solution y(t),

$$y_n(t)=\sum_{i=0}^{N} b_{ni}(t-t_{n-1})^i,\ t\in[t_n,t_{n+1}] \quad (11)$$

In fact, the multi-step DTM assumes the following solution,

$$y(t)=\begin{cases} y_1(t), & t\in[0,t_1] \\ y_2(t), & t\in[t_1,t_2] \\ \vdots & \\ y_N(t), & t\in[t_{N-1},t_N] \end{cases} \quad (12)$$

TRIGONOMETRIC CURVE FITTING

We consider data obtained from the multi-step DTM. The problem of data fitting consist of finding a function f that "best" represents the data which are subject to errors. A reasonable way to approach the problem is to plot data points in an xy-plane and try to recognize the shape of a guess function $f(t)$. Trigonometric fitting is a method to approximate a function f by series of trigonometric functions. Trigonometric functions in general form can be written.

$$\begin{aligned} f(t)=&A_1\cos(w_1t+\varphi_1)+A_2\cos(w_2t+\varphi_2) \\ &+\cdots+A_i\cos(w_it+\varphi_i)+\cdots+D \\ &(i=1,2,\dots,n) \end{aligned} \quad (13)$$

As a function approximately describing the data, in which

$$M=(A_1,w_1,\varphi_1,A_2,w_2,\varphi_2,\dots,A_i,w_i,\varphi_i,\dots,D)^T$$

is the unknown coefficients. Set is y_i is the found value at the point x_i (i = 1,2,….,n), y_i' is the found value at the point x_i, is the result of the calculation by the fitting function at point, the mean square error of the n data points

$$\delta=\sqrt{\sum_{i=1}^{n}(y_i-y_i')^2}$$

obviously the smaller the value of this function, the better results are. Under general conditions, according to the properties of boundary of trigonometric functions, the correlation between the maximum pure trigonometric function (A), the maximum(max) and minimum(min) of data is A≤(max+min)/2; as the periodic characteristics of the trigonometric function (period 2π) the $w_it+\varphi_i$ data will repeat every 2π every π positive and negative values will be changed. So, w_it and φ_I 's absolutely value should be less than π; considering the accuracy, the minimum spacing between numerical as T is determined to be maximum absolute value of w_i for the π/T. D range of value; min≤D≤max [49, 52-56].

APPLICATIONS

Example 1: (The motion of a rigid rod rocking back). In this section, we consider a complicated and practical case of the motion of a rigid rod rocking back and forth on the circular surface without slipping [50] as follow:

$$\begin{aligned} &\frac{d^2y}{dt^2}+\frac{3}{4}y^2\frac{d^2y}{dt^2}+\frac{3}{4}y\left(\frac{dy}{dt}\right)^2+\frac{3gycos(y)}{l}=0 \\ &y(0)=A,\ y'(0)=0 \end{aligned} \quad (14)$$

where A represents the amplitude of the oscillation. Motion is assumed to start from the position of maximum displacement with zero initial velocity. Eq. (14) can be expressed in the form of two simultaneous first-order differential equations in terms of y(t) and z(t), i.e.

$$\begin{aligned} &\frac{dy}{dt}=z \\ &\frac{dz}{dt}+\frac{3}{4}y^2\frac{dz}{dt}+\frac{3}{4}yz^2+\frac{3gycos(y)}{l}=0 \\ &y(0)=A,\ z(0)=0 \end{aligned} \quad (15)$$

We will apply classic DTM and the multistage DTM to nonlinear ordinary differential Eq. (15). Applying classic DTM for Eq. (15)

$$\begin{aligned} (k+1)Y(k+1)=&Z(k)(k+1)Z(k+1)+\frac{3}{4}\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}(k-k_2+1)Y(k_1)Y(k_2-k_1)Z(k-k_2+1) \\ &+\frac{3}{4}\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}Y(k_1)Z(k_2-k_1)Z(k-k_2)+\frac{3g}{l}\sum_{k_1=0}^{k}Y(k_1)G(k-k_1)=0,Y(0)=A,Z(0)=0 \end{aligned} \quad (16)$$

Trigonometric nonlinearity:

$$f(x) = \sin(x), g(x) = \cos(x)$$

By definition,

$$\begin{aligned} F(0) &= \left[\sin(x(t))\right]_{t=0} = \sin(x(0)) = \sin(X(0)) \\ G(0) &= \left[\cos(x(t))\right]_{t=0} = \cos(x(0)) = \cos(X(0)) \end{aligned} \tag{17}$$

To find other transformed functions, we differentiate

$$f(x) = \sin(x), g(x) = \cos(x)$$

obtaining:

$$\begin{aligned} \frac{df(x)}{dt} &= \cos(x)\left(\frac{dx(t)}{dt}\right) = g(x)\left(\frac{dx(t)}{dt}\right) \\ \frac{dg(x)}{dt} &= -\sin(x)\left(\frac{dx(t)}{dt}\right) = -f(x)\left(\frac{dx(t)}{dt}\right) \end{aligned} \tag{18}$$

Applying the differential transform to Eq. (18) obtain:

$$\begin{aligned} (k+1)F(k+1) &= \sum_{k_1=0}^{k}(k+1-k_1)G(k_1)X(k+1-k_1) \\ (k+1)G(k+1) &= -\sum_{k_1=0}^{k}(k+1-k_1)F(k_1)X(k+1-k_1) \end{aligned} \tag{19}$$

Similarly, replacing k -1 by k gives:

$$\begin{aligned} F(k) &= \sum_{k_1=0}^{k-1}\left(\frac{k-k_1}{k}\right)G(k_1)X(k-k_1),\ k \ge 1 \\ G(k) &= -\sum_{k_1=0}^{k-1}\left(\frac{k-k_1}{k}\right)F(k_1)X(k-k_1),\ k \ge 1 \end{aligned} \tag{20}$$

Combine Eqs.(17) and (20) to give the recursive relation:

$$F(k) = \begin{cases} \sin(X(0)), & k = 0 \\ \sum_{k_1=0}^{k-1}\left(\frac{k-k_1}{k}\right)G(k_1)X(k-k_1), & k \ge 1 \end{cases} \tag{21}$$

and

$$G(k) = \begin{cases} \cos(X(0)), & k = 0 \\ -\sum_{k_1=0}^{k-1}\left(\frac{k-k_1}{k}\right)F(k_1)X(k-k_1), & k \ge 1 \end{cases} \tag{22}$$

are given by [48].
where $Y_i(n)$, for n = 1…M, satisfy the following recurrence relations,

$$\begin{aligned} &(k+1)Y_i(k+1) = Z_i(k)(k+1)Z_i(k+1) + \frac{3}{4}\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}(k-k_2+1)Y_i(k_1)Y_i(k_2-k_1)Z_i(k-k_2+1) \\ &+\frac{3}{4}\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}Y_i(k_1)Z_i(k_2-k_1)Z_i(k-k_2) + \frac{3g}{l}\sum_{k_1=0}^{k}Y_i(k_1)G(k-k_1) = 0 \\ &Y_0(0) = A, Y_0(1) = 0, Y_{i+1}(0) = Y_i(t^*), Y_{i+1}(1) = Y_i(t^*), t \in [t_i, t_{i+1}], t^* = t_i, i = 0,\ldots,M-1 \end{aligned} \tag{23}$$

By applying the multistage DTM to Eq.(15) is obtained Eq.(24) as following:

$$y(t) = \begin{cases} \sum_{k=0}^{N} Y_0(k)t^k, t \in [t_0, t_1] \\ \sum_{k=0}^{N} Y_1(k)t^k, t \in [t_1, t_2] \\ \vdots \\ \sum_{k=0}^{N} Y_M(k)t^k, t \in [t_{M-1}, t_M] \end{cases} \tag{24}$$

On the function (13) taken n = 1, $t \in [0,7]$, g = l = 1, solution of curve fitting

$$y_{cf}(t) = 0.15737\cos(1.715664t + 0.0003702) - 0.14184e\text{-}4$$

is obtained and presented in the following in Fig. 1. RK4 method is also compared with trigonometric curve fitting solution.
In (13) expression, n = 2, $t \in [0,8]$ and g = l = 1 are taken, solution of

$$y_{cf}(t) = 0.47835\cos(1.5939t + 0.3643e\text{-}4) + 0.7651e\text{-}3\cos(6.6469t - 28.6595) - 0.20276e\text{-}3$$

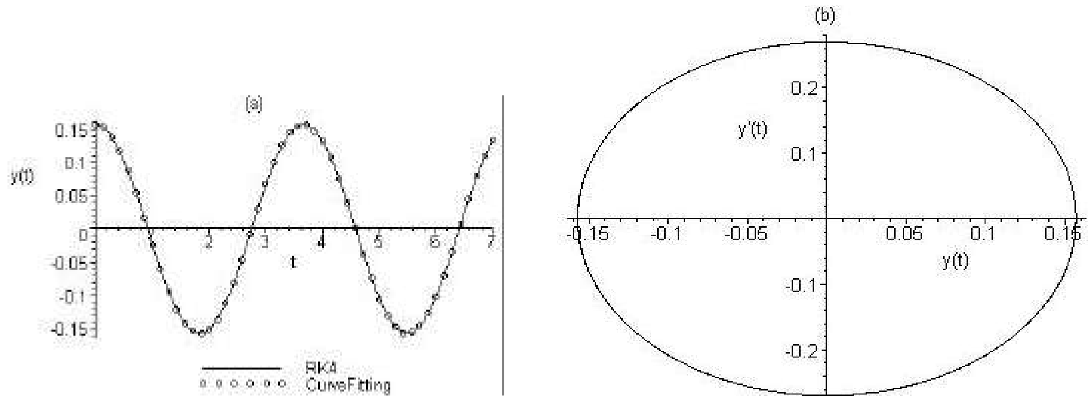

Fig. 1: Plots of (a) displacement y versus time t and (b) phase portrait for A = 0.05π

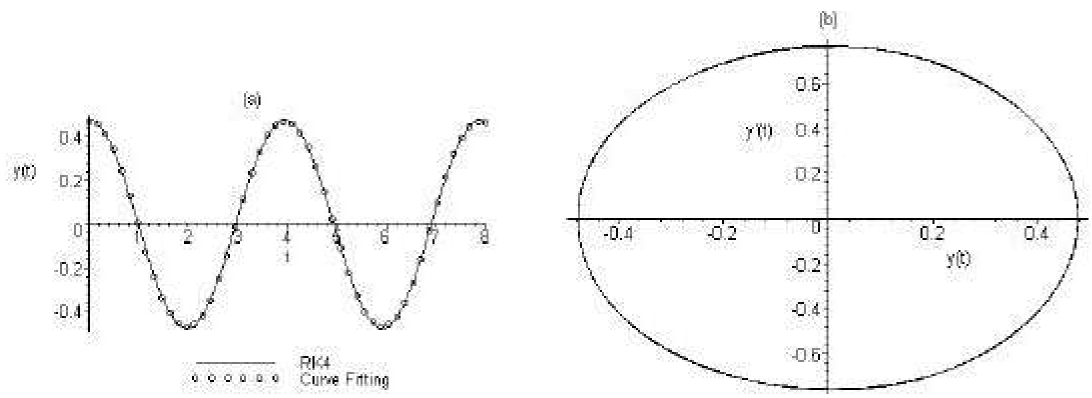

Fig. 2: Plots of (a) displacement y versus time t and (b) phase portrait for A = 0.05π

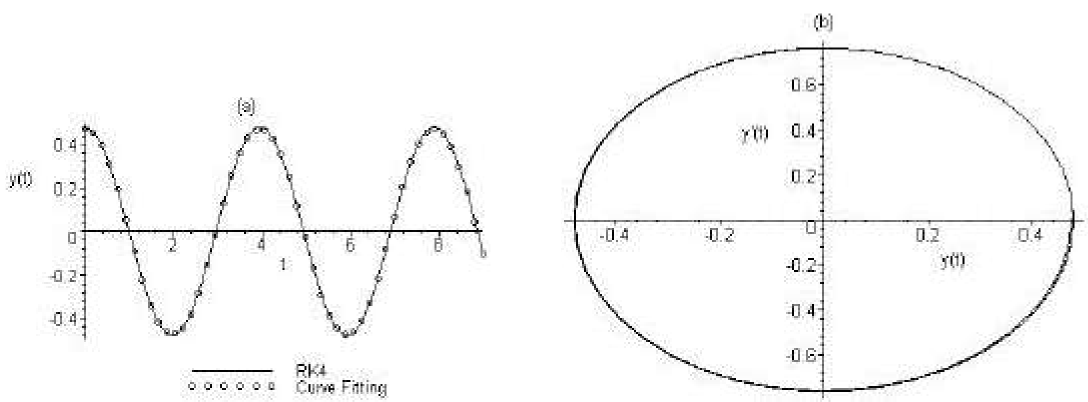

Fig. 3: Plots of (a) displacement y versus time t and (b) phase portrait for A = 0.05π

is obtained and presented in the following in Fig. 2. RK4 method is also compared with trigonometric curve fitting solution.

In (13) equation, $n = 3$, $t \in [0,9]$ and $g = l = 1$ are taken, solution of

$$y_{cf}(t) = 0.4781\cos(1.594t\text{-}0.00557) + 79.95\cos(0.000701t + 3.143) + 0.00228\cos(0.973t\text{-}2.961) + 79.95$$

is obtained and presented in the following in Fig. 3. RK4 method is also compared with trigonometric curve fitting solution.

Example 2: In this section, we consider physical model of nonlinear equation [1] as following showed in Fig. 4:

$$\frac{d^2y}{dt^2}+\frac{4r}{3m}\sin(y)-\frac{3F_0}{ml}\sin(w_0t)=0 \qquad (25)$$
$$y(0)=A,\ y'(0)=0$$

where A represents the amplitude of the oscillation. Motion is assumed to start from the position of maximum displacement with zero initial velocity. We will apply classic DTM and the mu ltistage DTM to nonlinear ordinary differential Eq. (25). Applying classic DTM for Eq. (25) and from the trigonometric nonlinear in the example 1 given above,

$$(k+1)(k+2)Y(k+2)=-\frac{4r}{3m}F(k)+\frac{3F_0w_0^k}{mlk!}\sin\left(\frac{k\pi}{2}\right) \qquad (26)$$
$$Y(0)=A,\ Y(1)=0$$

where $Y_i(n)$, for n = 1…M, satisfy the following recurrence relations,

$$(k+1)(k+2)Y_i(k+2)=-\frac{4r}{3m}F_i(k)+\frac{3F_0w_0^k}{mlk!}\sin\left(\frac{k\pi}{2}\right)$$
$$Y_0(0)=A,\ Y_0(1)=0,\ Y_{i+1}(0)=Y_i\left(t^*\right),\ Y_{i+1}(1)=Y_i\left(t^*\right) \qquad (27)$$
$$t\in\left[t_i,t_{i+1}\right],\ t^*=t_i,\ i=0,\ldots,M-1$$

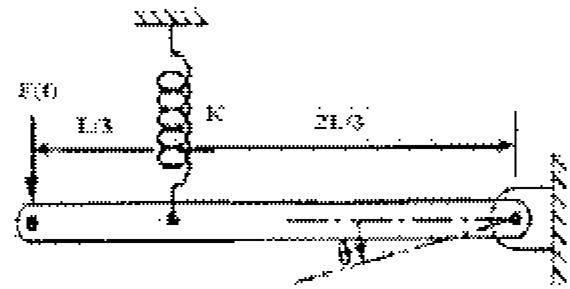

Fig. 4: The physical model of nonlinear equation [51]

By applying the multistage DTM to Eq. (25) is obtained Eq. (28) as following:

$$y(t)=\begin{cases}\sum_{k=0}^{N}Y_0(k)t^k,\ t\in\left[t_0,t_1\right]\\ \sum_{k=0}^{N}Y_1(k)t^k,\ t\in\left[t_1,t_2\right]\\ \vdots\\ \sum_{k=0}^{N}Y_M(k)t^k,\ t\in\left[t_{M-1},t_M\right]\end{cases} \qquad (28)$$

On the function (13) taken n = 1, t∈[0,4], r = 1000, m = 10, $F_0 = w_0 = l = 1$ and A = π/12 solution of

$$y_{cf}(t) = 0.2617993878*\cos(3.65979516413592\pi t +0.442004866524034105e\text{-}4)$$

is obtained and presented in the following in Fig. 5. RK4 method is also compared with trigonometric curve fitting solution.

In (13) expression, n = 1, t∈[0,5], r = 1000, m = 10, $F_0 = w_0 = l = 1$ and A = π/8 solution of

$$y_{cf}(t) = 0.3926990818\cos(3.64014563935407\pi t-0.612387145462451292e\text{-}4)$$

is obtained and presented in the following in Fig. 6. RK4 method is also compared with trigonometric curve fitting solution.

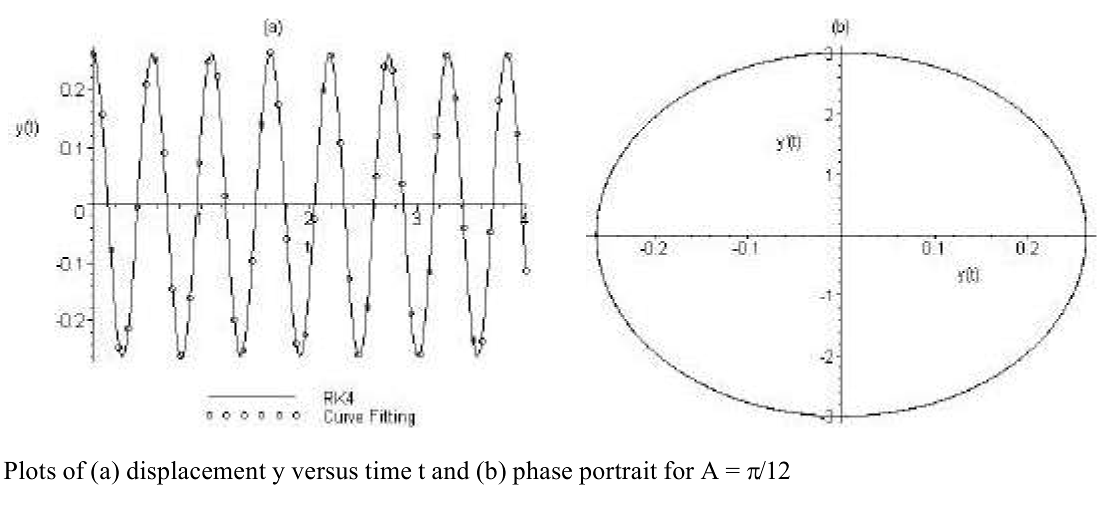

Fig. 5: Plots of (a) displacement y versus time t and (b) phase portrait for A = π/12

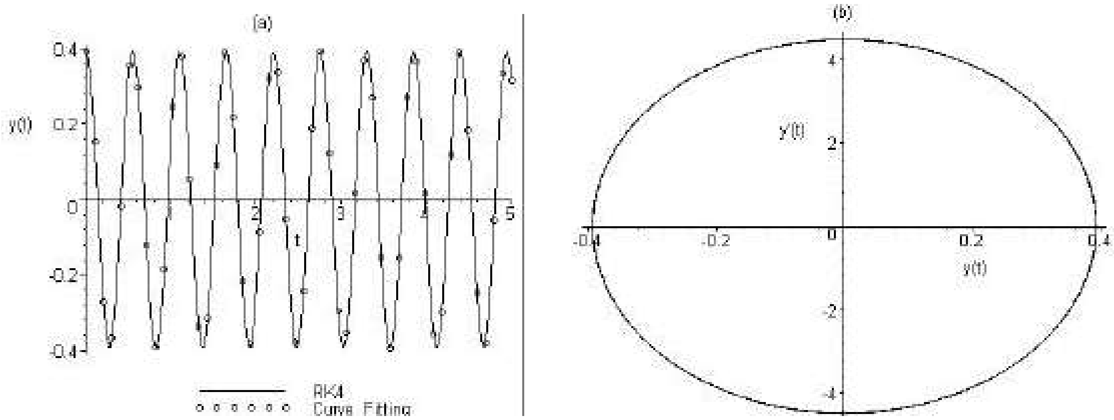

Fig. 6: Plots of (a) displacement y versus time t and (b) phase portrait for A = π/8

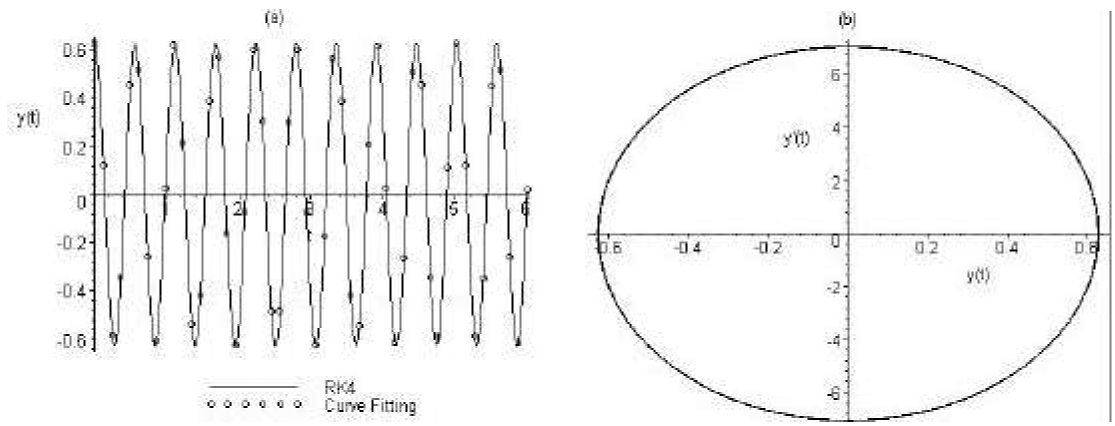

Fig. 7: Plots of (a) displacement y versus time t and (b) phase portrait for A = π/5

In (13) equation, $n = 1$, $t \in [0,6]$, $r = 1000$, $m = 10$, $F_0 = w_0 = l = 1$ and $A = \pi/5$ solution of

$$y_{cf}(t) = 0.6283185308\cos(3.58502086997380\pi t - 0.625140793577042644e\text{-}4)$$

is obtained and presented in the following in Fig. 7. RK4 method is also compared with trigonometric curve fitting solution.

Example 3: In this section, we have Duffing equation with constant coefficient [1] as following showed in Fig. 8:

$$\frac{d^2x}{dt^2} + \frac{K_1}{m}x + \frac{K_2}{2mh^2}x^3 = \frac{F_0}{ml}\sin(w_0 t),\ x(0) = A,\ x'(0) = 0 \tag{29}$$

where A represents the amplitude of the oscillation. Motion is assumed to start from the position of maximum displacement with zero initial velocity. We will apply classic DTM and the multistage DTM to nonlinear ordinary differential Eq. (29). Applying classic DTM for Eq. (29),

$$\begin{aligned}&(k+1)(k+2)X(k+2) = -\frac{K_1}{m}X(k) - \frac{K_2}{2mh^2}\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2} X(k_1)X(k_2-k_1)X(k-k_2) + \frac{F_0 w_0^k}{mk!}\sin\left(\frac{k\pi}{2}\right)\\ &X(0) = A, X(1) = 0\end{aligned} \tag{30}$$

where $Y_i(n)$, for $n = 1\ldots M$, satisfy the following recurrence relations,

$$\begin{aligned}&(k+1)(k+2)X_i(k+2)=-\frac{K_1}{m}X_i(k)-\frac{K_2}{2mh^2}\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}X_i(k_1)X_i(k_2-k_1)X_i(k-k_2)+\frac{F_0 w_0^k}{mk!}\sin\left(\frac{k\pi}{2}\right)\\&X_0(0)=A, X_0(1)=0, X_{i+1}(0)=X_i(t^*), X_{i+1}(1)=X_i(t^*)\\&t\in[t_i, t_{i+1}], t^*=t_i,\ i=0,\ldots,M-1\end{aligned} \tag{31}$$

By applying the multistage DTM to Eq.(29) is obtained Eq.(32) as following:

$$x(t)=\begin{cases}\sum_{k=0}^{N}X_0(k)t^k, t\in[t_0, t_1]\\ \sum_{k=0}^{N}X_1(k)t^k, t\in[t_1, t_2]\\ \vdots\\ \sum_{k=0}^{N}X_M(k)t^k, t\in[t_{M-1}, t_M]\end{cases} \tag{32}$$

On the function (13) taken n = 1, $t\in[0,3]$,

$$L=1\,m, h=0.9\,m, m=10\,kg, K_1=1000\,N/m, K_2=1100\,N/m, F_0=1\,N, w_0=1\,rad/s$$

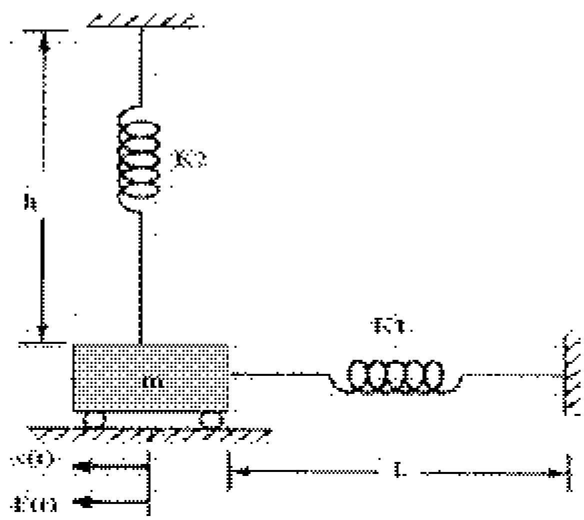

Fig. 8: The physical model of Duffing equation with constant coefficient [51]

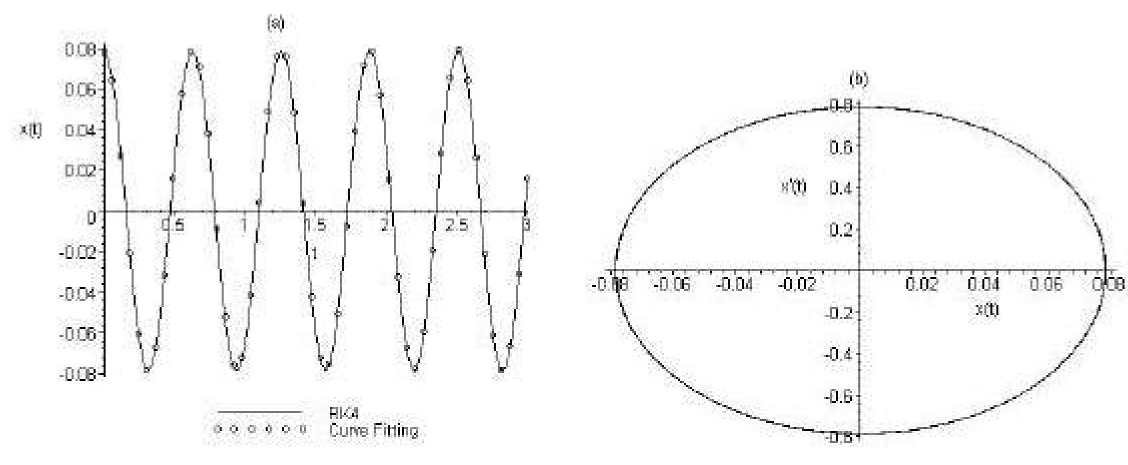

Fig. 9: Plots of (a) displacement x versus time t and (b) phase portrait for A = 0.9 tan (π/36)

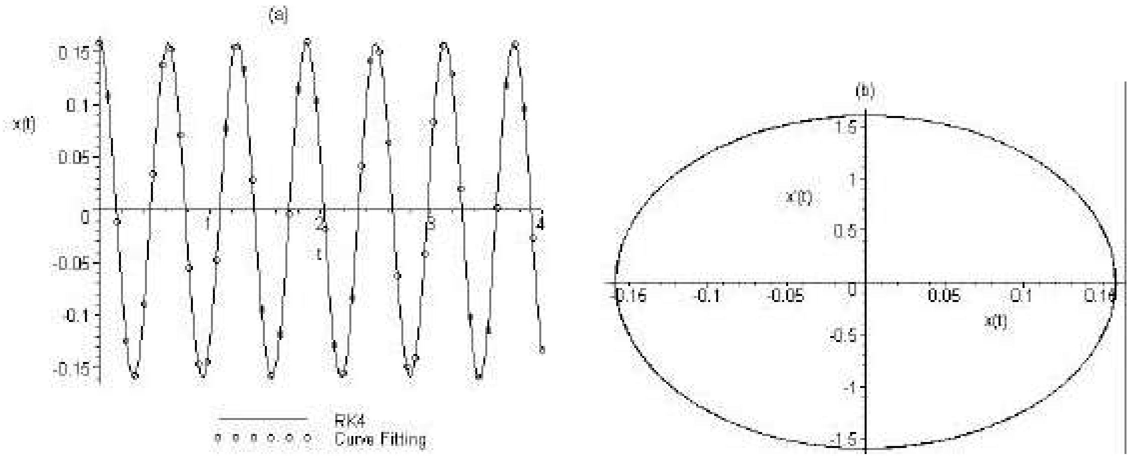

Fig. 10: Plots of (a) displacement x versus time t and (b) phase portrait for A = 0.9 tan (π/18)

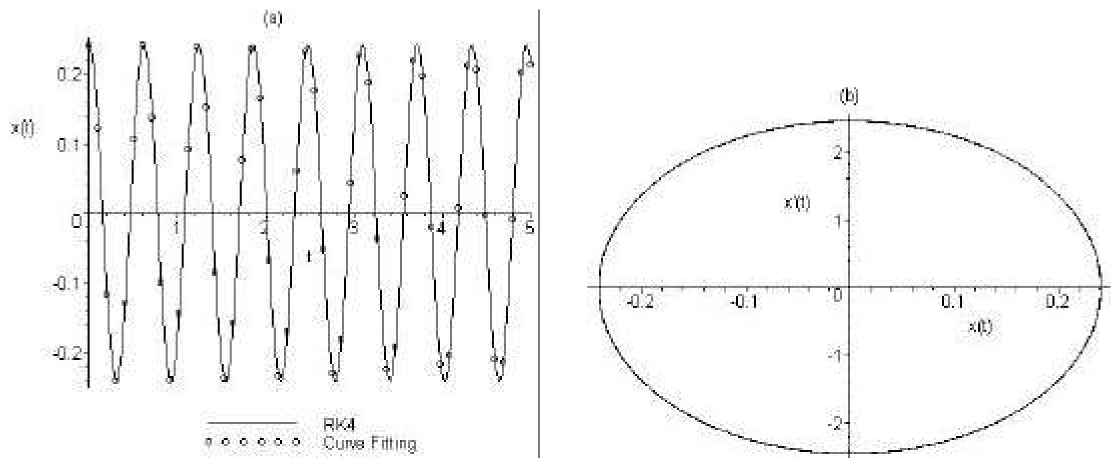

Fig. 11: Plots of (a) displacement x versus time t and (b) phase portrait for A = 0.9 tan (π/12)

and A = 0.9 tan (π/36) solution of

$$y_{cf}(t) = 0.7873979720\text{e-}1\cos(10.01653269t + 0.1005230648\text{e-}3)$$

is obtained and presented in the following in Fig. 9. RK4 method is also compared with trigonometric curve fitting solution.

In (13) expression, n = 1, $t \in [0,4]$,

$$L = 1\,m, h = 0.9\,m, m = 10\,kg, K_1 = 1000\,N/m, K_2 = 1100\,N/m, F_0 = 1\,N, w_0 = 1\,rad/s$$

and A = 0.9 tan (π/18) solution of

$$y_{cf}(t) = 0.1586942826\cos(10.06478006t + 0.3279565577\text{e-}3)$$

is obtained and presented in the following in Fig. 10. RK4 method is also compared with trigonometric curve fitting solution.

In (13) equation, n = 1, $t \in [0,5]$,

$$L = 1\,m, h = 0.9\,m, m = 10\,kg, K_1 = 1000\,N/m, K_2 = 1100\,N/m, F_0 = 1\,N, w_0 = 1\,rad/s$$

and A = 0.9 tan (π/12) solution of

$$y_{cf}(t)=0.2411542732\cos(10.14819210t+0.7173231348e\text{-}3)$$

is obtained and presented in the following in Fig. 11. RK4 method is also compared with trigonometric curve fitting solution.

Example 4: In this section, we consider the motion equation of the pendulum with harmonic stringer point [1] as following showed in Fig. 12:

$$\frac{d^2\theta}{dt^2}+\left(\frac{g}{l}-\frac{w_0^2Y}{l}\cos(w_0t)\right)\sin\theta=0 \qquad (33)$$
$$\theta(0)=A,\ \dot\theta(0)=0$$

where A represents the amplitude of the oscillation. Motion is assumed to start from the position of maximum displacement with zero initial velocity. We will apply classic DTM and the multistage DTM to nonlinear ordinary differential Eq. (33). Applying classic DTM for Eq. (33) and from the trigonometric nonlinear in the example 1 given above,

$$(k+1)(k+2)\Theta(k+2)=F(k)\left(\frac{g}{l}-\frac{Y\cos(0.5\pi k)w_0^{k+2}}{lk!}\right) \qquad (34)$$
$$\Theta(0)=A,\Theta(1)=0$$

where $\Theta_i(n)$, for n = 1…M, satisfy the following recurrence relations,

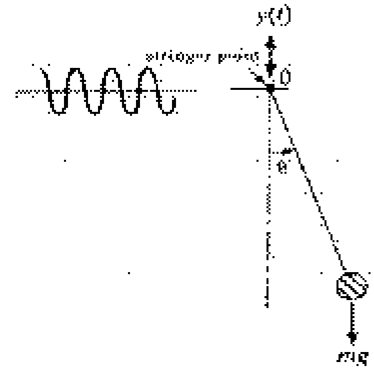

Fig.12: Pendulum with harmonic stringer point: y(t) = Y cos w_0t [51]

$$(k+1)(k+2)\Theta_i(k+2)=F(k)\left(\frac{g}{l}-\frac{Y\cos(0.5\pi k)w_0^{k+2}}{lk!}\right)$$
$$\Theta_0(0)=A,\Theta_0(1)=0,\Theta_{i+1}(0)=\Theta_i(t^*),\Theta_{i+1}(1)=\Theta_i(t^*) \qquad (35)$$
$$t\in[t_i,t_{i+1}],t^*=t_i,\ i=0,\dots,M-1$$

By applying the multistage DTM to Eq. (33) is obtained Eq. (36) as following:

$$\theta(t)=\begin{cases}\sum_{k=0}^{N}\Theta_0(k)t^k, t\in[t_0,t_1]\\ \sum_{k=0}^{N}\Theta_1(k)t^k, t\in[t_1,t_2]\\ \vdots\\ \sum_{k=0}^{N}\Theta_M(k)t^k, t\in[t_{M-1},t_M]\end{cases} \qquad (36)$$

On the function (13) taken $n=1,t\in[0,5],l=1\,m,w_0=1\,rad/s,Y=0.25m,g=9.81m/s^2$ and A = π/12 solution of

$$y_{cf}(t)=0.261895058654144197\cos(0.979981126902420784\pi t\text{-}0.431628482521539199e\text{-}4)$$

is obtained and presented in the following in Fig. 13. RK4 method is also compared with trigonometric curve fitting solution.

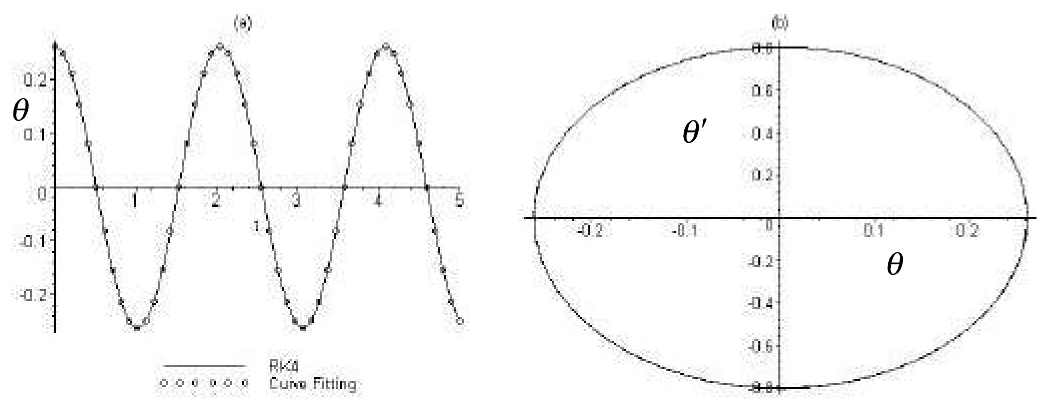

Fig. 13: Plots of (a) displacement θ versus time t and (b) phase portrait for A = π/12

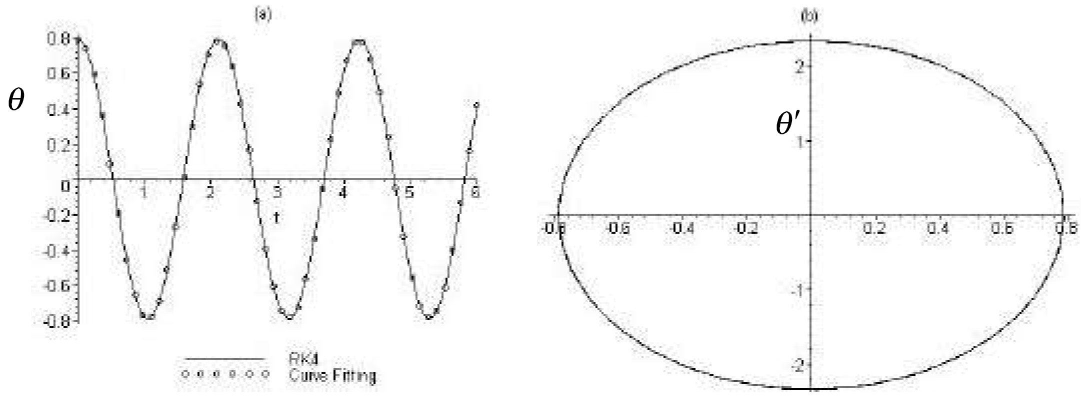

Fig. 14: Plots of (a) displacement θ versus time t and (b) phase portrait for A = π/4

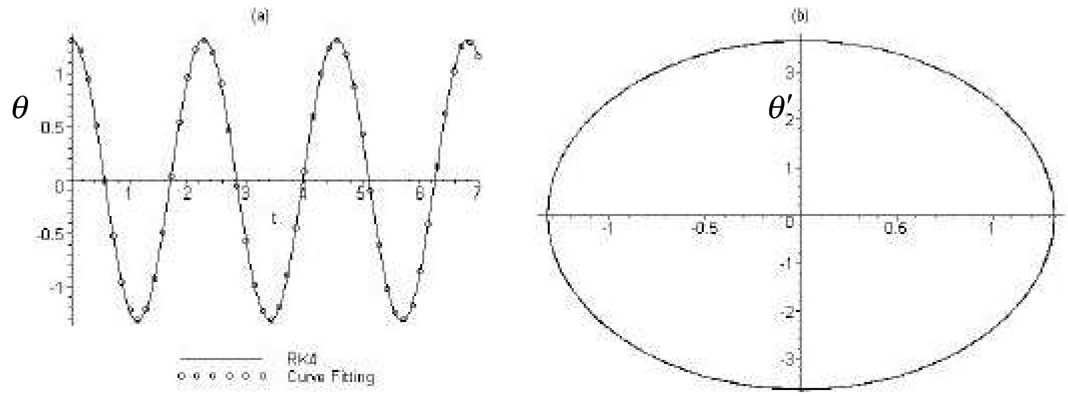

Fig. 15: Plots of (a) displacement θ versus time t and (b) phase portrait for A = 5π/12

In (13) expression, $n=1, t\in[0,6], l=1\,m, w_0=1\,rad/s$, $Y=0.25m, g=9.81m/s^2$ and A = π/4 solution of

$$y_{cf}(t)=0.788238375005162582\cos(0.946384386107354980\pi t+6.28291712268189606)$$

is obtained and presented in the following in Fig. 14. RK4 method is also compared with trigonometric curve fitting solution.

In (13) equation, $n=1, t\in[0,7], l=1\,m, w_0=1\,rad/s$, $Y=0.25m, g=9.81m/s^2$ and A = 5π/12 solution of

$$y_{cf}(t)=-1.32267015669864163\cos(0.879650367165896506\pi t-28.2751089644735885)$$

is obtained and presented in the following in Fig. 15. RK4 method is also compared with trigonometric curve fitting solution.

We can observe that local changes and the phase plane trajectories trigonometric curve fitting obtained using the multi-stage DTM are in high agreement with local changes and the phase plane trajectories obtained using RK4 method. In calculations of trigonometric curve fitting calculations were used command maple fit.

CONCLUSIONS

In this paper, we carefully applied the multistage DTM, a reliable modification of the DTM that improves the convergence of the series solution to the nonlinear vibrating equations. For data obtained from the multi-step DTM, generated the most appropriate trigonometric curve fitting. The method provides immediate and visible symbolic terms of analytic solutions, as well as numerical approximate solutions to both linear and nonlinear differential equations. The validity of the proposed method has been successful by applying it for the nonlinear vibrating equations. As understood from the above examples, the comparison between multistage DTM and RK4, both results obtained by both methods are accurate and close to each other.

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