Triplet Structure Model of Arithmetical Word Problems for Learning by Problem-Posing

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Abstract. We have been investigating and developing several interactive environments for learning by problem-posing targeting arithmetical word problems. In this paper, we describe "triplet structure" of an arithmetical word problem that is composed of two "single quantity sentences" and one "relative quantity sentence", as the base model of the design of these learning environments. We also report practice use of the interactive learning environments in usual class room by using tablet PCs.

Keywords: Triplet Structure Model, Learning by Problem-Posing, Arithmetical Word Problem.

1 Introduction

Learning by problem-posing is well known as an important way to promote learners to master the use of solution methods [1, 2]. We have investigating technology-enhanced learning environment for learning by problem-posing [3-10]. Learners are required to make arithmetical word problems by combining several sentence cards provided from the environment. We call this kind of problem posing “problem posing by sentence integration”. Because the meaning of each sentence can be described formally, it is possible for the environment to diagnose the problems posed by the sentences. We call this automatic diagnosis facility “agent-assessment” in contrast with “self-assessment”, “teacher-assessment”, and “peer-assessment” [11].

We believe that this problem-posing method is not only worth for realization of agent-assessment by system, but also is valuable for learning for learners. One benefit of the problem-posing of sentence integration is that it enables learners to concentrate learners to construct the structure of problem. In problem-posing with natural language, learners often take much time to write sentences themselves and don’t use enough time to think about problems themselves. One more benefit is that the combination of sentences expresses the problem structure of arithmetical word problems explicitly. So, it is expected that to deal with the sentences directly contributes to learn the problem structure.

In this paper, as a model to problem structure of arithmetical word problems, we describe "triplet structure model" that is composed of two "single quantity sentences" and one "relative quantity sentence". We also introduce interactive learning environment for problem-posing and its practical use in usual classroom with tablet PCs.

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2 Triplet Structure Model

An arithmetical word problem that is solved by one arithmetical operation is composed of three quantities: operand, operant and result quantity. Triplet structure model is a model of these three quantities considering meaning of the arithmetical word problem. In this model, all word problems that are solved by one of the four basic operations are composed of two "independent quantity sentences" and one "relative quantity sentence". Then, depending on combination of them, role of each sentence is changed. In this section, this model is introduced with examples. Relation between an arithmetic story and other problems are shown in Figure 1.

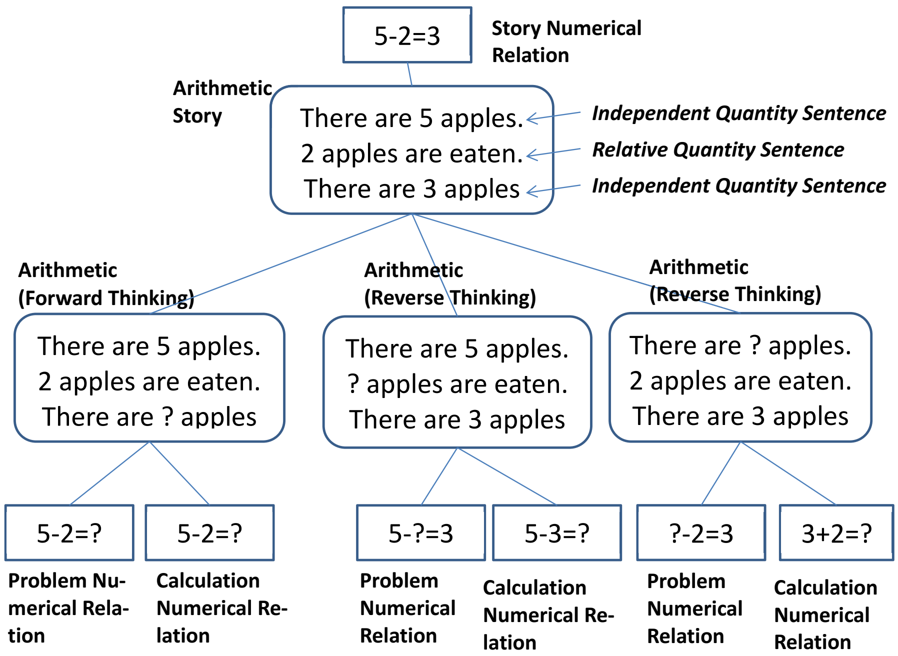

Fig. 1. Relation between Arithmetic Story and Problems

2.1 Story Numerical Relation and Calculation Numerical Relation

By using an answer of an arithmetical word problem, it is possible to make a numerical relation and a cover story composed of all known quantities. We call this cover story "arithmetical story" and this numerical relation "story numerical relation". Then, the numerical relation in the problem including unknown quantity is called "problem numerical relation". As one more numerical relation, there is a numerical relation used in calculation. We call this numerical relation "calculation numerical relation". In this framework, a problem is generated from a story by changing a known quantity to an unknown quantity.

Following is a typical arithmetical problem that is expressed by the triplet structure model.

{There are "?" apples. 2 apples are eaten. There are 3 apples.}

In this problem, the answer is 5 and calculation to derive the answer is 2+3. So, in the above framework, the story numerical relation is "5-2=3", the problem numerical relation is "?-2=3" and the calculation numerical relation is "2+3=5". Then, the arithmetical story is {There are 5 apples. 2 apples are eaten. There are 3 apples.}. In this problem, story numerical relation and calculation numerical relation are different. We call this kind of problem "reverse thinking problem". Reverse thinking problem is much harder than "forward thinking problem" where story numerical relation and calculation numerical relation are the same ones.

2.2 Triplet Structure of Addition and Subtraction

Addition story is usually categorized into two subcategories: increase story and combination story. Then, subtraction story is also usually categorized into two categories: decrease story and comparison story. Comparison story can be further divided into "more than" story and "less than" problem. Each story is composed of two independent quantity sentences and one relative quantity sentence. Although an independent quantity sentence can be used in any stories, a relative quantity sentence is used only one specific story. In this subsection, the four stories are explained respectively.

Increase story is composed of two independent quantity sentences and one relative quantity sentence. One independent quantity sentence describes quantity before increase, and the other independent quantity sentence describes quantity after the increase. Each independent quantity sentence only describes a quantity of an object. Relative quantity sentence describes the quantity of the increase. The relative quantity sentence expresses the relation between before quantity and after quantity of the increase. Triplet structure of increase story is composed of these three sentences.

Decrease story is also composed of two independent quantity sentences and one relative quantity sentence. One independent quantity sentence describes quantity before decrease, and the other independent quantity sentence describes quantity after the decrease. Each independent quantity sentence only describes a quantity of an object. Relative quantity sentence describes the quantity of the decrease. The relative quantity sentence expresses the relation between before quantity and after quantity of the decrease. Triplet structure of increase story is composed of these three sentences.

Composition story is also composed of two independent quantity sentences and one relative quantity sentence. One independent quantity sentence describes the number of an object, and the other independent quantity sentence describes number of another object. The relative quantity sentence expresses total number of the two objects. Triplet structure of combination story is composed of these three sentences.

Comparative story is also composed of two independent quantity sentences and one relative quantity sentence. One independent quantity sentence describes the number of an object, and the other independent quantity sentence describes number of another object. The relative quantity sentence expresses difference of the two objects. Triplet structure of comparative story is composed of these three sentences.

In these triple structures, although a relative sentence is specific one to each story, all independent quantity sentences has the same expressions and their roles in the stories are decided depending on other sentences. Framework of "learning by problem-posing as sentence integration" is designed based on this model. Multiplication and division story are also defined with the triples model composed of two general independent quantity sentences and one specific relative quantity sentence. In this paper, we also introduce an explanation of problem-solving and problem-posing process based on this model.

In Figure 2, one independent quantity sentence “there are 6 apples” is used in several stories with difference roles in each story. In the increase story, it is used as a smaller number of the apples in the story. In the decrease story, then, it is used as a larger number of the apples. By using with “there are 3 oranges”, “there are 6 apples” are use in a combination story, more than story and less than story. The independent quantity sentence is also used in multiplication by combining with relative quantity sentence of multiplication as shown in Figure 2.

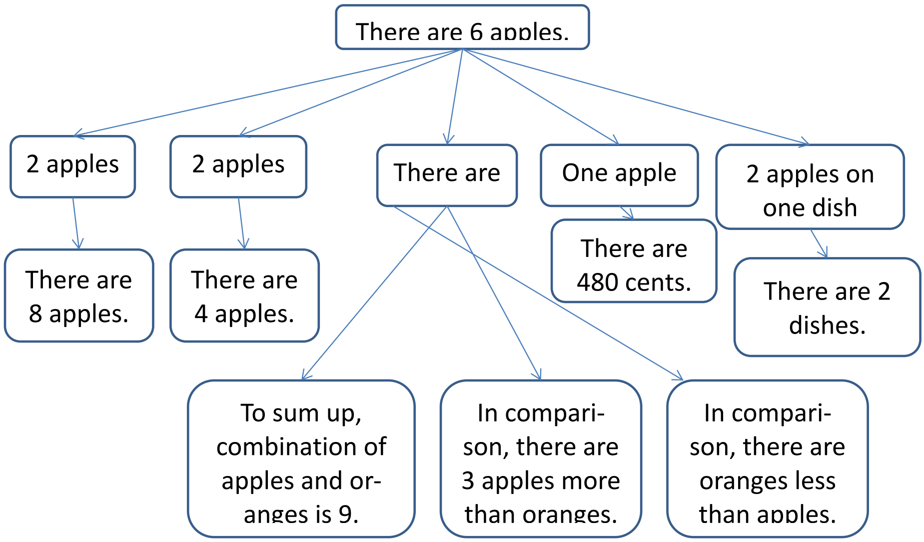

Fig. 2. Various Combination of Three Sentences

2.3 Basic Story Set

The four stories of addition and subtraction can be generated by 8 sorts of sentences because independent quantity sentence can be shared in difference stories. The set of seven sentences are composed of three independent quantity sentences and four relative quantity sentences. The set of sentences is shown in Figure 3. By changing object and numeric numbers, it is easily make many stories with the same characteristics in arithmetic.

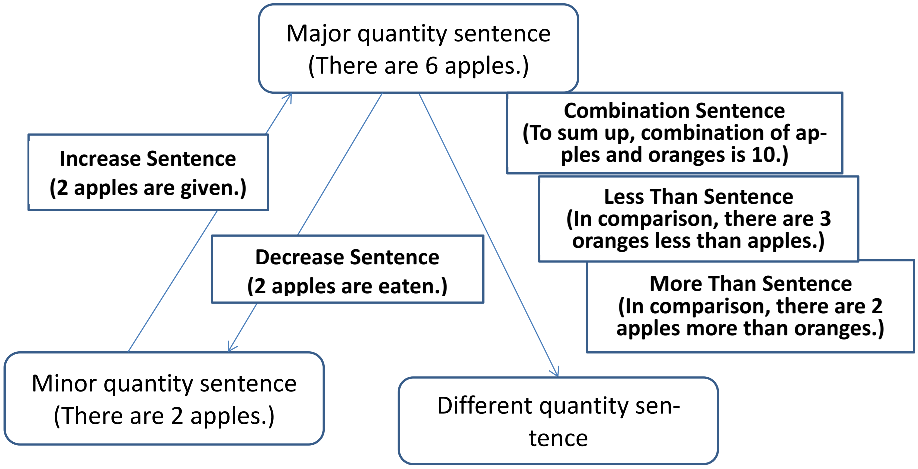

Fig. 3. The Basic Card Set Composed with 8 Sentences

2.4 Basic Problem Set

A problem is generated by changing a value in the arithmetic story to a variable. Because the arithmetic story is composed of three values, three problems are generated by one story. So, from one basic story set, it is possible to make 15problems. In order to make these 15 problems, it is necessary to add 7 sentence cards each of which has a variable.

3 MONSAKUN Touch for Addition and Subtraction

Based on the triplet structure model, we have developed an interactive environment for learning by problem-posing. Now, the environment can be used with a tablet PC. We call this “MONSAKUN touch”. In this section, practice use by the first grade students in an elementary school is reported. In the next section, practice use of MONSAKUN touch for multiplication for the second grade students is reported.

3.1 Interface of Problem-Posing of MONSAKUN Touch

In Figure 4, interface of problem-posing of MONSAKUN touch is shown. In the upper left side of the interface, a calculation numerical relation and arithmetic story are shown. A learner is required to pose a problem that can be solved the calculation numerical relation and belongs to the specified arithmetic story by using sentence cards provided in the right side of the interface. The set of sentence cards includes not only necessary ones but also unnecessary ones. The unnecessary sentence cards are called dummy cards. In the lower left side, there are three blanks where a leaner puts sentence cards in order to complete a problem. In Figure 4, two cards have been put in the blanks. In this case, required problem is composed of {"Tom has 3 pencils." "Tom buys several pencils." "Tom has 7 pencils."}. The scene of using MONSAKUN Touch is shown in Figure 5.

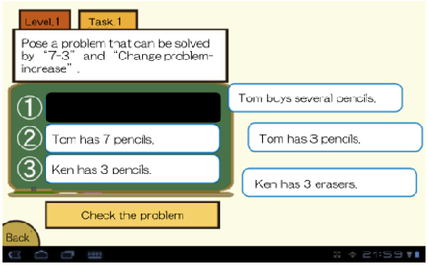

Fig. 4. Interface of MONSAKUN

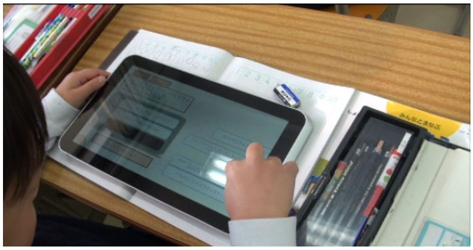

Fig. 5. Scene of Using MONSAKUN

3.2 Procedure of Practical Use of MONSAKUN Touch for Addition and Subtraction

In this experiment, 39 students in the first grade of an elementary school were subjects. Arithmetic word problems that are solve by one addition or subtraction are usually taught in the first grade of elementary schools, but the problem structures are not taught explicitly. In this practice, the problem structure of arithmetic word problems used in MONSAKUN Touch were taught explicitly by a teacher and problem-posing exercise with MONSAKUN Touch was carried out as an exercise to operate the problem structure. This practice composed of nine lesson times (45 minutes per lesson, 3 weeks, 9 days). The students took pretest before the practice period, and took posttest and questionnaire after the period. Each test took 45 minutes. Problem-posing exercises divided into 6 levels. The levels categorized by (1) forward-thinking or reverse-thinking, (2) story operation stricture given or calculation operation structure given, and (3) cover story. In a level, students were required to pose problems following provided story operation structure or calculation operation structure and cover story. Cover stories were excerpted from several textbooks. Also, if the student finishes problem-posing exercise in a level in a class, he/she repeats the same level exercise.

In this practical use, students used the MONSAKUN Touch as an introduction of new level problem-posing (5-10 min) at the beginning of a class. The students, then, are taught the problem structures by the teacher on blackboard (20-35 min). Finally, they used the MONSAKUN Touch as confirmation of teaching (5-10 min).

In pre- and post-test, we used the same problem solving test and problem-posing test. Problem solving test used to assess the students problem solving performance. In problem-posing test, the students are required to pose four problems by composing several sentence cards provided beforehand. This test is used to examine the student's problem-posing performance.

3.3 Log Data and Questionnaire

The number of students that finished posing problems in each level is shown in Table 1. The students performed level 1 and 2 during the 3rd day from the 1st day, level 3 and 4 during the 6th day from the 4th day, and then, level 5 at the 8th day. The teacher has taught the problem structure corresponding to level 5 in detail in the 7th day. The task in level 5 is very difficult for learners, because it requires them to pose reverse-thinking problems from calculation numerical structure. Then, problem-posing with MONSAKUN was not carried out in the 7th day and took almost double times for the exercise on the 8th day. These results suggested that teaching method about the task to present story operation structure was effective for understanding of forward thinking problem and reverse thinking problem. But it is necessary for teaching method about the task to present calculation operation structure to be improved.

Table 1. Number of Student who Finished Each Level of Posing Problems

Level	1	2	3	4	5	6
Number of students	39	39	39	38	39	23
Number of not finished students	3	1	11	0	17	16

The results of the questionnaire are shown in Table 2. Almost all students agreed that problem-posing exercise by using MONSAKUN and effective to learn, but, we supposed, because of level 5, many students answered the problem-posing is difficult. The teacher agreed that it is easy to teach problem-posing using a tablet PC in the general classroom, and he said that he want to use the MONSAKUN in his class. But, also he suggested that it is necessary to improve the sentence of feedback and to expand the kinds of feedback.

Table 2. Results of Questionnaire

	Strongly Agree	Agree	Disagree	Strongly Disagree
1.Do you enjoy posing problems in arthmetic?	35	3	0	0
2. Are arithmetic problems easy to pose?	8	7	19	4
4.Do you think that posing problems made it easier to solve problems?	20	17	1	0
7.Would you like to attend arithmetic classes where problem posing is used?	36	2	0	0

3.4 Pre-test and Post-test Comparison

The results of pre- and post-test are shown in Table 3. The full mark of problem-posing test is 4. The full mark of problem-solving of forward-thinking problems is 9 and the full mark of problem-solving of revers-thinking problems is 8. In the scores of problem-solving test shown in Table 3, there was a significant difference in the scores between pretest and posttest of reverse thinking problems (two sided p-values from

Wilcoxon matched-pairs signed-ranks test with correction for ties, p=.009), and effect size is medium (|r|=.45). These results suggest that explicit teaching of problem structures was effective to understand the reverse thinking problem. In problem-posing test, there was a significant difference in the between pre-test and post-test as for the number of correct problems at reverse thinking problems (two sided p-values from Wilcoxon matched-pairs signed-ranks test with correction for ties, p=.0006), and effect size is medium (|r|=.39). In contrast with this, the number of correct problems at forward thinking problems decreased. These results suggested that the students would be aware of the difference between the reverse thinking problems and forward thinking problems. Based on these results, we have judged that this teaching method with MONSAKUN Touch is a promising way to teach arithmetic word problems.

Table 3. Results of Problems Test (*1% significant)

		forward thinking problem	reverse thinking problem
pre-test	M	8.82	7.13*
	SD	0.6	0.65
post-test	M	8.71	7.66*
	SD	0.39	1.28

4 Conclusion Remarks

In this paper, as a model to problem structure of arithmetical word problems, we describe "triplet structure model" that is composed of two "single quantity sentences" and one "relative quantity sentence". We have developed an interactive learning environment for learning by problem-posing: MONSAKUN and we used it practical. In the practical use, the first grade students in an elementary school use the environment for 8 class times. By comparing pre-test scores and post-test scores, we have confirmed that learning by problem-posing with MONSAKUN is useful learning method.

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