ASTROBIOLOGY
Volume 5, Number 5, 2005

Research Paper

Trojans in Habitable Zones

RICHARD SCHWARZ,[1] ELKE PILAT-LOHINGER,[1] RUDOLF DVORAK,[1]
BALINT ÉRDI,[2] and ZSOLT SÁNDOR[2]

ABSTRACT

With the aid of numerical experiments we examined the dynamical stability of fictitious terrestrial planets in 1:1 mean motion resonance with Jovian-like planets of extrasolar planetary systems. In our stability study of the so-called "Trojan" planets in the habitable zone, we used the restricted three-body problem with different mass ratios of the primary bodies. The application of the three-body problem showed that even massive Trojan planets can be stable in the 1:1 mean motion resonance. From the 117 extrasolar planetary systems only 11 systems were found with one giant planet in the habitable zone. Out of this sample set we chose four planetary systems—HD17051, HD27442, HD28185, and HD108874—for further investigation. To study the orbital behavior of the stable zone in the different systems, we used direct numerical computations (Lie Integration Method) that allowed us to determine the escape times and the maximum eccentricity of the fictitious "Trojan planets." Key Words: Trojan planets—Exoplanets—Habitable zone—Dynamical astronomy. Astrobiology 5, xxx–xxx.

INTRODUCTION

In this paper, we present the results of a study of the dynamics of hypothetical Earth-like planets with an emphasis on their positions and respect to their "habitable zones" (HZs) [*i.e.*, the region where possible terrestrial planets can have (a) liquid water on the surface and (b) a stable atmosphere (shown in Fig. 1)]. With regard to terrestrial extrasolar planets in the HZ, it should be noted that we only have observational evidence for gas giants (GGs) with 14 Earth masses (like Uranus) and larger. This is why a study of dynamical stability of possible additional terrestrial planets (planets with a size comparable to Earth) is a hypothetical one. Thus, the question arises as to whether such planets exist in other systems. From the dynamical point of view, there are four possible configurations for terrestrial-like planets in an HZ:

1. The HZ lies outside the orbit of the giant planet. Most of the known GG planets are located very close to their star. From the dynamical point of view, there may exist terrestrial planets with stable orbits within the HZ and sufficiently small eccentricities over time scales long enough to facilitate the development of a biosphere.
2. In the solar configuration, a Jupiter-like planet has moved far enough from its central star to allow additional planets to move

[1]Institute for Astronomy, University of Vienna, Vienna, Austria.
[2]Institute for Astronomy, Eötvös University, Budapest, Hungary.

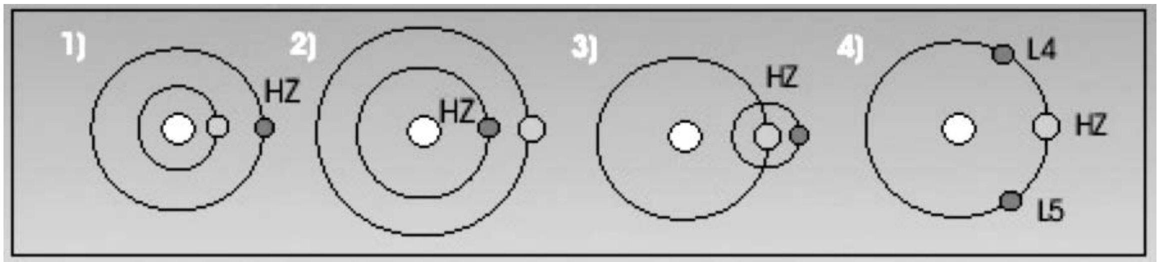

FIG. 1. **Four different classes of orbits in which possible terrestrial planets can exist.**

closer to the star in stable low eccentric orbits in the HZ.

3. If the GG moves into the HZ there are two possible motion scenarios:

- The satellite configuration. A terrestrial planet that orbits a GG in the HZ could potentially develop a biosphere.
- The Trojan configuration. When a GG moves into the habitable region, a terrestrial Trojan planet may move in a stable orbit around the Lagrangian equilibrium point L_4 or L_5.

Menou and Tabachnik (2003) quantified the dynamical habitability of extrasolar planetary systems in general via simulations of their orbital dynamics in the presence of potentially habitable terrestrial planets. The configuration where HZ lies outside the orbit of the giant planet and the satellite configuration (configurations 1 and 2) have been the subject of a number of investigations (*e.g.*, Dvorak *et al.*, 2003a,b; Érdi and Pál, 2003; Pál and Sándor, 2004). If the gravitational zone of a GG overlaps with that of a terrestrial planet in the HZ, gravitational perturbation can push the terrestrial planet out of the HZ. For this reason, we focus our work on the dynamical stability of the Trojan configuration (configuration 3, second scenario), in which possible terrestrial planets with a GG have a "1:1 mean motion resonance" (MMR). (We speak of 1:1 resonance when two planets orbit the sun with the same mean semi-major axis. The most popular example in our solar System is the Sun–Jupiter system, where numerous asteroids librate around the triangular Lagrangian points L_4 and L_5.)

Nauenberg (2002) found a stable configuration for motions in the 1:1 resonance, where the more massive planet has an almost circular orbit, while the smaller body has a high eccentric orbit. Further investigations of the Trojan configuration focused on Trojan planets in the HZ (Érdi and Sándor, 2004) and Trojan planets in low eccentric orbits (Dvorak *et al.*, 2004). Our interests are centered on Trojan planets in the 1:1 MMR with a GG that moves fully in the HZ. The main goal was to see how many orbits (of the Trojan planets) of the stable region are fully in the HZ after the calculation. These stable orbits are a main requirement for a possible formation of life.

Laughlin and Chambers (2002) considered the possibility of two planets in a 1:1 MMR as a result of an interaction with the protoplanetary accretion disc.

We emphasize that the discussion of habitable regions around a host star is an interdisciplinary one: Astrophysics is involved because the spectral type and age of the host star define the HZ (*e.g.*, Lammer *et al.*, 2003), atmospheric chemistry is fundamental when considering planetary habitability (*e.g.*, Kasting *et al.*, 1993), and astrodynamics is important with regard to the determination of orbital stability.

NUMERICAL SETUP

More than 120 extrasolar planetary systems have been discovered (The Extrasolar Planets Encyclopedia at http://www.obspm.fr/encycl/encycl.html, maintained by Jean Schneider), but only 11 systems have a giant planet in the HZ, a feature that depends on the spectral type of the star (Table 1). With respect to the requirement that a planet stay in the HZ during the calculations, we picked the following four systems: HD28185, HD17051, HD108874, and HD27442 (in Table 1, the selected systems are italicized).

To investigate the size of the stability region, we used direct numerical integrations of the equation of motion. The integration was carried out with the Lie integration method, which uses

TABLE 1. LIST OF GGS MOVING MAINLY OR PARTLY IN THE HZ OF ITS HOST STAR, DEPENDING ON ITS SPECTRAL TYPE

System	*Spectral type* M	[M_{sun}]	M × *sin(1)* [$M_{Jupiter}$]	*a (AU)*	*Eccentricity*	*Width of HZ (AU)*	*% of system in HZ*	*Argument of pericenter [°]*
HD134987	G5V	1.05	1.58	0.78	0.24	0.75–1.40	58	6
HD150706	G0	0.98	1	0.82	0.38	0.70–1.30	69	178
HD17051	*G0V*	*1.03*	*1.94*	*0.91*	*0.24*	*0.70–1.30*	*100*	*31.8*
HD177830	K0	1.17	1.28	1	0.43	0.93–1.80	58	191
HD28185	*G5*	*0.99*	*5.7*	*1.03*	*0.07*	*0.70–1.30*	*100*	*351*
HD108874	*G5*	*1*	*1.65*	*1.07*	*0.2*	*0.70–1.30*	*100*	*232*
HD27442	*K21Va*	*1.2*	*1.28*	*1.18*	*0.07*	*0.93–1.80*	*100*	*4*
HD114783	K0	0.92	0.99	1.2	0.1	0.65–1.25	50	—
HD20367	G0	1.05	1.07	1.25	0.23	0.75–1.40	76	83
HD141937`	G2/G3V	1	9.7	1.52	0.41	0.70–1.30	69	187.72
HD23079	(F8)/G0V	1.1	2.61	1.65	0.1	0.85–1.60	35	43

Data in italics are for the four systems discussed further in the text.

an adaptive step size (Hanslmeier and Dvorak, 1984; Lichtenegger, 1984).

The computations were carried out using the dynamical model of the elliptic restricted three-body problem that consists of a central star, a GG, and a hypothetical (massless) terrestrial planet.

The integration time was 10^6 years, and the two primaries were always started in their periastron position. For the terrestrial Trojan planets we have taken the following initial conditions: The semi-major axis of the massless terrestrial planet (starting at the fixed semi-major axis of the GG) was computed for a grid with a step size of Δa = 0.003 AU. Also, the eccentricity was fixed at the value of the GG, and the inclination was set to be 0. For the synodic longitude of the massless planet, we chose a range from $20° < \alpha < 140°$ with a grid size of $\Delta\alpha = 2°$. The longitude node was set to 0, and the argument of the pericenter has the same value as that of the GG (see Table 1). During the integration time, the largest value of the eccentricity of the massless planet was determined [a procedure also called the maximum eccentricity method = (MEM)]. The massless

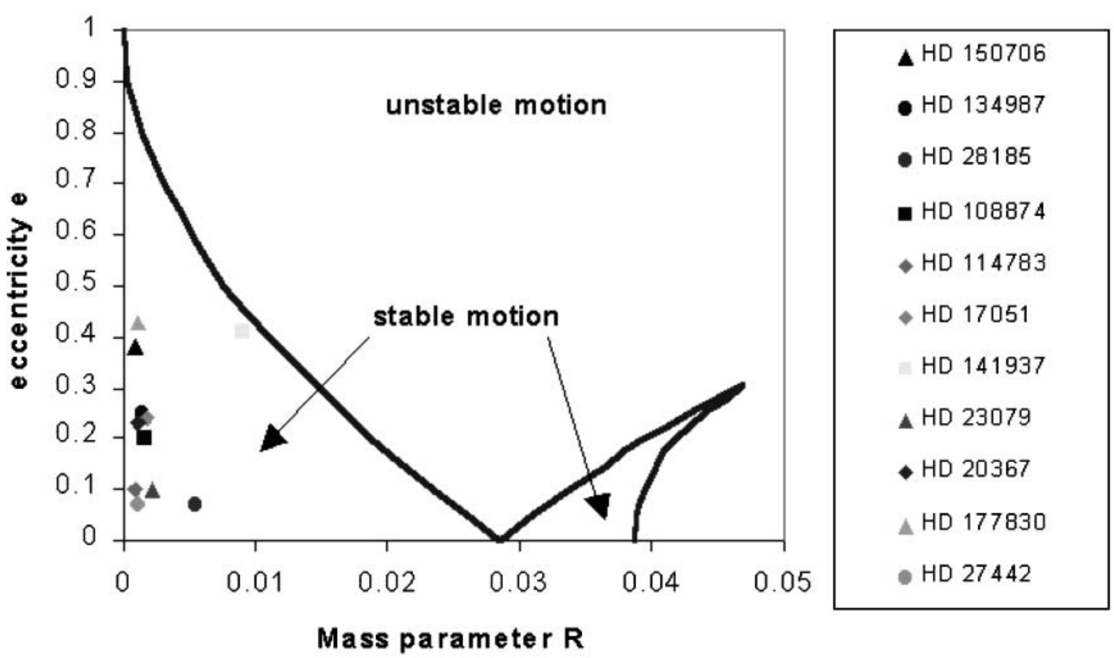

FIG. 2. Zone of first-order stability for elliptic Lagrangian motions of the mass parameter *R* and the eccentricity *e*.

planet is hereafter referred to as the hypothetical Trojan.

PREPARATIONS FOR THE CALCULATIONS

Before we started the calculations, we had to decide which systems have stable motion. Many investigations of the elliptic restricted three-body problem have been done, *e.g.*, Rabe (1967) and Lohinger and Dvorak (1993), who considered the stability of the Lagrangian points, which depend on the eccentricity of the orbit and the mass ratio of the primaries (as shown in Fig. 2). Furthermore, a study by Marchal (1991) addressed the general three-body problem (when $m_3 > 0$). These results were used to determine whether the Trojan planets are in the stable region. This is shown in Table 1, where the mass of the Trojan planet (m_3) is equal to 1 Earth mass.

We defined the mass parameter R as:

$$R = (m_2 + m_3)/M + m_2 \times m_3/m_1 + O \times (m_2^3 \times m_3/m_1^4)$$

rather than using the mass ratio equation, which had been used in the elliptic restricted three-body problem (Marchal, 1991). Figures 2 and 3 show that all four of the selected extrasolar systems lie within the zone of stable motion. Only HD141937, which has a planet 9.7 times greater than Jupiter mass, could display unstable motion. Consequently, all planetary systems with one planet in the HZ could have stable Lagrangian points (L_4 and L_5). Additionally, Fig. 3 illustrates how the position of the fictitious Trojan planet depends upon its mass. We determined that even for a very massive body (of Jupiter size, as shown in Fig. 3), which is not of concern in terms of the problem we addressed here, the triangular equilibrium point would be stable. As shown in Fig. 3, the differences between Jupiter-mass Trojans and Earth-mass Trojans are minimal. Therefore, the difference in stability between Earth mass and massless Trojans was disregarded. Thus, we calculated the further investigations of massless bodies. We concluded that orbits of hypothetical Trojan planets with a small initial R and e are stable.

This stability analysis does not reveal the nature of the extension of the stable region around the equilibrium points, information that requires numerical simulations for each extrasolar system under consideration (Table 1), as shown in the next section.

RESULTS

The system HD17051 is a G0V star of approximately 1 solar mass (1.03), which hosts a GG of

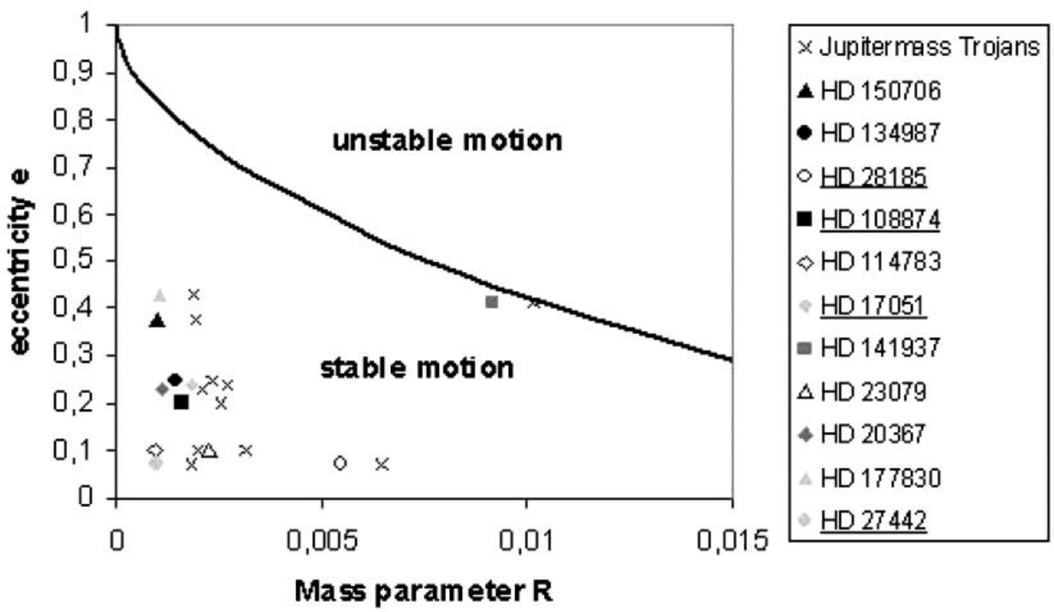

FIG. 3. Magnified image of the graph shown in Fig. 2. The crosses label the positions when m_3 (Trojan planet) is equal to 1 Jupiter mass.

TABLE 2. SYSTEM HD17051 FOR DIFFERENT CALCULATION TIMES AND INITIAL ECCENTRICITIES

Calculation time (× 1,000 years)	*Initial eccentricity*	e_{max} *of the stable region*	*Number of stable orbits/calculated orbits*	*Minimum of the perihelion (AU)*	*Maximum of the aphelion (AU)*	*% in the HZ after calculated time*
0.1	0.24	0.4–0.5	494/1,800	0.41	1.41	33
1	0.24	0.4–0.5	190/1,800	0.41	1.41	33
10	0.24	0.4–0.5	80/1,800	0.41	1.41	33
1,000	0.24	0.4–0.5	73/1,800	0.41	1.41	33
10	0.17	0.3–0.4	128/1,800	0.51	1.31	66
10	0.10	0.15–0.2	537/1,800	0.71	1.11	100

1.94 Jupiter masses ($M_{Jupiter}$) that revolves around its sun in an eccentric orbit ($e = 0.24$) with a semi-major axis of $a = 0.91$ AU. During the simulation, the semi-major axis was varied from 0.96 AU to 0.86 AU, and the synodic longitude (the difference between the mean longitude of the hypothetical Trojan planet and the GG) moved from 20° to 110° (initial conditions; see also Table 1). A numerical simulation for this system was calculated for four different periods—10^2, 10^3, 10^4, and 10^6 years—to determine how the stability region shrinks as a function of time (Table 2). To determine the number of stable orbits, it was necessary to calculate the value of e_{max} after 10^6 years (the new e_{max} ranged from 0.4 to 0.5) (Table 3). By using the new e_{max} we could set the upper limit for the Trojan's eccentricity so that it lies within the stable region of the system. We concluded from our simulation that the size of the stability region did not vary during a computation time of 10^4 years. However, there was no difference for the simulation between 10^4 and 10^6 years. Therefore, we used the smaller time period (10^4 years) when calculating additional simulations. By considering the actual orbit (see http://exoplanets.org/esp/hd17051/hd17051.shtml) of HD17051 the eccentricity indicated that it would have changed from 0.16 to 0.24 over the past 10^4 years. By calculating the initial eccentricity (e_{ini}) of the GG, we were able to illustrate how the structure of the stable region became larger with time (Fig. 4). The relative position of HD17051 within the stability region of the G0V star was computed for three different values: 0.24 the actual value, 0.17 the older value, and 0.1 the fictitious value of e_{ini}. The results for different values of e_{ini} showed that for the lowest e_{ini} (0.1) the stable domain was 6.7 times larger than for $e_{ini} = 0.24$, which explains the stable region shift out of the HZ, as shown in Table 2. Our calculations also revealed that the e_{max} of the stable region was twice as large as that of the e_{ini} (Table 2), a result that illustrates how the size of the stable region and the value of e_{max} depends on e_{ini}.

HD27442 is a K2IVa star with 1.2 solar masses. The GG (Table 3) of the system, with a mass of 1.28 $M_{Jupiter}$, orbits the central star at $a = 1.18$ AU on an almost circular orbit ($e = 0.07$). The orbital behavior of this system was also calculated for two different times: 10^4 years and 10^6 years. The MEM results for 10^4 years, a ring-like structure with a maximum eccentricity (e_{max}) of the orbits around L_4 close to the equilibrium point, are shown in Fig. 5. The ring-like structure disappeared after a calculation time of 1 Myr, as shown in Fig. 5, though the stable region was still large and extended from $40° < \alpha < 120°$ ($\Delta\alpha = 2°$) and in the semi-major axis from 1.14 AU $\leq a <$ 1.23 AU ($\Delta a = 0.003$AU). No significant reduction of the stability region was observed when we in-

TABLE 3. SIMULATED RESULTS FOR THE FOUR ITALICIZED SYSTEMS LISTED IN TABLE 1, WHICH ILLUSTRATES THE EXTENSION OF THE STABLE REGION OF THE TROJAN PLANETS AFTER 1 MYR

System	*New* e_{max} *of the stable region*	*Number of stable orbits/calculated orbits*	*Minimum of the perihelion (AU)*	*Maximum of the aphelion (AU)*	*% in the HZ after 1 Myr*
HD17051	0.4–0.5	73/1,800	0.41	1.41	33
HD28185	0.1–0.2	161/1,800	0.83	1.23	100
HD27442	0.1–0.15	926/2,000	1.03	1.33	100
HD108874	0.3–0.4	159/2,000	0.67	1.47	90

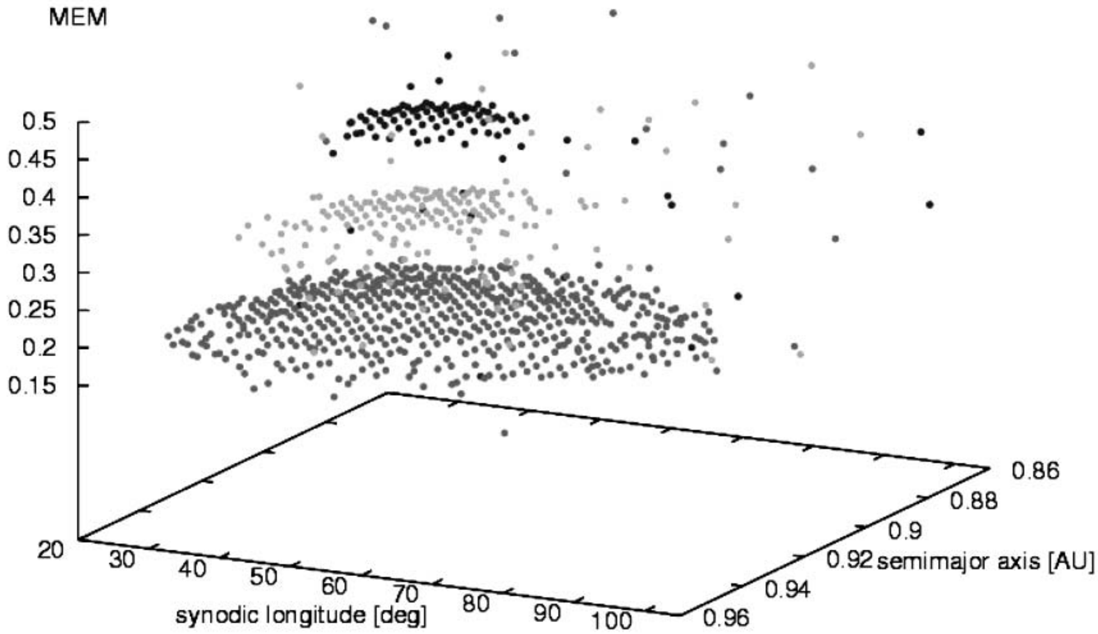

FIG. 4. The system HD17051 for different initial eccentricities: $e = 0.24$ (dark gray), 0.17 (light gray), and 0.10 (medium gray).

creased the calculation time from 10^4 years to 10^6 years: The number of stable orbits changed from 995 (of 2,000 calculated) at 10^4 years to 926 at 10^6 years. Table 3 shows the fraction of the dynamically stable region that lies within the HZ after a calculation time of 10^6 years. HD27442 remains in a stable orbital zone within the HZ even though the star left the main sequence (as indicated by its spectral type).

From an examination of the numerical simulations for the systems HD28185 and HD108874—both contain main sequence stars

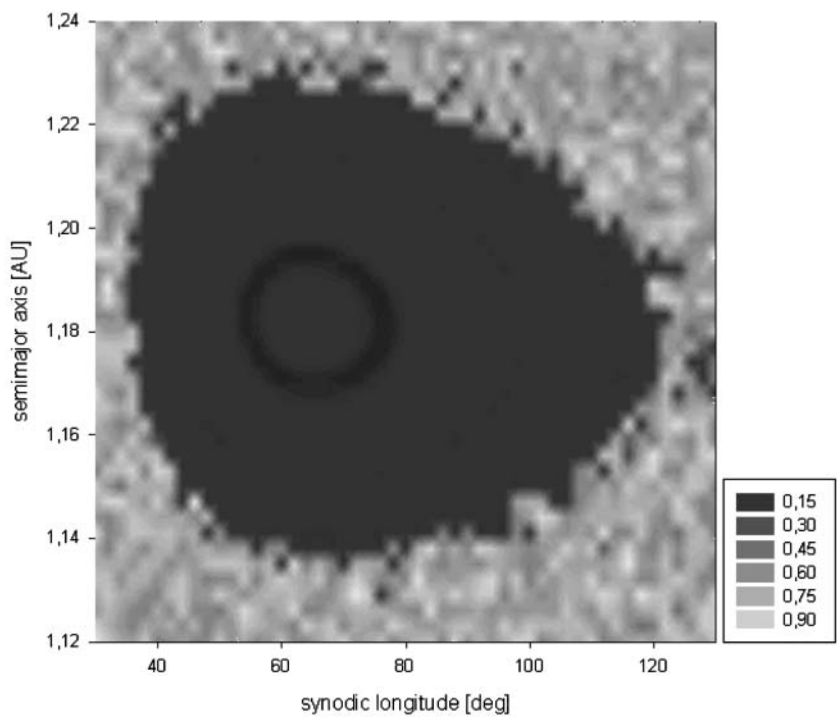

FIG. 5. System HD27442 for a computation time of 10,000 years. The dark region is the most stable, whereas the light region indicates chaotic motion.

(G5)—it follows that the Trojan planets of the GGs of these systems are mainly within the HZ, with their e_{max}, respectively, less than 0.4 (Table 3).

Our calculations demonstrated that all orbits of the Trojan planets in the stable region of three systems remained (after 10^6 years) within the HZ (HD28185, HD27442, and HD108874), but only two of them have Sun-like spectra. Future work should verify the size of the stability region and determine whether additional GGs exist.

CONCLUSIONS

We investigated Trojan-like motion in four extrasolar planetary systems with numerical simulations of the restricted three-body problem and the three-body problem with different mass ratios of the primary bodies. In this work, we studied the dynamical stability of the Trojan configuration where the GG is fully in the HZ.

With the method of Marchal (1991), we have confirmed the stability of the 1:1 MMR for the 11 extrasolar systems that were found with one planet in the HZ. We concluded that the stable zone of hypothetical Trojan planets does not depend on their mass, and that the region of the stable zone becomes smaller as the initial values of R and e increase.

We also studied the dynamical stability in the 1:1 MMR for four extrasolar planetary systems that lie entirely within the HZ (HD17051, HD27442, HD28185, and HD108874). With the MEM, we showed which objects move in stable orbits and which escape. Our studies showed that the e_{max} of the stable region was two times higher than e_{ini}. Hence, it follows that the size of the stable region and the value of e_{max} depends on e_{ini}. This means that, if the e_{ini} is too large, the width of the stable region of the system shrinks, and the stable region of the Trojan planets moves out of the HZ. Therefore, we checked whether all stable orbits of the Trojan planets stay fully within the HZ after the calculation. The probability of life formation on a terrestrial (Trojan) planet, which has a stable orbit and stays for a long time in the HZ, is large. We can conclude that three of these systems lie completely within the HZ (HD27442, HD28185, and HD108874), but only two of them have Sun-like spectra (HD28185 and HD108874).

ACKNOWLEDGMENTS

R.S. acknowledges support from the Austrian FWF (Project P16024). E.P.-L. thanks the Hertha Firnberg Project T122-N08 for support of this project.

ABBREVIATIONS

GG, gas giant; HZ, habitable zone; MEM, maximum eccentricity method; MMR, mean motion resonance.

REFERENCES

Dvorak, R., Pilat-Lohinger, E., Funk, B., and Freistetter, F. (2003a) Planets in habitable zones: A study of the binary Gamma Cephei. *Astron. Astrophys*. 398, L1–L4.

Dvorak, R., Pilat-Lohinger, E., Funk, B., and Freistetter, F. (2003b) A study of the stable regions in the planetary system HD 74156—can it host Earth like planets in habitable zones? *Astron. Astrophys*. 410, L13–L16.

Dvorak, R., Pilat-Lohinger, E., Schwarz, R., and Freistetter, F. (2004) Extrasolar Trojan planets close to habitable zones. *Astron. Astrophys*. 426, L37–L40.

Érdi, B. and Pál, A. (2003) Dynamics of resonant exoplanetary systems. In *Proceedings of the 3rd Austrian-Hungarian Workshop on Trojans and Related Topics*, edited by F. Freistetter, R. Dvorak, and B. Érdi, Eötvös University Press Budapest, Vienna, p. 3.

Érdi, B. and Sándor, Z. (2004) Stability of co-orbital motions in exoplanetary systems. *CMDA.* (in press).

Hanslmeier, A. and Dvorak, R. (1984) Numerical integration with Lie series. *Astron. Astrophys*. 132, 203–207.

Kasting, J.F., Whitmire, D.P., and Reynolds, R.T. (1993) Habitable zones around main sequence stars. *Icarus* 101, 108–128.

Lammer, H., Dvorak, R., Pilat-Lohinger, E., Funk, B., Freistetter, F., Ribas, I., Selsis, F., Guinan, E.F., Weiss, W.W., and Bauer, S.J. (2003) Atmosphere and orbital stability of exosolar planets orbiting gamma Cephei. In *EGS-AGU-EUG Joint Assembly, Abstracts*, Abstractbook [abstract 12443], Nice, France.

Laughlin, G. and Chambers, J.E. (2002) Extrasolar Trojans: The viability and detectability of planets in the 1:1 resonance. *Astron. J.* 124, 592–600.

Lichtenegger, H. (1984) The dynamics of bodies with variable masses. *CMDA.* 34, 357–368.

Lohinger, E. and Dvorak, R. (1993) Stability regions around L4 in the elliptic restricted problem. *Astron. Astrophys*. 280, 683–687.

Nauenberg, M. (2002) Stability and eccentricity for two planets in a 1:1 resonance, and their possible occurrence in extrasolar planetary systems. *Astron. J.* 124, 2332–2338.

Marchal, C. (1991) The three-body problem [book review]. *Observatory* 111, 192.

Menou, K. and Tabachnik, S. (2003) Dynamical habitability of known extrasolar planetary systems. *Astrophys. J.* 583, 473–488.

Pál, A. and Sándor, Z. (2003) Dynamical stability of the habitable zone of exoplanetary systems. In *Proceedings of the 3rd Austrian-Hungarian Workshop on Trojans and Related Topics*, edited by F. Freistetter, R. Dvorak, and B. Érdi, Eötvös University Press Budapest, Vienna, p. 25.

Rabe, E. (1967) Third-order stability of the long-period Trojan librations. *Astron. J.* 72, 10.

Sándor, Z. and Érdi, B. (2003) Symplectic mapping for Trojan-type motion in the elliptic restricted three-body problem. *Celestial Mechanics Dynamical Astron.* 86, 301–319.

Address reprint requests to:
Richard Schwarz
Institute for Astronomy
Türkenschanzstrasse 17
1180 Vienna, Austria

E-mail: schwarz@astro.univie.ac.at