and hence, by taking into account Eqs. (13a) and (13b), we have the alternative relation

$$F_K'(0)=F_\pi'(0) \text{ or } \langle r_K{}^2\rangle=\langle r_\pi{}^2\rangle . \quad (17)$$

From these results, Eqs. (13a) $\sim$ (17), it is concluded that the three-quark-system spreads over wider than the two-quark-system and the extension of the u-quark is larger than those of the d- and s-quark.

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Trouser-Type Universe Metric

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Kundt[1] proved the non-existence of the trouser type universe[1] under the condition that every geodesic is complete. Geroch[2] also proved the theorem of non-existence of the trouser type universe under globally hyperbolicity. Thus, if assuming the causality, in constructing a trouser type universe singularity is inevitable. In our construction, of course, there is a singular point and near the point the curvature tensor and scalar curvature become infinite. We do not derive the model from the Einstein equation, but after constructing the model we impose the Einstein equation on the constructed model. In this construction we use the cobordism theory.[3] We construct the model, first 2-dimensional case and second 4-dimensional case.

First case: The manifold we consider here is the following. Let M' be the subset of R^2 such that

$$M':=\{(x,y)\,\varepsilon R^2|\,-1\leq -x^2+y^2\leq 1|,\ |x\cdot y|\leq 1\}$$

and M be a quotient space of M' with respect to the equivalence relation; $(x, 1/x) \sim (x, -1/x)$, $(x,y)\sim(x,y)$; other points. The Riemannian metric on M is $h=dx^2+dy^2$. M has a Morse function f with index 1,[3] i.e., $f(x,y)=-x^2+y^2$. Next we can define a Lorentz metric g on M by using the above Morse function f. We first define a gradient-like vector field for f[3]

$$v_a:=\frac{\partial f}{\partial x_a}.$$

This vector field (v. f.) is non-singular except at one critical point of f, i.e., $(0, 0)\,\varepsilon M$. We can normalize $\|v\|=1$, except (0, 0), with respect to h, i.e.,

$$u_a:=\frac{v_a}{\sqrt{h_{ab}v^a v^b}}.$$

The Lorentz metric is defined as follows. $g_{ab}:=h_{ab}-\xi u_a u_b$, where ξ is a real function greater than one.[2),4)] The explicit form of g is

$$g=\left(1-\frac{\xi x^2}{x^2+y^2}\right)dx^2+\frac{2\xi xy}{x^2+y^2}(dxdy)$$

$$+\left(1-\frac{\xi y^2}{x^2+y^2}\right)dy^2.$$

Using the transformation, $-x^2+y^2=4X$, $xy=2Y$, g has the following form (this transformation can be carried out for four open charts $\{x>0\}$, $\{y>0\}$, $\{x<0\}$, $\{y<0\}$):*)

$$g=\frac{1}{\sqrt{X^2+Y^2}}[(1-\xi)dX^2+dY^2].$$

Choosing $\xi=2$ everywhere on M, g becomes the metric which is conformal to the Minkowski metric form.

$$g=\frac{1}{\sqrt{X^2+Y^2}}[-dX^2+dY^2].$$

In general if $\hat{g}=\Omega^2 g$, $\hat{R}_{ab}$ and $\hat{R}$ have the following form:

$$\begin{aligned}\hat{R}_{ab}-R_{ab}=&-\Omega^{-1}(n-2)\Omega_{;ab}\\&-\Omega^{-1}g_{ab}g^{cd}\Omega_{;cd}\\&+\Omega^{-2}(2n-4)\Omega_{;a}\Omega_{;b}\\&+\Omega^{-2}(3-n)g_{ab}g^{cd}\Omega_{;c}\Omega_{;d}\end{aligned}$$

and

$$\begin{aligned}\hat{R}-\Omega^{-2}R=&-2(n-1)\Omega^{-3}g^{cd}\Omega_{;cd}\\&-(n-1)(n-4)\Omega^{-4}g^{cd}\Omega_{;c}\Omega_{;d},\end{aligned}$$

where ; means that the covariant derivative with respect to g and n is the dimension of M. In our case $g=\eta=\mathrm{diag.}(-+)$ and $\Omega^2=1/\sqrt{X^2+Y^2}$, thus, $\Box:=\eta^{ab}\partial_a\partial_b$, $\nabla\Omega:=(\partial_a\Omega)$,

$$R_{ab}=\Omega^{-1}\eta_{ab}(-\Box\Omega+\Omega^{-1}\nabla\Omega\cdot\nabla\Omega), \tag{1}$$

$$R=2\Omega^{-3}(-\Box\Omega+\Omega^{-1}\nabla\Omega\cdot\nabla\Omega). \tag{2}$$

The Einstein equation $R_{ab}-\frac{1}{2}g_{ab}R=\kappa T_{ab}$ has the form $R_{ab}-\frac{1}{2}g_{ab}R=0$. That is to say, our model is the solution of the 2-dimensional vacuum Einstein equation. From (2) R has a singularity at $(X,Y)=(0,0)$.

*) Coordinate systems near the points $\{|x|\times|y|=1\}$ are chosen such that each level of f has constant value of one of the coordinate.

Second case: Now we construct 4-dimensional case analogous to the 2-dimensional case. Let M' denote the subset of R^4 such that

$$M':=\{(\boldsymbol{x},y)\in R^3\times R|-1\leq-|\boldsymbol{x}|^2+y^2\leq 1,\ \times|\boldsymbol{x}|\cdot|y|\leq 1\}$$

and M be a quotient space with respect to the following equivalence relation: $(\boldsymbol{x},1/|\boldsymbol{x}|)\sim(\boldsymbol{y},1/|\boldsymbol{y}|)$, $(\boldsymbol{x},-1/|\boldsymbol{x}|)\sim(\boldsymbol{y},-1/|\boldsymbol{y}|)$; for $|\boldsymbol{x}|=|\boldsymbol{y}|$, and $(\boldsymbol{x},y)\sim(\boldsymbol{x},y)$: other points. The Morse function f and the normalized gradient-like v.f. u_a are $f(\boldsymbol{x},y)=-|\boldsymbol{x}|^2+y^2$ and $u_a=1/\sqrt{|\boldsymbol{x}|^2+y^2}(-\boldsymbol{x},y)$. The Lorentz metric $g_{ab}:=\delta_{ab}-\xi u_a u_b$ has the following form:

$$\begin{aligned}g=&d\boldsymbol{x}\cdot d\boldsymbol{x}+dy\cdot dy\\&-\xi\frac{1}{|\boldsymbol{x}|^2+y^2}(-\boldsymbol{x}d\boldsymbol{x}+ydy)^2.\end{aligned}$$

Using the transformation on each open chart,*) $-|\boldsymbol{x}|^2+y^2=4x_0$, $|\boldsymbol{x}|\cdot y=2x'$, θ, ϕ, where θ,ϕ are spherical angles of $\boldsymbol{x}$, and choosing $\xi=2$,

$$\begin{aligned}g=&\frac{1}{\sqrt{x_0^2+x'^2}}[-dx_0^2+dx'^2]\\&+2(-x_0+\sqrt{x_0^2+x'^2})\\&\times(d\theta^2+\sin^2\theta d\phi^2).\end{aligned}$$

Further we can deform this metric without losing the trouser property, i.e., the following $\tilde{g}$ is also the trouser metric:

$$\begin{aligned}\tilde{g}:=&\frac{1}{\sqrt{x_0^2+x'^2}}[-dx_0^2+dx'^2\\&+x'^2(d\theta^2+\sin^2\theta d\phi^2)].\end{aligned}$$

This metric is conformal to the Minkowski metric with conformal factor $\Omega^2=1/\sqrt{x_0^2+x'^2}$. The curvature tensor and the scalar curvature are

$$\begin{aligned}R_{ab}=&-2\Omega^{-1}\Omega_{;ab}-\Omega^{-1}\eta_{ab}\eta^{cd}\Omega_{;cd}\\&+4\Omega^{-2}\Omega_{;a}\Omega_{;b}-2\Omega^{-2}\eta_{ab}\eta^{cd}\Omega_{;c}\Omega_{;d}\end{aligned}$$

*) See the footnote of the first case.

and

$$R = -6\Omega^{-3}\eta^{cd}\Omega_{;cd}.$$

In this case the Einstein equation $R_{ab} - \frac{1}{2}\tilde{g}_{ab}R = 8\pi T_{ab}$ does not represent the vacuum solution. Whether there exists a metric g on M which is the vacuum solution or not is an open question here. Our model corresponds to colliding two cylinder $(S^3 \times R)$ universes and forming one cylinder. This model is not causally continuous[5] and for this point we will discuss elsewhere.

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Stability of Vacuum in the Curved Space-Time of Relativistic Star

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Spontaneous particle creation in the Kerr-Newman space-time is one of the most interesting effects which are derived from the quantized field theory in curved space-time.[1] By this effect, the black-hole space-time becomes secularly unstable. Stimulated by this result, we now consider what kind of quantum effect is expected in the curved space-time of an equilibrium star instead of a black hole.

In the space-time of an equilibrium star, the event horizon does not exist, which plays an essential role for the particle creation in the black hole. As we shall show later, the physical aspect of this problem is relevant with a stationary external interaction of quantized field in the flat space-time. The study of this problem teaches us the following:[2)~4)] For sufficiently strong external field or sufficiently deep potential, a bound level begins to exist. But, there is a critical strength above which the level energy becomes complex and the particles can be produced in this critical level infinitely.[3] However, more physical implication of such phenomena is that a strong polarization of the vacuum sets in and a screening field due to the produced particles prevents the level from approaching this critical value, allowing for the repulsive interaction among the produced particles. This is the argument given by Migdal and he expected a phase transition of the vacuum into the polarized vacuum above such critical strength.[4] This phenomena occurs only for boson and not for fermion. In this note, we investigate whether the strength of gravity of the equilibrium star is strong enough to cause such phase transition or not. For some limiting cases, it will be shown that the gravity is not strong enough to cause such new effect.

We consider the simplest case; the space-time is spherically symmetric and the matter field is scalar and transparent in the