Truss optimization with natural frequency bounds using improved symbiotic organisms search

Short title: Improved SOS for truss optimization

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Abstract:

Many engineering structures are subjected to dynamic excitation, which may lead to undesirable vibrations. The multiple natural frequency bounds in truss optimization problems can improve dynamic behaviour of structures. However, shape and size variables with frequency bounds are challenging due to its characteristic, which is non-linear, non-convex, and implicit with respect to the design variables. As the main contribution, this work proposes an improved version of a recently proposed Symbiotic Organisms Search (SOS) called an Improved SOS (ISOS) to tackle the above-mentioned challenges. The main motivation is to improve the exploitative behaviour of SOS since this algorithm significantly promotes exploration which is a good mechanism to avoid local solution, yet it negatively impacts the accuracy of solutions (exploitation) as a consequence. The feasibility and effectiveness of ISOS is studied with six benchmark planar/space trusses and thirty functions extracted from the CEC2014 test suite, and the results are compared with other meta-heuristics. The experimental results show that ISOS is more reliable and efficient as compared to the basis SOS algorithm and other state-of-the-art algorithms.

Key words

Structural optimization; Shape and size optimization; Frequency; Meta-heuristics; Exploration; Exploitation; CEC2014

1. Introduction

The optimal engineering truss subjected to dynamic behaviour is a challenging area of study that has been an active research area. Thus, optimal truss design subjected to frequency bounds has seen much consideration in the past decades. natural frequencies of a truss are really useful considerations to improve the dynamic behaviour of the truss (Pholdee and Bureerat, 2014; Savsani *et al.* 2017). Therefore, natural frequencies of the truss should be constrained to avoid resonance with an external excitation. In addition, engineering structures should be as light as possible. On the other hand, mass minimization conflicts with frequency bounds and increases complexity in truss optimization. As such, an efficient optimization method is required to design the trusses subjected to fundamental frequency constraints and continuous efforts are put by researchers in this aspect.

Size optimization, shape optimization, and topology optimization are fundament types of truss optimization. In size optimization, the final goal is to obtain the best bar sections, whereas shape optimization works to search the best nodal positions of predefine nodes of the truss structure. The effect of shape and sizing on objective function and constraints are in conflict. Therefore, simultaneous shape and sizing with natural frequency bounds adds further complexity and often lead to divergence. Several researchers have been using different optimization algorithms, yet this research area has not been fully investigated so far.

Truss optimization with frequency bound was firstly addressed by Bellagamba and Yang (1981) since proposal many scholars have been investigating further into this research area. Lin *et al.* (1982) presented a bi-factor algorithm. Grandhi and Venkayya (1988) and Wang *et al.* (2004) tested an optimality criterion (OC). Wei *et al.* (2005) presented a niche genetic hybrid algorithm (NGHA). Particle swarm optimization (PSO; Kennedy and Eberhart, 1995) tested by Gomes (2011). Kaveh and Zolghadr (2011) used a charged system search (CSS; Kaveh and Talatahari, 2010) and enhanced CSS. Wei *et al.* (2011) applied a parallel genetic algorithm (GA). Kaveh and Zolghadr (2012) addressed a hybridized CSS and a big bang-big crunch (CSS-BBBC). Miguel and Miguel (2012) tested a harmony search (HS; Geem *et al.,* 2001) and a firefly algorithm (FA). Kaveh and Zolghadr (2014a) utilized a democratic PSO (DPSO). Kaveh and Zolghadr (2014b) investigated nine recent optimization algorithms. Pholdee and Bureerat (2014) investigated twenty-four advanced algorithms. Zuo *et al.* (2014) applied a hybrid OC-GA. Khatibinia and Naseralavi (2014) presented an orthogonal multi-gravitational search algorithm (GSA). Kaveh and Mahdavi (2014) studied a colliding-bodies optimization (CBO). Tejani *et al.* (2016b) suggested a modified sub-population teaching-learning-based optimization (MS-TLBO) and Farshchin *et al.* (2016) used Multi-Class TLBO (MC-TLBO) for trusses subjected to frequency bounds. Kaveh and Zolghadr (2017) used tug of war optimization (TWO), whereas Kaveh and Ilchi Ghazaan (2017) used vibrating particles system (VPS). On the other hand, truss subjected to both static and dynamic bounds has been investigated by few scholars (Xu *et al.*, 2003; Kaveh and Zolghadr, 2013; Savsani *et al.,* 2016; 2017).

In the second test, thirty benchmark functions extracted from the CEC2014 test suite are solved using the proposed technique and the results are compared with state-of-the-art algorithms. The comparative algorithms are selected from different categories as follows: Invasive Weed Optimization (IWO) (Mehrabian and Lucas, 2006), Biogeography-Based Optimization (BBO) (Simon, 2008), GSA (Rashedi *et al.,* 2009), Hunting Search (HuS) (Oftadeh *et al.,* 2010), Bat Algorithm (BA) (Yang, 2010), and Water Wave Optimization (WWO) (Zheng, 2014).

All these studies proved the efficacy of stochastic optimization algorithms in handling a large number of difficulties when solving structure design problems. According to the No Free Lunch theorem in the field of optimization, however, there is no algorithm to solve all optimization problems. This means that a new adapted algorithm has potential to solve a group of problems (*e.g.* structures design) better than the current algorithms while they still perform equal considering all optimization problems. This motivated our attempts to improve the performance of the recently proposed symbiotic organisms search (SOS) algorithm and adapt it better for structure design problems.

Cheng and Prayogo (2014) proposed the SOS algorithm works on cooperating behaviour among species in the society. SOS simulates symbiotic living behaviours. SOS is a population-based method, where species of the society is assumed to be a population. SOS has been equipped with a minimum number of controlling parameters: population size and number of generations. This makes this algorithm more convenient to use compare to GA which requires mutation, crossover, selection rate etc., PSO which needs inertia weight, social, and cognitive parameters, and HS which should be tested with setting harmony memory rate, pitch adjusting rate, and improvisation rate (Cheng *et al.,* 2015; Tejani *et al.,* 2016a).

The SOS algorithm has been applied to a large number of constrained and unconstrained problems and proved to be a very competitive algorithm (Cheng and Prayogo, 2014; Cheng *et al.,* 2015). In 2015, Cheng *et al.* proposed a discrete version of SOS to optimize multiple-resources levelling problems. Capability of SOS in truss optimization is still under research, although Cheng and Prayogo (2014) and Tejani *et al.* (2016a) have investigated SOS for some structural optimization problems. Another interesting work in the literature has been conducted by Tran *et al.* (2015), in which a multi-objective SOS was proposed and applied to multiple work shifts problems in construction projects. As another improvement, Tejani *et al.* (2016a) introduced an adaptive search mechanism called Adaptive benefit factor (ABF) in the mutualism phase of SOS. Adaptive versions of SOS were called as a SOS-ABF1 incorporates ABF1 and BF2, a SOS-ABF2 incorporates BF1 and ABF2, and a SOS-ABF1&2 incorporates ABF1 and ABF2. These motivated our attempt to improve the performance of SOS.

Regardless of the successful application of SOS, this algorithm estimates the global optimum of a given problem in three phases: the mutualism phase, commensalism phase, and parasitism phase.

In the parasitism phase, parasite vector is produced by a fusion of host design variables and randomly generated variables, therefore this phase works mainly in order to improve exploitation capabilities of the search process. The highly heuristic nature of the phase leads solution to jump into non-visited regions (exploration) and permits local search of visited regions (exploitation) as well. However, the exploitation capability of this phase is considerably low as compared to exploratory capability. Thus, the acceptance rate of new solution obtained by the parasitism phase reduces rapidly with function evaluations (*FEs*) or number of generations. This action consumes a large number of unused *FEs* later in the parasitism phase. Moreover, it seems that the literature lacks efficient methods to improve exploration to improve the convergence speed and exploitation. Also, adaptive mechanisms are required to balance exploration and exploitation since either of these will not guarantee the success of SOS. In other works, a propose balance of these two phases is essential to avoid local solutions and find an accurate estimation of the global optimum for a given optimization problem. To alleviate these drawbacks, an improved SOS (ISOS) algorithm is equipped with an improved parasitism phase to boosts exploitation capability of the algorithm.

This study intends to devise a method to establish a good balance between exploration and exploitation of the search space using SOS. In addition, several considerations are made in the paper to solve structure design problems using ISOS.

2. The symbiotic organisms search algorithm

The SOS algorithm, proposed by Cheng and Prayogo (2014), is a simple and powerful meta-heuristic. SOS works on the biological dependency seen among organisms in the nature. Some organisms live together because they are reliant on other species for survival and food. The reliance between two discrete organisms is known as symbiotic. In this context, mutualism, commensalism, and parasitism are the most common symbiotic relations found in the nature. An interdependency between two different species benefits to each other is called mutualism. A relationship between two different species benefits to one of them without affecting other is called commensalism. Whereas, a relationship between two different species benefits to one of them with aggressively harm another is called parasitism.

SOS starts with a randomly generated population, where the system has *'n'* number of organisms (population size) in the ecosystem. In the next stage, the population is updated in each generation „*g*‟ by „the mutualism phase‟, „the commensalism phase‟, and „the parasitism phase‟ respectively. Moreover, updated solution in each phase is accepted only if it has better objective value. These steps are repeated until a termination criterion is satisfied. In this optimization method, the better solution can be achieved the symbiotic relations between the current solution and either of other random solution and the best solution from population.

The detailed description of all three phases and modification of SOS is explained in the subsequent sections:

2.1 The mutualism phase

A relationship between two organisms of different species results in individual benefits of the symbiotic interaction is called mutualism. The symbiotic interaction between bee and flower is a classic example of this phenomenon. Bees fly from one flower to another and collect nectar that is produced into honey. This activity also benefits to result in the formation of seeds as the bee acts as the vehicle to move pollen for plant. In this way, this symbiotic association benefits both individuals from the exchange. Therefore, this relationship is called a mutually beneficial symbiotic (Cheng and Prayogo, 2014).

In the mutualism phase, the design vector (X_i) of the organism „i" (i.e. population) interacts with another design vector (X_k) of a randomly selected organism „k" of the ecosystem (where k ≠ i). The interaction between these organisms results in a mutualistic relationship, which improves individual functional values of the organisms in the ecosystem. Therefore, new organisms are governed by a Mutual Vector (MV) and Benefit Factors (BF_1 and BF_2). The mutual vector (the average of two organisms) signifies the mutual connection between organisms „X_i" and „X_k"(Equation 3). The benefit factors are decided by a heuristic step and so it is decided randomly with equal probability as either 1 or 2 (Equations 4 and 5). Therefore, the benefit factors signify two conditions where organisms „X_i" and „X_k" benefit partially or fully from the interaction respectively. The organism with the best functional value is considered as the best organism (X_{best}) of the ecosystem. In this phase, organisms „X_i" and „X_k" also interact with the best organism. Therefore, this phase keeps a good balance between exploitation and exploitation of the search space. The organisms are updated only if their new functional value $\left(F(X_i) \; or \; F(X_k)\right)$ is fitter than existing. The mathematical formulation of the new populations is given in Equations 1 and 2.

$$X_i = X_i + rand * (X_{best} - MV * BF_1) \qquad (1)$$

$$X_k = X_k + rand * (X_{best} - MV * BF_2) \qquad (2)$$

$$MV = \frac{X_i + X_k}{2} \qquad (3)$$

$$BF_1 = 1 + round\ [rand] \qquad (4)$$

$$BF_2 = 1 + round\ [rand] \qquad (5)$$

Where, $i = 1,2, \dots, n$; k is a randomly selected population; $k \neq i$; $k \in (1,2, \dots, n)$; *rand* is a random number; $rand \in [0, 1]$.

2.2 The commensalism phase

A relationship establishes by an organism with another organism of different species is beneficial to the species itself but have no influence to the other organism such symbiotic interaction is called commensalism. The commensalism relationship between the remora fish and sharks is a classic example of this phenomenon (Cheng and Prayogo, 2014). The remora fish rides shark to get food or other benefits. On the other hand, the shark is neither damaging nor benefiting by the remora fish.

In this phase, the design vector (X_i) of the organism „i" (i.e. population) interacts with another design vector (X_k) of a randomly selected organism „k" of the ecosystem (where k ≠ i). The interaction between these organisms results in a commensalism relationship, which improves the functional value of the organism „i". However, the organism „k" has neither benefits nor loss from the relationship. Moreover, the organism „X_i" also interacts with the best organism of the ecosystem. The organism is updated only if its new functional value, $F(X_i)$ is fitter than existing. Therefore, this phase keeps a good exploitation promising region near the best organism of the search space and works to improve convergence speed of the algorithm. Mathematical formulation of new population is given in Equation 6.

$$X_i = X_i + rand(-1,1) * (X_{best} - X_k) \qquad (6)$$

Where, $i = 1,2, \dots, n$; k is a randomly selected population; $k \neq i$; $k \in (1,2, \dots, n)$; *rand* is a random number in the range [–1, 1].

2.3 The parasitism phase

A relationship establishes by an organism with another organism of different species either benefit or harms the other organism such symbiotic interaction is called parasitism. The symbiotic interaction between plasmodium parasite, and anopheles mosquito is an example of this phenomenon. The anopheles mosquito passes the plasmodium parasite between human hosts. The parasite thrives and breeds inside the human body, as a result the human host suffers disease. If the human host fits to fight with the parasite, he will benefit immunity from the parasite and the parasite will no longer be able to live in that ecosystem otherwise the human host may die. In this way, this symbiotic association benefit or harm other organism from the exchange (Cheng and Prayogo, 2014).

In this phase, the design vector (X_i) of the organism „i" (i.e. population) is assumed to be the anopheles mosquito. The anopheles mosquito produces an artificial parasite called Parasite_Vector. Parasite vector is produced by changing values of some randomly selected design variables of the organism „X_i", the randomly selected design variables are modified using a random generated number within its bounds. Therefore, Parasite_vector is a fusion of design variables of the organism „i" and randomly generated design variables. The design vector (X_k) of a randomly selected organism „k" of the ecosystem (where k ≠ i) works as a human host to the parasite vector. The interaction between these organisms results in a parasitism relationship. If parasite vector has better functional value than functional value of organism „k", the parasite will kill organism „k" and acquire its position in the ecosystem. If the functional value of organism „k" is better, organism „k" will have immunity from the parasite and the parasite will die. Therefore, the parasitism phase improves the exploration and exploitation of the search space as parasite vector is generated by a fusion of host design variables and randomly generated variables. Schematic diagram of SOS and its variants is shown in Figure 1. The figure signifies various stages of the proposed algorithms like initialization, mutualism phase, commensalism phase, parasitism phase, and termination criteria. The detail pseudo code to generate the parasite vector of i^{th} population is given as follows:

Parasite_Vector = X_i'
for *j=1:m* ***do*** */* j is design variable /**
Generate a random number (r_j) / r_j is 0 or 1 /**
if *r_j = 0* ***then***
$Parasite\_Vector_{i,j} = L_{i,j} + rand_{i,j} * (U_{i,j} - L_{i,j})$
end if
end for
/ If parasite vector is fitter than the organism 'k', parasite will kill organism 'k' and acquire its position in the ecosystem. /**
if $F(Parasite\_Vector) < F(X_k)$ ***then*** */* 'k' is a randomly selected population, k ≠ i /**
$X_k = Parasite\_Vector$
end if

3. Improvements in the SOS algorithm

In the parasitism phase of the basic SOS algorithm, parasite vector is produced by a fusion of host design variables and randomly generated variables, therefore this phase works mainly to improve exploitation capabilities of the search process. Exploration is the process of finding non-visited regions of a search space, whereas exploitation refines visited regions with a local search. Mere exploration reduces the precision of the optimization algorithm but improves its capacity to avoid local solutions.

On the other hand, a high level of exploitation improves the existent population in order to find accurate solutions. Therefore, the effectiveness of an optimization algorithm to search a global optimal solution highly depends on its ability to set a good balance between the exploitation and the exploration of the search space. The stochastic components in the parasitism phase mainly focuses search to jump into non-visited regions and also allows local search of visited regions. In this way, this phase has an additional characteristic to avoid local optima trap and maintains diversity of the population. However, exploitation capability of this phase is considerable low as compared to exploration capability. As exploitation contributes to speed up the convergence rate of an optimization algorithm. Whereas exploitation oriented algorithm can have but at additional computational cost.

The main reason for this improvement is that the parasitism phase is good at exploration but poor at exploitation because of its search mechanism. In SOS, the acceptance rate of new solution obtained by the parasitism phase reduces rapidly with *FEs*. As the exploration is over focused in SOS and many *FEs* are wasted for some inferior results in the parasitism phase. This investigation is mainly focused on improving the local search procedure to accelerate exploitation of local search without declining global exploration capability of the algorithm. Thus, the parasitism phase of the basic SOS is changed to improved parasitism phase to improve the convergence ability and set a good balance between exploration and exploitation.

The new search mechanism encourages exploration during the first *q %* of FE_{max} and exploitation during remaining *FEs,* where *q* is a parasitism parameter and it depends on acceptance rate of the improved parasite vector obtained by the parasitism phase. In this way, the proposed improvement boosts exploitation capability of the algorithm. On the other hand, the modification of SOS is still under research. These aspects encouraged us to propose ISOS and to test its effect on truss design.

Improved_Parasite_Vector = X_i'
if *FEs* < (*q* % of FE_{max}) **then** /* *q is the parasitism parameter;* $q \in [1, 100]$ % /*
    $Improved\_Parasite\_Vector_{i,j} = L_{i,j} + rand_j * (U_{i,j} - L_{i,j})$ /* *j is a randomly selected variable* /*
else
  for *j=1:m* **do**
    Generate a random number (r_j) /* r_j *is 0 or 1* /*
    if *rand[0,1] < 0.5* **then**
      $Improved\_Parasite\_Vector_{i,j} = L_{i,j} + rand_{i,j} * (U_{i,j} - L_{i,j})$
    end if
  end for
end if
/* *If improved parasite vector is fitter than the organism 'k', parasite will kill organism 'k' and acquire its place.* /*
if $F(Improved\_Parasite\_Vector) < F(X_k)$ **then** /* *'k' is a randomly selected population, k ≠ i* /*
    $X_k = Improved\_Parasite\_Vector$
end if

The following sections investigate the efficiency of SOS and ISOS with respect to the truss optimization problems.

4. The formulation of the design problem

The goal of the design optimization of truss is mass minimization by considering frequency bounds. Therefore, the mass of truss (neglect lumped masses at nodes) is the objective function, whereas nodal coordinates and bar sections are the design variables. The formulation of the problem can be done mathematically as follows:

Find, $X = \{A, N\}$, $where\ A = \{A_1, A_2, \ldots, A_m\}\ and\ N = \{N_1, N_2, \ldots, N_n\}$ *(7)*

$to\ minimize, Mass\ of\ truss,$

$$F(X) = \sum_{i=1}^{m} A_i \rho_i\, L_i$$

$Subjected\ to:$

$g_1(X)$: $f_q - f_q^{min} \geq 0$

$g_2(X)$: $f_r - f_r^{max} \leq 0$

$g_3(X)$: $A_i^{min} \leq A_i \leq A_i^{max}$

$g_4(X)$: $N_j^{min} \leq N_j \leq N_j^{max}$

$where, i = 1,2, \ldots, m;\ j = 1,2, \ldots, n$

Where, $A_{i,}\ \rho_{i,}\ and\ L_i$ signify the sectional area, density, and length of the bar *'i'* respectively. N_j presents nodal coordinate (x_j, y_j, z_j) of node *'j'*. $f_q\ and\ f_r$ are *'qth'* and *'rth'* natural frequencies respectively. The superscripts, *,max'* and *,min'* signify the maximum and minimum permissible bounds respectively. The finite element method is applied to calculate fundamental Eigen values and natural frequencies the truss.

The objective function is penalized to handle frequency bounds. There is no penalty for non-violation of the bounds; otherwise, the penalty function is considered as follows (Kaveh and Zolghadr, 2013):

$$Penalized\ F(X) = \begin{cases} F(X), & if\ no\ violation\ of\ bound \\ F(X) * F_{penalty}, & otherwise \end{cases} \tag{8}$$

$$F_{penalty} = (1 + \varepsilon_1 * C)^{\varepsilon_2}, C = \sum (C_q + C_r),\ C_q = \left|1 - \frac{|f_q - f_q^{min}|}{f_q^{min}}\right|, C_r = \left|1 - \frac{|f_r - f_r^{max}|}{f_r^{max}}\right| \tag{9}$$

The parameters ε_1 and ε_2 are selected by considering their nature. In this investigation, values of ε_1 and ε_2 are set as 3 by investigating its effect. Schematic diagram of construction of the truss optimization problem is shown in Figure 2.

5. Truss problems and discussions

Six distinct trusses (Figures 3 and 4) of shape and sizing with multiple natural frequency bounds are considered to evaluate feasibility and validity of ISOS. The trusses are designed with continuous sections. The results obtained are compared with the previous results obtained using OC, GA, FA, TLBO, CBO, CSS, SOS, TWO, VPS, etc. The results and discussions of the test problems are explained in the following sections:

5.1 The 10-bar truss

The 10-bar truss is shown in Figure 3 (A). This truss has been examined by several researchers (Table 2). The design parameters are given in Table 1. Sizing with ten continuous design variables is considered for this truss. Moreover, a lumped mass of 454.0 kg is added at all free nodes (nodes 1–4) as presented in Figure 3 (A).

In this section, ISOS is investigated to test its effects on size optimization by considering population size and FE_{max} as 20 and 4000 respectively. Graphical representation of the accepted solutions" counts in the parasitism phase of SOS and the improved parasitism phase of ISOS are presented in the Figures 5 and 6 respectively. It is observed from the Figure 5 that the acceptance rate of new solutions obtained in the parasitism is at the maximum level in the early stages and gradually approaches to zero nearly at 50 % of FE_{max}. Thus, q is assumed as 50 % of FE_{max} for ISOS in all the problems. ISOS works as per the original parasitism phase during first 50 % of FE_{max} and it works as per the improved parasitism phase for the remaining FEs. It can be observed from the Figure 6 that the acceptance of the acceptance rate of new solutions obtained in the improved parasitism phase of ISOS is improved significantly compared to the parasitism of SOS.

Table 2 highlights size variables, best mass, mean mass, standard deviation (STD) of mass, *FEs,* and frequency responses obtained for 100 runs. The results show that SOS and ISOS find the best mass of 525.2789 and 524.7341 kg respectively. The results show that SOS and ISOS give better results as compared to related results stated in the literature. Moreover, ISOS ranks first among considered meta-heuristics. Therefore, the results of ISOS are compared with the results of the other meta-heuristics. The mass benefit for ISOS is 18.0159, 13.2459, 10.4059, 7.2159, 4.5159, 10.2559, 6.5459, 4.3559, 10.9959, 7.4019, 7.3169, 7.4959, 6.0359, 0.5448, 0.1933, 0.0948, and 0.5361 kg as compared to those obtained from the NHGA, PSO, NHPGA, CSS, enhanced CSS, HS, FA, CSS-BBBC, hybrid OC-GA, TLBO, MC-TLBO, TWO, VPS, SOS, SOS-ABF1, SOS-ABF2, and SOS-ABF1&2 algorithms respectively.

SOS and ISOS give mean mass as 531.4033 and 530.0286 kg respectively. Moreover, ISOS gives best mean mass among the considered algorithms except SOS-ABF1, SOS-ABF2, and SOS-ABF1&2. The results show that SOS and its variants give better mean mass as compared to related results stated in the literature. The mean mass benefit for ISOS is 10.8614, 6.3614, 8.5014, 7.6514, 5.0414, 5.0904, 3.2034, 5.5214, 5.6114, and 1.3747 kg as compared to those obtained from the PSO, CSS, enhanced CSS, HS, FA, TLBO, MC-TLBO, TWO, VPS, and SOS algorithms respectively.

SOS and ISOS give STD of mass as 4.2243 and 3.4763 respectively. It can be seen from the results that that ISOS gives better result as STD of mass with SOS. It should be noticed that maximum number of *FEs* used in the proposed algorithm is fairly small as compared to the HS, FA, hybrid OC-GA, TLBO, MC-TLBO, and VPS algorithms. This study indicates that the results of SOS and its variants are more reliable and superior as compared to the other results reported in the literature. Moreover, it is found from the results that ISOS is more efficient than SOS.

5.2 The 37-bar truss

The 37-bar truss, simply supported bridge, is depicted in Figure 3 (B). Wang *et al.* (2004) initially considered this truss and later it was investigated by many researchers (Table 3). Table 1 presents design parameters for this problem. A lumped mass of 10 kg is attached at all free nodes of the lower chord. The lower chord bars are assumed to have a fixed rectangular sections of 0.4 cm^2, whereas the remaining bars are clustered into fourteen groups by considering structure symmetry about the middle vertical plane. Upper nodes can shift vertically by considering structural symmetry, whereas the lower nodes are fixed. Therefore, this problem has fourteen sizing and five shape variables.

In this study, ISOS is tested by considering population size and FE_{max} as 20 and 4000 respectively. The obtained results are presented in Table 3. It can be seen from the results that SOS and ISOS give the best mass for 4000 *FEs* as 360.8658 and 360.7432 kg respectively. The results show that TLBO gives better results as compared to related results stated in the literature. However, maximum number of *FEs* used in the proposed algorithm is fairly small as compared to the PSO, HS, FA, CBO, DPSO, TLBO, MC-TLBO, and VPS algorithms.

SOS and ISOS give mean mass as 364.8521 and 363.3978 kg respectively. The SOS and ISOS algorithms give STD of mass as 4.2278 and 2.6642 respectively. It can be seen from the results that that ISOS gives better result as mean and STD of mass among the proposed algorithm for 4000 *FEs*. Moreover, it is also observed that ISOS is more efficient than SOS.

5.3 The 72-bar truss

Figure 4 presents the third benchmark truss. This truss was investigated by many scholars (Table 4) as a large-scale, sizing problem. The design considerations are summarized in Table 1. The bars are grouped into sixteen by seeing symmetry as reported in the previous study. A lumped mass of 2770 kg is added at all top nodes (nodes 1–4) as shown in Figure 4.

In this problem, the ISOS is tested by considering population size and FE_{max} as 20 and 4000 respectively. From the results shown in Table 4, the best mass achieved by SOS and ISOS are 325.5585 and 325.0682 kg respectively. The results show that ISOS presents better results as compared to related results stated in the literature (except results for CBO and SOS-ABF2). However, it observed that maximum number of *FEs* used by the CBO, TLBO, MC-TLBO, and VPS algorithms is significantly higher as compared to the proposed algorithm. Moreover, ISOS performs better among considered meta-heuristics for 4000 *FEs*. Therefore, we compared the results of ISOS with the results of the other meta-heuristics. The results signify that the mass benefit for SOS-ABF2 is 3.7458, 3.3248, 2.4388, 2.4998, 2.5068, 3.7618, 2.5808, 0.4903, 0.0178, 0.1635 and 0.1635 kg compared to those obtained from the CSS, enhanced CSS, CSS-BBBC, TLBO, MC-TLBO, TWO, VPS, SOS, SOS-ABF1, and SOS-ABF1&2 algorithms respectively.

The results signify that SOS and ISOS give mean mass as 331.1228 and 329.4699 kg respectively. SOS and ISOS give STD of mass as 4.2278 and 2.6642 respectively. It can be seen from the results that that ISOS gives best result as mean and STD of mass among the proposed algorithm for 4000 *FEs*. This study indicates that the results of ISOS is more reliable and proficient as compared to the results of the other meta-heuristics.

5.4 The 52-bar truss

The 52-bar dome truss is selected as the fourth problem, shown in Figure 3 (C). This problem was first studied by Lin *et al.* (1982) and followed by several others (Table 5) for sizing and shape optimization. Table 1 illustrations design considerations. A lumped mass of 50 kg is attached at all free nodes. The bars are linked into eight groups by considering symmetry about the z-axis, whereas the free nodes can shift ±2 m in each direction of the vertical plane in to keep the dome symmetric.

In this study, ISOS is used by considering population size and FE_{max} as 20 and 4000 respectively. Table 5 presents the results of the considered algorithms with other optimization methods. The results indicate that SOS and ISOS propose trusses with the optimum mass of 195.4969 and 194.7483 kg respectively. TLBO and MC-TLBO rank first among the considered meta-heuristics respectively. However, it observed that maximum number of *FEs* used by TLBO, and MC-TLBO is 3.75 time higher as compared to the proposed algorithm. Moreover, ISOS ranks second among considered algorithms. The mass benefit for ISOS is 103.2517, 41.2977, 33.6327, 10.4887, 2.5887, 20.1917, 2.7817, 2.5607, 0.6027, 0.7486, 0.0606, 0.4247, and 3.5147 kg compared to those obtained from the bi-factor algorithm, NGHA, PSO, CSS, enhanced CSS, HS, FA, CSS-BBBC, DPSO, SOS, SOS-ABF1, SOS-ABF2, and SOS-ABF1&2 algorithms respectively.

The results signify that SOS and ISOS give mean mass as 214.6676 and 207.5498 kg respectively. The results indicate that ISOS gives better mean mass as compared to other algorithms stated in the literature except the results of the enhanced CSS, DPSO, TLBO and MC-TLBO algorithms. However, maximum number of *FEs* used in the proposed algorithm is fairly small as compared to the PSO, HS, FA, DPSO, TLBO, and MC-TLBO algorithms. SOS and ISOS give STD as 15.1499 and 8.7354 respectively. It can be seen from the results that that SOS-ABF1 gives better result as STD of mass as compared SOS. This study indicates that the results of SOS and ISOS are more reliable and proficient as compared to the results of the other considered meta-heuristics. Moreover, ISOS performs more efficiently as compared to SOS.

5.5 The 120-bar truss

Figure 3 (D) presents the fifth benchmark. This 3-D dome truss was initially optimized by Kaveh and Zolghadr (2012) for size optimization. The design considerations are tabulated in Table 1. A lumped mass is added as 3000 kg at node 1, 500 kg at nodes 2 to 13, and 100 kg at the rest of the free nodes. The bars are grouped into seven by assuming symmetry about the z-axis.

In this test, ISOS is used population size and FE_{max} as 20 and 4000 respectively. Table 6 presents the obtained results using the proposed algorithm and other meta-heuristics. The results present that SOS and ISOS give the trusses with the best mass of 8713.3030 and 8710.0620 kg respectively. The results show that SOS and ISOS give better results as compared to related results stated in the literature. Moreover, ISOS ranks first among considered meta-heuristics. ISOS gives mass benefit as 494.448, 336.278, 179.0683, 461.868, 180.418, 178.678, 3.241, 2.048, 0.268, and 6.885 kg compared to those obtained from the CSS, CSS-BBBC, CBO, PSO, DPSO, VPS, SOS, SOS-ABF1, SOS-ABF2, and SOS-ABF1&2 algorithms respectively.

Mean mass for SOS and ISOS are of 8735.3452 and 8728.5951 kg respectively. Moreover, ISOS gives better mean mass among the considered algorithm except SOS-ABF1 and SOS-ABF2. The mean mass benefit for ISOS is 162.65890, 523.24490, 167.39490, 167.44490, 6.75010, and 62.10100 kg as compared to those obtained from the CBO, PSO, DPSO, VPS, SOS, and SOS-ABF1&2 algorithms respectively. It is seen clearly that the ISOS gives better mean mass as compared to related results stated in the literature.

SOS and ISOS give STD of mass as 17.9011 and 14.2296 respectively. CBO and DPSO stand first and second respectively in terms of STD of mass. Moreover, it is noticed that maximum number of *FEs* used in the proposed algorithm is fairly small as compared to the CBO, PSO, DPSO, and VPS algorithms. This study indicates that the results of ISOS is more reliable and proficient as compared to the results of the literature.

5.6 The 200-bar truss

The sixth benchmark truss, illustrated in Figure 3 (E), is considered as a large-scale, sizing problem. Table 1 presents design considerations for this problem. A lumped mass of 100 kg is added at all top nodes (nodes 1–5), whereas all bars are grouped into twenty-nine by seeing symmetry of the structure.

SOS and its variants are considered with population size and FE_{max} as 20 and 10000 respectively. Table 7 presents the comparative results. The best masses for SOS and ISOS are 2180.3210 and 2169.4590 kg respectively. The results show that SOS and ISOS give better results as compared to similar results reported in the literature (except the results of TLBO, MC-TLBO, SOS-ABF1, and SOS-ABF2). However, it observed that maximum number of *FEs* used by TLBO, and MC-TLBO is 2.3 times higher as compared to the proposed algorithm. The table shows that ISOS gives mass benefit of 90.401, 129.151, 33.753, 19.621, 10.862, and 38.4290 kg as compared to those obtained from the CSS, CSS-BBBC, CBO, 2D-CBO, SOS, and SOS-ABF1&2 algorithms respectively.

The results show that SOS and ISOS give the mean mass of 2303.3034 and 2244.6372 kg respectively. ISOS gives better mean mass than SOS for 10000 *FEs*. SOS and ISOS give STD of mass as 83.5897 and 43.4808 respectively. It can be seen from the results that that ISOS gives better result as STD of mass than SOS for 10000 *FEs*. This study specifies that the results of ISOS are more reliable and proficient as compared to the results of the literature.

Result summary of SOS and ISOS is presented in Table 8. It can be seen from the summary table, ISOS outperforms SOS for all of the truss optimization problems in terms best mass, mean mass, and STD of mass respectively.

6. The thirty benchmark functions of the CEC2014

In this section, the thirty benchmark functions proposed in the CEC2014 special session on single objective real-parameter numerical optimization (Liang *et al.,* 2014) are used to demonstrate effectiveness of the proposed algorithms. The benchmark functions are summarized in Table 9 and are divided into four categories: unimodal functions (f1–f3), multimodal functions (f4–f16), hybrid functions (f17–f22), and composition functions (f23–f30). For results verification, the comparison is made between 10 different optimization algorithms (IWO, BBO, GSA, HuS, BA, WWO, SOS, and ISOS). In this study, 30-dimensional functions are used with search ranges as [-100, 100]. Population size is considered as 50 and FE_{max} are taken as 150000 for proposed algorithm whereas q is assumed to be 50. All results are collected from 60 independent runs on each test function.

Comparative mean and STD of fitness values over the 60 runs are presented in Tables 10 and 11 respectively. Statistical tests are essential to check significance improvements by a proposed method over existing methods. Thus, the Friedman rank test on the results of ISOS, SOS, and other state-of-the-art algorithms. The test is performed on the minimum and STD of functional values obtained. The tables also present the rank sum of the algorithms over the test functions for median value. The results signify that ISOS and WWO performs best for unimodal functions, WWO gives best results for multimodal functions and hybrid functions, and ISOS ranks first for composition functions among the considered algorithms. Moreover, ISOS ranks better compared to SOS for unimodal, multimodal, hybrid, and composition functions.

The overall performance of ISOS is second best among the considered algorithms whereas WWO performs the best on the benchmark functions of unimodal, multimodal, hybrid, and composition functions. These results confirm the merits of the proposed algorithms once more.

7. Conclusion

In this study, the SOS and improve SOS algorithms are proposed to design optimum planar and space trusses subjected to multiple natural frequency bounds. The improved parasite vector is proposed in the basic SOS algorithm in order to improve exploitation ability of SOS in the search process. This mechanism aims to achieve better control of the exploration and exploitation. The effectiveness of the proposed algorithm is investigated on six widely used truss problems of sizing and shape optimization. In addition, three unimodal functions, thirteen multimodal functions, six hybrid functions, and eight composition functions of the CEC2014 are also investigated. Design variables such as nodal coordinates and cross-sectional areas are of extensively diverse characteristics, and their simultaneous use often leads to divergence. In addition, the implicit relationship between the frequencies and design variables induces more complexity.

This study compared performance of ISOS with the original SOS and other meta-heuristics such as NHGA, NHPGA, CSS, enhanced CSS, HS, FA, CSS-BBBC, OC, GA, hybrid OC-GA, CBO, 2D-CBO, PSO, DPSO, TLBO, and MC-TLBO. It was observed that in all the problems, ISOS has a better capability for obtaining results based on the best mass, mean mass, and STD of mass as compared to the results of SOS. Another finding was high exploration capability during the initial function evaluations and high exploitation during the last function evaluations, which plays a significant role in global exploration and exploitation of the search space. Both ISOS and SOS outperformed the current approaches, yet the superiority of ISOS were more substantial on the majority of case studies. In order to evaluate the performance of the proposed algorithms in benchmark functions, the results of SOS and ISOS are compared with the results of the IWO, BBO, GSA, HuS, BA, and WWO algorithms for the thirty benchmark functions proposed in the CEC2014 competition. Overall, ISOS has a better or competitive for obtaining results based on the mean and SD of functional values obtained over the stated runs as compared to SOS. A possible direction for future work would be to extend the proposed approaches to investigate simultaneously the size, shape, and topology optimization of truss structures.

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Tables

Table 1. Design considerations of the test problems

	The 10-bar truss	The 37-bar truss	The 72-bar truss	The 52-bar truss	The 120-bar truss	The 200-bar truss
Design variables	$A_i, i = 1,2,\dots,10$	$A_i, i = 1,2,\dots,14$; $y_j, j = 3,5,7,9,11$	$G_i, i = 1,2,\dots,16$	$G_i, i = 1,2,\dots,8$; z_A, z_B, z_F, x_B, x_F	$G_i, i = 1,2,\dots,7$	$G_i, i = 1,2,\dots,29$
Bounds	$f_1 \geq 7\ Hz$, $f_2 \geq 15\ Hz$, $f_3 \geq 20\ Hz$	$f_1 \geq 20$, $f_2 \geq 40$, $f_3 \geq 60$	$f_1 \geq 4\ Hz$, $f_3 \geq 6\ Hz$	$f_1 \leq 15.916\ Hz$, $f_2 \geq 28.648\ Hz$	$f_1 \geq 9\ Hz$, $f_2 \geq 11\ Hz$	$f_1 \geq 5\ Hz$, $f_2 \geq 10\ Hz$, $f_3 \geq 15\ Hz$
Size variables	$A_i \in [0.645, 50]\ cm^2$	$A_i \in [1, 10]\ cm^2$	$A_i \in [0.645, 30]\ cm^2$	$A_i \in [1, 10]\ cm^2$	$A_i \in [1, 129.3]\ cm^2$	$A_i \in [0.1, 30]\ cm^2$
Shape variables	-	$y_j \in [0.1, 3]\ m$	-	±2 m	-	-
Density	$\rho = 2770.0\ kg/m^3$	$\rho = 7800\ kg/m^3$	$\rho = 2770\ kg/m^3$	$\rho = 7800\ kg/m^3$	$\rho = 7971.81\ kg/m^3$	$\rho = 7860\ kg/m^3$
Modules of elasticity	$E = 6.98 \times 10^{10}\ Pa$	$E = 2.1 \times 10^{11}\ Pa$	$E = 6.98 \times 10^{10}\ Pa$	$E = 2.1 \times 10^{11}\ Pa$	$E = 2.1 \times 10^{11}\ Pa$	$E = 2.1 \times 10^{11}\ Pa$

Table 2. Optimal design parameters for the 10-bar truss, where cross-sectional areas are in cm^2

	Wei *et al.* (2005)	Gomes (2011)	Wei at al. (2011)	Kaveh and Zolghadr (2011)		Miguel and Miguel (2012)		Kaveh and Zolghadr (2012)	Zuo *et al.* (2014)	Farshchin *et al.* (2016)		Kaveh and Zolghadr (2017)	Kaveh and Ilchi Ghazaan (2017)	Tejani *et al.* (2016a)				Proposed work
Design variable	NHGA	PSO	NHPGA	CSS	enhanced CSS	HS	FA	CSS-BBBC	hybrid OC-GA	TLBO	MC-TLBO	TWO	VPS	SOS	SOS-ABF1	SOS-ABF2	SOS-ABF1&2	ISOS
A_1	42.234	37.712	36.630	38.811	39.569	34.282	36.198	35.274	37.284	36.0171	35.8507	34.544	35.1471	35.3794	34.4523	35.3013	36.4206	35.2654
A_2	18.555	9.959	13.043	9.0307	16.740	15.653	14.030	15.463	9.445	15.0926	14.8424	15.148	14.6668	14.8826	14.9767	14.8119	14.3010	14.6803
A_3	38.851	40.265	34.229	37.099	34.361	37.641	34.754	32.11	35.051	35.1797	35.5768	37.088	35.6889	35.7321	36.1675	34.9522	34.1835	34.4273
A_4	11.222	16.788	15.289	18.479	12.994	16.058	14.900	14.065	19.262	14.8551	14.9305	14.813	15.0929	14.3069	14.6638	14.9436	15.5395	14.9605
A_5	4.783	11.576	0.645	4.479	0.645	1.069	0.645	0.645	2.783	0.6495	0.645	0.646	0.645	0.6450	0.6680	0.6450	0.6450	0.6450
A_6	4.451	3.955	4.8472	4.205	4.802	4.740	4.672	4.880	5.450	4.6192	4.6249	4.613	4.6221	4.7142	4.5484	4.5828	4.6247	4.5927
A_7	21.049	25.308	22.140	20.842	26.182	22.505	23.467	24.046	19.041	24.2147	23.9816	24.373	23.5552	24.1569	23.9613	23.5712	22.2793	23.3417
A_8	20.949	21.613	27.983	23.023	21.260	24.603	25.508	24.340	27.939	23.8069	24.2358	23.72	24.468	23.6047	23.4914	23.5602	24.8589	23.8236
A_9	10.257	11.576	15.034	13.763	11.766	12.867	12.707	13.343	14.950	12.9309	12.6977	12.318	12.7198	12.1590	12.0449	11.9314	12.9163	12.8497
A_{10}	14.342	11.186	10.216	11.414	11.392	12.099	12.351	13.543	10.361	12.3585	12.3319	12.618	12.6845	12.0061	12.4632	13.0401	11.8151	12.5321
Mass (kg)	542.75	537.98	535.14	531.95	529.25	534.99	531.28	529.09	535.73	532.136	532.051	532.23	530.77	525.2789	524.9274	524.8289	525.2702	524.7341
f_1 (Hz)	7.008	7.000	7.0003	7.000	7.000	7.0028	7.0002	7.000	7.0007	7.0001	7.0000	7.0000	7.0000	7.0005	7.0001	7.0003	7.0007	7.0001
f_2 (Hz)	18.148	17.786	16.080	17.442	16.238	16.7429	16.1640	16.119	17.030	16.1777	16.1837	16.1599	16.1599	16.2484	16.2437	16.1997	16.2072	16.1703
f_3 (Hz)	20.000	20.000	20.002	20.031	20.000	20.0548	20.0029	20.075	20.156	20.0001	20.0001	20.000	20.0000	19.9999	20.0064	20.0022	19.9996	20.0024
FEs	-	2000	-	4000	4000	20000	5000	4000	8000	10000	10000	-	30000	4000	4000	4000	4000	4000
Mean	-	540.89	-	536.39	538.53	537.68	535.07	-	-	535.119	533.232	535.55	535.64	531.4033	528.6291	528.5501	528.7075	530.0286
STD	4.864	6.84	-	3.32	5.97	2.49	3.64	-	-	3.219	2.179	3.24	2.55	4.2243	3.4999	2.9827	2.8779	3.4763

Table 3. Optimal design parameters for the 37-bar truss, where size variables are in cm^2 and shape variables are in m

	Wang *et al.* (2004)	Wei *et al.* (2005)	Gomes (2011)	Wei at al. (2011)	Kaveh and Zolghadr (2011)		Miguel and Miguel (2012)		Kaveh and Mahdavi (2014)	Kaveh and Zolghadr (2014a)	Farshchin *et al.* (2016)		Kaveh and Zolghadr (2017)	Kaveh and Ilchi Ghazaan (2017)	Tejani *et al.* (2016a)				Proposed work
Design variable	OC	GA	PSO	NHPGA	CSS	enhanced CSS	HS	FA	CBO	DPSO	TLBO	MC-TLBO	TWO	VPS	SOS	SOS-ABF1	SOS-ABF2	SOS-ABF1&2	ISOS
y_3,y_{19}	1.2086	1.1998	0.9637	1.09693	0.8726	1.0289	0.8415	0.9392	0.9562	0.9482	0.9639	0.9830	1.0039	0.9042	0.9598	0.9168	0.9413	0.9060	0.9257
y_5,y_{17}	1.5788	1.6553	1.3978	1.45558	1.2129	1.3868	1.2409	1.3270	1.3236	1.3439	1.3551	1.3803	1.3531	1.2850	1.3867	1.2980	1.3393	1.2665	1.3188
y_7,y_{15}	1.6719	1.9652	1.5929	1.59539	1.3826	1.5893	1.4464	1.5063	1.5037	1.5043	1.5338	1.5645	1.5339	1.5017	1.5698	1.4777	1.5434	1.4834	1.4274
y_9,y_{13}	1.7703	2.0737	1.8812	1.76551	1.4706	1.6405	1.5334	1.6086	1.6318	1.6350	1.6367	1.6871	1.6768	1.6509	1.6687	1.6046	1.6744	1.6004	1.5806
y_{11}	1.8502	2.3050	2.0856	1.87413	1.5683	1.6835	1.5971	1.6679	1.6987	1.7182	1.7052	1.7590	1.7728	1.7277	1.7203	1.6596	1.7571	1.6397	1.6548
A_1,A_{27}	3.2508	2.8932	2.6797	2.62463	2.9082	3.4484	3.2031	2.9838	2.7472	2.6208	2.9055	2.9913	2.8892	3.1306	2.9038	2.8448	2.9344	3.3609	2.6549
A_2,A_{26}	1.2364	1.1201	1.1568	1.0000	1.0212	1.5045	1.1107	1.1098	1.0132	1.0397	1.0012	1.0005	1.0949	1.0023	1.0163	1.0785	1.0256	1.0203	1.0383
A_3,A_{24}	1.0000	1.0000	2.3476	1.00176	1.0363	1.0039	1.1871	1.0091	1.0052	1.0464	1.0001	1.0042	1.0213	1.0001	1.0033	1.0000	1.0095	1.0169	1.0000
A_4,A_{25}	2.5386	1.8655	1.7182	2.07586	3.9147	2.5533	3.3281	2.5955	2.5054	2.7163	2.5598	2.5958	2.6776	2.5883	3.1940	2.8906	2.5838	2.6772	3.0083
A_5,A_{23}	1.3714	1.5962	1.2751	1.22071	1.0025	1.0868	1.4057	1.2610	1.1809	1.0252	1.2523	1.2139	1.1981	1.1119	1.0109	1.0335	1.1569	1.0166	1.0024
A_6,A_{21}	1.3681	1.2642	1.4819	1.48922	1.2167	1.3382	1.0883	1.1975	1.2603	1.5081	1.2141	1.1423	1.1387	1.2599	1.5877	1.2119	1.2548	1.2244	1.4499
A_7,A_{22}	2.4290	1.8254	4.6850	2.30847	2.7146	3.1626	2.1881	2.4264	2.7090	2.3750	2.3851	2.3170	2.6537	2.6743	2.4104	3.1886	2.5104	2.7056	3.1724
A_8,A_{20}	1.6522	2.0009	1.1246	1.43236	1.2663	2.2664	1.2223	1.3588	1.4023	1.4498	1.3881	1.5100	1.4171	1.3961	1.3864	1.3435	1.4626	1.5535	1.2661
A_9,A_{18}	1.8257	1.9526	2.1214	1.64678	1.8006	1.2668	1.7033	1.4771	1.4661	1.4499	1.5235	1.5172	1.3934	1.5036	1.6276	1.7247	1.5245	1.4833	1.4659
A_{10},A_{27}	2.3022	1.9705	3.8600	2.87072	4.0274	1.7518	3.1885	2.5648	2.6107	2.5327	2.6065	2.2722	2.7741	2.4441	2.3594	2.6980	2.4586	2.4032	2.9013
A_{11},A_{17}	1.3103	1.8294	2.9817	1.50405	1.3364	2.7789	1.0100	1.1295	1.1764	1.2358	1.1378	1.2112	1.2759	1.2977	1.0293	1.1401	1.1888	1.0000	1.1537
A_{12},A_{15}	1.4067	1.2358	1.2021	1.31328	1.0548	1.4209	1.4074	1.3199	1.3767	1.3528	1.3078	1.2739	1.2776	1.3619	1.3721	1.2840	1.3765	1.4982	1.3465
A_{13},A_{16}	2.1896	1.4049	1.2563	2.32277	2.8116	1.0100	2.8499	2.9217	2.6809	2.9144	2.6205	2.4934	2.1666	2.3500	2.0673	2.3044	2.2341	2.7480	2.6850
A_{14}	1.0000	1.0000	3.3276	1.04258	1.1702	2.2919	1.0269	1.0004	1.0064	1.0085	1.0003	1.0000	1.0099	1.0000	1.0000	1.0000	1.0007	1.0072	1.0000
Mass(kg)	366.50	368.84	377.20	363.032	362.84	362.38	361.50	360.05	359.9239	360.40	359.88	359.966	360.27	359.94	360.8658	360.4260	359.9050	360.5007	360.7432
f_1 (Hz)	20.0850	20.0013	20.0001	20.0819	20.0000	20.0028	20.0037	20.0024	20.0031	20.0194	20.0001	20.0001	20.0279	20.0002	20.0366	20.0230	20.0052	20.0023	20.0119
f_2 (Hz)	42.0743	40.0305	40.0003	40.0961	40.0693	40.0155	40.0050	40.0019	40.0060	40.0113	40.0005	40.0005	40.0146	40.0005	40.0007	40.0394	40.0048	40.0363	40.0964
f_3 (Hz)	62.9383	60.0000	60.0001	60.0321	60.6982	61.2798	60.0082	60.0043	60.0033	60.0082	60.0066	60.0066	60.0946	60.0000	60.0138	60.0339	60.0077	60.0065	60.0066
FEs	-	-	12500	-	4000	4000	20000	5000	6000	6000	12000	12000	-	30000	4000	4000	4000	4000	4000
Mean	-	-	381.2	-	366.77	365.75	362.04	360.37	360.4463	362.21	360.803	360.839	363.75	360.23	364.8521	363.3662	363.0816	363.6336	363.3978
STD	-	9.0325	4.26	-	3.742	3.461	0.52	0.26	0.35655	1.68	0.633	0.496	2.48	0.22	2.9650	2.1704	1.8304	2.0771	1.5675

Table 4. Optimal design parameters for the 72-bar truss, where size variables are in cm^2

	Kaveh and Zolghadr (2011)		Kaveh and Zolghadr (2012)	Kaveh and Mahdavi (2014)	Farshchin *et al.* (2016)		Kaveh and Zolghadr (2017)	Kaveh and Ilchi Ghazaan (2017)	Tejani *et al.* (2016a)				Proposed work
Design variable	CSS	enhanced CSS	CSS-BBBC	CBO	TLBO	MC-TLBO	TWO	VPS	SOS	SOS-ABF1	SOS-ABF2	SOS-ABF1&2	ISOS
A_1–A_4	2.528	2.522	2.854	3.3699	3.5491	3.4188	3.380	3.5017	3.6957	4.1820	3.6273	3.8745	3.3563
A_5–A_{12}	8.704	9.109	8.301	7.3428	7.9676	7.9263	8.086	7.9340	7.1779	7.8990	7.9416	7.6185	7.8726
A_{13}–A_{16}	0.645	0.648	0.645	0.6468	0.6450	0.6450	0.647	0.6450	0.6450	0.6450	0.6460	0.6450	0.6450
A_{17}–A_{18}	0.645	0.645	0.645	0.6457	0.6450	0.6450	0.646	0.6450	0.6569	0.6450	0.6450	0.6957	0.6450
A_{19}–A_{22}	8.283	7.946	8.202	8.0056	8.1532	8.0143	8.89	8.0215	7.7017	8.0149	7.5653	8.4112	8.5798
A_{23}–A_{30}	7.888	7.703	7.043	8.0185	7.9667	7.9603	8.136	7.9826	7.9509	8.1772	8.0171	7.7833	7.6566
A_{31}–A_{34}	0.645	0.647	0.645	0.6458	0.6450	0.6450	0.654	0.6450	0.6450	0.6450	0.6714	0.6450	0.7417
A_{35}–A_{36}	0.645	0.6456	0.645	0.6457	0.6450	0.6450	0.647	0.6450	0.6450	0.6450	0.6450	0.6450	0.6450
A_{37}–A_{40}	14.666	13.465	16.328	12.4585	12.9272	12.7903	13.097	12.8175	12.3994	12.4516	13.4781	12.0976	13.0864
A_{41}–A_{48}	6.793	8.250	8.299	8.1211	8.1226	8.1013	8.101	8.1129	8.6121	7.7290	7.6531	7.7086	8.0764
A_{49}–A_{52}	0.645	0.645	0.645	0.6460	0.6452	0.6450	0.663	0.6450	0.6450	0.6525	0.6450	0.6450	0.6450
A_{53}–A_{54}	0.645	0.646	0.645	0.6459	0.6450	0.6473	0.646	0.6450	0.6450	0.6450	0.6450	0.6450	0.6937
A_{55}–A_{58}	16.464	18.368	15.048	17.3636	17.0524	17.4615	16.483	17.3362	17.4827	16.8203	16.6583	16.9516	16.2517
A_{59}–A_{66}	8.809	7.053	8.268	8.3371	8.0618	8.1304	7.873	8.1010	8.1502	7.9846	8.1609	8.7289	8.1703
A_{67}–A_{70}	0.645	0.645	0.645	0.6460	0.6450	0.6450	0.651	0.6450	0.6740	0.6742	0.6450	0.6450	0.6450
A_{71}–A_{72}	0.645	0.646	0.645	0.6476	0.6450	0.6451	0.657	0.6450	0.6550	0.6450	0.6450	0.6450	0.6450
Mass (kg)	328.814	328.393	327.507	324.7552	327.568	327.5750	328.83	327.649	325.5585	325.086	324.6897	325.2317	325.0682
f_1 (Hz)	4.000	4.000	4.000	4.0000	4.000	4.000	4.000	4.000	4.0023	4.0045	4.0013	4.0016	4.0000
f_3 (Hz)	6.006	6.004	6.004	6.0000	6.000	6.000	6.000	6.000	6.0020	6.0019	6.0002	6.0003	6.0008
FEs	4000	4000	4000	6000	15000	15000	-	30000	4000	4000	4000	4000	4000
Mean	337.70	335.77	-	330.4154	328.684	327.6930	336.1	327.670	331.1228	328.6582	328.4621	334.9979	329.4699
STD	5.42	7.20	-	7.7063	0.73	0.1250	5.8	0.018	4.2278	2.7948	2.4600	6.0566	2.6642

Table 5. Optimal design parameters for the 52-bar truss, where size variables are in cm^2 and shape variables are in m

	Lin *et al.* (1982)	Wei *et al.* (2005)	Gomes (2011)	Kaveh and Zolghadr (2011)		Miguel and Miguel (2012)		Kaveh and Zolghadr (2012)	Kaveh and Zolghadr (2014a)	Farshchin *et al.* (2016)		Kaveh and Zolghadr (2017)	Tejani *et al.* (2016a)				Proposed work
Design variable	Bi-factor algorithm	NGHA	PSO	CSS	enhanced CSS	HS	FA	CSS-BBBC	DPSO	TLBO	MC-TLBO	TWO	SOS	SOS-ABF1	SOS-ABF2	SOS-ABF1&2	ISOS
z_A	4.3201	5.8851	5.5344	5.2716	6.1590	4.7374	6.4332	5.331	6.1123	6.0026	5.9531	6.012	5.7624	5.9650	6.0120	5.8950	6.1631
x_B	1.3153	1.7623	2.0885	1.5909	2.2609	1.5643	2.2208	2.134	2.2343	2.2626	2.2908	1.598	2.3239	2.3240	2.4250	2.4237	2.4224
z_B	4.1740	4.4091	3.9283	3.7093	3.9154	3.7413	3.9202	3.719	3.8321	3.7452	3.7037	4.287	3.7379	3.7002	3.7000	3.7030	3.8086
x_F	2.9169	3.4406	4.0255	3.5595	4.0836	3.4882	4.0296	3.935	4.0316	3.9854	3.9660	3.641	3.9842	3.9636	4.0201	3.9926	4.1080
z_F	3.2676	3.1874	2.4575	2.5757	2.5106	2.6274	2.5200	2.500	2.5036	2.5000	2.5001	2.888	2.5121	2.5000	2.5000	2.5000	2.5018
A_1–A_4	1.00	1.0000	0.3696	1.0464	1.0335	1.0085	1.0050	1.0000	1.0001	1.0000	1.0002	2.1245	1.0988	1.0000	1.0000	1.0000	1.0074
A_5–A_8	1.33	2.1417	4.1912	1.7295	1.0960	1.4999	1.3823	1.3056	1.1397	1.1210	1.0962	1.1341	1.0031	1.1797	1.0000	1.0000	1.0003
A_9–A_{16}	1.58	1.4858	1.5123	1.6507	1.2449	1.3948	1.2295	1.4230	1.2263	1.2113	1.2252	1.187	1.1956	1.2109	1.1280	1.0000	1.1982
A_{17}–A_{20}	1.00	1.4018	1.5620	1.5059	1.2358	1.3462	1.2662	1.3851	1.3335	1.4486	1.4555	1.318	1.4563	1.4800	1.4466	1.5759	1.2787
A_{21}–A_{28}	1.71	1.911	1.9154	1.7210	1.4078	1.6776	1.4478	1.4226	1.4161	1.4156	1.4172	1.3637	1.3773	1.3977	1.4298	1.4046	1.4421
A_{29}–A_{36}	1.54	1.0109	1.1315	1.0020	1.0022	1.3704	1.0000	1.0000	1.0001	1.0000	1.0003	1.0299	1.0055	1.0229	1.0032	1.0000	1.0000
A_{37}–A_{44}	2.65	1.4693	1.8233	1.7415	1.6024	1.4137	1.5728	1.5562	1.5750	1.5434	1.6204	1.3479	1.7397	1.6747	1.7686	1.6494	1.4886
A_{45}–A_{52}	2.87	2.1411	1.0904	1.2555	1.4596	1.9378	1.4153	1.4485	1.4357	1.4034	1.3296	1.4446	1.3084	1.3033	1.2770	1.5664	1.4990
Mass (kg)	298.0	236.046	228.381	205.237	197.337	214.94	197.53	197.309	195.351	193.185	193.185	194.25	195.4969	194.8089	195.1730	198.2630	194.7483
f_1 (Hz)	15.22	12.81	12.751	9.246	11.849	12.2222	11.3119	12.987	11.315	11.4613	11.5924	9.265	12.7144	11.8992	12.2594	12.8140	12.5459
f_2 (Hz)	29.28	28.65	28.649	28.648	28.649	28.6577	28.6529	28.648	28.648	28.6480	28.6480	28.667	28.6540	28.6478	28.6576	28.7301	28.6518
FEs	-	-	11270	4000	4000	20000	10000	4000	6000	15000	15000	-	4000	4000	4000	4000	4000
Mean	-	-	234.3	213.101	205.617	229.88	212.80	-	198.71	200.300	197.876	214.25	214.6676	210.7033	211.5683	224.5050	207.5498
STD	-	37.462	5.22	7.391	6.924	12.44	17.98	-	13.85	15.4816	5.7905	12.64	15.1499	11.8339	12.7871	17.8552	8.7354

Table 6. Optimal design parameters for the 120-bar truss, where size variables are in cm^2

	Kaveh and Zolghadr (2012)		Kaveh and Mahdavi (2014)	Kaveh and Zolghadr (2014a)		Kaveh and Ilchi Ghazaan (2017)	Tejani *et al.* (2016a)				Proposed work
Group no.	CSS	CSS-BBBC	CBO	PSO	DPSO	VPS	SOS	SOS-ABF1	SOS-ABF2	SOS-ABF1&2	ISOS
G_1	21.710	17.478	19.6917	23.494	19.607	19.6836	19.5203	19.5449	19.5715	19.3806	19.6662
G_2	40.862	49.076	41.1421	32.976	41.290	40.9581	40.8482	40.9483	39.8327	40.4230	39.8539
G_3	9.048	12.365	11.1550	11.492	11.136	11.3325	10.3225	10.4482	10.5879	11.1095	10.6127
G_4	19.673	21.979	21.3207	24.839	21.025	21.5387	20.9277	21.0465	21.2194	21.2086	21.2901
G_5	8.336	11.190	9.8330	9.964	10.060	9.8867	9.6554	9.5043	10.0571	9.9200	9.7911
G_6	16.120	12.590	12.8520	12.039	12.758	12.7116	12.1127	11.9362	11.8322	11.3161	11.7899
G_7	18.976	13.585	15.1602	14.249	15.414	14.9330	15.0313	14.9424	14.7503	14.7820	14.7437
Mass (kg)	9204.51	9046.34	8889.1303	9171.93	8890.48	8888.74	8713.3030	8712.1100	8710.3300	8716.9470	8710.0620
f_1 (Hz)	9.002	9.000	9.0000	9.0000	9.0001	9.0000	9.0009	9.0011	9.0006	9.0012	9.0001
f_2 (Hz)	11.002	11.007	11.0000	11.0000	11.0007	11.0000	11.0005	11.0003	11.0002	11.0023	10.9998
FEs	4000	4000	6000	6000	6000	30000	4000	4000	4000	4000	4000
Mean	-	-	8891.2540	9251.84	8895.99	8896.04	8735.3452	8727.4267	8725.3075	8790.6961	8728.5951
STD	-	-	1.7926	89.38	4.26	6.65	17.9011	16.5503	10.6402	55.7294	14.2296

Table 7. Optimal design parameters for the 200-bar truss, where size variables are in cm^2

		Kaveh and Zolghadr (2012)		Kaveh and Mahdavi (2015)		Farshchin *et al.* (2016)		Tejani *et al.* (2016a)				Proposed work
Group no.	Bars	CSS	CSS-BBBC	CBO	2D-CBO	TLBO	MC-TLBO	SOS	SOS-ABF1	SOS-ABF2	SOS-ABF1&2	ISOS
G_1	1,2,3,4	1.2439	0.2934	0.3268	0.4460	0.3030	0.3067	0.4781	0.2822	0.3058	0.3845	0.3072
G_2	5,8,11,14,17	1.1438	0.5561	0.4502	0.4556	0.4479	0.4450	0.4481	0.5014	0.5196	0.8524	0.5075
G_3	19,20,21,22,23,24	0.3769	0.2952	0.1000	0.1519	0.1001	0.1000	0.1049	0.1071	0.1000	0.1130	0.1001
G_4	18,25,56,63,94,101,132,139,170,177	0.1494	0.1970	0.1000	0.1000	0.1000	0.1001	0.1045	0.1002	0.1092	0.1000	0.1000
G_5	26,29,32,35,38	0.4835	0.8340	0.7125	0.4723	0.5124	0.5077	0.4875	0.5277	0.5238	0.5084	0.5893
G_6	6,7,9,10,12,13,15,16,27,28,30,31,33,34,36,37	0.8103	0.6455	0.8029	0.7543	0.8205	0.8241	0.9353	0.8248	0.7956	0.8885	0.8328
G_7	39,40,41,42	0.4364	0.1770	0.1028	0.1024	0.1000	0.1001	0.1200	0.1300	0.1003	0.1000	0.1431
G_8	43,46,49,52,55	1.4554	1.4796	1.4877	1.4924	1.4499	1.4367	1.3236	1.4016	1.3119	1.2170	1.3600
G_9	57,58,59,60,61,62	1.0103	0.4497	0.1000	0.1000	0.1001	0.1000	0.1015	0.1000	0.1056	0.1356	0.1039
G_{10}	64,67,70,73,76	2.1382	1.4556	1.0998	1.6060	1.5955	1.5787	1.4827	1.4657	1.6178	1.5477	1.5114
G_{11}	44,45,47,48,50,51,53,54,65,66,68,69,71,72,74,75	0.8583	1.2238	0.8766	1.2098	1.1556	1.1587	1.1384	1.1327	1.1954	1.0568	1.3568
G_{12}	77,78,79,80	1.2718	0.2739	0.1229	0.1061	0.1242	0.1000	0.1020	0.1196	0.1615	0.4552	0.1024
G_{13}	81,84,87,90,93	3.0807	1.9174	2.9058	3.0909	2.9753	2.9573	2.9943	3.0262	2.9102	3.4433	2.9024
G_{14}	95,96,97,98,99,100	0.2677	0.1170	0.1000	0.7916	0.1000	0.1000	0.1562	0.2527	0.1134	0.1000	0.1000
G_{15}	102,105,108,111,114	4.2403	3.5535	3.9952	3.6095	3.2553	3.2569	3.4330	3.3267	3.5156	3.6060	3.4120
G_{16}	82,83,85,86,88,89,91,92,103,104,106,107,109,110,112,113	2.0098	1.3360	1.7175	1.4999	1.5762	1.5733	1.6816	1.5963	1.6227	1.4460	1.4819
G_{17}	115,116,117,118	1.5956	0.6289	0.1000	0.1000	0.2680	0.2675	0.1026	0.2417	0.3687	0.1893	0.2587
G_{18}	119,122,125,128,131	6.2338	4.8335	5.9423	5.2951	5.0692	5.0867	5.0739	4.8557	4.6196	5.1791	4.8291
G_{19}	133,134,135,136,137,138	2.5793	0.6062	0.1102	0.1000	0.1000	0.1004	0.1068	0.1001	0.1543	0.2666	0.1499
G_{20}	140,143,146,149,152	3.0520	5.4393	5.8959	4.5288	5.4281	5.4551	6.0176	5.4975	5.6545	5.8750	5.5090
G_{21}	120,121,123,124,126,127,129,130,141,142,144,145,147,148,150,151	1.8121	1.8435	2.1858	2.2178	2.0942	2.0998	2.0340	2.0829	2.2106	2.5624	2.2221
G_{22}	153,154,155,156	1.2986	0.8955	0.5249	0.7571	0.6985	0.7156	0.6595	0.8522	0.6688	0.7535	0.6113
G_{23}	157,160,163,166,169	5.8810	8.1759	7.2676	7.7999	7.6663	7.6425	6.9003	7.5480	7.4241	7.9706	7.3398
G_{24}	171,172,173,174,175,176	0.2324	0.3209	0.1278	0.3506	0.1008	0.1049	0.2020	0.1279	0.1187	0.3324	0.1559
G_{25}	178,181,184,187,190	7.7536	10.98	7.8865	7.8943	7.9899	7.9352	6.8356	7.6278	7.5955	7.3386	8.6301
G_{26}	158,159,161,162,164,165,167,168,179,180,182,183,185,186,188,189	2.6871	2.9489	2.8407	2.8097	2.8084	2.8262	2.6644	3.0233	2.7572	3.0958	2.8245
G_{27}	191,192,193,194	12.5094	10.5243	11.7849	10.4220	10.4661	10.4388	12.1430	10.3024	11.1467	9.1512	10.8563
G_{28}	195,197,198,200	29.5704	20.4271	22.7014	21.2576	21.2466	21.2125	22.2484	21.4034	21.4328	20.7230	20.9142
G_{29}	196,199	8.2910	19.0983	7.8840	11.9061	10.7340	10.8347	8.9378	10.4810	9.8690	12.1258	10.5305
Mass		2259.86	2298.61	2203.212	2189.08	2156.541	2156.639	2180.3210	2164.8840	2165.8010	2207.8880	2169.4590

(kg)												
f_1 (Hz)		5.000	5.010	5.0010	5.0016	5.0000	5.0000	5.0001	5.0001	5.0000	5.0000	5.0000
f_2 (Hz)		15.961	12.911	12.5247	13.3868	12.2171	12.2306	13.4306	12.1388	12.3327	13.3064	12.4477
f_3 (Hz)		16.407	15.416	15.1845	15.1981	15.0380	15.0259	15.2645	15.1284	15.1454	15.5397	15.2332
FEs		10000	10000	10000	10000	23000	23000	10000	10000	10000	10000	10000
Mean		-	-	2481.492	2308.443	2157.547	2157.447	2303.3034	2186.5744	2187.2517	2405.3479	2244.6372
STD		-	-	250.8259	132.5148	1.545	0.528	83.5897	15.2711	16.9436	128.1578	43.4808

Table 8. Result summery of truss problems

Test problem	Algorithm	Best mass	Worse mass	Mean mass	STD	FEs
The 10-bar truss	SOS	525.2789	543.0061	531.4033	4.2243	4000
	ISOS	524.7341	537.8942	530.0286	3.4763	4000
The 37-bar truss	SOS	360.8658	378.3619	364.8521	2.9650	4000
	ISOS	360.7432	368.5105	363.3978	1.5675	4000
The 72-bar truss	SOS	325.5585	349.0252	331.1228	4.2278	4000
	ISOS	325.0682	336.9062	329.4699	2.6642	4000
The 52-bar truss	SOS	195.4969	258.2653	214.6676	15.1499	4000
	ISOS	194.7483	241.7603	207.5498	8.7354	4000
The 120-bar truss	SOS	8713.3030	8791.5830	8735.3452	17.9011	4000
	ISOS	8710.0620	8770.8110	8728.5951	14.2296	4000
The 200-bar truss	SOS	2180.3210	2580.6030	2303.3034	83.5897	10000
	ISOS	2169.4590	2349.7180	2244.6372	43.4808	10000

Table 9. The CEC2014 benchmark functions

Test function	optimum	Test function	optimum
f1: Rotated high conditioned elliptic function	100	f16: Shifted and rotated Expanded Scaffer's f6 function	1600
f2: Rotated bent cigar function	200	f17: Hybrid function1 (f9, f8 ,f1)	1700
f3: Rotated discus function	300	f18: Hybrid function2 (f2, f12, f8)	1800
f4: Shifted and rotated Rosenbrock function	400	f19: Hybrid function3 (f7, f6, f4, f14)	1900
f5: Shifted and rotated Ackley's function	500	f20: Hybrid function4 (f12, f3, f13, f8)	2000
f6: Shifted and rotated Weierstrass function	600	f21: Hybrid function5 (f14, f12, f4, f9, f1)	2100
f7: Shifted and rotated Griewank's function	700	f22: Hybrid function6 (f10, f11, f13, f9, f5)	2200
f8: Shifted Rastrigin function	800	f23: Composition function1 (f4, f1, f2, f3, f1)	2300
f9: Shifted and rotated Rastrigin's function	900	f24: Composition function2 (f10, f9, f14)	2400
f10: Shifted Schwefel function	1000	f25: Composition function3 (f11, f9, f1)	2500
f11: Shifted and rotated Schwefel's function	1100	f26: Composition function4 (f11, f13, f1, f6, f7)	2600
f12: Shifted and rotated Katsuura function	1200	f27: Composition function5 (f14, f9, f11, f6, f1)	2700
f13: Shifted and rotated HappyCat function	1300	f28: Composition function6 (f15, f13, f11, f16, f1)	2800
f14: Shifted and rotated HGBat function	1400	f29: Composition function7 (f17, f18, f9)	2900
f15: Shifted and rotated Expanded Griewank's plus Rosenbrock's function	1500	f30: Composition function8 (f20, f21, f22)	3000

Table 10. Comparative mean of fitness values of the CEC2014 (The results of first six algorithms are as per Zheng, 2015)

Function	WWO	BA	Hus	GSA	BBO	IWO	SOS	ISOS
f1	628064.7331	316593399.3261	5555804.7723	14413625.1299	27262607.1812	1463430.6695	1026753.8150	982235.8503
f2	330.4397	25714756385.1935	10068.1285	8771.2239	4012004.0764	17672.1722	213.1503	205.2755
f3	526.8209	72001.7161	502.0203	45384.2492	13100.3120	8167.4657	938.9390	779.0035
Friedman value of f1–f3	6	24	11	17	20	15	9	6
Friedman rank of f1–f3	1	8	4	6	7	5	3	1
f4	417.0105	3697.5439	506.9362	676.4360	538.7936	500.3255	468.2918	459.8026
f5	519.9999	520.9716	520.7029	519.9990	520.1556	520.0140	520.5639	520.3440
f6	605.9873	636.3693	623.0650	619.5872	613.9623	602.2138	610.8746	610.4609
f7	700.0037	910.6678	700.0407	700.0001	701.0283	700.0337	700.0161	700.0156
f8	801.1436	1070.3076	940.1063	800.4991	877.4573	843.7475	852.1217	814.7215
f9	961.0930	1250.0944	1011.9988	1059.7399	951.4286	946.0714	970.5093	954.3446
f10	1581.5778	6426.1095	2253.5001	4392.2443	1002.1744	2565.2591	2107.2343	1156.5243
f11	3349.4633	8152.1644	3302.9108	5099.2681	3247.3542	2887.3064	4017.4845	2882.3048
f12	1200.0995	1202.5771	1200.1870	1200.0011	1200.2257	1200.0355	1200.6611	1200.2345
f13	1300.2617	1304.0199	1300.3921	1300.2972	1300.5091	1300.2789	1300.4233	1300.3774
f14	1400.2169	1473.1361	1400.2377	1400.2540	1400.4439	1400.2360	1400.3309	1400.2711
f15	1503.2828	194533.2621	1517.0308	1503.2887	1514.6242	1503.6932	1517.6988	1510.6991
f16	1610.4351	1612.9981	1611.7074	1613.6691	1609.9125	1610.4324	1610.6564	1609.2194
Friedman value of f4–f16	31	103	71	55	60	38	68	42
Friedman rank of f4–f16	1	8	7	4	5	2	6	3
f17	26618.6801	4641277.7674	198099.0415	578588.7550	4299306.6650	86437.0037	143235.1725	176709.8548
f18	2026.3758	121880897.9466	3780.5580	2289.6856	28418.2340	5787.0752	8320.0810	5689.7268
f19	1907.7291	2004.9297	1931.0413	1995.2919	1928.4718	1907.9130	1923.3954	1907.7915
f20	5363.8611	19356.8922	38657.3368	22421.9064	31411.1843	2992.6053	5770.2949	6983.0969
f21	38673.7809	1095231.5294	60455.7923	170612.9594	485593.2936	39074.3102	68597.8240	91120.6969
f22	2481.9864	3134.0717	3072.5807	3161.1458	2722.8879	2346.3986	2496.3689	2475.0983
Friedman value of f17–f22	9	44	31	35	38	14	24	21
Friedman rank of f17–f22	1	8	5	6	7	2	4	3
f23	2615.3339	2589.2348	2616.4306	2563.8030	2617.4563	2615.3912	2615.2440	2615.2440
f24	2631.3935	2601.3984	2658.1593	2600.0628	2635.3078	2617.7220	2600.0069	2600.0066
f25	2708.1025	2706.4776	2725.1243	2700.2992	2711.6826	2704.7821	2700.0000	2700.0000

f26	2700.2599	2703.3054	2785.4129	2800.0236	2705.5728	2700.2802	2700.4248	2700.3838
f27	3092.5281	3320.5881	4965.1704	3719.8566	3397.2649	3080.5244	3266.2256	3242.6389
f28	3888.1193	4534.8709	5415.1985	5281.7471	3801.1221	3692.6907	3846.7017	3768.4085
f29	4094.1268	4611072.2249	2078150.7960	52153.5350	149151.6886	16399.9243	1723986.6548	573330.0771
f30	5652.2988	198139.5111	16723.0281	19137.7225	16205.1664	9196.3092	5940.8351	5376.7276
Friedman value of f23–f30	28	43	59	39	46	25	28	20
Friedman rank of f23–f30	3	6	8	5	7	2	3	1
Overall Friedman value	74	214	172	146	164	92	129	89
Overall Friedman rank	1	8	7	5	6	3	4	2

Table 11. Comparative STD of fitness values of the CEC2014 (The results of first six algorithms are as per Zheng, 2015)

Function	WWO	BA	Hus	GSA	BBO	IWO	SOS	ISOS
f1	244526.8140	104690309.2627	2620084.7953	13187933.1609	16720012.8760	571747.0082	732930.2258	705470.8598
f2	202.2221	7553596375.4800	6012.6897	2903.3044	1549219.3214	8673.4818	20.2822	17.1804
f3	184.6450	17548.6717	540.6109	10432.6453	12764.8742	2692.8884	527.5818	623.8666
f4	36.3636	1973.8532	36.6181	51.5149	38.3545	28.7968	31.7519	35.7188
f5	0.0007	0.0481	0.0783	0.0006	0.0422	0.0038	0.0801	0.0667
f6	2.6204	1.5591	2.1784	1.8319	2.3542	1.1219	2.5681	2.3958
f7	0.0063	32.3193	0.0556	0.0010	0.0264	0.0121	0.0214	0.0183
f8	2.3361	25.6476	12.7304	0.2063	20.6917	10.1117	12.3208	3.3488
f9	11.0977	44.1294	25.9919	17.4329	11.4372	11.3933	24.0796	13.4374
f10	361.6122	518.6548	433.1531	360.9861	0.6800	380.0190	344.1052	40.8420
f11	289.2180	362.2389	465.5429	567.3467	511.5523	447.7160	835.0838	454.0984
f12	0.0561	0.3339	0.0777	0.0010	0.0562	0.0148	0.1833	0.0573
f13	0.0641	0.5483	0.0650	0.0665	0.1061	0.0650	0.0864	0.0710
f14	0.0441	13.9463	0.0474	0.0423	0.1992	0.1191	0.1296	0.0512
f15	0.7753	140338.9490	3.2695	0.7297	4.2976	0.8484	3.7981	3.7171
f16	0.4667	0.1904	0.7249	0.3428	0.5923	0.6144	0.6059	0.7314
f17	12403.5374	1789909.2516	160518.8631	219949.3460	4192494.2708	68473.6644	159023.3392	164458.2001
f18	125.1962	100285357.3457	2246.5148	377.9286	19674.9440	3690.0554	10313.3555	5150.4892
f19	1.3780	20.3164	33.1485	34.3190	27.6885	1.6545	26.6142	1.7869
f20	3177.0847	10283.6255	8492.7252	13860.3564	17604.9005	700.4102	3295.2452	3393.2462
f21	35555.5716	750680.8765	42428.1036	65285.4119	334571.5390	23011.1766	80093.3096	107774.8931
f22	142.8952	205.4095	267.2685	250.0137	234.4393	73.3907	151.5147	145.4101
f23	0.1447	128.3564	0.8448	64.5044	1.3178	0.0795	0.0000	0.0000
f24	6.8854	1.1996	12.4866	0.0171	5.9741	10.7664	0.0013	0.0015
f25	2.0009	14.9765	6.2686	1.3194	3.0104	0.8076	0.0000	0.0000
f26	0.0650	0.5372	35.3282	0.0054	22.0234	0.0543	0.0862	0.0955
f27	59.0084	64.6177	682.5243	350.5128	63.5282	35.0337	146.2996	136.2682
f28	360.7000	592.9149	461.3382	715.2876	93.3415	41.2055	190.1300	126.6216
f29	359.5639	2830613.7427	7704687.2363	378105.7595	1114430.5187	5140.1597	3468336.1382	2146439.0984
f30	738.0508	91057.1920	6582.6405	18411.7936	6076.3255	2078.6364	3248.5105	1107.0801

Overall Friedman value	77	195	169.5	128	168	87.5	137	118
Overall Friedman rank	1	8	7	4	6	2	5	3

Figures

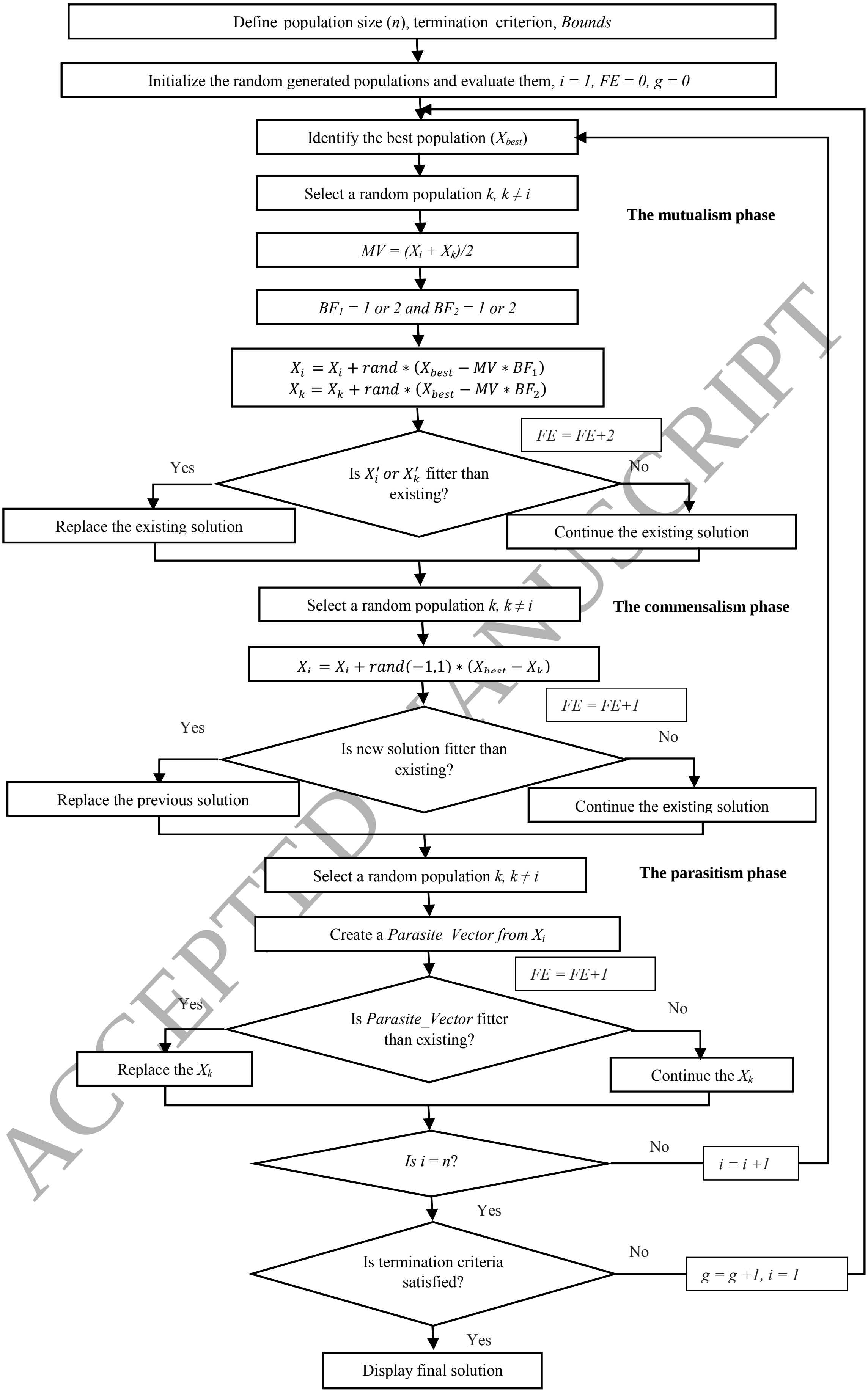


Figure 1. Schematic diagram of the SOS algorithm

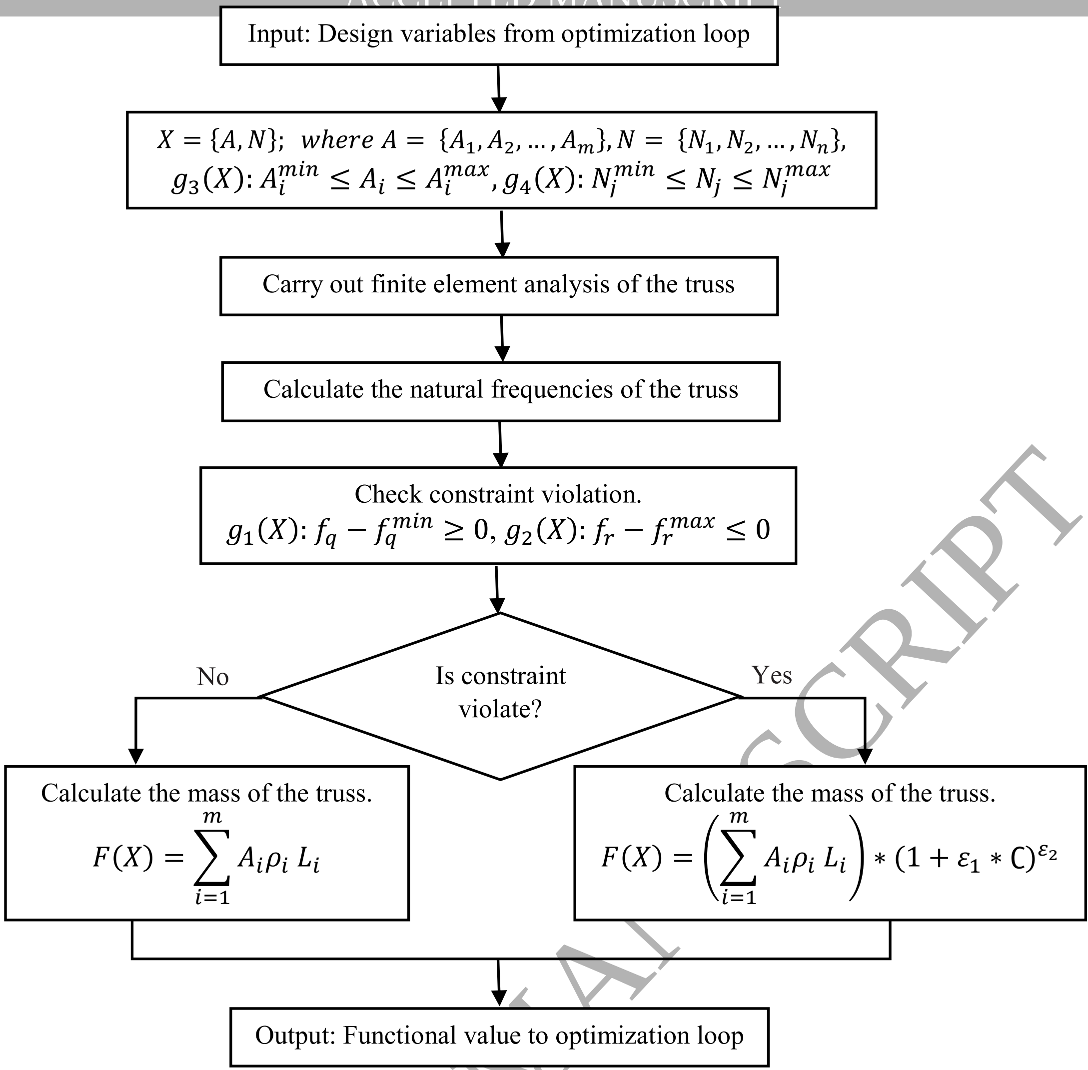


Figure 2. Schematic diagram of the truss optimization problem

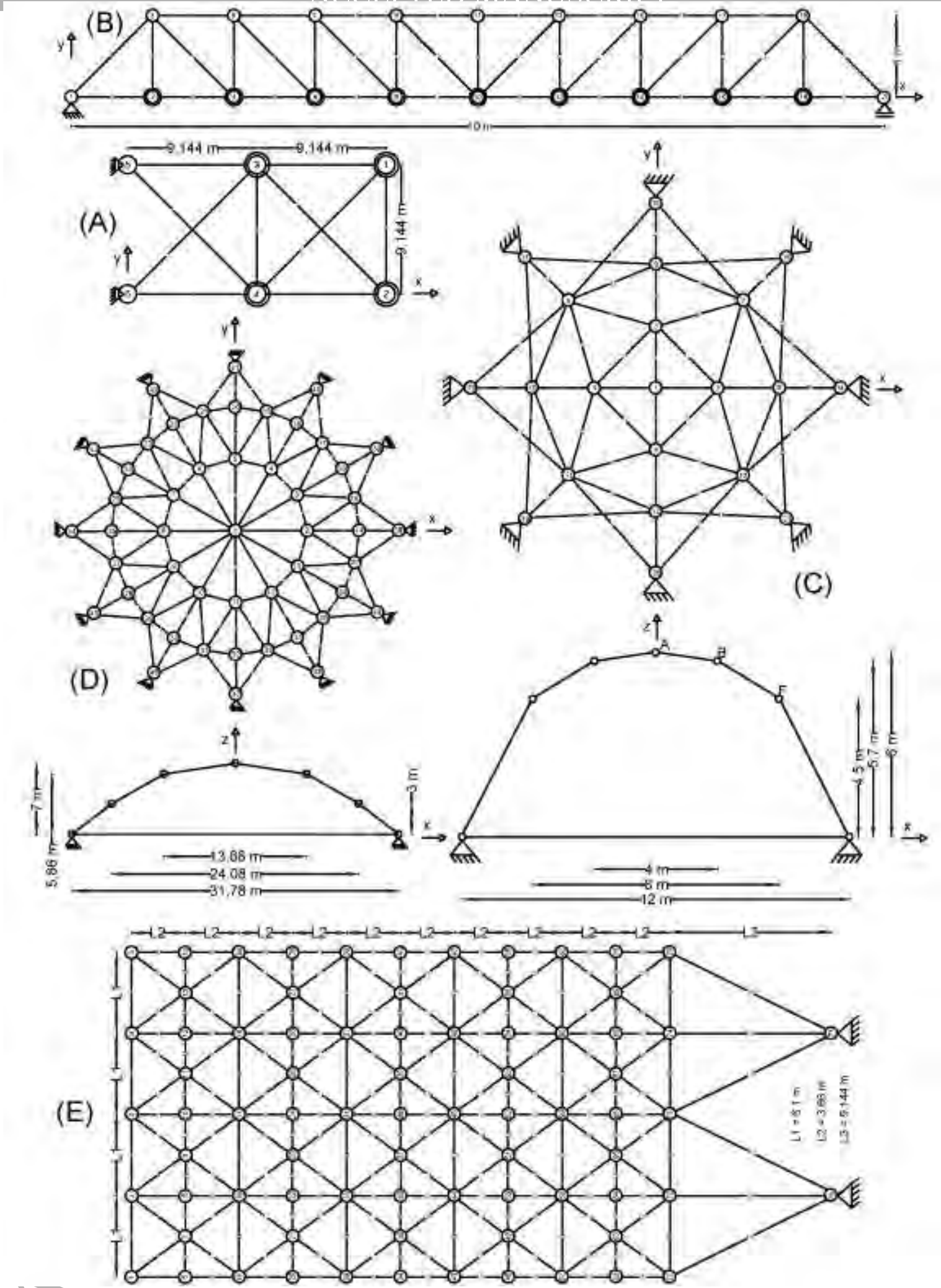


Figure 3. Test problems: (A) The 10-bar truss, (B) The 37-bar truss, (C) The 52-bar truss, (D) The 120-bar truss and (E) The 200-bar truss

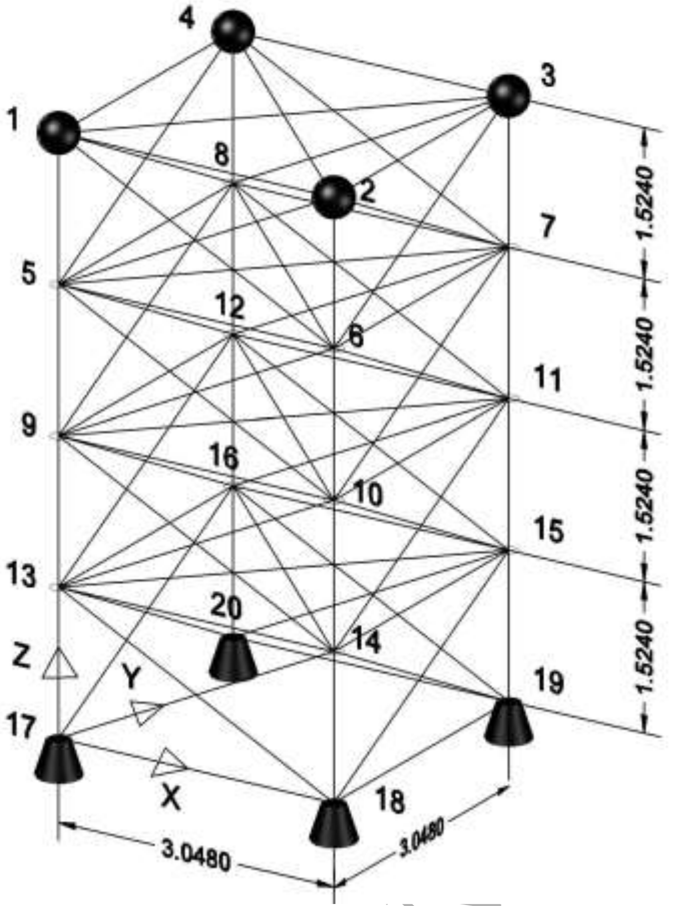


Figure 4. Test problem: The 72-bar truss

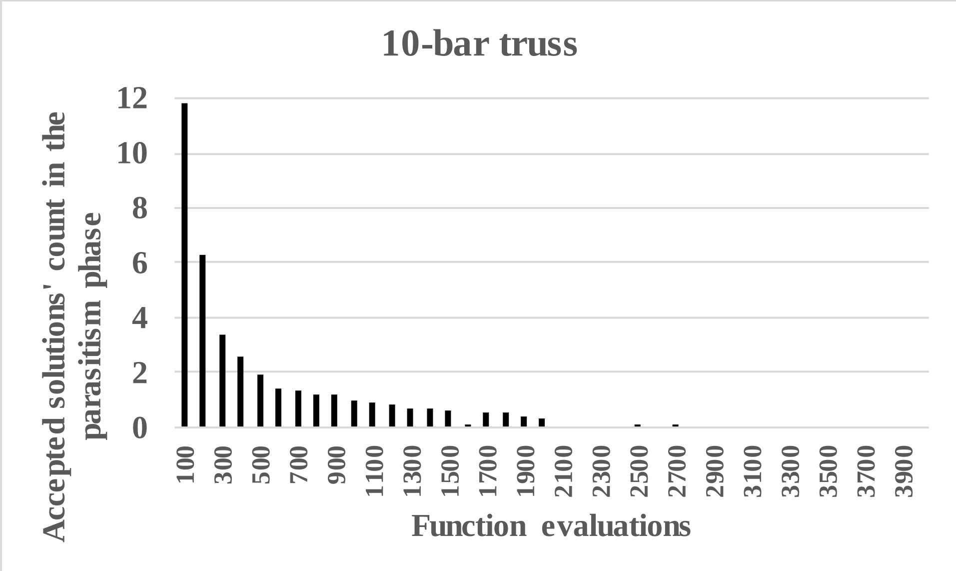


Figure 5. Accepted solution‟ count in the modified parasitism phase of SOS

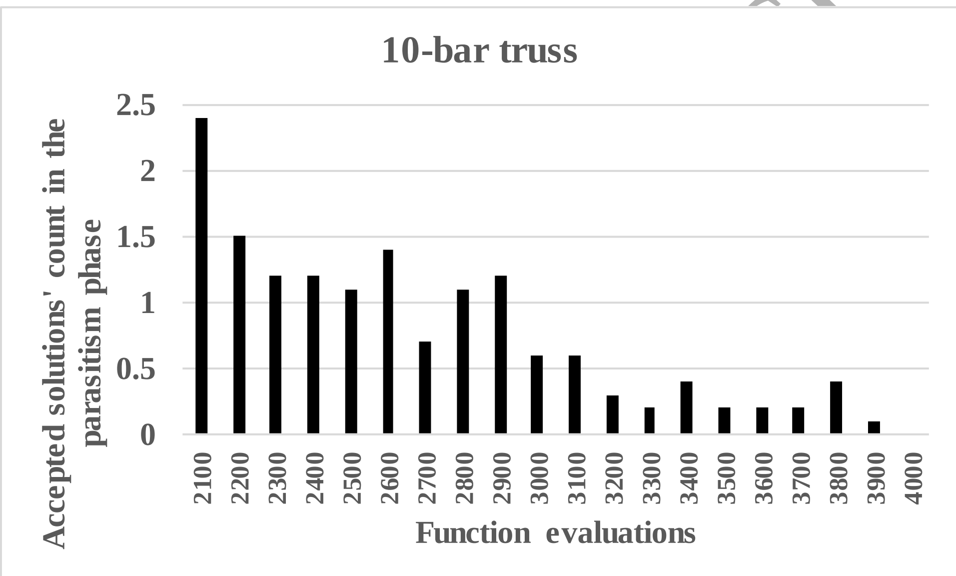
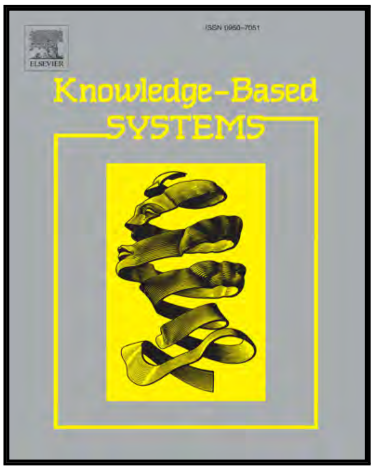


Figure 6. Accepted solution‟ count in the improved parasitism phase of ISOS



Truss optimization with natural frequency bounds using improved symbiotic organisms search

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