Trust and Discretion in Agency Contracts

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ABSTRACT. We extend the standard agency framework to allow for complex information, trustworthiness of the principal, and incomplete contracts and show that contractual incompleteness arises endogenously when there is enough complexity and trust. Several predictions of the standard model break down in our more general construction: trust plays a crucial role in the design of optimal contracts; not all the relevant, valuable information on the agent's choice of action is incorporated in the equilibrium contract; and, even when inference is perfect, the principal may only be able to implement the low cost effort. We conclude that one main function of agency contracts is to protect the agent from possible opportunistic behavior of the principal.

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1. Introduction

The economic literature on agency studies the design of optimal contracts in settings where the parties are opportunistic and the informational content of the observable and verifiable signals on the agent's choice of action can be captured by *complete contracts*, wage schedules that specify one and only one payment for each conceivable outcome.¹ In this setting, the agent does not need to trust the principal because the contract confers no discretion. In standard agency, trust and contractual incompleteness are worthless.

Nevertheless, real agency contracts are characterized by a substantial degree of incompleteness. The Law of Agency, for example, contains provisions informing how the agent should be paid when the contract leaves compensation unspecified.² According to the Law, when there is no written agreement for a definite amount, the agent should receive 'a fair value for his services.' Of course, in the standard model of the principal-agent relationship this is a vacuous statement because the wage schedule *is* the contract.

We propose three variations to the standard model and show that in our more general framework, contractual incompleteness arises endogenously: we introduce a notion of complex information; we require that contracts are written down; and we allow for some degree of trustworthiness on the part of the principal.

The literature assumes that the environment is sufficiently stable so that complete ex ante contracts can capture all useful information on the agent's choices: the information contained in the outcomes varies sufficiently smoothly from out-

¹See, for example, Berhold (1971), Ross (1973), Holmstrom (1979), Shavell (1979), Grossman and Hart (1983).

²See American Law Institute (1957) Restatement of the Law, Second, Agency, pg. 343: "If the amount of compensation is not otherwise agreed upon, as where no specific amount is stated and there is no customary rate for the services, it is inferred that, in a transaction in which some compensation is due, the parties have agreed that the agent is to receive the reasonable value of his services. In determining this, evidence of what other agents receive for similar services is competent, together with other factors, including the reputation of the agent, the skill with which the work is done, and the difficulty or danger of the task."

come to outcome so that contracts can be made sensitive to every minor detail of the signals.

We depart from this 'smoothness' assumption and allow information to vary drastically from outcome to outcome. We define complexity in these terms: roughly, information is *complex* if 'close by' outcomes lead to very different inferences on the agent's choice of action. Intuitively, an environment is complex if 'details matter.' Complexity arises not because of the number of possible outcomes per se is large, but because of the number of independent pieces of information that must be taken into account in writing optimal contracts. In contrast, in a *simple* environment the informational content of outcomes does not vary much for close by outcomes, just as in the standard model.

In order for contracts to be enforceable, they must be written down unambiguously. To formally model this idea, we describe the outcomes in terms of their objective features. We let the outcome space to have a product structure where each level of "depth" represents a different feature. Contracts specify payments contingent on sets of features. Since contracts have to be of finite length, they will necessarily condition on *finitely many features*.

One main consequence of the coupling of complexity and finite definability is that no complete contract can fully capture the richness and variability of the information contained in the outcomes. Under complexity and finite definability, complete contracts fall short from being second best because useful information must necessarily be left out from the written document.

We also depart from previous work on agency in that we do not focus exclusively on written contracts as the only possible method of governance. We allow for trustworthiness on the part of the principal. The principal is trustworthy if she can commit not to take advantage of the agent when the contract grants her discretion. A trustworthy principal compensates the agent for the full cost of effort, for the risk derived from the stochastic nature of outcomes, and, when applicable, for the risk ensuing from the belief that the principal may in fact be opportunistic. Of course, an opportunistic principal pays the agent as little as possible.

The presence of trust, in a world of complex information and finite contracts, allows for the emergence of *incomplete contracts*. A contract is incomplete if it is set-valued. When the contract is incomplete, the principal has discretion to choose the wage after the outcome has been realized, within the limits set by the contract. We show that equilibrium contracts consist of a simple lower bound below which the principal is not allowed to pay and are open above. In equilibrium, the realized compensation is always above the lower bound. Trust restores flexibility when written contracts cannot capture the full variability of the environment.

We show that trust is necessary for the working of incomplete contracts and that there is a monotone relationship between the principal's level of trustworthiness and her expected profit.³ Trust reduces the agent's risk bearing and, thus, it results in larger total surplus of the relationship. Since the principal makes a take-it-or-leave-it offer, she appropriates the gains.

We also show that in our more general setting, the sufficient statistic result breaks down. (See Holmstrom (1979).) When information is complex, some informative signals have to be left out of the written contract. If contracts could be of infinite length, then the result would be restored.

In standard agency, whenever the densities associated with each level of effort have different supports, the first best can be achieved by sufficiently punishing the agent if the realized outcome is impossible under high effort. In our framework, this result also breaks down. Again, if contracts could be of infinite length or, alternatively, if there was sufficient trust, the result would be reinstated.

In conventional agency models, contracts trade off incentives and risk. In our model, one main function of written contracts is protecting the agent from the possibility that the principal may be opportunistic. If the principal was completely trustworthy, then the optimal contract would be no contract at all. It is the principal's imperfect trustworthiness that makes written contracts necessary in the first place.

³Throughout the paper we will refer to the principal as *she* and to the agent as *he*.

1.1. Literature review

The first work formalizing complexity considerations in contracting is Dye (1985). He considers a contracting problem with exogenously given cost of contracting on each contingency. This approach was more recently taken by Battigalli and Maggi (2000). One difficulty with such approach is that conclusions are sensitive to the way the cost of contracting is modeled. It is also hard using this approach to account for the possibility that initially complicated contingent actions/compensations can be codified in standard-form contracts with minimal writing cost. In other words, if the contingent actions are sufficiently routine in nature, one would expect dramatic reduction in writing costs using, for instance, standard-form contracts. One would thus like to capture incompleteness that persists after contracting parties exhausted all reasonable possibilities of reducing contracting cost by hardwiring repetitive aspects that may initially appear as a complicated contingent action.

In response to these difficulties, Anderlini and Felli (1994) proposed a different approach where contracts are restricted only to be finitely defined. This formulation, which we largely follow in this paper, may be viewed as a way to capture the limit of positive but vanishingly small cost of contracting, without making any specific assumptions about the exact form of this cost.

A problem with Anderlini and Felli's model is that they obtain an approximation result: in their setting the first best contract can be approximated arbitrarily closely by finitely-defined contracts. This clearly undermines the potential of such model to understand incomplete contracting.

An alternative approach has been proposed by Al-Najjar, Casadesus-Masanell, and Ozdenoren (1998). These authors introduce complexity in a model of the continuum resembling that of Anderlini and Felli (1994). In a somewhat different contracting setting they show that in a complex environment, the expost optimal course of action cannot be approximated arbitrarily closely by finitely-defined exante complete contracts. Their model of complexity has the attractive feature that,

just as in the original Anderlini and Felli (1994), is built directly on the continuum and does not need to resort to discrete (countable) versions of the state-space to introduce a meaningful notion of complexity that circumvents the approximation result.

Recently, Al-Najjar, Anderlini, and Felli (2000) have suggested a different approach to break Anderlini and Felli (1994)'s result, using a modelling device introduced in Al-Najjar (1999). The idea there, as in the present paper, is to use a discrete state space instead of the continuum. The discrete state space allows more concreteness in modelling complex objects such as functions and correspondences. Krasa and Williams (1999) provide yet another avenue to evade Anderlini and Felli's approximation theorem.

Al-Najjar, Casadesus-Masanell, and Ozdenoren (1998) present a behavioral foundation for decision making in a complex environment. This foundation can be easily extended to justify behavior under complexity in the discrete setting that we consider.

We adopt the feature structure of Anderlini and Felli (1994) and the discrete state space model introduced in Al-Najjar (1999). However, the economic questions we address are very different. Our focus is on trust and discretion in the classic principal-agent problem.⁴ The central features of our model, namely moral hazard and delegation under asymmetric information, are absent in the works above which focus on co-insurance problems. In contrast to the endogenous model of agent's trustworthiness in Casadesus-Masanell (1999), we study the role of exogenous trust of the principal for the lay out of optimal incentive contracts in complex environments.

Section 2 reviews the standard agency model and characterizes optimal contracts. In Section 3 we introduce three variations to the standard model. First, we reinterpret the set of verifiable outcomes as collections of objective features; second, we allow for incomplete contracts; and third, we let the principal be trust-

⁴For excellent surveys on the principal-agent literature, see Levinthal (1988), Sappington (1991), Gibbons (1998), and Prendergast (1999).

worthy. We show that in the standard model, none of these variations affect the shape of the optimal contract. Section 4 introduces our notion of complexity. In Section 5 we restate the principal-agent problem now in a potentially complex environment and characterize optimal contracts as a function of the primitives of the agency problem. Section 6 presents concluding remarks.

2. The Standard Agency Problem

Consider the classic contracting problem between a risk neutral principal and a risk averse agent. The agent's action choice (his effort level) determines, probabilistically, an outcome x from some finite outcome space $X_N = \{x_1, x_2, ..., x_N\}$. Outcomes are assumed to be observable and verifiable so that enforceable contracts can be written on them.

The agent takes one of two actions (or effort levels) $e \in \{H, L\}$. Effort is unobservable and costly. The cost of e is c_e , with $c_H > c_L$. We normalize $c_L = 0$. Each effort level induces a probability distribution on the set of outcomes, X_N . If the agent takes action e, then the outcomes are distributed according to density $\pi_e: X_N \to \mathbb{R}_+$. That is, $\pi_e(x_n)$ is the probability of x_n given e.

Incentives to work are provided by means of a take-it-or-leave-it contract $\alpha: X_N \to \mathbb{R}$ specifying a payment for each outcome, $x \in X_N$.

The contract is designed so that the agent is willing to enter the relationship: if \bar{U} is the agent's reservation utility, then α must satisfy the participation constraint:

$$E_H[u(\alpha(x))] - c_H \ge \bar{U}. \tag{1}$$

The contract also needs to be *incentive compatible*; it must give incentives to take the action that the principal wants to implement. In particular, if the principal wants the agent to choose e = H, then α must be such that

$$H \in \arg\max_{e} \{ E_e[u(\alpha(x))] - c_e \}. \tag{2}$$

Given contract α and outcome x, the principal obtains benefit $B(x) - \alpha(x)$. To make the problem interesting, we assume that function B is such that the principal would like to implement high effort. The expected cost to the principal of contract α that implements e = H is

$$E_H \alpha(x)$$
. (3)

The timing of the game is as follows: The principal makes a take-it-or-leave-it contract offer. The agent accepts or rejects the offer. If the contract is accepted, the agent exerts effort $e \in \{H, L\}$. If rejected, the game ends. An outcome x is realized according to π_e . Payments are made as prescribed by the contract α .

Optimal contracts are designed by minimizing (3) subject to (1) and (2). There is a trade-off between incentives and risk bearing. On the one hand, because risk bearing reduces the total surplus and the principal is risk neutral and the agent risk averse, the principal would ideally like to assume all risk by offering a constant wage schedule. On the other hand, if the wage schedule is constant, the agent chooses e = L because $c_L < c_H$. Thus, the principal needs to carefully trade incentives and risk off when designing the incentives scheme.

Standard results by Holmstrom (1979), Grossman and Hart (1983), and others show that in equilibrium, constraints (1) and (2) are binding. The first order conditions characterizing the optimal contract are

$$\frac{1}{u'(\alpha(x_n))} = \lambda + \mu \left[1 - \frac{\pi_L(x_n)}{\pi_H(x_n)} \right] \text{ for all } x_n \in X_N,$$
 (4)

where λ and μ are the Kuhn-Tucker multipliers associated to the Individual Rationality and Incentive Compatibility constraints, respectively. Because $\mu > 0$, the presence of moral hazard is costly: the principal would be strictly better off under symmetric information.

Condition (4) implies that the equilibrium wage varies as a function of the likelihood ratio, $\frac{\pi_L(x_n)}{\pi_H(x_n)}$. The likelihood ratio indicates the precision by which outcome x_n signals that e = H. The lower the likelihood ratio, the stronger the signal that the agent chose high effort.

3. Discretion, trust, and complexity considerations in standard agency

In this paper, we focus on issues of discretion, trust, and complexity in contracting. We formalize these concepts in the context of agency contracts, then show that the standard model presented in Section 2 does not (cannot) take these considerations into account. This will be the motivation for considering an expanded model of the type introduced in Section 5.

3.1. Outcomes and verifiability

In standard agency, outcomes are assumed to be costlessly enforceable by court. This assumption reflects the view that the description of these outcomes is unambiguous to the contracting parties and to outside enforcement mechanisms, such as a court. We want to model more explicitly what it means for an outcome to be verifiable.

We will think of verifying an outcome as a process of checking combinations of elementary features. Each such feature is unambiguous, in the sense that there is no room for disagreement as to whether a given outcome does or does not have this feature. For example, one feature may represent a profit threshold, so we may use 1 to indicate that the threshold has been exceeded, and 0 otherwise. Another feature may be 'change in market share,' and so on.

Formally, there is an infinite number of binary features. Outcome x's j^{th} feature, x^j , takes values in set $\{0,1\}$. Thus, an outcome may be viewed as a point in the product space $\{0,1\}^{\mathbb{N}}$.

It may help the reader to distinguish between features that are payoff relevant, and those that have a purely informational content. The payoff relevant features affect the principal's payoff from the relationship directly. For example, these features may correspond to the principal's profits. A change in payoff-irrelevant features (keeping the others fixed) has no effect on the principal's benefit, the agent's costs, etc. Nevertheless, these may have a valuable informational content, and the contract may well condition on them to provide better incentives.

A key point to keep in mind is that, although we might reasonably expect a linear ordering on the payoff-relevant features, there is no reason to expect any such ordering on features that have purely informational content. Our analysis hinges on the possibility that these pieces of information may interact in a complex way to determine the inference that can be drawn about the agent's actions.

3.2. Incompleteness and discretion

We now introduce two variations to the model. First, we let contracts be incomplete, and second, we extend the principal's preferences to allow for trustworthy (or non self-interested) behavior. We then analyze how these two variations affect the form of optimal contracts in the standard model.

To model incomplete contracts, we generalize the wage schedule α to be a correspondence:

$$\alpha: X_N \twoheadrightarrow \mathbb{R}.$$

⁵For convenience, one may view this as as the interval [0,1] where one identifies real numbers with their binary expansions. There are, however, important differences. First, $\{0,1\}^{\mathbb{N}}$ involves duplication as (countable many) real numbers may have different equivalent binary expansions. Second, the interval [0,1] includes a metric structure which implicitly assumes a particular ordering of the features. Thus, in [0,1] two outcomes that agree on the *first* K features are very close when K is large. $\{0,1\}^{\mathbb{N}}$ presumes no such ordering by importance.

The interpretation is that for every outcome, x, the contract specifies a range of possible payments from which the principal can choose *after* outcome x has been realized. Formally,

DEFINITION 1: A contract is complete if it prescribes a single wage for each outcome x. A contract is incomplete if it is multiple-valued at some x.

To formalize the principal's ex post choice, given a contract α we let s_{α} be a selection of α ; that is, $s_{\alpha}: X \to \mathbb{R}$ such that $s_{\alpha}(x) \in \alpha(x)$ for every x.

EXAMPLE 1: An example of a contract is $\alpha(x) = [m_1, m_2]$ for all $x \in X_N$ with $m_1 < m_2$. In such a contract, after the agent has chosen the action and outcome x has been realized, the principal offers payment $s_{\alpha}(x) \in [m_1, m_2]$. This is an incomplete (or discretionary) contract because ex post the principal has some discretion to decide how much to pay the agent.

Example 2: A second example is $\alpha(x) = \begin{cases} m_1 & \text{if } 0 \leq x < \frac{1}{2} \\ m_2 & \text{otherwise} \end{cases}$. This contract is complete; it gives no discretion to the principal as it specifies an exact payment for each possible outcome.

3.3. Trust

When α is not single-valued (the contract is incomplete), the contract no longer determines the principal's behavior in the relationship. We allow the principal to be of one of two types. She may be opportunistic or trustworthy. The opportunistic principal pays the agent the lowest allowed under the contract. The selection rule under opportunism is denoted by $\underline{s}_{\alpha}(x)$. In contrast, given a contract α the trustworthy principal compensates the agent so that his expected utility is as close as possible to \overline{U} , the agent's reservation utility. The trustworthy principal covers the full cost of effort; she also covers the risk ensuing from the stochastic relation between actions and outcomes; in addition, the trustworthy principal pays a

premium to compensate for the ex ante risk that she may turn out to be opportunistic when time comes to remunerate the agent. The selection rule under trust is denoted by $\bar{s}_{\alpha}(x)$.

To abstract from signaling considerations, we assume that the contract is written before the principal's type is realized. With probability τ she will act trustworthily and with complementary probability she will behave opportunistically.⁶

The expected cost to the principal of contract α that implements e = H is

$$E_H[\tau \overline{s}_{\alpha}(x) + (1 - \tau)\underline{s}_{\alpha}(x)]. \tag{5}$$

In this model, the agent bears two types of risk. First, for any given level of effort, e, the exact realization of x is random. Second, the type of the principal is unknown: with probability τ she is trustworthy (i.e. unwilling to take advantage of the agent) and with probability $1 - \tau$ she is opportunistic (i.e. self-interest seeker with guile). The modified individual rationality and incentive compatibility constraints are

$$\tau E_H[u(\overline{s}_{\alpha}(x))] + (1 - \tau)E_H[u(\underline{s}_{\alpha}(x))] - c_H \ge \bar{U}$$
(6)

and

$$H \in \arg\max_{e} \{ \tau E_e[u(\overline{s}_{\alpha}(x))] + (1 - \tau) E_e[u(\underline{s}_{\alpha}(x))] - c_e \}. \tag{7}$$

With these variations, the timing of the game is as follows: The principal makes a take-it-or-leave-it contract offer (which may allow for discretion). The agent accepts or rejects the offer. If the contract is accepted, the agent exerts effort $e \in \{H, L\}$. If rejected, the relationship ends. An outcome x is realized according to the distribution induced by π_e . Nature chooses the type of the principal. If the

⁶One interpretation for the lack of signaling content in the contract is that contracts are set by convention. We may also allow the principal to signal her type through the choice of contract. We would then be centering attention to a pooling equilibrium where the agent believes that if the contract is different from the one characterized below then the principal is opportunistic with probability one.

contract allows for discretion, then the principal chooses a payment in the allowed set, $\alpha(x)$. If she is opportunistic, she chooses $\underline{s}_{\alpha}(x)$, and if she is trustworthy, she chooses $\overline{s}_{\alpha}(x)$.

3.4. Incomplete contracts and trust in the standard model

The following proposition shows that in the standard model of agency there is no role for contractual incompleteness, no matter how trustworthy the principal is. In other words, given any feasible incomplete contract, there always is a complete contract that does just at least as well.

PROPOSITION 1: In the standard model of agency incompleteness and trust have no value.

PROOF: Let (P1) be the program that minimizes (3) subject to (1) and (2) and let V_{P1} be the value of (P1). Fix $\tau \in (0,1]$. Let (P3) be the program that minimizes (5) subject to (6) and (7) and let V_{P3} be the value of (P3). We need to show that if α^* solves (P1) then it must also solve (P3). Suppose α^* solves (P1). Note that α^* is in the feasible set of (P3) and that $V_{P1}(\alpha^*) = V_{P3}(\alpha^*)$. Towards a contradiction suppose that there is another contract $\hat{\alpha}$ such that $V_{P3}(\hat{\alpha}) \geq V_{P3}(\alpha)$ for all α in the feasible set of (P3) and $V_{P3}(\hat{\alpha}) > V_{P3}(\alpha^*)$. The complete randomized contract $\hat{\alpha}'$ for (P1) that assigns wage $\bar{s}_{\hat{\alpha}}(x)$ with probability τ and $\underline{s}_{\hat{\alpha}}(x)$ with probabilities than contract $\hat{\alpha}$ for (P3) when the principal is of type τ . Therefore, we must have that $\hat{\alpha}'$ is feasible for (P1) and that $V_{P1}(\hat{\alpha}') = V_{P3}(\hat{\alpha})$. But then,

$$V_{P3}(\alpha^*) < V_{P3}(\hat{\alpha}) = V_{P1}(\hat{\alpha}') \le V_{P1}(\alpha^*) = V_{P3}(\alpha^*),$$

a contradiction.

As a simple corollary notice that the principal's expected payoff is always the same, no matter how trustworthy she is. The reason is that the optimal contract

is *complete* and thus trust and discretion are never used.

In the following sections we show that contractual incompleteness emerges endogenously in the limit of a sequence of simple finite models like the one we just presented.

4. The Infinite Model

4.1. Regular sequences of outcome spaces

For reasons that will become clear later, we will work with infinite versions of the finite model just presented. Suppose a sequence of finite outcome spaces. The N^{th} outcome space has the form:

$$X_N = \{x_1, \dots, x_{\#X_N}\} \subset \{0, 1\}^{\mathbb{N}}, \quad N = 1, 2, \dots$$

For the N^{th} model, $X_N = \{x_1, \dots, x_{\#X_N}\}$, the uniform distribution λ_N assigns to each subset $B \subset X_N$ a measure equal to its relative frequency:

$$\lambda_N(B) \equiv \frac{\#B}{\#X_N}.$$

We would like to consider the limiting model of the sequence of agency problems with outcome spaces $\{X_N\}$, as N goes to infinity. We define the outcome space of the limiting model to be $X = \bigcup_{N=1}^{\infty} X_N$. We would also like to have a counterpart on X for the uniform distribution λ_N in the finite models.

Define $A_m = \{x : x^m = 1\}$. That is, A_m is the set of outcomes whose mth feature is 1. Let \mathcal{A} be the algebra of subsets of X generated by A_m , $m = 1, 2, \ldots$ We call sets in \mathcal{A} finitely defined or finitely describable, because each such set can be fully pinned down by verifying a fixed finite number of features.

Definition 2: A sequence of outcome spaces $\{X_N\}_{N=1}^{\infty}$ is regular if:

- 1. $X_N \subset X_{N+1}$ for every N;
- 2. $\lim_{N\to\infty} \lambda_N(A)$ converges for every $A \in \mathcal{A}$;
- 3. There is $\epsilon > 0$ such that $\epsilon < \lim_{N \to \infty} \lambda_N(A_m) < 1 \epsilon$;

EXAMPLE 3: Here is a 'canonical' example of a regular sequence: Let \tilde{z}^j be a random variable taking values 0 and 1 with probability $p=\frac{1}{2}$. Consider random sequences of the form $\tilde{z}=\{\tilde{z}^j\}_{j=1}^{\infty}$. Each \tilde{z} is an element in $\{0,1\}^{\mathbb{N}}$. Let P^{∞} be the distribution on the elements of $\{0,1\}^{\mathbb{N}}$ constructed by taking infinite products of p; that is, $P^{\infty}=\times_{j=1}^{\infty}p$. Fix N>0. The N^{th} model X^N is generated by drawing N outcomes from $\{0,1\}^{\mathbb{N}}$ using distribution P^{∞} . The $N+1^{th}$ model X^{N+1} is the X^{th} model plus one extra draw. Let X^{th} are resulting sequence is regular.

Note that given $A \in \mathcal{A}$, we have, by definition of λ_N

$$\lambda_N(A) \equiv \lambda_N(X_N \cap A) \equiv \frac{\#(X_N \cap A)}{\#X_N}$$

Then, condition 2 in the definition simply states that the relative frequency of a set $A \in \mathcal{A}$ in the Nth model settles down to some limiting value. The idea here is that the sequence of outcome spaces, although increasing in size, resemble each other in terms of the distribution of the features in them. Condition 3 is a non-degeneracy condition whose role is to eliminate superfluous features that have zero limiting mass.

There generally, let Z denote the set of all sequences $\{\tilde{z}_i\}_{i=1}^{\infty}$ where $\tilde{z}_i \in \{0,1\}^{\mathbb{N}}$. Because the way P^{∞} was constructed, random elements $\{\tilde{z}_1,\tilde{z}_2,...\}$ are i.i.d. with mean $\frac{1}{2}$. By the law of large numbers, for any set $A \in \mathcal{A}$, there is $Z_A \subset Z$, with $P^{\infty}(Z_A) = 1$, such that if $\{\tilde{z}_i\}_{i=1}^{\infty} \in Z_A$ then $\lim_{I \to \infty} \frac{1}{I} \sum_{i=1}^{I} 1_A(\tilde{z}_i) = P^{\infty}(A)$. Since \mathcal{A} is countable, $Z' = \bigcap_{A \in \mathcal{A}} \{Z_A\}$ also has probability 1. In particular, it is non-empty. Any $X \in Z'$ satisfies assumptions in Definition 2.

LEMMA 1: Fix any sequence of finite models $\{X_N\}_{N=1}^{\infty}$ and let $X = \bigcup_{N=1}^{\infty} X_N$. Then there is a finitely additive probability measure λ on 2^X such that:

$$\lambda(B) = \lim_{N \to \infty} \lambda_N(B) \tag{8}$$

for every $B \subset X$ for which the limit exists.

Proof: See Appendix. ■

For the rest of the paper we fix outcome space X (the limit of a regular sequence satisfying the conditions in Definition 2). We let λ be a finitely additive probability measure on X given by Lemma 1.

4.2. Integration

Given X, and λ we are interested in integrating (averaging out) functions of the form $f: X \to \mathbb{R}$. We will use such integrals to compute expected values.

The integral $\int_X f d\lambda$ with respect to a finitely additive probability λ is well-defined and extensively studied; its development closely follows that of the usual integral.

For our purposes it suffices to define the integral of a *simple* function f (i.e., a function with finite range $\{y_1, \ldots, y_K\}$):

$$\int_{X} f \, d\lambda \equiv \sum_{k=1}^{K} y_k \, \lambda(\{x : f(x) = y_k\}). \tag{9}$$

That is, the integral of a simple function is the average of its values weighted by their frequencies, $\lambda(\{x: f(x)=y_k\})$. This integral is well-defined for any simple function, since every subset of X is measurable.⁸

⁸The appendix (section A.2) contains the definition of the integral for more general functions.

4.3. Complexity

4.3.1. Finite definability. Consider a function $f: X \to \mathbb{R}$ that represents a contract between two parties. It is natural to require that any such contract can be written down in terms of the elementary, objective features. An example of a function that can be written down is

$$f(x) = \begin{cases} 0 & \text{if first feature} = 0 \text{ and second feature} = 0, \\ 1 & \text{if first feature} = 0 \text{ and second feature} = 1, \\ 2 & \text{if first feature} = 1 \text{ and second feature} = 0, \\ 3 & \text{if first feature} = 1 \text{ and second feature} = 1. \end{cases}$$

$$(10)$$

A difficulty one faces in formalizing the idea that 'a contract can be written down' is where to draw the line between what can and cannot be written down. Is a contract that conditions on, say, 10 features complex or simple? Rather than resort to ad hoc criteria based on the number of features or writing cost, we simply allow the principal to write *any* contract she wants as long as it conditions on a *finite* number of features.

DEFINITION 3: A function $f: X \to \mathbb{R}$ is finitely defined if it is A-measurable.

Finitely defined functions can be written down. We let \mathcal{F} be the set of all finitely defined functions.

4.3.2. Complexity defined. Complex functions vary more than what what finitely defined sets can capture. Formally,

DEFINITION 4: A function $g: X \to \mathbb{R}$ is complex over $A \in \mathcal{A}$ if

$$\sup_{f \in \mathcal{F}} \int_A |f(x) - g(x)| \ d\lambda > 0.$$

A function $g: X \to \mathbb{R}$ is simple over A if it is not complex over A.

Note that the limits of finitely defined functions are not necessarily finitely defined but they are not complex. This captures the intuition that there is nothing complex about continuous functions such as g(x) = x. For a function to be complex it should not be possible to approximate it arbitrarily closely by a sequence of finitely-defined functions.

4.3.3. Complex information. Information is complex if the likelihood ratio (as a function of x) is complex over some finitely defined set A. Clearly, complex information implies that at least one of the density functions π_e is complex, but the converse is not necessarily true.

4.4. Expected payoffs

In the agency context, each action induces a probability distribution on the set of outcomes X. It is convenient to represent this distribution (as we did in the finite case) using a density function π , where π will, of course, depend on the action the agent takes.

The following assumption is a natural extension of the usual definition of a density function:

DEFINITION 5: A function $\pi: X \to \mathbb{R}$ is a density function if $\pi(x) \geq 0$ for every x, and $\int_X \pi(x) d\lambda = 1$.

For example, if

$$\pi(x) = \begin{cases} 2 & \text{if } \frac{1}{2} \le x \le 1\\ 0 & \text{otherwise} \end{cases}$$

then, the probability of the event $A=\left[0\,,\frac{3}{4}\right]$ is $\int_0^{\frac{3}{4}}\pi(x)\,d\lambda=\int_{\frac{1}{2}}^{\frac{3}{4}}2d\lambda=\frac{1}{2}.$

⁹Continuity of functions on X is defined analogously to continuity of functions on the continuum, [0,1]. That is, g is continuous at x if and only if given $\epsilon > 0$ there exists $\delta > 0$ such that $|g(y) - g(x)| < \epsilon$ for all y satisfying $|y - x| < \delta$.

The central point of our model would be to show that not all densities have to take this simple form. Our model allows a density π as in the following example: π , takes values in $\{0,2\}$ such that for each set $A \in \mathcal{A}$ with $\lambda(A) > 0$, $\frac{1}{\lambda(A)}\lambda\{x : \pi(x) = 0\} = \frac{1}{2}$.

Given a bounded function $f: X \to \mathbb{R}$ representing a state-contingent payoff, say, its expected payoff is given by

$$E_{\pi}f \equiv \int_{X} f(x)\pi(x) d\lambda.$$

Even as we allow for very general density functions on the limiting outcome space X, we would like to restrict attention to densities with averages that settle down on finitely-defined sets.

Assumption 1: There is an \mathcal{A} -measurable finite partition of X, $\mathcal{P}(X) = \{A^1, ..., A^N\}$, such that if $A^n \in \mathcal{P}(X)$ and $\{A_1^n, A_2^n\}$ is a finitely defined partition of A^n with $\lambda(A_i^n) > 0$, $i \in \{1, 2\}$, then

$$\frac{1}{\lambda(A_1^n)} \int_{A_1^n} \pi_e(x) d\lambda = \frac{1}{\lambda(A_2^n)} \int_{A_2^n} \pi_e(x) d\lambda$$

for $e \in \{H, L\}$.

For expository convenience, we also assume that the densities are finitely valued functions.

Assumption 2: Densities π_e , $e \in \{H, L\}$ take on finitely many values.

Let $\lambda \left\{ x \in A^n : \pi_e(x) = \pi_e^j(A^n) \right\} \equiv \lambda_e^j(A^n)$ and let $\pi_e^j(A^n)$ be π_e 's j^{th} value on $A^n \in \mathcal{P}(X), j = 1, ..., J_e^n$ is an arbitrary ordering.

With this notation, there is a simple expression for the probability that finitely-defined event A^n will occur if the agent's effort is e:

$$\int_{A^n} \pi_e(x) \, d\lambda = \sum_{j=1}^{J_e^n} \lambda_e^j(A^n) \pi_e^j(A^n). \tag{11}$$

We define $\varphi_e(x)$ to be the irreducible averages of $\pi_e(x)$. That is, on all $x \in A^n$ with $\lambda(A^n) > 0$,

$$\varphi_e(x) = \frac{1}{\lambda(A^n)} \int_{A^n} \pi_e(x) \, d\lambda.$$

Clearly, since π is a density, so is φ .¹⁰ We will sometimes refer to φ_e as the 'smoothed' density of π_e . Likewise, $\frac{\varphi_L(x)}{\varphi_H(x)}$ is the 'smoothed' likelihood ratio corresponding to $\frac{\pi_L(x)}{\pi_H(x)}$. Finally, notice that if f is finitely defined, then

$$E_{\pi_e}f(x) = E_{\varphi_e}f(x).$$

EXAMPLE 4: Suppose that $\mathcal{P}(X)$ consists of two sets $\{A^1, A^2\}$. Suppose that the only difference between the elements of A^1 and A^2 is that on all $x \in A^1$ the first feature is 0 and on A^2 it is 1. On A^1 , $\pi_H(x)$ is a 50/50 distribution on values $\frac{1}{4}$ and $\frac{1}{2}$ and $\pi_L(x) = 1$ for all x. On A^2 , $\pi_H(x)$ is a 50/50 distribution on values $\frac{1}{4}$ and 3 and $\pi_L(x)$ is again 1 for all x. Assume π_H and π_L are uncorrelated. It is easy to check that $\pi_H(x)$ and $\pi_L(x)$ are well defined densities satisfying Assumptions 1 and 2.

The likelihood ratios are

$$\frac{\pi_L(x)}{\pi_H(x)} = \begin{cases} 50/50 \text{ distribution on 4 and 2, for } x \in A^1 \\ 50/50 \text{ distribution on 4 and } \frac{1}{3}, \text{ for } x \in A^2. \end{cases}$$

Therefore, information is complex (see Section 4.3.3).

It is easy to derive the 'smoothed' densities φ . On A^1 , $\varphi_H(x) = \frac{3}{8}$ and on A^2 , $\varphi_H(x) = \frac{13}{8}$. $\varphi_L(x) = 1$ for all $x \in X$. The smoothed likelihood ratios are

$$\frac{\varphi_L(x)}{\varphi_H(x)} = \begin{cases} \frac{8}{3}, & \text{for } x \in A^1 \\ \frac{8}{13}, & \text{for } x \in A^2. \end{cases}$$

Let $f: X \to \mathbb{R}$ be as in equation (10), a finitely defined function. Then,

$$E_{\pi_H} f(x) = E_{\varphi_H} f(x) = \frac{17}{8},$$

 $E_{\pi_L} f(x) = E_{\varphi_L} f(x) = \frac{3}{2}.$

4.4.1. Complexity and the density functions. Note that when information is complex, at least one of the densities must itself be complex. On the other hand, there are situations where even if the densities are complex, the likelihood ratio is a simple function of x.

5. The agency problem revisited

5.1. Complete contracts

The agency problem in the infinite outcome model may be written just as the standard problem of Section 2. That is,

$$\min_{\alpha} E_{\pi_H} \alpha(x)$$

$$subject \ to$$
(IC) $H \in \arg \max_{e} E_{\pi_e} \alpha(x) - c_e$
(IR) $E_{\pi_H} \alpha(x) - c_H \ge \bar{U}$

We solve for the optimal contract by minimizing pointwise the expected wage subject to the (IC) and (IR) constraints. Just as in the finite case, the optimal schedule is characterized by the likelihood ratios (eq. 4). Let α^* be the solution to Problem (P1).

Even though α^* is an optimal contract, the principal may have trouble enforcing it because she may not be able to write it down. To see this, consider the following example.

EXAMPLE 5: Suppose that for $e \in \{H, L\}$, π_e takes values in $\{0, 2\}$ so that for each set $A \in \mathcal{A}$ with $\lambda(A) > 0$, $\frac{1}{\lambda(A)}\lambda\{x : \pi_e(x) = 0\} = \frac{1}{2}$. Further, suppose that for each $A \in \mathcal{A}$ and $k \in \{0, 2\}$, $\{x \in A : \pi_H(x) = k\} \cap \{x \in A : \pi_L(x) = k\} = \emptyset$. Clearly, after observing the outcome, the principal knows with certainty whether the agent chose high or low effort. The optimal contract α^* gives the agent his reservation utility plus c_H if the observed outcome belongs to the outcome subset generated by e = H and less than his reservation utility plus c_L if the observed outcome comes from e = L. This contract cannot be written down because on every $A \in \mathcal{A}$, there is a 50/50 distribution of outcomes from H and L.

In this example, the likelihood ratios characterizing the optimal contract (and, thus, the information contained in the outcomes) vary more than what the most finely grained finitely-defined contract can capture.

In what follows we show that if the principal is trustworthy, then, depending on the complexity of the environment, the optimal contract is incomplete.

5.2. Finite definability

For a contract to be enforceable, it must be written down. We capture this intuition by requiring contracts to be *finitely defined*.

DEFINITION 6: A contract α is finitely-defined if it is \mathcal{A} -measurable.

Finitely-defined contracts can be written down, communicated, and reproduced. Intuitively, finitely-defined contracts wind up conditioning on intervals: if J is the index of the last feature included in the contract, then the schedule is effectively dividing the outcome space into 2^J intervals and can possibly assign a different compensation to each one of these intervals.

In the standard agency model of Section 2, contracts are finitely defined and, thus, they can be written down. When the environment is complex, the solution to Problem (P1), α^* , is not finitely defined.¹¹

Let $\mathcal C$ be the set of all finitely-defined contracts. The finite definability constraint (FD) is

 $\alpha \in \mathcal{C}$.

5.2.1. Contractual incompleteness. We now extend Definition 1 to the infinite outcome space.

DEFINITION 7: A contract is complete if it is complete at almost every x. A contract is incomplete if it is incomplete on a set $B \in X$ with $\lambda(B) > 0$.

It is important not to confuse finite-definability with incompleteness. A contract can be finitely defined but complete and not-finitely-defined but incomplete. Finite definability means that contracts need be written down. Incompleteness, on the other hand, confers discretion to choose.

5.2.2. Complexity and finite-definability. By definition of simplicity and complexity, the likelihood ratios characterizing the optimal contract are (are not) finitely defined when the environment is simple (complex). Therefore,

REMARK 1: $\alpha^* \in \mathcal{C}$ if and only if the information is simple over the entire outcome space.

5.2.3. Ex ante planning and ex post contractual obligations. Under complexity, the agency problem looks very differently if assessed from an ex ante rather than an ex post point of view. Ex post, after a specific outcome has been realized, the principal has a clearer idea as to what has been the agent's action.¹²

¹¹In fact, α^* is not even be the limit of a sequence of finitely defined contracts.

¹²In some cases, as in Example 5, the principal may know *exactly* the agent's action after the fact. Of course, our model allows for intermediate cases where the principal, after observing the outcome, has a better understanding as to what has been de agent's action, but she still does not know for sure.

Ex ante, the principal would like to introspect and write a complete contract that captures her full ex post knowledge about the agent's actions at each instance. But the contract needs be written down. Effectively, having to write down a complete contract means that many contingencies will have to be bunched together (will have to have the same ex post payment).

As we formally argue below, the coupling of complexity and contractual finitedefinability ends up adding 'noise' to the agency relationship.

5.3. Finitely defined complete contracts

We are interested on the solution to Problem (P1) when the additional finite definability constraint is imposed. Ideally, the principal would like to write down α^* and if the information is simple, she can do it. However, under complex information, the optimal finitely defined contract will be obtained by solving a version of (P1) where the expectations are taken using densities φ_e (instead of π_e). Thus,

Proposition 2: The solution to

$$\min_{\alpha} \{ E\alpha(x) \text{ subject to (IC), (IR), and (FD)} \}$$
 (P2)

where expectations are taken using π , coincides with the solution to

$$\min_{\alpha} \{ E\alpha(x) \text{ subject to (IC) and (IR)} \}$$
 (P2')

where expectations are taken using φ .

PROOF: Let $\tilde{\alpha}$ be the solution to Problem (P2'). That is, $V_{P2'}(\tilde{\alpha}) \geq V_{P2'}(\alpha)$, for all contract α in the feasible set of (P2'). Because φ_e , $e \in \{H, L\}$ are finitely defined, so are the induced likelihood ratios and $\tilde{\alpha}$. Recall that the expectations of finitely defined functions with respect to π are equal to those taken using the corresponding φ . Therefore, $\tilde{\alpha}$ is in the feasible set of (P2). Towards a contradiction, suppose that

there is another contract $\check{\alpha}$ different from $\tilde{\alpha}$ that satisfies all the constraints in (P2) but yields lower expected cost to the principal, so that $V_{P2}(\check{\alpha}) < V_{P2}(\check{\alpha})$. Notice that $\check{\alpha}$ is in the feasible set of (P2'). Since $\check{\alpha}$ is finitely defined, the expectations under φ must coincide with those under π . But then,

$$V_{P2'}(\check{\alpha}) = V_{P2}(\check{\alpha}) > V_{P2}(\tilde{\alpha}) = V_{P2'}(\tilde{\alpha})$$

a contradiction.

Intuitively, the optimal complete contract will condition on as many finitely defined sets as it is useful to. Assumption 1 guarantees that there is a last feature j that it is valuable conditioning on. Since the expectations of finitely defined functions are the same under π and φ , a contract satisfying (IC) and (IR) in Problem (P2'), will also satisfy (IC) and (IR) in Problem (P2). Further, since φ_e are finitely defined, the solution to (P2') will also satisfy (FD). The argument is complete by noticing that the objective functions are same in both problems.

It is useful to think of Problem (P2) as the 'ex ante problem' and of Problem (P1) as the 'ex post problem.' If information is complex, the optimal ex ante complete contract is coarse. The principal would have liked to offer a finer schedule, but the finite-definability constraint precluded such contract from being written down even if what needs to be written down can be thought of by the parties before the ex post stage is reached.

Let $\tilde{\alpha}$ be the solution to (P2) and recall that V is the principal's expected utility. Clearly, $V(\alpha^*) \geq V(\tilde{\alpha})$ with strict inequality when information is complex. This is so because (P2) is (P1) with an additional constraint (finite definability) that is binding whenever the problem is complex.

The finite definability restriction translates into noise: the contract cannot be as finely grained as the principal would like it to be. Technically, the contract is based on the ex ante likelihood ratios that reflect the extra noise added by complexity. The best a complete contract can do is to prescribe ex post payments that are efficient on average over the range of contingencies covered by its clauses. If the

efficient compensation is sensitive to details, then the optimal ex post remuneration is likely to differ form the payment stipulated in the contract. An incomplete contract, on the other hand, allows for finer adjustment to the specific features of the contingencies that arise ex post.

5.4. Endogenous incompleteness

In this section we show that under complexity, trust and contractual incompleteness are valuable.

The extended model where the principal may be trustworthy is:

$$\min_{\alpha} \tau E_H[\overline{s}_{\alpha}(x)] + (1 - \tau)E_H[\underline{s}_{\alpha}(x)]$$

subject to

(IC)
$$H \in \arg\max_{e} \{ \tau E_e[u(\overline{s}_{\alpha}(x))] + (1 - \tau) E_e[u(\underline{s}_{\alpha}(x))] - c_e \}$$
 (P3)

- (IR) $\tau E_H[u(\overline{s}_{\alpha}(x))] + (1 \tau)E_H[u(\underline{s}_{\alpha}(x))] c_H \ge \overline{U}$
- (FD) $\alpha \in \mathcal{C}$

5.4.1. Finite definability constraint. By an argument mimicking that in the proof of Proposition 1, it can be easily seen that if we did not require the contract to be finitely defined, there would be no role for trust and contractual incompleteness. Absent (FD),

$$V_{P3}(\alpha^*) \ge V_{P3}(\alpha)$$

for all α , complete or incomplete (where α^* is the solution to (P1).)

5.4.2. Optimality of complete contracts. Complete contract $\tilde{\alpha}$ (the solution to (P2)), is feasible for Problem (P3). In fact, when the problem is simple, $\tilde{\alpha}$ is optimal for (P3).¹³ Moreover, if there is no trust ($\tau = 0$), then, regardless of the complexity of the problem, $\tilde{\alpha}$ is optimal.¹⁴

¹³This can be shown by an argument similar to that in the proof of Proposition 2.

¹⁴As we show below, when $\tau > 0$ but small, the optimal contract may still be $\tilde{\alpha}$.

When the problem is complex, $\tilde{\alpha}$ is generally not optimal for (P3). To see this, suppose that the principal is completely trustworthy ($\tau = 1$), then the ex post optimal schedule α^* can be achieved de facto by a fully incomplete contract (or no contract at all): give discretion to the principal to choose any payment she wishes ex post, after the outcome has been realized. Because $\tau = 1$, the agent knows that he will be paid according to the ex post likelihood ratios characterizing α^* , $\frac{\pi_L(x)}{\pi_H(x)}$. By construction, this schedule satisfies (IC), (IR), and (FD) in (P3). Because under complexity $V(\alpha^*) > V(\tilde{\alpha})$, the principal will be strictly better off with the incomplete contract than with the contract that solves (P2). Thus,

REMARK 2: When $\tau = 1$, the fully incomplete contract achieves the highest expected payoff to the principal.

Notice that if the information is simple, complete contract $\tilde{\alpha}$ will do just as well.

5.4.3. Optimal incomplete contracts. We now characterize optimal contracts for intermediate levels of trustworthiness $(0 < \tau < 1)$. Optimal contracts consist of a finitely defined lower bound and are open above. The lower bound protects the agent from the possibility that the principal may be opportunistic. The openness above allows the trustworthy principal to use her ex post knowledge to compensate the agent. Trust acts as a substitute for written contracts.

PROPOSITION 3: When τ is sufficiently large and information is complex, the contract α^{τ} that solves Problem (P3) is partially incomplete: it gives the principal discretion to choose payment expost from a set of allowed wages.

PROOF: To characterize the optimal contract, we transform Problem (P3) into an equivalent problem that is tractable using standard methods.

By Assumption 2, $\pi_e(x)$ ($e \in \{H, L\}$) take finitely many values on each $A^n \in \mathcal{P}(X)$. Thus, the likelihood ratios will also take finitely many values on A^n , n = 1

1, ..., N. Because the likelihood ratios indicate the informational content of outcomes, there are finitely many *informational equivalence classes* of outcomes.

Formally, let $\hat{\pi}_L$ and $\hat{\pi}_H$ be two numbers. Consider the ratio

$$R(\hat{\pi}_H, \hat{\pi}_L) = \frac{\hat{\pi}_L \lambda \left\{ x \in A^n : \pi_L(x) = \hat{\pi}_L \right\}}{\hat{\pi}_H \lambda \left\{ x \in A^n : \pi_H(x) = \hat{\pi}_H \right\}}.$$

Because π_e may take only finitely many values, there are finitely many possible values of R. Let $J^n \in \mathbb{N}$ be the number of values that R takes on A^n . Each x in A^n has a value R^j for some $j \in \{1, ..., J^n\}$ associated with it. With some abuse of notation, we let this value be R(x). We say that two outcomes y and z in A^n are informationally equivalent if R(y) = R(z). Informationally equivalent outcomes will induce the same wage. As a consequence, we need only compute finitely many ex post payments.

Consider the subpartition of A^n into sets of informationally equivalent outcomes

$${x \in A^n : R(x) = R^j, j = 1, ..., J^n}.$$

The components of the subpartition will not be finitely defined (a consequence of Assumption 1). Let, $\pi_H^j(A^n)$ and $\pi_L^j(A^n)$ be the values of π_H and π_L on the j^{th} element of the subpartition.

Let $\lambda^j(A^n) \equiv \lambda \left\{ x \in A^n : R(x) = R^j \right\}$. Then, $\Pi_e^j(n) \equiv \pi_e^j(A^n) \lambda^j(A^n)$ is the probability that an outcome in equivalence class j in A^n will ensue if the agent chooses effort e. By definition (eq. 11), $\sum_{j=1}^{J^n} \Pi_e^j(n) = \Pr\{x \in A^n | e\}$. Clearly, $\sum_{n=1}^N \sum_{j=1}^{J^n} \Pi_e^j(n) = 1$, $e \in \{H, L\}$. Notice that probability distributions Π_e , $e \in \{H, L\}$ reflect the risk associated with not knowing with certainty the equivalence class j and the finitely defined set A^n where the outcome x will fall after effort has been chosen. However, these probability distributions do not capture the risk associated with not knowing the principal's type; the fact that she may turn out to be opportunistic. Below, we construct probability distributions $\bar{\Pi}_e$, $e \in \{H, L\}$ that reflect such additional risk.

Within each A^n , we fix the ordering j so that

$$\frac{\Pi_L^j(n)}{\Pi_H^j(n)} < \frac{\Pi_L^{j+1}(n)}{\Pi_H^{j+1}(n)}$$

for all $j = 1, ..., J^n - 1$.

Let $\alpha^{j}(n)$ be the wage that the trustworthy principal pays the agent if the j^{th} informational equivalence class on A^{n} is realized and consider the following problem:

$$\min_{\alpha} \sum_{n=1}^{N} \sum_{j=1}^{J^{n}} \bar{\Pi}_{H}^{j}(n) \alpha^{j}(n)$$

$$subject \ to$$

$$(IC) \quad H \in \arg\max_{e} \{\sum_{n=1}^{N} \sum_{j=1}^{J^{n}} \bar{\Pi}_{e}^{j}(n) u(\alpha^{j}(n)) - c_{e}\}$$

$$(IR) \quad \sum_{n=1}^{N} \sum_{j=1}^{J^{n}} \bar{\Pi}_{H}^{j}(n) u(\alpha^{j}(n)) - c_{H} \ geq\bar{U}$$

$$(P3')$$

where

$$\bar{\Pi}_{e}^{j}(n) = \begin{cases} \tau \Pi_{e}^{j}(n) & \forall n \text{ and } j = 1, ..., J^{n} - 1\\ \sum_{j=1}^{J^{n}-1} (1 - \tau) \Pi_{e}^{j}(n) + \Pi_{e}^{J^{n}}(n) & \forall n \text{ and } j = J^{n}. \end{cases}$$
(12)

This is a standard agency problem that can be solved applying standard techniques. The likelihood ratios characterizing the optimal contract are for each n

$$\frac{\bar{\Pi}_{L}^{j}(n)}{\bar{\Pi}_{H}^{j}(n)} = \frac{\Pi_{L}^{j}(n)}{\Pi_{H}^{j}(n)}$$
(13)

for $j = 1, ..., J^n - 1$ and

$$\frac{\bar{\Pi}_{L}^{J^{n}}(n)}{\bar{\Pi}_{H}^{J^{n}}(n)} = \frac{\sum_{j=1}^{J^{n}-1} (1-\tau) \Pi_{L}^{j}(n) + \Pi_{L}^{J^{n}}(n)}{\sum_{j=1}^{J^{n}-1} (1-\tau) \Pi_{H}^{j}(n) + \Pi_{H}^{J^{n}}(n)}.$$
(14)

Assume that τ is sufficiently large (hypothesis in the Proposition) so that expression (14) is strictly larger than (13). With this assumption, the lowest equilibrium wage on A^n corresponds to informational equivalence class J^n .

Let $\alpha^{\tau'}$ be the solution to (P3'). Notice that $\alpha^{\tau'}$ is a standard complete contract.

Consider the following *incomplete* contract:

$$\alpha^{\tau}(x) = \begin{cases} \{\alpha^{1}(1), \alpha^{2}(1), ..., \alpha^{J^{1}}(1)\} & \text{if } x \in A^{1} \\ \{\alpha^{1}(2), \alpha^{2}(2), ..., \alpha^{J^{2}}(2)\} & \text{if } x \in A^{2} \\ \vdots & \vdots & \vdots \\ \{\alpha^{1}(N), \alpha^{2}(N), ..., \alpha^{J^{N}}(N)\} & \text{if } x \in A^{N} \end{cases}$$

$$(15)$$

where $\alpha^{j}(n)$ are given by contract $\alpha^{\tau'}$. Contract α^{τ} is incomplete as it gives the principal freedom to choose compensation between the possible payments on each A^{n} : the principal places the realized x in the corresponding finitely defined set, A^{n} , and if trustworthy, she chooses

$$\bar{s}_{\alpha^{\tau}}(x) = \alpha^{j}(n)$$

where j is the realized informational equivalence class. If she is opportunistic, she chooses

$$\underline{s}_{\alpha^{\tau}}(x) = \alpha^{J^n}(n).$$

CLAIM 1: α^{τ} solves (P3).

To see this, note that by construction, α^{τ} is finitely defined and is in the feasible set of (P3). Towards a contradiction, suppose that there is another contract $\check{\alpha}^{\tau}$ different from α^{τ} that satisfies the constraints in (P3) but yields lower expected cost to the principal, so that $V_{P3}(\check{\alpha}^{\tau}) > V_{P3}(\alpha^{\tau})$. Without loss of generality we may assume that $\check{\alpha}^{\tau}$ is constant on the components of \mathcal{P} (this follows from Assumption 1). Let $\check{\alpha}^{\tau'}$ be the complete contract (on informational equivalence classes) that when equivalence class j in finitely defined set A^n is realized, assigns with probability τ the wage paid by the trustworthy principal under $\check{\alpha}^{\tau}$ when x belongs to informational equivalence class j in finitely defined set A^n and with complementary probability assigns the lowest allowed wage by $\check{\alpha}^{\tau}$ on A^n . By

construction, $\check{\alpha}^{\tau\prime}$ is in the feasible set of (P3') and $V_{P3'}(\check{\alpha}^{\tau\prime}) = V_{P3}(\check{\alpha}^{\tau})$. But then,

$$V_{P3'}(\check{\alpha}^{\tau'}) = V_{P3}(\check{\alpha}^{\tau}) > V_{P3}(\alpha^{\tau}) = V_{P3'}(\alpha^{\tau'})$$

a contradiction. \blacksquare

Thus, incomplete contract α^{τ} (eq. 15) is the solution to Problem (P3): the agent is willing to accept the contract and feels compelled to choose high effort, the contract is finitely defined and it minimizes expected cost to the principal. Notice that when the density functions π_e take many values, it may be time consuming to write contract α^{τ} (because there may be many informational equivalence classes). There is, though, a very simple way to write α^{τ} : just specify the lowest bound and leave it open above.

COROLLARY 1: If there is a cost $\delta > 0$ to write contractual clauses, then α^{τ} consists of lowest bound under which the principal is not allowed to pay and is left open above.

PROOF: If there is a cost $\delta > 0$ to writing down in a piece of paper each contingency and/or payment, the 'cheapest' way to write down contract α^{τ} is to just specify the lower bound for each element of \mathcal{P} . That is,

$$\alpha^{\tau}(x) = \begin{cases} \left[\alpha^{J^{1}}(1), \infty\right) & \text{if } x \in A^{1} \\ \vdots & \vdots \\ \left[\alpha^{J^{N}}(N), \infty\right) & \text{if } x \in A^{N} \end{cases}$$

The selection rule $\{\bar{s}_{\alpha^{\tau}}, \underline{s}_{\alpha^{\tau}}\}$ guarantees α^{τ} 's optimality.

5.4.4. Trustworthiness and the principal's expected utility. We now analyze the relationship between the principal's trustworthiness and her expected gains from the agency relationship. The following proposition shows that the principal is better off the more trustworthy she is. Therefore, if the principal could credibly commit to acting in a trustworthy manner, she would.

PROPOSITION 4: Suppose $\tau > \tau'$, that τ' is large enough so that the construction in Proposition 3 is valid, and that information is complex. Then, if $\Pi_L \succ_{FOSD} \Pi_H$ within each A^n ,

$$V(\alpha^{\tau}) > V(\alpha^{\tau'}).$$

PROOF: We want to show that given τ and τ' with $\tau > \tau'$, the likelihood ratio distribution under τ' is a mean-preserving spread of that under τ . Then, by Kim (1995)'s Proposition 1, the value to the principal of the τ problem is strictly larger than that of the τ' problem:

$$V(\alpha^{\tau}) > V(\alpha^{\tau'}).$$

Let

$$L_{\tau}(z) = \Pr \left[\frac{\bar{\Pi}_L}{\bar{\Pi}_H} = z \middle| e = H \right].$$

Notice that the expectation of the likelihood ratio distribution on A^n ,

$$E\left[L_{\tau}(z)\right] = \frac{\Pi_{L}^{1}(n)}{\Pi_{H}^{1}(n)}\Pi_{H}^{1}(n)\tau + \frac{\Pi_{L}^{2}(n)}{\Pi_{H}^{2}(n)}\Pi_{H}^{2}(n)\tau + \dots + \frac{\Pi_{L}^{J^{n}-1}(n)}{\Pi_{H}^{J^{n}-1}(n)}\Pi_{H}^{J^{n}-1}(n)\tau +$$

$$+ \frac{\sum_{j=1}^{J^{n}-1}(1-\tau)\Pi_{L}^{j} + \Pi_{L}^{J^{n}}(n)}{\sum_{j=1}^{J^{n}-1}(1-\tau)\Pi_{H}^{j} + \Pi_{H}^{J^{n}}(n)} \sum_{j=1}^{J^{n}-1}(1-\tau)\Pi_{H}^{j} + \Pi_{H}^{J^{n}}(n)$$

$$= \tau \sum_{j=1}^{J^{n}-1}\Pi_{L}^{j} + \sum_{j=1}^{J^{n}-1}(1-\tau)\Pi_{H}^{j} + \Pi_{H}^{J^{n}}(n) = \Pr\left[A^{n} \mid e = H\right],$$

is independent of τ . Thus, the mean is preserved as τ changes.

To see that the likelihood ratio distribution spreads out as τ increases, note that the set of informationally equivalent outcomes with likelihood ratio $\frac{\bar{\Pi}_L}{\bar{\Pi}_H} = \frac{\Pi_L}{\Pi_H}$, does not change as τ increases or decreases. Thus, the probability of the set containing the first k informationally equivalent outcomes $(k < J^n)$ is $\tau \sum_{j=1}^k \Pi_H^j$, an increasing function of τ .

The only problem arises for the z such that the set of informationally equivalent outcomes with $\frac{\bar{\Pi}_L}{\bar{\Pi}_H} \leq z$ contains all outcomes $j=1,...,J^n$ (because the value of

 $\frac{\bar{\Pi}_L}{\bar{\Pi}_H}$ for the last outcome depends on τ).

Recall that

$$\frac{\bar{\Pi}_L^{J^n}}{\bar{\Pi}_H^{J^n}} = \frac{\sum_{j=1}^{J^n-1} (1-\tau) \Pi_L^j + \Pi_L^{J^n}(n)}{\sum_{j=1}^{J^n-1} (1-\tau) \Pi_H^j + \Pi_H^{J^n}(n)}.$$

Therefore,

$$\frac{\partial}{\partial \tau} \left(\frac{\bar{\Pi}_{L}^{J^{n}}}{\bar{\Pi}_{H}^{J^{n}}} \right) = \frac{-\sum_{j=1}^{J^{n}-1} \Pi_{L}^{j} \Pi_{H}^{J^{n}}(n) + \sum_{j=1}^{J^{n}-1} Pi_{H}^{j} \Pi_{L}^{J^{n}}(n)}{\left(\sum_{j=1}^{J^{n}-1} (1-\tau) \Pi_{H}^{j} + \Pi_{H}^{J^{n}}(n)\right)^{2}},$$

which is negative because of our assumption that $\Pi_L \succ_{FOSD} \Pi_H$. This takes care of the highest value of z.

We can then safely conclude that as τ increases, so does $\Pr\left[\frac{\bar{\Pi}_L}{\bar{\Pi}_H} \leq z \middle| e = H\right]$ and, by Kim's Proposition 1,

$$V(\alpha^{\tau}) > V(\alpha^{\tau'}).$$

proving the result.

The intuition is clear. As the principal's trustworthiness increases, the agent faces less risk because it is less likely that the principal will use her discretion to take advantage of the agent. As risk bearing diminishes, the principal needs offer less expected compensation to induce the agent to accept the contract and the principal can capture more of the total surplus generated.

A simple observation lets us conclude that

COROLLARY 2: Under complex information,

$$V(\alpha^*) > V(\alpha^{\tau}) > V(\widetilde{\alpha}).$$

PROOF: Follows from $\lim_{\tau \to 1} \alpha^{\tau} = \alpha^*$ and $\lim_{\tau \to 0} \alpha^{\tau} = \widetilde{\alpha}$.

5.5. An example

It is interesting to provide a fully worked out example to illustrate Propositions 3 and 4.

5.5.1. Setup. Let the agent's vNM utility function be $u(\alpha(x)) = \sqrt{\alpha(x)}$ and

$$\mathcal{P}(X) = \left\{ \begin{array}{ll} A^1 = \left\{x \in X : x^1 = 0\right\} & (roughly \ [0, \frac{1}{2}]) \\ A^2 = \left\{x \in X : x^1 = 1\right\} & (roughly \ [\frac{1}{2}, 1]) \end{array} \right.$$

Assume further that the agent's reservation utility is $\bar{U} = 50$ and that $c_H = 10$ and $c_L = 0$.

Let the densities be as in Example 4: On A^1 , $\pi_H(x)$ is a 50/50 distribution on values $\frac{1}{4}$ and $\frac{1}{2}$ and $\pi_L(x) = 1$ for all x. On A^2 , $\pi_H(x)$ is a 50/50 distribution on values $\frac{1}{4}$ and 3 and $\pi_L(x)$ is again 1 for all x. Assume π_H and π_L are uncorrelated.

5.5.2. Ex post optimal complete contract. By using the likelihood ratios computed in Example 4, one can easily compute the ex post optimal complete contract (the solution to (P1)):

$$\alpha^*(x) = \begin{cases} 1,685.66, \text{ for all } x \in A^1 \text{ with } \frac{\pi_L(x)}{\pi_H(x)} = 4\\ 2,881.01, \text{ for all } x \in A^1 \text{ with } \frac{\pi_L(x)}{\pi_H(x)} = 2\\ 1,685.66, \text{ for all } x \in A^2 \text{ with } \frac{\pi_L(x)}{\pi_H(x)} = 4\\ 4,123.10, \text{ for all } x \in A^2 \text{ with } \frac{\pi_L(x)}{\pi_H(x)} = \frac{1}{3}. \end{cases}$$

If α^* was implementable, the expected cost to the principal would be 3,663.16. The problem is, of course, that this contract is not enforceable because it does not describe the outcomes in terms of their features.

5.5.3. Optimal finitely defined contract in the absence of trust. On A^1 , $\varphi_H(x) =$.375 and on A^2 , $\varphi_H(x) = 1.625$. $\varphi_L(x) = 1$ on all $x \in X$. Thus, the optimal

finitely-defined, complete contract in the absence of trust is

$$\tilde{\alpha}(x) = \begin{cases} 1,156, \text{ for all } x \in A^1\\ 4,356, \text{ for all } x \in A^2. \end{cases}$$

This results in a cost to the principal of 3,756. Therefore, $V(\alpha^*) > V(\tilde{\alpha})$.

5.5.4. Optimal incompleteness. We now allow the principal to be trustworthy $(\tau > 0)$ and solve (P3).

Notice that within finitely defined set A^1 , there are two informational equivalence classes: those outcomes with likelihood ratio 4 and those with likelihood ratio 2. Similarly for A^2 . Therefore, the principal needs only compute four numbers: (a) a minimum bound in case the realized outcome falls in A^1 , (b) a minimum bound in case x falls in A^2 , (c) the wage to be paid if the outcome falls in A^1 and the likelihood ratio suggests that the agent took high effort, and, finally, (d) the wage if the outcome falls in A^2 and the likelihood ratio suggests that the agent took high effort.

This will be an incomplete contract that will give discretion to the principal: If x belongs to A^1 , then the principal chooses between (a) and (c) and if it belongs to A^2 , then she chooses between (b) and (d). The opportunistic principal will always choose either (a) or (c), but the trustworthy principal will choose 'fairly.'

Suppose that $\tau = \frac{1}{2}$, then the optimal incomplete contract is

$$\alpha^{\tau = \frac{1}{2}}(x) = \begin{cases} \{1,083.52, 2,157.60\}, & \text{if } x \in A^1\\ \{3,833.63, 4,767.94\}, & \text{if } x \in A^2. \end{cases}$$

The expected cost to the principal is 3,735.48. Thus,

$$V(\alpha^*) > V(\alpha^{\tau = \frac{1}{2}}) > V(\tilde{\alpha}).$$

Finally, note that if there is a cost $\delta > 0$ of writing each number in the contract,

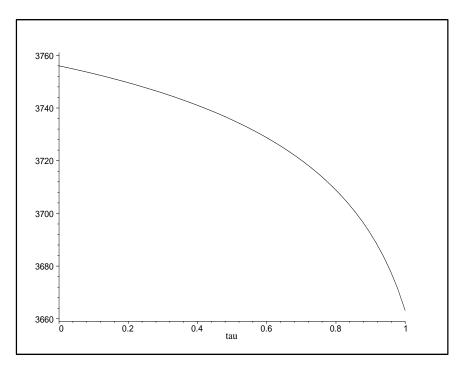


Figure 1: Expected cost of implementing e=H as a function of τ

the optimal incomplete contract will specify two numbers only:

$$\alpha^{\tau = \frac{1}{2}}(x) = \begin{cases} [1, 083.52, \infty), & \text{if } x \in A^1 \\ [3, 833.63, \infty), & \text{if } x \in A^2. \end{cases}$$

Figure 1 illustrates Proposition 4. It shows that the expected cost to implement e=H is a decreasing function of the principal's trustworthiness.

6. Concluding remarks

6.1. Trust, gains from trade, and implementable actions

The presence of trust allows for exchange to occur in situations where its absence would preclude trade.¹⁵

An illustration is Example 5. There, because the best complete contract is constant, the principal can only motivate the agent to undertake low effort. If the agent fully trusted the principal, then high effort is easily implemented: the principal pays the agent $\alpha(x) = \overline{U} + c_H$ if the observed outcome reflects e = H and $\alpha(x) = \overline{U} - \delta$, for some $\delta > 0$ otherwise.

Nevertheless, trust does not work miracles. In order for certain actions to be implementable there must be sufficient trust. In Example 5, as $\tau \to 0$, we have that $V(\alpha^{\tau}) \to -\infty$. Thus, if trust is sufficiently low, the principal is better off staying out of the relationship: she cannot provide incentives to implement the high action at an economically viable cost.

6.2. Distinct support result breaks down

One main result from standard agency is that if the supports of π_H and π_L are different, then the first best can be obtained by punishing the agent sufficiently harshly if the realized outcome could only have accrued under e = L.

Example 5 shows how in our more general setting this result breaks down. Here, the full support of π_H and π_L are distinct, yet in the absence of trust, not only the first best is not achievable, but, as we just discussed, the principal can only implement e = L.

¹⁵For the same conclusion in a model where trust is placed on the agent, see Casadesus-Masanell (1999).

The problem, of course, is that when information is complex, the schedule that would implement the first best is not finitely defined. When the finite definability constraint is imposed, only constant contracts are 'optimal' and with these, only low effort is implementable.

6.3. Sufficient statistic result breaks down

Another landmark result in the standard model of agency is that the optimal contract should condition on all available *informative* signals.¹⁶ (See Holmstrom (1979).)

Example 5 shows that this intuition does not apply to agency problems with complex information. The most that a written contract can do is to condition on finitely many features, even if ex post the agent's choice of action is obvious.

There is valuable information in the signal x that is being obviated by the optimal finitely defined contract. The problem is that under complexity this valuable information varies more than what the more sophisticated finitely defined contract can capture.

6.4. Immunity to Maskin-Tirole's critique

In our model, contractual incompleteness arises because complexity prevents parties from writing detailed ex ante contracts. One critique of this modelling approach is that contracting parties may be able to eliminate the source of incompleteness by playing 'message games.' That is, they may be able to commit ex ante to play a game such that the equilibrium of that game leads to implement ex

¹⁶Formally, the principal should condition the wage on a sufficient statistic for all the signals. An additional signal y is informative about the agent's action if and only if the original signal x is not a sufficient statistic for (x, y). An informative signal y conveys information about the agent's choice of action in addition to that which is already in x.

post the outcome they would have liked to contract on ex ante (but weren't able to because of complexity, or other impediments to contracting).

Maskin and Tirole (1999) formulated this critique, focusing mainly on the buyer-seller model. They also note that hidden-action principal-agent models, like the one we consider here, are not vulnerable to this critique. The reason, roughly, is that implementation mechanisms work by exploiting ex post differences in agent preferences across states. In the hidden-action agency model considered here, the principal and the agent have the same utility functions independent of the outcome realized, and so no sorting is possible.

6.5. The value of written contracts

Written agency contracts exist to protect the agent from the possible opportunism of the principal as well as to provide incentives for performance. If the principal was completely trustworthy, there would be no point in having written contracts. The agent would feel compelled to work hard because he would know that the wage would be fair. It is because of imperfect trust that contracts need be written down. If τ is close to 1, incentives come, mainly, from the expectation that the wage will be fair ex post. However, the agent needs protection because the principal may turn out to be opportunistic. At the other extreme, if τ is close to 0, incentives cannot come from the expectation of a fair wage. In this case, incentives have to be written down. The written contract also limits the extent of the principal's opportunism.

APPENDIX

A.1. Proof of Lemma 1

Let l_{∞} denote the set of all bounded sequences with the supremum norm. That is, for $x = (x_1, \ldots), ||x|| = \sup_n x_n$. We use the fact that there exists a function $T: l_{\infty} \to \mathbb{R}$ that is:

- 1. linear: T(cx + dy) = cT(x) + dT(y);
- 2. non-negative: $T(x) \ge 0$ if $\inf_n x_n \ge 0$;
- 3. preserves identity: T(1, 1, 1, ...) = 1;
- 4. translation invariant: $T(x_1, x_2, ...) = T(x_2, x_3, ...)$.

Any such function is known as a Banach limit (see Rao and Rao (1983, p.39-41)). One may think of T as a generalized limit. In fact, the above reference shows that:

$$\liminf_{n} x_n \le T(x) \le \limsup_{n} x_n;$$

so, in particular, $T(x) = \lim_{n \to \infty} x_n$ whenever the limit exists.

Fix one such function T. For an arbitrary set $A \subset X$, define

$$\lambda(A) = T(\lambda_1(A), \lambda_2(A), \ldots).$$

It is clear that λ thus defined is a finitely additive probability on X. Furthermore, Equation 8 must hold since T assigns to every convergent sequence its limit.

A.2. Integration with respect to a finitely additive measure

The integral of simple functions is given in Section 4.2. For more general functions, we have:

DEFINITION 8: The integral of a bounded function $f: X \to \mathbb{R}$ is

$$\int_X f \, d\lambda = \lim_{n \to \infty} \int_X f_n(x) \, d\lambda,$$

where $f_n: X \to \mathbb{R}$, $n \in \mathbb{N}$, is any sequence of simple functions that converges uniformly to f^{17} .

This integral is well-defined for all bounded functions and does not depend on the particular approximating sequence $\{f_n\}$.

Integration with respect to λ_N reduces to taking the simple average of its values over X_N :¹⁸

$$\int_X f \, d\lambda_N \equiv \frac{1}{\# X_N} \sum_{x \in X_N} f(x).$$

Notational convention:

It is convenient to think of λ_N as probability distribution on X with support X_N so λ_N and λ are defined on the same space. With this convention, expression $\lambda_N(A)$, $A \in \mathcal{A}$ has an unambiguous interpretation as $\lambda_N(X_N \cap A)$. Thus, we may use λ_N and X_N interchangeably to talk about the Nth model, since X_N is just the support of λ_N . Furthermore, given $f: B \to \mathbb{R}$, it is legitimate to write:

$$\int_A f \, d\lambda_N \quad \text{instead of} \quad \int_{X_N \cap A} f \, d\lambda_N \quad \text{whenever} \quad A \subset B \subset X.$$

For example, $\int_A f d\lambda_N$ for an $A \in \mathcal{A}$ is meaningful for $f: B \to \mathbb{R}$.

¹⁷That is, for every $\epsilon > 0$ there is N such n > N implies that $|f(x) - f_n(x)| < \epsilon$ for every $x \in X$.

¹⁸The integral f on a subset $A \in \mathcal{A}$ is covered by this notation, since $\int_A f d\lambda \equiv \int_X \chi_A f d\lambda$, where χ_A is the indicator function of A.

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