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Tunable spatial filtering with a Fabry-Perot etalon

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A Fabry-Perot etalon is used in the image plane as a means of selecting arbitrary directions of propagation in the angular spectrum of an object. This provides a simple widely tunable spatial filter that could be useful in a number of image processing and pattern recognition applications.

I. Introduction

Spatial filtering of pictorial inputs is usually performed by selecting some spatial frequency bands with masks in the Fourier plane of a coherent processor. The output of such a device represents the spatial frequency content of the input in these chosen bands. The same operation can be performed in the input plane with any device capable of selecting plane waves with chosen directions of propagation.

In this paper, the use of a Fabry-Perot etalon as an image plane angular filter is investigated. It is shown that a large range of spatial filters (high, low, and bandpass) can be obtained by varying the finesse of the etalon and the spacing between its mirrors. The versatility of such a tunable spatial angular filter is experimentally demonstrated, and some applications in image processing, feature extraction, pattern recognition, and pseudocolor encoding of spatial frequency are suggested.

II. Angular Filtering

The equivalence between spatial frequency filtering in the Fourier plane and angular filtering in the image plane is immediately recognized if one recalls that an input of transmittance $f(x,y)$ illuminated by a quasi-monochromatic plane wave can be written as a superposition of plane waves of strength $A(\alpha,\beta)$ propagating with direction cosines (α,β).

$$A(\alpha,\beta) = \int\int_{-\infty}^{\infty} f(x,y) \exp[-2i\pi(\alpha x + \beta y)/\lambda]dxdy \qquad (1)$$

is the angular spectrum of plane waves of the input.[1] Since the directions cosines are proportional to the input spatial frequencies,

$$(\alpha,\beta) = \lambda(u,v), \qquad (2)$$

the angular spectrum is a scaled version of the Fourier transform $F(u,v)$ of the input.

An angular filter is a device that, placed in the input plane, is capable of selecting a set of plane waves with arbitrarily chosen direction cosines. The output of such a filter is then equivalent to the output of a spatial frequency filter selecting (with masks) a corresponding set of spatial frequencies in the input spectrum. The use of a Fabry-Perot as an angular filter presents several desirable features. It is essentially a 2-D device that can be used to perform orientation-independent filtering operations. Furthermore, the width and the position of the angular bandpass can be conveniently chosen and tuned by changing the finesse of the etalon and varying the spacing between its mirrors.

The selectivity of some periodic structures such as Bragg gratings could also be used as angular filters. Case has recently demonstrated 1-D high-pass filtering using a volume hologram.[2] The use of a Fabry-Perot as a means of providing feedback to an optical system has been reported by Lee *et al.*[3] Another kind of feedback technique, using a long feedback loop, has also been used by Händler and Röder to perform flexible bandpass filtering.[4]

III. Fabry-Perot Angular Filter

Because of the rotational symmetry of the device it is convenient to shift to cylindrical coordinate systems (r,θ) in the image plane and (ρ,ϕ) in the Fourier plane. One further defines a direction of propagation by the angle θ between the propagation vector and the z axis:

$$\sin\theta = (\alpha^2 + \beta^2)^{1/2} = \lambda\rho. \qquad (3)$$

The angular filtering setup is shown in Fig. 1, which is

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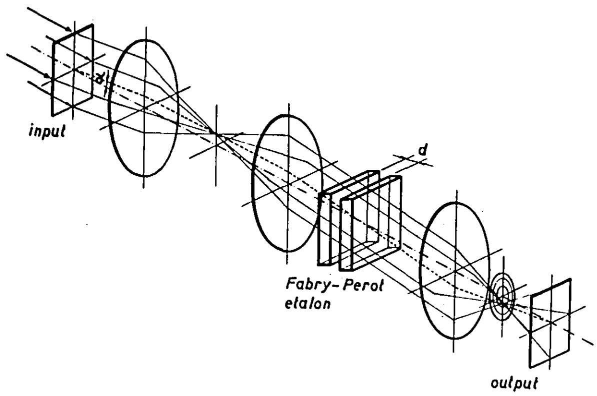

Fig. 1. Experimental setup for tunable angular filtering.

a simple imaging system with an etalon in an intermediate image plane. The irradiance transmittance of the etalon is given by

$$T(\theta) = \left(1 - \frac{A}{1-R}\right)^2 \left(1 + \frac{4F^2}{\pi^2}\sin^2\delta/2\right)^{-1}, \tag{4}$$

where A and R are the absorption and reflectance of the mirrors (in the following, we assume $A = 0$ for simplicity); F is the finesse of the etalon:

$$F = \pi R/(1-R), \tag{5}$$

and the phase angle δ is given by

$$\delta = \frac{4\pi d}{\lambda}\cos\theta, \tag{6}$$

where d is the spacing between mirrors.

As a function of δ, Eq. (4) describes the well-known Fabry-Perot transmittance curve with peaks of value $T = 1$, width

$$\Delta\delta = 2/F, \tag{7}$$

and located at

$$\delta = 2\pi M \ (M \text{ integer}). \tag{8}$$

In the paraxial and quasi-monochromatic approximation, this can be translated into a transmittance-vs-radial spatial frequency curve $T(\rho)$. Using the Eqs. (3), (6), and (8) and the small angle approximation, the condition for a maximum becomes

$$\delta = \frac{4\pi d}{\lambda}(1 - \lambda^2\rho^2/2) = 2\pi M. \tag{9}$$

To display explicitely its small tunable part, the spacing d is written as

$$d = (N + \epsilon)\lambda/2. \tag{10}$$

Combining Eqs. (9) and (10) and using $\epsilon \ll N$ lead to the determination of the spatial frequencies of maximum transmittance:

$$\rho_k = [2(k+\epsilon)/N]^{1/2}\,1/\lambda, \tag{11}$$

where $k = N - M$ is the order of the maxima (starting at $k = 0$ for $M = N$ and $\epsilon = 0$). The width of the peaks are found from Eqs. (3), (6), and (7) to be

$$\Delta\rho_k (d\delta/d\rho)^{-1}\,\Delta\delta = (2\lambda d F \rho_k)^{-1} \tag{12}$$

IV. Theoretical Results

The results of the preceding section can be summarized as follows: All parameters being fixed, the effect of the etalon is to select and transmit some bands of radial spatial frequencies centered on the resonant frequencies ρ_N. For a diffuse input containing all spatial frequencies, the transmitted bands are displayed in the Fourier plane where they appear as the usual concentric rings characteristic of Fabry-Perot fringes. Equations (7) and (9) also clearly show how the central frequency of the transmitted band can be varied by tuning the spacing ϵ and how the width of the band depends on the finesse of the etalon.

There is a limit, however, to how narrow the bandpass can be made. (Although it is rarely desirable to have an extremely narrow spatial filter.) Narrowing the bandpass is accomplished by either increasing the finesse or the spacing d. A higher finesse results in a larger effective number of beams reflected back and forth in the cavity, while a larger spacing increases the phase shift between two successive beams. As these beams are actually carrying images of the input, the effect, in both cases, is to superpose images increasingly out of focus. This clearly degrades the lateral resolution of the output.

The periodic nature of Eq. (4) leads to the definition of a free (spatial) spectral range (FSR) as the distance between two successive radial spatial frequencies transmitted by the etalon. For large spatial frequencies ($\rho \gg$ FSR), the free spectral range is found by writing Eq. (9) for two successive maxima: ρ_k and $\rho_{k+1} = \rho_k$ + FSR. One finds

$$\text{FSR} = (2\lambda d\rho)^{-1} = F\Delta\rho(\rho \gg \text{FSR}). \tag{13}$$

In spatial spectrum analysis, however, one is often interested in filtering bands of relatively low spatial frequencies ($\rho \ll$ FSR). In this case, the FSR is found by Eq. (9) for $\rho_0 = 0$ and ρ_1 = FSR. One finds

$$\text{FSR} = (\lambda d)^{-1/2}, \quad (\rho \ll \text{FSR}). \tag{14}$$

As an example, let us assume that a single passband, tunable from $\rho = 0$ to $\rho = \rho_{\max}$, is desired. The single band condition implies that the free spectral range be larger than $\rho_{\max}$. Together with Eq. (11) this fixes the etalon spacing to a value

$$d \leq 1/\lambda\rho_{\max}^2. \tag{15}$$

For a given finesse, the width of the transmitted band is therefore specified to be (at least for $\rho \neq 0$)

$$\Delta\rho \approx (1/2F)\rho_{\max}^2/\rho. \tag{16}$$

The tunability of the filter further requires that the spacing be continually variable from d min = $N\lambda/2$ to d max = $(N + 1)\lambda/2$, where $N = 2/\lambda^2\rho_{\max}^2$. The following values were chosen as being relatively close to the experimental conditions described in Sec. V:

wavelength $\lambda \approx 500$ nm;
mirror reflectance $R \approx 75\%$;
finesse $F \approx 10$;
mirror spacing $d \approx 0.5$ mm ($N \approx 2 \times 10^3$).

A series of transmission curves obtained for different values of ϵ and demonstrating the essential features of the tunable filter is shown in Fig. 2. These curves can also be regarded as transfer functions of the imaging system of Fig. 1.

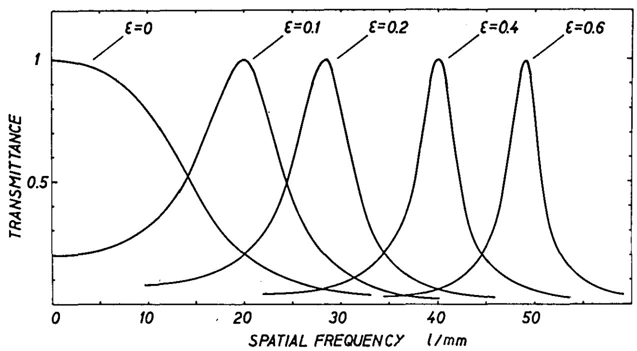

Fig. 2. Irradiance transmittance of the etalon as a function of spatial frequency for spacings $(2 \times 10^3 + \epsilon)\lambda/2$, $\epsilon = 0 - 0.6$. Equivalently, the curves represent the tunable transfer function of the optical system.

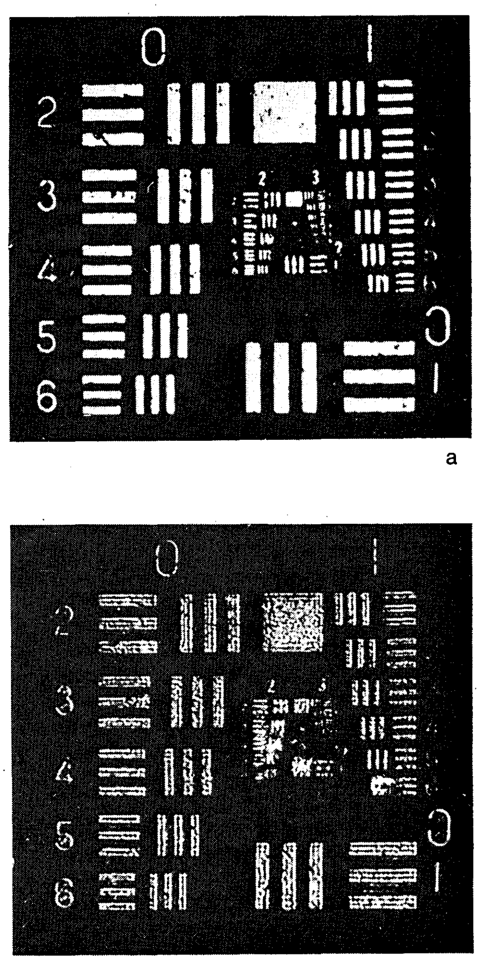

Fig. 3. Example of high-pass filtered image.

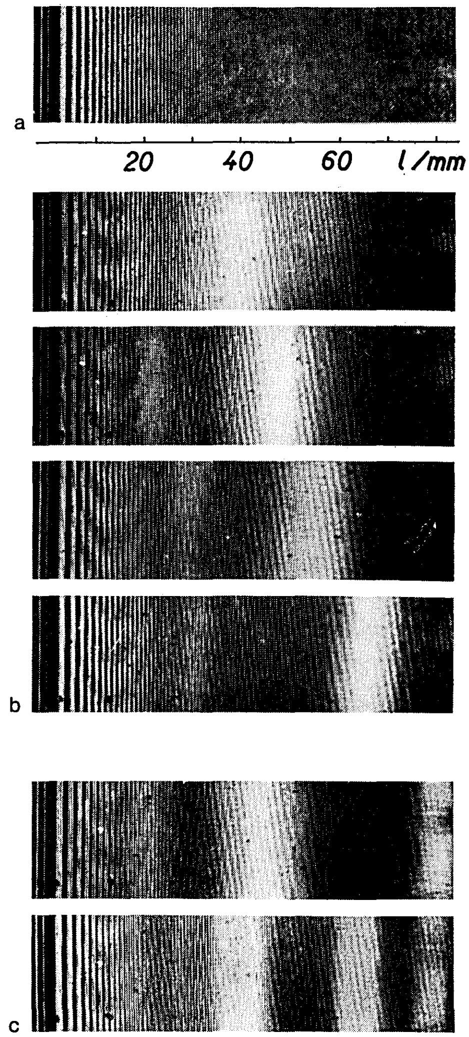

Fig. 4. (a) Input, (b) outputs with the bandpass tuned to several positions (etalon spacing ~0.5 mm), (c) outputs with etalon spacing about 0.75 and 1 mm.

V. Experimental Results

The experimental results of Figs. 3–5 were obtained with a setup similar to the one sketched in Fig. 1. The Fabry-Perot etalon consisted of two flat half-silvered mirrors of 75% reflectivity, one of them placed in an adjustable mount. (Ideally a piezo-aligner/translator would be used.) A typical example of high-pass filtered image is shown in Fig. 3. In Fig. 4, a variable spatial frequency chart is used as input [Fig. 4(a)] to illustrate the tunability of the angular filter. Figure 4(b) shows several outputs obtained with different settings of the mirror spacing. The spacing is slightly varied around a nominal value of about 0.5 mm. The moiré effect observable on the results of Fig. 4 is due to spurious reflections on the back faces of the mirror substrates, which were slightly wedged. For larger mirror spacing, the free spectral range decreases, and several spatial

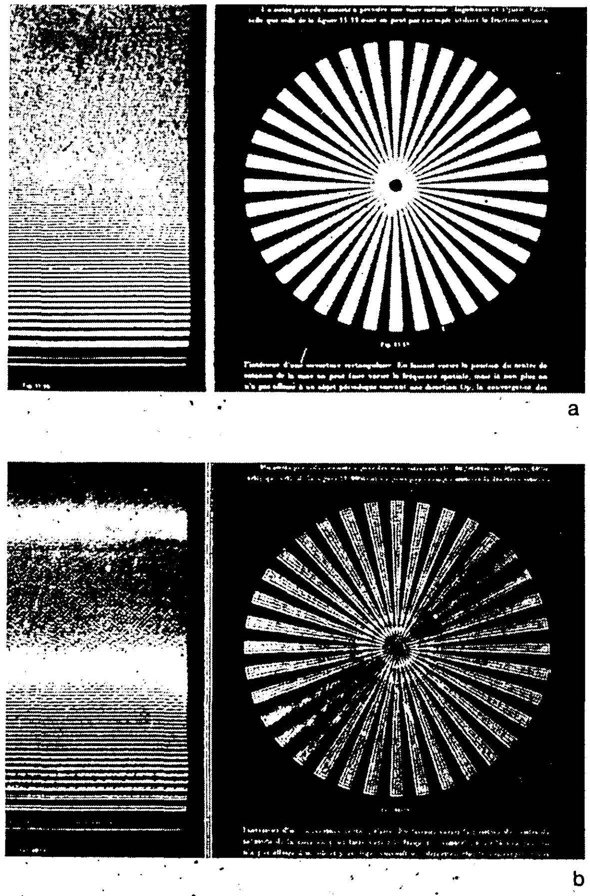

Fig. 5. (a) Input, (b) output with filter set to select two frequency bands near 30 and 70 liters/mm.

frequency bands are eventually transmitted by the system. Examples of such behavior is demonstrated in Fig. 4(c) for spacing of about 0.75 and 1 mm. As a final example, Fig. 5 shows the filtering of two frequency bands near 30 and 70 mm^{-1}. The two filtered rings are readily visible in the central part of the radial target.

VI. Applications and Conclusions

A Fabry-Perot etalon placed in an image plane of an optical system has been shown to behave as a filter for the angular frequencies of the input. The characteristics of such a filter can conveniently be varied by changing the spacing and finesse of the etalon.

The application of such a device in image processing (edge enhancement, smoothing, etc. is immediate. The technique allows arbitrary high-pass, low-pass, single or multiple bandpass to be performed with a single tunable filter.

With a polychromatic source the device offers the possibility of color coding the spatial frequency content of an image. This type of processing could be used, for example, in feature extraction or recognition, a very powerful tool in image analysis.

With a continually scanned etalon, the technique provides a means of performing rotation insensitive pattern recognition. The energy transmitted by the system is, at any one time during the scan, a measure of the spatial frequency content of the input in a ring centered on a radial frequency ρ, that sweeps the entire frequency range as the etalon is scanned. The output of a photodetector measuring this energy has a form

$$S(t) = \int_0^{2\pi} \int_{\rho(t)-\Delta\rho/2}^{\rho(t)+\Delta\rho/2} R(\rho,\phi)\rho d\rho d\phi, \tag{17}$$

where $\rho(t)$ can be computed from the scanning function, and $R(\rho,\phi)$ is the input power spectrum. This signal can be used as a rotation invariant signature to characterize the input. It can eventually be compared with the signals produced by known patterns for recognition purposes.

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