# Tuning of Fractional PID Controllers Using Adaptive Genetic Algorithm for Active Magnetic Bearing System

LONG-YI CHANG HUNG-CHENG CHEN

Department of Electrical Engineering, National Chin-Yi University of Technology 35, Lane 215, Sec. 1, Chungshan Road, Taiping, Taichung, Taiwan <a href="mailto:lychang@ncut.edu.tw">lychang@ncut.edu.tw</a>

Department of Electrical Engineering, National Chin-Yi University of Technology 35, Lane 215, Sec. 1, Chungshan Road, Taiping, Taichung, Taiwan hcchen@ncut.edu.tw

Abstract: - This paper proposes a novel adaptive genetic algorithm (AGA) for the multi-objective optimization design of a fractional PID controller and applies it to the control of an active magnetic bearing (AMB) system. Different from PID controllers with three constants, the fractional PID controller's parameters are composed of proportional constant, integral constant, derivative constant, derivative order and integral order. The fractional PID controller is more flexible and gives the possibility of adjusting more carefully the closed-loop system characteristics. However, its design becomes more complex than that of conventional integer order PID controller. An adaptive genetic algorithm is proposed to design the fractional PID controller. The five parameters of the fractional PID controller are selected as parameters to be determined. The dynamic model of an AMB system for axial motion is also presented. The simulation results of this AMB system show that a fractional PID controller designed via the proposed AGA has good performance.

Key-Words: - Active Magnetic Bearing, Adaptive Genetic Algorithms, Fractional PID Controller

#### 1 Introduction

The active magnetic bearing (AMB) systems with controlled-permanent magnet electromagnets have been reported elsewhere. It supports a rotating body without direct contact and will be used widely for various purposes due to its significant feature. It offers a number of practical advantages over conventional bearings such as higher speeds, lower rotating losses, elimination of the lubrication system and lubricant contamination of the process, operation at temperature extremes and in vacuum, and longer life [1-3]. However, AMB applications often require the solution of very interesting and formidable control problems because of the inherent instability and the nonlinear relationship between the lift force and the air gap distance [4, 5]. The controller is one of key techniques of AMB system, and its performance affects directly whether magnetic bearing can work stably and unfailingly or not. Large numbers of control strategies have been studied inside and outside, e.g., acceleration feedforward control [6], sliding control [7], switching control [8], LQ control [9], and nonlinear control [10].

In the past decades, conventional PID controllers are widely applied in industry process control. This is mainly because PID controllers have simple control structures, and are simple to maintain [11, 12]. However, a conventional PID controller may have poor control performance for nonlinear and/or complex systems. Since the PID gains are fixed, the main disadvantage is that they usually lack in flexibility and capability. Recently, many researchers revealed that factional order differential equations could model various materials more adequately than integer order ones. Especially, controllers consisting of factional order derivatives and integrals could achieve better performance and robustness results than those obtained with conventional (integer order) controllers [13-15]. Expanding derivatives and integrals to fractional orders can control system's response directly and continuously. This great capability makes it possible to design more robust control system. A fractional PID controller has five design parameters. Unfortunately, it is quit difficult to optimize the parameter settings of fractional PID controllers because AMB systems have serious nonlinearities and strong couplings. There is a need for effective and efficient global optimal approach to optimize the parameter settings of robot fractional PID controllers automatically.

Genetic algorithms (GA) have received much interest in recent years [16-18]. The basic operating principles of GA are based on the principles of natural evolution. GA requires little knowledge of the problem itself and need not require that the search space is differentiable or continuous. Therefore, it can solve nonlinear multi-objective optimization problems. The basic form of GA is simple genetic algorithm (SGA). SGA searches global optimum solution possibly, but premature convergence and random roam can easily take place [16, 19]. On this issue, more efforts should be made especially for industrial control applications.

In this paper, we propose a novel multi-objective optimization method for the parameter tuning of fractional PID controller based on adaptive genetic algorithm (AGA) to solve the control problem of an AMB system. By using adaptive crossover and mutation operators, the global searching ability and the convergence speed of the genetic algorithm are significantly improved. With the incorporating of both the transient performance index of dynamic response and control input into the fitness function and properly weighting these terms, the overall performance of the fractional PID controller is optimized to satisfaction. The performance of the optimized fractional PID based on proposed AGA is also shown superior to the one base on SGA.

### 2 Analysis of System Dynamic Model

Fig. 1 shows the schematic of the controlled AMB system. It consists of a levitated object (rotor) and a pair of opposing E-shaped controlled-PM electromagnets with coil winding. An attraction force acts between each pair of hybrid magnet and extremity of the rotor. The attractive force each electromagnet exerts on the levitated object is proportional to the square of the current in each coil and is inversely dependent on the square of the gap. The entire system becomes only one degree of freedom of one axis, namely the axial position. Assuming a minimum distance to the length of the

axis, the two attraction forces assure the restriction of radial motions of the axis in a stable way. The rotor position in axial direction is controlled by a closed loop control system, which is composed of a noncontact type gap sensor, a fractional PID controller and an electromagnetic actuator (power amplifier). This control is necessary since it is impossible to reach the equilibrium only by permanent magnets.

To model the AMB system, few simplifications are assumed: (a) the rotor maintains symmetry around the rotating axis, (b) deviation around the normal operating point is small, and (c) the magnetic axial attraction force and the electromagnetic force are linearized around the operation point. The rotor with mass m is suspended. Two attraction forces  $F_1$  and  $F_2$  are produced by the hybrid magnets. The applied voltage E from power amplifier to the coil will generate a current i which is necessary only when the system is subjected to an external disturbance w. Equations governing the dynamics of the system are

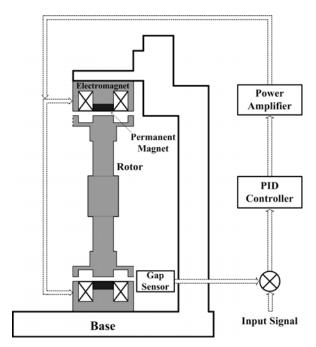


Fig. 1 The schematic of the controlled AMB system.

$$F_1(y,i) + F_2(y,i) - mg + w = m\frac{d^2y}{dt^2}$$
 (1)

$$E = Ri + N \frac{d}{dt} (\phi_1(y, i) + \phi_2(y, i))$$
 (2)

Where y is the distance from gap sensor to bottom of rotor. R and N are the resistance and number of turns of the coil.  $\phi_1$  and  $\phi_2$  are the flux of the top and bottom air gap, respectively. Under small disturbance, the above equation becomes

$$\Delta E = R\Delta i + N \frac{d}{dt} (\phi_1(y, i) + \phi_2(y, i))$$

$$= R\Delta i + N (\frac{\partial (\Delta \phi_1 + \Delta \phi_2)}{\partial \Delta y} \frac{d\Delta y}{dt} + \frac{\partial (\Delta \phi_1 + \Delta \phi_2)}{\partial \Delta i} \frac{d\Delta i}{dt})$$
(3)

If the weight of rotor is equal to the sum of these two attraction forces, the rotor will rotate on specific gap. According to (2), the disturbance equation at specific gap is calculated as follows

$$\Delta F_1(\Delta y, \Delta i) + \Delta F_2(\Delta y, \Delta i) + w = m \frac{d^2 \Delta y}{dt^2}$$
 (4)

and

$$\Delta F_1(\Delta y, \Delta i) = \frac{\partial \Delta F_1}{\partial \Delta y} \Delta y + \frac{\partial \Delta F_1}{\partial \Delta i} \Delta i$$
 (5)

$$\Delta F_2(\Delta y, \Delta i) = \frac{\partial \Delta F_2}{\partial \Delta y} \Delta y + \frac{\partial \Delta F_2}{\partial \Delta i} \Delta i \tag{6}$$

We denote  $\phi = \phi_1 + \phi_2$  and  $F = F_1 + F_2$ . The system is linearized at the operation point  $(y=y_o, i=0)$  and described as follows

$$\frac{d^2 \Delta y}{dt^2} = \frac{1}{m} \frac{\partial \Delta F}{\partial \Delta y} \Big|_{(y_0,0)} \Delta y + \frac{1}{m} \frac{\partial \Delta F}{\partial \Delta i} \Delta i \tag{7}$$

$$\frac{d\Delta i}{dt} = -\frac{R}{L}\Delta i - \frac{N}{L}\frac{\partial\Delta\phi}{\partial y}\bigg|_{(y_o,0)}\frac{d\Delta y}{dt} + \frac{1}{L}\Delta E \qquad (8)$$

Then

$$\frac{d}{dt} \begin{bmatrix} \Delta y \\ \Delta \dot{y} \\ \Delta i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ a_{21} & 0 & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta \dot{y} \\ \Delta i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} E + \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix} w \tag{9}$$

where

$$a_{21} = \frac{1}{m} \frac{\partial \Delta F}{\partial y}$$

$$a_{23} = \frac{1}{m} \frac{\partial \Delta F}{\partial \Delta i}$$

$$a_{32} = -\frac{N}{L} \frac{\partial \Delta \phi}{\partial y}$$

$$a_{33} = -\frac{R}{L}$$
(10)

$$b = \frac{1}{L}$$

$$d = \frac{1}{m}$$

$$L = N \frac{\partial \Delta \phi}{\partial \Delta i}$$
(11)

The partial derivatives are calculated from the experimental characteristics at the normal equilibrium operating point. It can be seen from the characteristic roots that the system is unstable. This system has to be stabilized by a controller with appropriate controller parameters tuning.

# 3 Fractional Order PID Controllers

Fractional controllers are characterized by differential equations that have an integral and/or a derivative of fractional-order in the control algorithm. These operators are defined by irrational continuous transfer functions, in the Laplace domain, or infinite dimensional discrete transfer functions, in the Z domain. We often encounter evaluation problems in the simulations. Therefore, when analyzing fractional systems, we usually adopt continuous or discrete integer-order approximations of fractional-order operators.

The mathematical definition of a fractional derivative and integral has been the subject of several different approaches [20, 21]. One commonly used definition is given by the Riemann-Liouville expression ( $\alpha > 0$  and  $n-1 < \alpha < n$ ):

$${}_{a}D_{t}^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^{n}}{dt^{n}} \int_{a}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau$$
 (12)

where f(t) is the applied function and  $\Gamma(x)$  is the Gamma function of x. Another widely used definition is given by the Grünwald-Letnikov approach ( $\alpha \in R$ ):

$${}_{a}D_{t}^{\alpha}f(t) = \lim_{h \to 0} \frac{1}{h} \sum_{k=0}^{\lfloor \frac{t-a}{h}\rfloor} (-1)^{k} {\binom{\alpha}{k}} f(t-kh)$$
(13)

where h is the time increment and [x] means the integer part of x. The "memory" effect of these operators is demonstrated by (9) and (10), where the convolution integral in (9) and the infinite series in (10). These definitions reveal the unlimited memory of these kinds of operators, ideal for modeling hereditary and memory properties in many physical systems and materials.

The most usual way of making use, both in simulations and hardware implementations, of transfer functions involving fractional powers of *s* is to approximate them with usual (integer order) transfer functions with a similar behavior. So as to perfectly mimic a fractional transfer function, an integer transfer function would have to include an infinite number of poles and zeroes. Nevertheless, it is possible to obtain reasonable approximations with a finite number of zeroes and poles. One of the best-known approximations is proposed by Manabe and Oustaloup, which is called Crone approximation. This approximation uses a recursive distribution of *N* poles and *N* zeros leading to a transfer function as follows [22]:

$$s^{\nu} \approx k \prod_{n=1}^{N} \frac{1 + (s/\omega_{zn})}{1 + (s/\omega_{pn})}, \quad \nu > 0$$
 (14)

The approximation is valid in the frequency range  $[\omega_l; \omega_h]$ . Gain k is adjusted so that both sides of (11) shall have unit gain at 1 rad/s. The number of poles and zeroes N is chosen beforehand, and the good performance of the approximation strongly depends thereon. Frequencies of poles and zeroes in (14) are given by

$$\alpha = (\omega_h / \omega_n)^{\nu/N} \tag{15}$$

$$\eta = (\omega_h / \omega_n)^{(1-\nu)/N} \tag{16}$$

$$\omega_{z1} = \omega_l \sqrt{\eta} \tag{17}$$

$$\omega_{pn} = \omega_{z(n-1)}\alpha, \quad n = 1, 2, \dots, N$$
 (18)

$$\omega_{zn} = \omega_{p(n-1)}\eta, \quad n = 2, 3, \dots, N$$
(19)

The case v<0 may be dealt with inverting (11). But if |v|>1, these approximations become unsatisfactory [22]. For that reason, it is usual to split fractional powers of s as

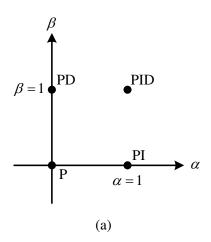
$$s^{\nu} = s^n s^{\beta} , \quad \nu = n + \beta$$
 (20)

where  $n \in \mathbb{Z}$  and  $\beta \in [0; 1]$ . In this manner only the latter term has to be approximated.

The generalized PID controller  $G_c(s)$  has a transfer function of the form:

$$G_c(s) = K_P + K_I \frac{1}{s^{\alpha}} + K_D s^{\beta}$$
 (21)

where  $\alpha$  and  $\beta$  are the orders of the fractional integrator and differentiator, respectively. As shown in Fig. 2, the fractional order  $PI^{\alpha}D^{\beta}$  controller generalizes the conventional integer order PID controller and expands it from point to plane. The constants  $K_P$ ,  $K_I$ , and  $K_D$  are correspondingly the proportional constant, the integral constant and the derivative constant. Clearly, taking  $(\alpha, \beta) = \{(1, 1), (1, 0), (0, 1), (0, 0)\}$  we get the classical {PID, PI, PD, P} controllers, respectively. The  $PI^{\alpha}D^{\beta}$  controller is more flexible and gives the possibility of adjusting more carefully the closed-loop system characteristics.



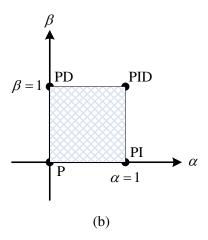


Fig. 2 PID controllers with fractional order (a) traditional PID controllers (b) fractional PID controllers.

# 4 AGA-Based Optimal Fractional PID Controller Design

As a mathematical means for optimization, GA can naturally be applied to the optimal-tuning of fractional PID controllers. The design of fractional PID controller could be treated as a multi-objective optimization problem, which is to compromise the rapidity, stability and accuracy of system control. It is difficult for the general adjustment of fractional PID parameters to satisfy the overall the performance at the same time. Therefore, this paper describes the application of GA to the fine-tuning of the parameters for fractional PID controllers. The novel multi-objective optimization method for parameter tuning of fractional PID controller based on adaptive genetic algorithm is proposed, which consists of the following five steps:

#### **Step 1: Representation of Parameters**

For most applications of genetic algorithms to optimization problems, the real coding technique is used to represent a solution to a given problem [23]. In real coding implementation, each chromosome is encoded as a vector of real numbers, of the same lengths as the solution vector. According to control objectives, five parameters  $K_P$ ,  $K_I$ ,  $K_D$ ,  $\alpha$ , and  $\beta$  of a fractional PID controller are required to be designed in this research. In this way, the k chromosome of i generation could be represented as  $X_k^i = [x_{k1}^i, x_{k2}^i, x_{k3}^i, x_{k4}^i, x_{k5}^i]$ . Each

chromosome  $X_k^i$  is corresponding to five tuned parameters of the fractional PID controller, i.e.  $K_P = x_{k1}$ ,  $K_I = x_{k2}$ ,  $K_D = x_{k3}$ ,  $\alpha = x_{k4}$ , and  $\beta = x_{k5}$ , where  $x_{jmin} \le x_{kj} \le x_{jmax}$ ,  $j = 1, 2, \cdots, 5$ ,  $x_{jmin}$  and  $x_{jmax}$  are the upper and lower limits of the  $j^{th}$  gene value respectively.

## **Step 2: Design of Fitness Function**

To evaluate the controller performance and get the satisfied transient dynamic, the fitness function includes not only the four main transient performance indices, overshoot, rise time, settling time and cumulative error, but also the quadratic term of control input to avoid that the control energy became too big. The fitness function is designed as

$$J = \frac{1}{\int_0^\infty [\omega_1 t e^2(t) + \omega_2 u^2(t)] dt + \omega_3 t_r + \omega_4 \sigma + \omega_5 t_s}$$
 (22)

where e(t) is the system error, u(t) is the controller input,  $t_r$  is the rise time,  $\sigma$  is the maximal overshoot,  $t_s$  is the settling time with 5% error band,  $\omega_1, \omega_2, \omega_3, \omega_4, \omega_5$  are weighting coefficients. For a practical fractional PID design issue, one could adjust all the weighting coefficients in the fitness function based on the specific requests such as rapidity, accuracy and stability of the system. For example if a system with little overshoot value is required,  $\omega_4$  would be increased appropriately; if a system with fast dynamic responses is required, then  $\omega_3$  would be increased appropriately. This research picked weighting coefficients has the  $\omega_i = 0.2$ , i = 1, 2, ..., 5 to cover all the performance indices completely.

# **Step 3: Selection**

In proportional selection procedure, the selection probability of a chromosome is proportional to its fitness. This simple scheme exhibits some undesired properties. To maintain a reasonable differential between relative fitness ratings of chromosomes and to prevent a too-rapid takeover by some super chromosomes, the exponential ranking fitness assignment is selected in fitness calculations of

reproduction operator, because its simplicity and robustness [23, 24]. The idea is straightforward: Sort the population from the best to the worst and assign the selection probability of each chromosome according to the ranking but not its raw fitness. Normalized geometric select is a ranking selection function based on the normalized geometric distribution, which is utilized in this research.

#### **Step 4: Crossover**

Crossover used here is single-point method. Setting two randomly selected chromosomes at i generation as  $X_k^i = [x_{k1}^i, x_{k2}^i, x_{k3}^i, x_{k4}^i, x_{k5}^i]$  and  $X_l^i = [x_{l1}^i, x_{l2}^i, x_{l3}^i, x_{l4}^i, x_{l5}^i]$ , the genetic values at the crossover point of these two chromosomes are  $x_{kj}^i$  and  $x_{lj}^i$  respectively. Two new chromosomes would be created after the crossover operation. The genetic values before and after crossover point remain the same, while the genetic value of the crossover point is

$$x_{kj}^{i'} = r_c x_{kj}^i + (1 - r_c) x_{lj}^i$$
  

$$x_{lj}^{i'} = r_c x_{lj}^i + (1 - r_c) x_{kj}^i$$
(23)

where  $r_c$  is the randomly generated constant between 0 and 1. Crossover operation is the major technique to generate new individual in genetic algorithm, and the crossover rate would generally pick the larger value. However if the crossover rate is picked too large, it might damage the good pattern of the population; if the value is too small, then the speed to generate the new individual is too slow. Furthermore, the less diversity of the population is the major cause for the instability and premature of GA [19, 25]. One should take measures before the diversity of population is getting poor. Therefore this paper puts forward the adaptive method which took the diversity of the population as the controlled variable and also adjusted the individual crossover rate based on the fitness value of itself. The adaptive crossover rate of an individual is defined as

$$p_{c} = \begin{cases} \frac{k_{c}}{(f_{\text{max}} - f_{avg})/f_{avg}} + p_{c1} e^{\frac{c}{\tau_{c}}(f_{c} - f_{avg})} f_{c} \ge f_{avg} \\ \frac{k_{c}}{(f_{\text{max}} - f_{avg})/f_{avg}} + p_{c1} f_{c} < f_{avg} \end{cases}$$
(24)

where

$$\tau_c = \frac{f_{max} - f_{avg}}{\ell n(p_{c1} / p_{c2})}$$

 $f_{max}$  is the maximal fitness value of the present population.  $f_{avg}$  is the average fitness value of the present population.  $f_c$  is the larger fitness value of the two individual who would intersect;  $k_c$ ,  $p_{c1}$  and  $p_{c2}$  are the crossover coefficients,  $p_{c1} > p_{c2}$  and they are the constants between 0 and 1, c the crossover amplitude coefficient.

### **Step 5: Mutation**

Mutation used here is non-uniform method. Set the mutation operation individual as  $X_k^i = [x_{k1}^i, x_{k2}^i, x_{k3}^i, x_{k4}^i, x_{k5}^i]$ , after the mutation operation, the genetic value of the individual which is not mutated remains the same, while the gene  $x_{kj}^{i'}$  on the mutated one is

$$x_{k_{j}}^{i'} = \begin{cases} x_{k_{j}}^{i} + \delta(i, x_{j max} - x_{k_{j}}^{i}) & r_{m} \ge 0.5 \\ x_{k_{j}}^{i} - \delta(i, x_{k_{j}}^{i} - x_{j min}) & r_{m} < 0.5 \end{cases}$$
(25)

where  $r_m$  is a random number between 0 and 1.  $\delta(i, y)$  represents a random number within the range of [0, y], which is varying with evaluation generation. The expression of  $\delta(i, y)$  is

$$\delta(i, y) = y(1 - r^{\left(1 - \frac{i}{G}\right)^{b}}) \tag{26}$$

where r is a random number between 0 and 1. i is the present evolution generation. G is the set maximal evolution generation. b is the coefficient that determines the dependency of stochastic disturbance on evolution generation i, which is generally determined by the experience, one would pick b=2 in

this research. Mutation rate has an important effect on the parametric optimization. If it is too large, the optimization procedure would not converge; if it is too small, then the GA might lead to prematurity. In the same way, the variation of diversity of the population is also the major cause for prematurity of GA [19, 25]. One should take measures before the diversity of population is getting poor. Therefore this paper puts forward the adaptive method which took the diversity of the population as the controlled variable and also adjusted the individual mutation rate based on the fitness value of itself. The adaptive mutation rate of an individual is defined as

$$p_{m} = \begin{cases} \frac{k_{m}}{(f_{\text{max}} - f_{avg})/f_{avg}} + p_{ml} e^{\frac{m}{\tau_{m}}(f_{m} - f_{avg})} & f_{m} \ge f_{avg} \\ \frac{k_{m}}{(f_{\text{max}} - f_{avg})/f_{avg}} + p_{ml} & f_{m} < f_{avg} \end{cases}$$
(27)

where

$$\tau_m = \frac{f_{max} - f_{avg}}{\ell n(p_{m1} / p_{m2})}$$

 $f_m$  is the fitness value of the individual that would undergo mutation operation.  $k_m$ ,  $p_{m1}$  and  $p_{m2}$  are the mutation coefficients.  $p_{m1}$  and  $p_{m2}$  are the constants between 0 and 1, and  $p_{m1} > p_{m2}$ . m is the mutation amplitude coefficient.

#### 5 Simulations and Discussion

To demonstrate the feasibility of the proposed approach to dynamic systems, the AMB system shown in Fig. 1 is used for illustration. Both the fractional PID and the conventional PID controllers are designed based on the proposed AGA. The overall flowchart of fractional PID controllers tuning using adaptive genetic algorithm for active magnetic bearing system is depicted in Fig. 3. After 20 generations of genetic operation, the searched optimal parameters are shown in Table 1. The Simulink module frame of the derived AMB system model in (9) with the fractional PID controller is depicted in Fig. 4 for simulation. In the simulation,

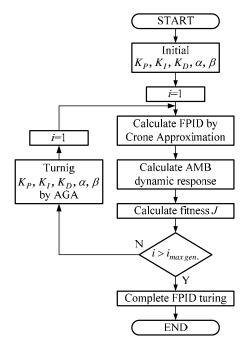


Fig. 3 The overall flowchart of fractional PID controllers tuning using AGA for an AMB system.

the goal is to use the proposed approach to tune the fractional PID gains in (21) such that the output response of the AMB system can be driven within the user's specification. The step responses of rotor position from the gap sensor in the AMB system using the optimized fractional PID controller and the optimized conventional PID controller are shown in Fig. 5. It shows that the fractional PID controller has remarkably reduced the overshoot and settling time compared with the optimized conventional PID controller. The fractional PID controller has achieved good performances in both transient and steady state periods. The fractional PID controller has more flexibility and capability than the conventional ones.

To illustrate how the proposed AGA works well than that of the SGA, the variation of the best and mean fitness values for both cases is plotted in Fig. 6. The population size and the generation size are all 20. On comparing the two plots, we observe that the mean fitness of the population increases gradually for the proposed AGA while it increases rapidly for the best SGA. A careful observation of Fig. 6 reveals that, in the first 7 generations the mean fitness for the SGA increases rapidly, remains rather flat until the last generation. The relatively flat zone occurs the SGA has yet located and gotten stuck at a locally

optimal solution with a fitness of 0.528. In contrast with the SGA, the best solution of the proposed AGA in each population is being propagated to the subsequent generation with a final fitness of 1.272. The best fitness is increasing with time. The higher fitness value of the proposed AGA indicates that the population has remained scattered in the solution space and has not gotten stuck at any local optimum. Such a simple but general approach, having ability for global optimization and with good robustness, is weakness effective to overcome some conventional approaches and to be more acceptable for industrial practices.

Table 1 The optimal parameters of the fractional PID controller and the conventional PID controller based on the proposed AGA.

Parameters	$K_P$	$K_{I}$	$K_D$	α	β
Controller					
Conventional PID	3.0956	1.2634	0.0651	1	1
Fractional PID	5.2193	3.8287	0.0909	0.7270	0.9921

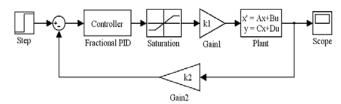


Fig. 4 Simulink module frame of the derived AMB system.

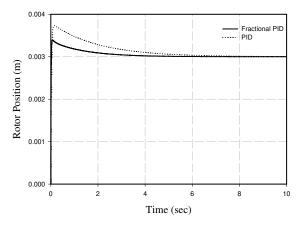
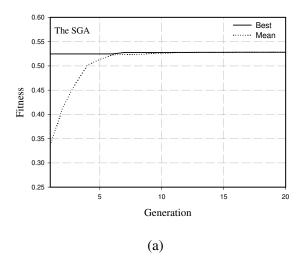


Fig. 5 The step responses of the rotor position from the gap sensor in the AMB system using the optimized fractional PID controller and the optimized conventional PID controller.



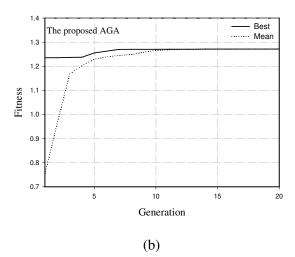


Fig. 6 Comparison of the best and mean fitness values at each generation during optimization process (a) the SGA (b) the proposed AGA.

# 6 Conclusions

This paper has proposed an improved adaptive the multi-objective genetic algorithm for optimization design of a fractional PID controller and applies it to the control of an AMB system. The proposed algorithm has better performance of convergence speed and better stability in the global optimum result. Another merit of the proposed method is the way to define the fitness function based on the concept of multi-objective optimization. This method allows the systematic design of all major parameters of a fractional PID controller and then enhances the flexibility and capability of the PID controller. The simulation results of this AMB

system show that a fractional PID controller designed via the proposed AGA has good performance.

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