Turbulence, Coherent Structures, Dynamical Systems and Symmetry

Turbulence pervades our world, from weather patterns to the air entering our lungs. This book describes methods that reveal its structures and dynamics. Building on the existence of coherent structures – recurrent patterns – in turbulent flows, it describes mathematical methods that reduce the governing (Navier–Stokes) equations to simpler forms that can be understood more easily.

This Second Edition contains a new chapter on the balanced proper orthogonal decomposition: a method derived from control theory that is especially useful for flows equipped with sensors and actuators. It also reviews relevant work carried out since 1995.

The book is ideal for engineering, physical science, and mathematics researchers working in fluid dynamics and other areas in which coherent patterns emerge.

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SECOND EDITION

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Preface to the first edition

On physical grounds there is no doubt that the Navier–Stokes equations provide an excellent model for fluid flow as long as shock waves are relatively thick (in terms of mean free paths), and in such conditions of temperature and pressure that we can regard the fluid as a continuum. The incompressible version is restricted, of course, to lower speeds and more moderate temperatures and pressures. There are some mathematical difficulties – indeed, we still lack a satisfactory existence-uniqueness theory in three dimensions – but these do not appear to compromise the equations' validity. Why then is the "problem of turbulence" so difficult? We can, of course, solve these nonlinear partial differential equations numerically for given boundary and initial conditions, to generate apparently unique turbulent solutions, but this is the only useful sense in which they *are* soluble, save for certain non-turbulent flows having strong symmetries and other simplifications. Unfortunately, numerical solutions do not bring much understanding.

However, three fairly recent developments offer some hope for improved understanding: (1) the discovery, by experimental fluid mechanicians, of coherent structures in certain fully developed turbulent flows; (2) the suggestion that strange attractors and other ideas from finite-dimensional dynamical systems theory might play a rôle in the analysis of the governing equations; and (3) the introduction of the statistical technique of Karhunen–Loève or proper orthogonal decomposition. This book introduces these developments and describes how the three threads can be drawn together to weave low-dimensional models that address the rôle of coherent structures in turbulence generation.

We have uppermost in our minds an audience of engineers and applied scientists wishing to learn about some new methods and ways in which they might contribute to an understanding of turbulent flows. Additionally, applied mathematicians and dynamical systems theorists might learn a little fluid mechanics here, and find in it a suitable playground for their expertise.

The fact that we are writing for a mixed audience will probably make parts of this book irritating to almost all our readers. We have tried to strike a reasonable balance, but experts in turbulence and dynamical systems may find our treatments of their respective fields superficial.

Our approach will be somewhat schizophrenic. On the one hand we hope to suggest a broad strategy for modeling turbulent flows (and, more generally, other spatio-temporally complex systems) by extracting coherent structures and deriving, from the governing

Navier–Stokes equations, relatively small sets of ordinary differential equations that describe their dynamical interactions. We freely admit that there is much speculation and there are few firm results in this, although a number of (partial) successes have been achieved. In collecting our thoughts and those of others, we hope to stimulate research which might ultimately put some of these ideas on a firmer footing. This is the "vision" side of the book. In contrast, and since we need these methods to analyze our low-dimensional models, we provide a brief introduction, with many simple examples, to relevant and well-established ideas from dynamical systems theory. This is the "technical manual" side of the book. We occasionally switch from vision to technical mode, or vice versa, with scant warning. We guarantee that we have a mode to annoy every reader.

Our (tedious) working of simple examples may make for impatience on the part of those hurrying to get to the main attraction, or attractor. In our defense we remark that full appreciation of an "application" as complex as turbulence must rest on a firm understanding of simpler cases. Equally, our use of the symbolic and abstract notation of dynamical systems theory may be a stumbling block for some. We encourage them to stagger on to the examples. (A glossary of technical terms and notations is provided at the end of Chapter 1.)

But while we may irritate, we hope not to confuse. The term "low-dimensional model" is already problematic. Our models ideally contain enough "modes" to permit reasonable spatial as well as temporal behavior of the larger scales in the flow: those dominant in the sense of average turbulent kinetic energy. Our models do not contain, nor shall we be concerned with, the inertial or dissipative ranges. We have in mind sets of ordinary differential equations containing perhaps 10–100 dependent variables: substantially larger than that of, say, Lorenz. This is drastically low in comparison to the number of modes (or nodes) necessary even in a large eddy simulation, let alone a direct numerical simulation, but it is high in the context of dynamical systems, in which we have relatively complete understanding only of systems of dimension ≤ 2! We approach the analysis of such "low but high"-dimensional models by building on yet lower dimensional models, for which more complete analyses are possible. In this, one of our prominent illustrative examples is provided by the heteroclinic attractor: a strongly nonlinear type of solution that occurs robustly in systems possessing certain symmetries. Heteroclinic attractors lead to "bursting" behavior in which systems exhibit relatively long quasisteady phases involving few modes, interrupted by violent events in which other groups of modes become active. The reader should not interpret our emphasis to mean that we think turbulence *is* a heteroclinic attractor, although such attractors do appear to represent some key features of the burst/sweep cycle in the boundary layer. Rather, the study of these attractors provides a nice example of the power of qualitative methods applied to equations which are, in dynamical systems terms, of high dimension (≥ 4).

A second area of potential confusion is in our use of linear spaces and linear analysis for the description and study of nonlinear objects. This is, of course, a quite normal tactic. Linear theory is well developed, relatively complete, and powerful. There is no contradiction in defining a nonlinear differential equation on a linear state space, or in representing a spatio-temporal field $u(x, t)$ as a linear combination of basis functions or "modes" $\varphi_j(x)$ multiplied by suitable time-dependent coefficients $a_j(t)$. The Fourier representation is a prime example. Of course, if $u(x, t)$ is a solution of a nonlinear partial differential equation, linear superposition will fail in the sense that the sum of two solutions will not generally produce a third, but we can still represent individual solutions via such series. Moreover, in spite of all the recent advances in nonlinear analysis, the tools of linear operator theory, including Fourier analysis and linearization of partial and ordinary differential equations, are still crucial in the study of nonlinear systems. The proper orthogonal or Karhunen–Loève decomposition, one of our major tools, also relies on linear theory and produces representations of functions and fields in linear spaces, within which we may then construct strongly nonlinear dynamical systems whose attractors quite happily display their nonlinear character.

The book falls into four parts. In the first – Turbulence – we introduce our general strategy and recall some key ideas from fluid mechanics and "classical" turbulence theory, which establish basic properties of some canonical turbulent flows. We describe coherent structures from the viewpoint of an experimental observer and follow this with a description of the Karhunen–Loève decomposition, with sufficient mathematical detail that the reader can appreciate the advantages and limitations of the low-dimensional, optimal representations of turbulent flows in terms of empirical eigenfunctions that it affords. We conclude this part by describing how the Navier–Stokes equations can be projected onto subspaces spanned by a few empirical eigenfunctions to yield a low-dimensional model, and outlining some of the additional "modeling" that must be done to account for modes and effects neglected in such radical truncations.

The second part – Dynamical systems – contains a review of some aspects of dynamical systems theory that are directly useful in the analysis of low-dimensional models. We discuss local and global bifurcations and important ideas such as structural stability and strange attractors. Symmetries play a central rôle in our ideas, and we devote a chapter to showing how they influence dynamical and bifurcation behaviors. We then gather our methods for a dry run: a study of the Kuramoto–Sivashinsky partial differential equation which, while much simpler than the Navier–Stokes equations, displays some of the same features and allows us to illustrate our techniques. The final chapter introduces some basic ideas from the theory of stochastic differential equations: ideas that we need in dealing with systems subject to (small) random disturbances. This part is a fairly relentless essay in the technical manual mode.

The third part – The boundary layer – returns to the Navier–Stokes equations and a specific class of models of the turbulent boundary layer. This is the problem to which our approach was first applied and it is probably still the most widely studied from this viewpoint. It is certainly the one that we understand best, shortcomings and all. We offer this description and critical commentary partly in the hope that it may help others avoid our mistakes.

In the final part – Other applications and related work – we briefly review a number of applications of this and similar strategies to other turbulent open flow problems. We do not consider applications in other areas, such as pattern-forming chemical reactions, flame dynamics, etc., although many such are now appearing. We close by speculating on the kind of understanding of turbulence that this approach is likely to yield, and on how some recent developments, such as inertial manifolds, are related to it.

We promise not to mention the word "fractal" in this book.

Preface to the second edition

Much work has been done on low-dimensional models of turbulence and fluid systems in the 16 years since the first edition of this book appeared. In preparing the second edition, we have not attempted a comprehensive review: indeed, we doubt that this is possible, or even desirable. Rather, we have added one chapter and several sections and subsections on some new developments that are most closely related to material in our first edition. We have also made minor corrections and clarifications throughout, and added comments in several places, as well as correcting a number of errors that readers have pointed out. Here, to orient the reader, we outline the major changes.

Clancy Rowley (the new member of our team) has contributed a chapter on balanced truncation, a technique from linear control theory that chooses bases that optimally align inputs and outputs. Over the past ten years this has led to the method of balanced proper orthogonal decomposition (BPOD), which is especially useful for systems equipped with sensors and actuators. Since low-dimensional models provide a computational means for studying control of turbulence, we feel that BPOD has considerable potential. This new chapter (5) now closes the first part of the book (readers familiar with the first edition must therefore remember to add 1 to correctly identify the following eight chapters). The only other entirely new sections are 7.5, a discussion of traveling modes in translation-invariant systems, 12.6, a review of work on coherent structures in internal combustion engines, and 12.7, which gathers a miscellany of recent results.

New materials also appear in Chapter 3, where we modestly generalize the derivation of the POD in Section 3.1, adding subsections on specific function spaces, and in Section 3.4, where the relationship between the method of snapshots and the classical singular value decomposition is described, where we introduce an inner product for compressible flows, and where we comment on using a fixed set of empirical eigenfunctions to represent data over a range of parameter values (e.g. Reynolds numbers). In Chapter 4 we now provide more details on Galerkin projection (Section 4.1), give an example of a PDE with time-dependent boundary conditions and explain how quadratic nonlinearities, such as those in the Navier–Stokes equations, permit analytical determination of coefficients in the projected ODEs (Section 4.2). We also describe the important notion of shift modes in Section 4.4. Section 8.4 now ends with remarks on spatially-localized models of the Kuramoto–Sivashinsky equation, Section 10.7.2 notes a model that uses shift modes to couple a time-varying "mean" flow and secondary modes, and in Section 12.4

we summarize low-dimensional models of unsteady wakes behind cylinders. We have also revised Section 13.4 to reflect the fact that the results on spatially-localized models with pressure rather than velocity boundary conditions described there are incomplete and do not completely resolve well-posedness of the Navier–Stokes equations with mixed velocity and pressure boundary conditions. Finally, the index has been substantially expanded and improved, and we have added over 80 references.

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Among our immediate colleagues, John Guckenheimer and Sidney Leibovich have been particularly helpful. Steve Pope helped with some of the probabilistic ideas in Chapters 3 and 13. At a greater distance, Keith Moffatt and Larry Sirovich have been useful critics, forcing us to examine our assumptions more closely. Ciprian Foias, Mark Glauser, and Dietmar Rempfer shared their expertise, and explained their insights and results to us. The opportunity to give lectures and short courses on this work has also clarified our understanding and, we believe, improved our presentation. PH would like to thank Klaus Kirchgässner, Jean-Claude Saut, John Brindley, Colin Sparrow, and Silvina Ponce Dawson and Gabriel Mindlin for arranging courses at Universität Stuttgart, Université de Paris-Sud, the University of Leeds, the Newton Institute, Cambridge, and the Fourth Latin American Workshop on Nonlinear Phenomena, San Carlos de Bariloche, Argentina, respectively. JLL would like to thank Yousuff Hussaini and Jean-Paul Bonnet for arranging courses at NASA Langley Research Center and The International Center for the Mechanical Sciences, Udine, respectively. CWR would like to thank Tim Colonius, the late Jerry Marsden, and Richard Murray for introducing him to this field.

The manuscript was typed in LaTeX, much of it by ourselves, but with the able assistance of Gail Cotanch and Phebe Tarassov. Alison Woolatt of CUP also helped us with LaTeXand CUP formats. Harry Dankowicz computed some of the figures in Chapter 8. Teresa Howley patiently redrew and improved all the figures. Jonathan Mattingly and Ralf Wittenberg read the manuscript and suggested numerous corrections and improvements. Jo Clegg's patient copyediting kept us on the (fairly) straight and narrow. Our thanks go to all of them.

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Finally, we are grateful to Simon Capelin of the Cambridge University Press, who encouraged us to complete this book almost from its beginning, and who invited us to prepare a second edition.

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