Turbulent Transport Near the Ground in Stable Conditions

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ABSTRACT

The variation of the nondimensional transfer coefficients, k^* and K^*, and the Monin-Obukhov function ϕ in strong stability, are examined. The exchange coefficients for heat, water vapor and momentum are shown to be approximately equal even at large Ri. The relationships suggest that the decay of turbulent motion in the lower atmosphere begins at $\mathrm{Ri}=0.1$, and completely ceases at $\mathrm{Ri}>0.3$. In these nonturbulent conditions the profiles of wind and water vapor show very little variation with height, but the temperature profile reflects the increasing importance of infrared radiation.

1. Introduction

Micrometeorology has a fairly precise knowledge of the processes governing the transport of heat, water vapor and momentum in the lowest layers of the atmosphere under conditions of neutral stability. However, it is the exception rather than the rule to have adiabatic conditions. The lowest layers of the atmosphere are usually thermally stratified, being characterized by large lapse conditions during the daytime and inversi ns at night. Unfortunately, the flux-gradient relationships derived for neutral conditions do not apply in these more realistic situations.

Nevertheless, as a result of studies such as those of Monin and Obukhov (1954), Priestley (1959), Panofsky (1961), Swinbank (1964, 1968) and Webb (1965), a c nsiderable amount *is* known about the fluxes of heat, water vapor and momentum, and the corresponding profiles of temperature, humidity and wind under superadiabatic conditions. On the other hand, our knowledge of the inversion case is very poorly documented. Kazanski and Monin (1954), Ellison (1957), McVehil (1964) and Webb (1970) are among the few who have made significant contributions to our understanding of the stable regime.

This paper presents some results of work largely carried out under conditions of extreme stability. As such it provides a mainly descriptive insight into the nature of the processes and the resulting profile forms when atmospheric motion is damped.

2. Theory

In neutral conditions the horizontal shearing stress τ and the vertical fluxes of heat (H) and water vapor (E) can be represented by

$$\tau=\rho K_M\frac{\partial u}{\partial z}, \qquad (1)$$

$$H=-\rho c_p K_H\frac{\partial \Theta}{\partial z}, \qquad (2)$$

$$E=-\rho K_E\frac{\partial q}{\partial z}, \qquad (3)$$

where K_M, K_H and K_E are the eddy diffusivities for momentum, heat and water vapor, respectively, and u, Θ, and q are the wind speed, potential air temperature and specific humidity at each height z. The constants and units are standard. The fluxes are assumed constant with height and therefore equal to their surface value, and the adiabatic lapse rate has been neglected. Assuming fully rough flow, it can be shown from Eq. (1) that

$$\frac{\partial u}{\partial z}=\frac{u_*}{kz}, \qquad (4)$$

where $u_*=(\tau/\rho)^{\frac{1}{2}}$ is the friction velocity, and k is von Kármán's constant (0.4). Integration of Eq. (4) with height gives the well known log-law wind profile. From (1) and (4) it is also possible to show that

$$K_M=k^2z^2(\partial u/\partial z)=ku_*z. \qquad (5)$$

As has already been noted the neutral log-law does not represent the form of the wind profile under nonneutral conditions. Of the many methods derived to correct for stability effects one of the most common approaches has been to modify Eq. (4) by introducing dimensionless mixing coefficients k^* or K^* which are

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defined by

$$k_M^* \equiv \frac{K_M}{u_* z}, \quad (6)$$

$$K_M^* \equiv \frac{K_M}{z^2(\partial u/\partial z)}. \quad (7)$$

Similar coefficients (k_H^*, k_E^*, K_H^*, K_E^*) may be defined for heat and water vapor transfer.

The similarity between Eqs. (6) and (7) and Eq. (5) is readily apparent, and Webb (1965) shows the relation between k^* [introduced by Priestley (1959)] and K^* [introduced by Pasquill (1949)] to be

$$K_M^* = (k_M^*)^2, \quad K_H^* = (k_M^* k_H^*), \quad K_E = (k_M^* k_E^*). \quad (8)$$

Thus, in this method, Eq. (4) is modified to read

$$\frac{\partial u}{\partial z} = \frac{u_*}{k_M^* z} = \frac{u_*}{(K_M^*)^{\frac{1}{2}} z}. \quad (9)$$

The mixing coefficients k^* and K^* may be viewed as dimensionless forms of the eddy diffusivities K_M, K_H and K_E, in the same manner as H^* and E^* are seen to be the dimensionless forms of H and E, as used by Priestley (1959) and Crawford (1965). In this way the adiabatic dependence on wind speed and height has been removed. This is especially useful in this study where interest is centered on the ability of turbulence to transfer entities down their concentration gradients as expressed in Eqs. (1)–(3). Using the dimensionless height ratio z/L proposed by Monin and Obukhov, where L is

$$L \equiv u_*^3 c_p \rho T/(kgH), \quad (10)$$

Eq. (4) becomes

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz} \phi_M(z/L), \quad (11)$$

where ϕ_M is an unknown function to be determined by experiment. Analogous functions ϕ_H and ϕ_E may also be introduced to describe the non-neutral temperature and humidity profiles. For small values of z/L, Eq. (11) may be expanded in a Taylor series so that by retaining only the linear term we have the log-linear law

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz}[1 + \alpha(z/L)], \quad (12)$$

where α is a constant.

Turbulence can either be generated thermally or mechanically. Thus, a convenient stability parameter for use in this work is the Richardson number (Ri) whose gradient form approximates the ratio between the forces producing thermal and mechanical turbulence, i.e.,

$$\mathrm{Ri} = \left(\frac{g}{\Theta'}\right)\left(\frac{\partial \Theta}{\partial z}\right) \Big/ (\partial u/\partial z)^2, \quad (13)$$

where g is the acceleration of gravity, and Θ' the mean potential temperature of the layer. Ri and z/L are related through the flux form of the Richardson number (Rf), i.e.,

$$\mathrm{Rf} = \mathrm{Ri}\frac{K_H}{K_M} = \frac{z}{\phi L} = \frac{k^*}{k}(z/L). \quad (14)$$

In neutral conditions, when k^*/k and K_H/K_M are unity, $z/L = \mathrm{Ri} = 0$. It should also be noted from Eq. (14) that the dimensionless terms are related; thus,

$$\phi = k/k^* = k/(K^*)^{\frac{1}{2}}. \quad (15)$$

Also since $\phi = (1 + \alpha z/L)$, it may be written in terms of Ri as

$$\phi = (1 - \alpha \mathrm{Ri} K_H/K_M)^{-1}. \quad (16)$$

3. Instrumentation and data

The instrumentation and site for this study have been described elsewhere (Oke,1970), and only a summary is presented here. The site was an extensive bare-soil surface located near Hamilton, Ontario. The site was relatively smooth ($z_0 = 0.01$ cm) and flat (slope $< 0.5\%$), and gave a height: fetch ratio > 1:200 in all directions. It was felt that observati ns from this site during anticyclonic conditions at night would provide results in a relatively steady-state boundary layer.

Unshielded, non-aspirated No. 26 gage copper-constantan thermocouples were used as dry-and wet-bulb thermometers at 2.5, 5, 10, 15, 25, 50 and 100 cm. Additional dry bulbs were exposed at 0, 1, 7.5 and 20 cm, and all signals monitored on potentiometric recorders. Horizontal wind speeds were measured at 25, 50 and 100 cm with Thornthwaite sensitive cup anemometers. These anemometers give a good response to gustiness and have a stalling speed of 7–15 cm sec^{-1}. Vertical wind speeds were measured by a Thornthwaite vertical velocity anemometer at 50 cm. All wind observations were averaged over a 15-min period.

Table 1. Details of studies whose data are plotted in Figs. 1–3.

Author	Flux measured	Flux measurement technique
Cramer and Record (1953)	τ, H	Eddy correlation, hot wire anemometer, bivane and thermocouples
Rider (1954)	τ	Drag plate
Swinbank (1955)	τ	Eddy correlation, hot wire anemometers
Field (1963)	τ	Eddy correlation, flux meter
Pasquill (1949)	E, H	Evaporimeter, solarimeters, soil thermometers
Pruitt and Aston (1963)	E, H	Lysimeter, net radiometer, soil heat flux plates

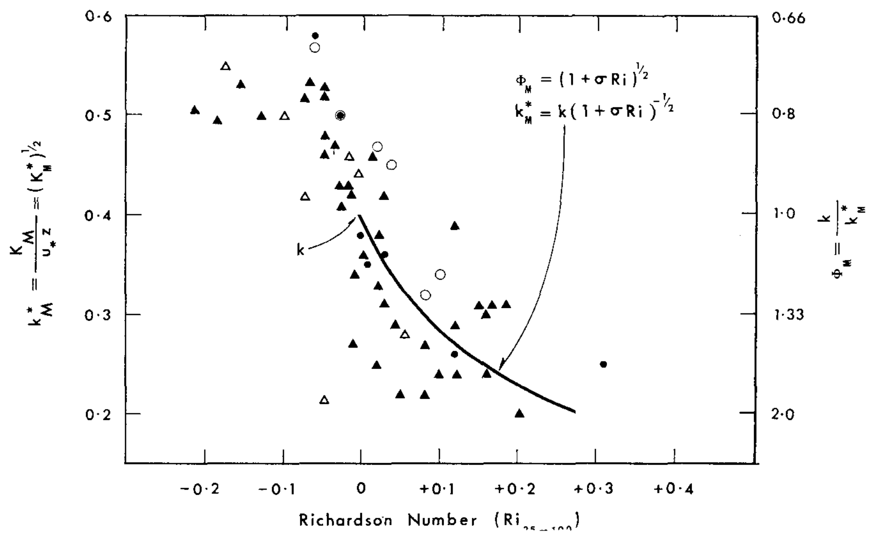


FIG. 1. Variation of the nondimensional mixing coefficient k_M* for momentum transfer and the function ϕ_M with stability, from the results of Cramer and Record (1953), open circles; Rider (1954), solid circles; Swinbank (1955), open triangles; and Field (1963), solid triangles.

The gradients $\Delta\Theta$ and Δu used in computations represent the interval 25 to 100 cm. Values of u_* were computed using the diabatic formulation of Kao (1959) which is applicable in the range $-0.5 \leqslant \mathrm{Rf} < 0.5$ (Kao, personal communication). The standard deviation σ_w of vertical velocity was calculated from the equation proposed by Blackadar (1962). Since σ_w is linearly related to u, the standard deviation of vertical angle $\sigma_\theta = \sigma_w/u$ (Lumley and Panofsky, 1964) was used here as an indicator of vertical motion. Because the spectrum of turbulence enters higher frequencies near the ground under strong stability, the anemometer's response was reduced, but relative values are useful. Values of σ_θ provide a method of examining vertical transfer without reference to flux-gradient expressions. All results presented here are hourly averages.

4. Nondimensional transfers

The stability variation of the dimensionless mixing coefficients for momentum (k_M* and K_M*) and the Monin-Obukhov function (ϕ_M) are shown in Fig. 1. The results are from studies (Table 1) where the shearing stress was measured directly. Fig. 1 is essentially the same as Fig. 8 in Priestley (1959) and Fig. 2 in Webb (1965), differing only in that it includes the data of Field (1963) and that it shows the relation between the three nondimensional parameters. It will be noted,

TABLE 2. Hourly average wind and stability parameters.

Date (1966)	Hour	Cloud	Ri[a]	u_{25} (cm sec^{-1})	$\Delta u/\Delta z$[b] (sec^{-1})	K_M*	ϕ_M	S[c]
23–24 June	00–01	nil	0.129	112.0	0.42	0.042	1.95	1.43
24–25 June	22–23	nil	0.139	39.0	0.21	0.094	1.30	1.17
	23–00	nil	0.163	48.5	0.25	0.085	1.37	1.20
	00–01	nil	0.101	73.0	0.38	0.083	1.39	1.28
	01–02	nil	0.043	62.5	0.21	0.147	1.04	1.38
27–28 June	21–22	0.1 Ci	0.145	49.0	0.22	0.023	2.64	1.97
	22–23	0.1 Ci	0.246	55.0	0.25	0.022	2.69	1.70
	00–01	nil	0.215	65.5	0.35	0.008	4.47	1.87
	04–05	nil	0.104	70.0	0.22	0.027	2.43	2.03
29–30 June	21–22	nil	0.688	42.5	0.25	0.012	3.65	1.94
	22–23	nil	0.348	53.5	0.28	0.008	4.47	1.89
	02–03	nil	0.110	70.0	0.35	0.005	5.65	1.74
	03–04	nil	0.224	73.5	0.33	0.006	5.16	1.82
	04–05	haze	0.070	75.5	0.31	0.027	2.43	1.77
30 June–1 July	22–23	0.1 Ci	0.234	41.5	0.27	0.022	2.70	2.50
1–2 July	22–23	nil	0.539	42.0	0.21	0.064	1.58	1.74
	23–00	nil	0.346	37.0	0.19	0.033	2.20	1.82
	02–03	nil	0.530	61.5	0.32	0.002	8.94	1.97

[a] Measured over height interval 100-25 cm: all values positive.
[b] $\Delta z = 100-25 = 75$ cm.
[c] Defined in Section 5.

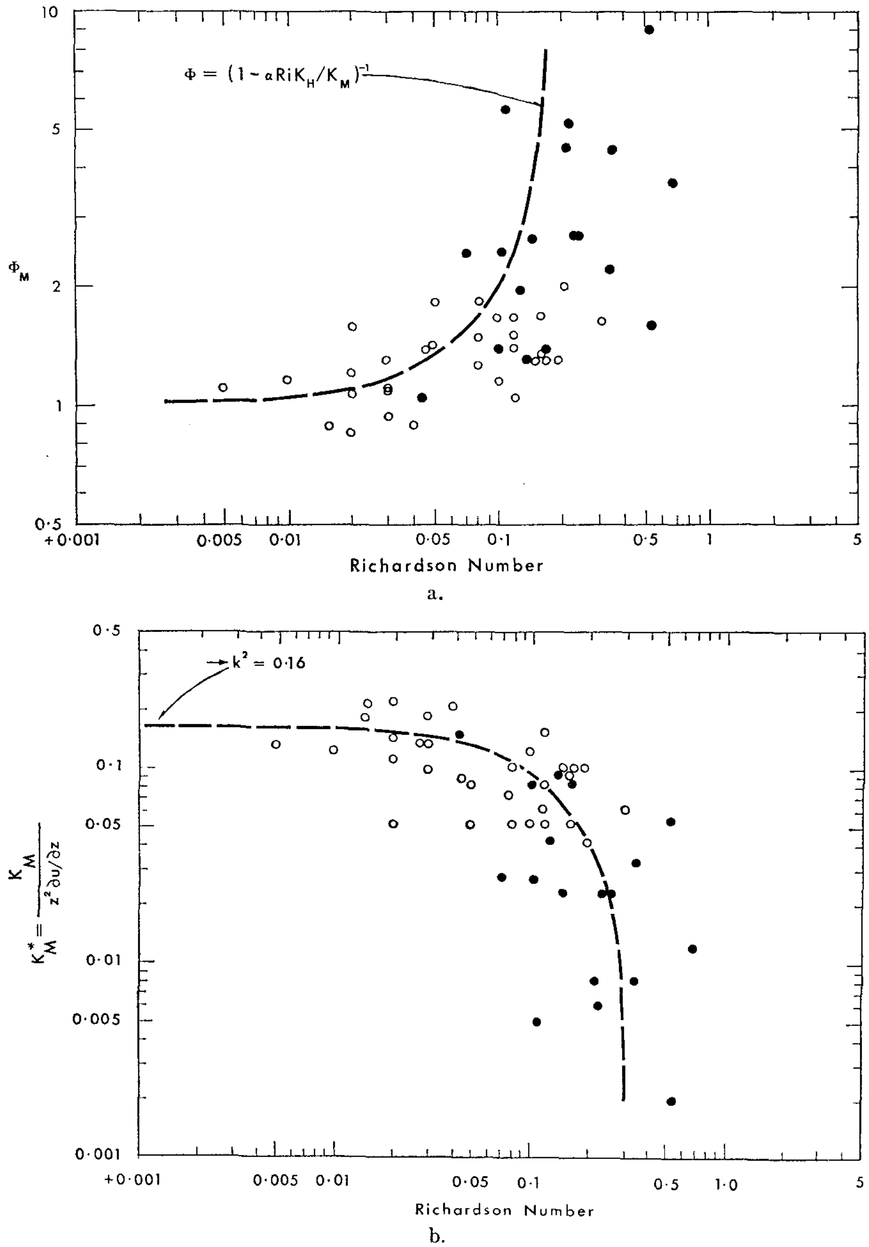


FIG. 2. Variation of the nondimensional parameters ϕ_M, a., and K_M^*, b., with Ri, including the data from Fig. 1, open circles, and this study, solid circles. The dashed line in a. represents Eq. (16) with $K_H/K_M=1$ and $\alpha=5$; in b. the line is an eye-fit to the data.

therefore, in neutral conditions (Ri zero) that

$$\left.\begin{array}{l} k_M^*=k=0.4 \\ K_M^*=(k_M^*)^2=0.16 \\ \phi_M=1 \end{array}\right\}.$$

For the unstable regime (Ri negative), Holzman (1943) proposed the empirical relation

$$k_M^*=k(1-\sigma\mathrm{Ri})^{\frac{1}{4}}, \tag{17a}$$

from which we have

$$\phi_M=(1-\sigma\mathrm{Ri})^{-\frac{1}{4}}. \tag{17b}$$

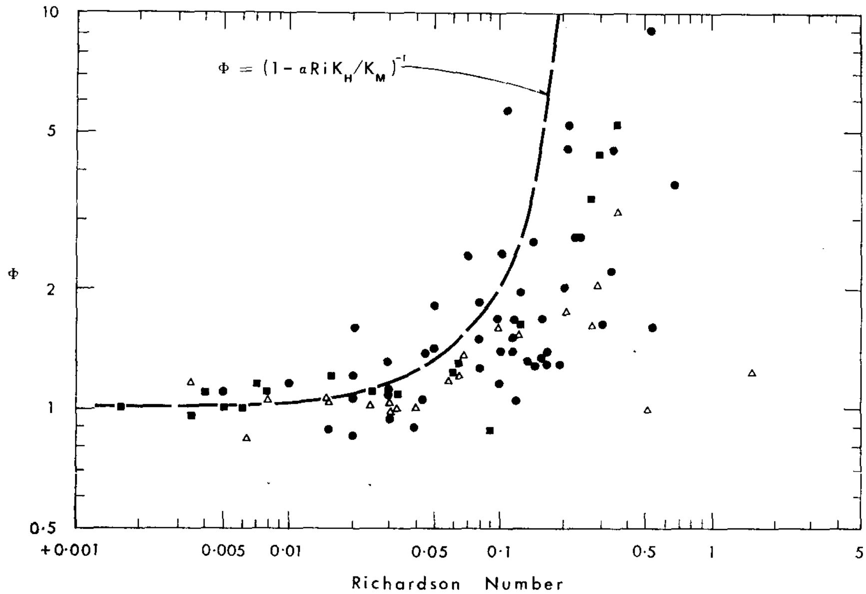
This is very similar to the relation suggested earlier by Rossby and Montgomery (1935) for stable conditions (Ri positive), i.e.,

$$k_M^*=k(1+\sigma\mathrm{Ri})^{-\frac{1}{2}}, \tag{18a}$$

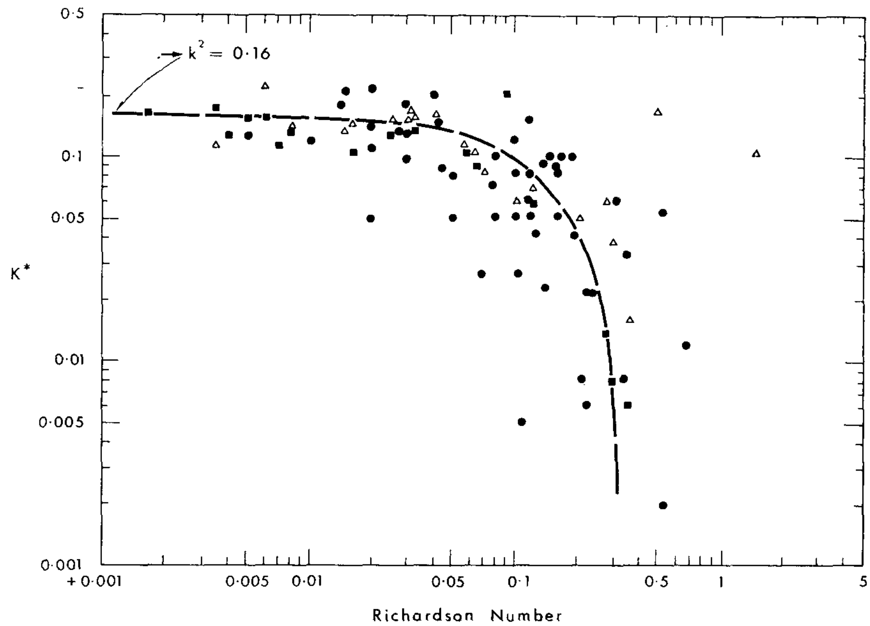
resulting in

$$\phi_M=(1+\sigma\mathrm{Ri})^{\frac{1}{2}}, \tag{18b}$$

where σ is a proportionality factor equal to $\sim$10 (Monteith, 1957), and $\sim$9 in the KEYPS formulation (Lumley and Panofsky, 1964). A value of 10 is used in the equation of the line fitting the stable data in Fig. 1.



a.



b.

FIG. 3. Variation of the nondimensional parameters ϕ, a., and K^*, b., with Ri. The data for momentum, solid circles, are from Fig. 2, and those for heat, open triangles, and water vapor, solid squares, are from Pasquill (1949) and Pruitt and Aston (1963). The dashed line in a. represents Eq. (16) with $K_H/K_M=1$ and $\alpha=5$; in b. the line is an eye-fit to the data.

In the range Ri=0–0.1 the fit is good, but lack of data prevents analysis in more stable conditions. As Webb (1965) points out there is little information beyond Ri=0.2. It is this region of uncertainty that constitutes the main concern of this paper.

Data from this study were used to compute values of ϕ_M and K_M^* (Table 2). These data, and those referenced in Fig. 1, are shown in Fig. 2. Both the Monin-Obukhov function (Fig. 2a) and the dimensionless mixing coefficient (Fig. 2b) are presented because the former is important in log-linear wind profile formulation, while the latter provides a more easily interpreted variable in terms of transfer efficiency in the lower atmosphere [Eqs. (6) and (7)]. From Table 2 it will be seen that the data from this study mainly occur at Ri>0.1, i.e., under conditions of extreme stability which have been rarely reported. From Fig. 2 it is gratifying to note the apparently good agreement between these results and those of the other authors. The line through the data is the best-fit line drawn by eye with the criterion that at small Ri the line asymptotically approaches the value for neutral conditions outlined previously. The points at Ri>0.1 are more scattered and may reflect the difficulty of observation under these light wind conditions ($u_{100}<100$ cm sec^{-1}).

From Fig. 2 it appears that the nondimensional variables show little change over the range from neutral stability to Ri=0.1, at which point there is an abrupt change in conditions. From Table 2 and Eq. (7) it can be seen, as stability increases, that the sharp decrease in K_M^*, combined with smaller values of $\partial u/\partial z$, will give very weak eddy viscosity values, approaching that due to molecular diffusion (e.g., $K_{M50}=10^{-1}$–10 cm^2 sec^{-1}). Hence, the turbulent transfer of horizontal momentum appears to decrease rapidly beyond Ri=0.1.

In Fig. 3 these momentum data are compared with the corresponding nondimensional variables for heat and water vapor from the results published by Pasquill (1949) and Pruitt and Aston (1963). It is apparent that the relationship between the function for each entity and stability is similar. This agreement is striking in the range Ri=0.001–0.1. Beyond this value the relationship becomes scattered, but suggests a sharp change in the transfer coefficients.

There are two very important points to draw from Fig. 3. First, there does not appear any reason to differentiate between the functions ϕ_M, ϕ_H and ϕ_E or the coefficients K_M^*, K_H^* and K_E^* (except for two heat values at Ri=0.50 and 1.54 which will be discussed later). Thus, one may tentatively conclude that $K_M=K_H=K_E$ throughout the large range of stable conditions presented here. This somewhat surprising conclusion is, however, in agreement with Webb (1970), who shows that K_H/K_M and K_E/K_M remain constant and equal to unity out to Ri≈0.2.

Second, there is an obvious change in transfer values beyond Ri=0.1. This supports the view that there is a critical Richardson number ($\mathrm{Ri}_{\mathrm{crit}}$) beyond which turbulence begins to decay. Eventually, at even stronger stabilities, a value ($\mathrm{Ri}_{\mathrm{max}}$) is reached where turbulent motion ceases entirely, and is replaced by laminar flow governed by molecular, rather than eddy diffusion. In Fig. 3b the K^* values appear to approach an absolute minimum in the vicinity of Ri=0.3.

Recent estimates of the threshold values of $\mathrm{Ri}_{\mathrm{crit}}$ and $\mathrm{Ri}_{\mathrm{max}}$ in the lowest layers (<8 m) are given in Table 3. These values are much lower than the value of unity expected by Richardson (1920) when he derived his stability criterion. This could be the result of two conditions. First, Ri is related it its more fundamental flux form (Rf) by assuming the ratio K_H/K_M to be equal to unity [see Eq. (14)]. The results presented here tend to confirm this equality of the exchange coefficients in stable air, but the problem still remains largely controversial. Even so, it appears very unlikely that the ratio is greater than unity, which would be necessary to give small $\mathrm{Ri}_{\mathrm{crit}}$ values. Second, even if the mechanical forces are greater than the thermal ones, turbulence may not be maintained because of dissipation into heat. It is this process which probably produces the small values of $\mathrm{Ri}_{\mathrm{crit}}$ (0.05–0.15) and $\mathrm{Ri}_{\mathrm{max}}$ (0.20–0.35) reported in the literature and confirmed here in Fig. 3. It should be noted however that as Lyons *et al.* (1964) and Zdunkowski *et al.* (1967) have pointed out, since the dissipation rate decreases with height, $\mathrm{Ri}_{\mathrm{crit}}$ may be expected to increase with height. For this reason caution should be exercised in comparing the threshold values from studies where the height intervals are significantly different.

TABLE 3. Estimates of $\mathrm{Ri}_{\mathrm{crit}}$ and $\mathrm{Ri}_{\mathrm{max}}$ from studies in the lowest layers of the atmosphere.

Author	$\mathrm{Ri}_{\mathrm{crit}}$	$\mathrm{Ri}_{\mathrm{max}}$
Deacon (1949)	0.15	
Rider and Robinson (1951)	0.40	1.4
Ellison (1957)	0.15	
Davis (1957)	0.15	
Townsend (1958)	0.08	0.30
Businger (1959)		0.20
Portman *et al.* (1962)		0.35
Lyons *et al.* (1964)	0.18	0.50
McVehil (1964)	0.14	
Crawford (1965)	0.05	0.35
Kaimal and Izumi (1965)		0.25

It is interesting to note that under inversion conditions, we have the log-linear form

$$\mathrm{Ri}=(z/L)(1+\alpha z/L)^{-1},$$

where the constant α has been found by observation to be ~5 (Webb, 1970) or ~7 (McVehil, 1964). In strongly stable conditions when z/L becomes large, the limiting value $\mathrm{Ri}_{\mathrm{crit}}=\alpha^{-1}$, or ~0.14–0.20. Similarly, in the log-linear range

$$K_M=ku_*z(1-\alpha\mathrm{Ri}),$$

so that K_M approaches zero at $\mathrm{Ri}_{\mathrm{crit}}$, which is in agreement with the values quoted earlier.

The upper limit for the cessation of turbulence is also confirmed theoretically in Miles' (1961) theorem which shows that a sufficient condition for stability in a parallel, stratified inviscid fluid is Ri>0.25 everywhere. Winn-Nielsen (1965) has shown that Miles' theorem holds for hydrostatic compressible fluids as well as for non-hydrostatic incompressible fluids.

It should be pointed out that values of ϕ_M and K_M^* presented here possess the obvious weakness that u_* values have been calculated using a diabatic correction formula for the wind profile. Ideally, values should come from direct measurement of τ. Similarly, the heat and water vapor values require direct measurements of H and E without resort to flux-gradient relations. This can be achieved through the use of techniques such as the eddy correlation method, involving drag plates or lysimetry (Table 1). In this study the vertical anemom-

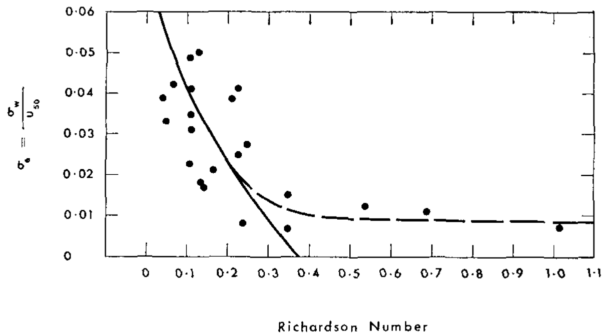


FIG. 4. Variation of the standard deviation (σ_θ) of vertical angle with Ri for stable conditions. The solid line is from Deacon (1959), the dashed line an eye-fit to the present data.

eter data provide an independent verification of the magnitude of turbulent mixing for Ri>0.05 where doubt may be cast upon the applicability of diabatic expressions for the wind profile.

Hourly average values of σ_θ are plotted in Fig. 4 as a function of Ri. The solid line is an extrapolation from the results of Deacon (1959), and shows a good link-up at slight stability with the data here. It is interesting to note that the extrapolation of this line to $\sigma_\theta=0$ closely corresponds to a value of $\mathrm{Ri}_{\max}\approx 0.35$. In nature, as the dashed line fitting the data indicates, complete cessation of vertical turbulence does not occur. But the relationship revealed by Fig. 4 is wholly consistent with the conditions depicted by Figs. 1–3 and agrees almost exactly with the results of Kaimal and Izumi (1965). Vertical transport decays rapidly at Ri>0.05, and remains negligibly small at Ri>0.3.

5. Profiles under strong stability

In the light of the conclusion that at Ri>0.1 the atmosphere is essentially quasi- or nonturbulent, it is interesting to examine the coincident profiles of wind speed, temperature and water vapor, since these are normally considered to be a direct function of eddy diffusion.

The wind profile in the near-neutral case is simply given by Eq. (4), requiring no correction. In the log-linear framework the progressive deviation from this simple log fit is accounted for by the function ϕ_M. Hence, in Figs. 2 and 3 the ϕ values increase from unity in near-neutral conditions to higher values at greater stability. The line fitting these data is Eq. (16), assuming K_H/K_M equals unity and using $\alpha=5$ after Webb (1970). The fit is good out to $\mathrm{Ri}_{\mathrm{crit}}$, but less useful in the quasi-turbulent region beyond. A lower α value would give a better, but still not acceptable fit.

Webb (1970) notes this breakdown of the log-linear form for $\mathrm{Ri}>\mathrm{Ri}_{\mathrm{crit}}$, and suggests a transition to a simple logarithmic form. As as test of this he plots shape indicator ratios S vs Ri, where

$$S=(u_c-u_b)/(u_b-u_a).$$

For the Australian data from the Kerang and Hay sites, Webb uses the heights 16, 4 and 1 m. His results show approximate agreement with curves representing Eq. (12) with $\alpha=5$ out to $\mathrm{Ri}_{200}\approx 0.15$. Although the data are sparse, at stronger stability Webb feels the data agree to a simple logarithmic form given by

$$\frac{\partial u}{\partial z}=\frac{u_*}{kz}(1+\alpha). \qquad (19)$$

Similar S values were computed for this study, using the heights 100, 50 and 25 cm (Table 2). As with Webb's data the S values here conform to a log-linear form with $\alpha=5$, up to $\mathrm{Ri}_{\mathrm{crit}}=0.1$. At greater stability the results sharply break from this form; however, they show no systematic tendency to follow a form such as is given by Eq. (19). Presumably the flow is then governed by processes other than turbulence, such as katabatic drift or gravity waves (Winn-Nielsen, 1965). Under very stable conditions, $\partial u/\partial z$ becomes very small (Table 2); on certain occasions, in fact, the data were not usable because completely laminar flow existed ($\partial u/\partial z=0$), or because inverted profiles ($\partial u/\partial z<0$) were recorded with all anemometers continuously turning. The same phenomenon has been reported by Halstead (1951), Rider and Robinson (1951), Deacon (1953) and Davis (1957).

Under conditions of weak stability, with $u_{25}>110$ cm sec^{-1}, the water vapor profile showed a decrease with height, resulting in a vapor transport which was generally directed upward. In stronger stability ($\mathrm{Ri}\leqslant 0.1$), the vapor pressure increased with height above the surface, and dew was common. However, if $u_{25}<70$ cm sec^{-1} and/or Ri>0.1, the vapor profile exhibited either very little or erratic variation in the vertical and dew was absent.

The temperature profile, on the other hand, is marked by strong gradients in the lowest layers. In the range Ri=0–0.1, the normal surface-based inversion is characteristic, and is especially well developed with clear skies. Beyond Ri=0.1, however, Oke (1970) has shown that the temperature profile exhibits a raised minimum with the base of the inversion situated from 1–25 cm above the bare soil surface. These strong gradients cannot be the result of sensible heat transfer by eddy diffusion. Thus, it is postulated that in these nonturbulent conditions the temperature profile is governed by radiative effects. Funk (1960) has demonstrated that at night in strong stability the infrared radiative flux is not constant with height, and produces large cooling or warming rates in the lower atmosphere. Radiation exerts no control on the fluxes of water vapor or momentum. Radiative effects may well explain the two anomalous results of Pruitt and Aston (1963) in Fig. 3. In fact, closer scrutiny of their data on these two occasions reveals clear skies with exceptionally light wind speeds ($u_{25}=25$–50 cm sec^{-1}), and in one case a raised minimum profile with the 12-cm level colder than at 6 cm.

6. Conclusions

There are two fundamental conclusions to be drawn from this study. First, it appears in agreement with Webb (1970) that the transfer coefficients K_M, K_H and K_E are approximately equal under very stable conditions in the lower atmosphere. Second, there is a critical Richardson number at ~ 0.1 beyond which the atmosphere cannot be said to be fully turbulent. For conditions closer to neutral the slightly stable form of the wind profile, and probably the profiles of temperature and water vapor, is given by the log-linear framework with $\alpha = 5$. In the quasi- or even nonturbulent conditions beyond $\mathrm{Ri}_{\mathrm{crit}}$, there may be no definitive forms for the profiles of wind, temperature and water vapor. It is likely that radiative and turbulent heat flux divergence will be evident in this region, thus raising fundamental objections to the use of the main body of micrometeorological theory and measurement techniques. At the present time we have no sound method of incorporating these divergence effects into practical transfer approaches.

In view of these problems it may be reasonable to take a purely empirical approach to estimating fluxes in the region beyond $\mathrm{Ri}_{\mathrm{crit}}$. For example, the average curve drawn through the data in Fig. 3b could be used to estimate a transfer coefficient from observations of Ri via Eq. (7). Then, by assuming similarity of the transfer coefficients, the fluxes may be computed from Eqs. (1)–(3) from gradients of the entities. However, it would seem that in view of the very small K values the fluxes will be negligible, as evidenced, for example, by the lack of dew beyond $\mathrm{Ri}_{\mathrm{crit}}$. In addition, one should point out that Ri itself is open to flux divergence effects, and hence becomes of questionable value as a stability parameter (Crawford, 1965).

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