TURBULENT TWO-PHASE FLOW

by
L. van Wijngaarden
Twente University, Enschede, The Netherlands.

1. General

Turbulent two-phase flows are of great importance in industry, in particular the chemical process industry. There is a wide variety of topologies: annular flow, slug flow, dispersed flow, mist flow and so on. These flows are almost always highly turbulent.

What industry would like us to provide is a theory applicable to all turbulent two-phase flows and materialized in a users friendly CFD code. What we, fluid dynamicists, can do for them in that department is not very much. We can consider each type of turbulent two-phase flow by itself. Even then, an additional difficulty is that in most industrial turbulent two-phase flows the volume concentration of each phase is of order unity. In situations in which we have some insight this concentration is either close to zero or close to unity.

With particles suspended in a turbulent flow, for example, hydrodynamic interaction between particles can be neglected at a volume concentration of the dispersed phase well below 1% . For such very dilute suspensions it is sufficient to consider only one particle and study its behaviour in a turbulent flow.
Knowledge about the dynamics of a turbulent particle or bubble dispersion can be obtained from averaging over many trajectories of particles or bubbles. This connection between one particle behaviour and suspension hydrodynamics has proven to be very fruitful in non-turbulent two-phase flow (Biesheuvel & Van Wijngaarden 1984). We will return to this in Section 4.

With increasing concentration, hydrodynamic interaction between particles or bubbles become important. Formally, an averaged quantity is defined as ensemble average

S. Gavrilakis et al. (eds.), Advances in Turbulence VI, 535-541.

over the ensemble of all possible configurations C_N of N particles. If the volume concentration is denoted with α, it is wellknown (e.g. Batchelor 1972) that accuracy in α is obtained when a configuration exists of only one particle, whereas accuracy in α^2 requires configurations consisting of two particles etc. In this way a theory for moderately dilute dispersions, can be obtained. In non-turbulent flow there are many examples, but for turbulent flow such calculations have not been made, as yet.

Direct numerical calculation must be attempted when multiple interactions are important. This must for solid particles be restricted to low Reynolds number flows, and to simplifying approximations for the flow along a bubble, in the case of bubbly flows.

For turbulent dispersed flow at high Reynolds numbers and high concentrations, the prevailing circumstances in industrial applications, DNS is beyond possibility. A time honoured way of dealing with these flows, both in the turbulent and non- or pseudo-turbulent case, is to consider a mixture as consisting of two fluids, each with its own set of conservation equations. These equations are subjected to Reynolds averaging. But then, one needs closure relations. In one-phase turbulent flow these are restricted to the Reynolds stresses (see e.g. Launder 1992). In two-phase flow many more are needed, since with the bubbles or particles new sets of scales and times appear. Usually the closure models are variatons on the k-ε theme. Good examples of what is achieved in this way are Elghobashi & About-Arab (1983) and Besnard & Harlow (1988). A recent survey can be found in Crowe, Troutt and Chung (1996).

Quite apart from the difficulties in single phase turbulent flow, there are additional ones, posed by the terms meant to describe the interaction between the phases. The great variety in scales, so typical for turbulence, is enriched or worsened, as you please, by an additional number of characteristic scales and times. Among these are particle (or bubble) size, wake size, relaxation time and so on.

2. Solid particles in turbulent flow

At low concentration by volume of particles, hydrodynamic interaction between these can be neglected. Let the particle be spherical with radius a. We denote the kinematic viscosity of the continuous phase with ν, its density with ρ, that of the particles with ρ_p and the acceleration of gravity with $\mathbf{g}$. An important parameter is the relaxation time τ, needed for the particle to adjust, at time t and position $\mathbf{X}(t)$,its velocity $\mathbf{v}$ to that of the fluid $\mathbf{u}\{\mathbf{X}(t),t\}$,

$$\tau \sim \frac{a^2\rho_p}{\rho\nu} . \tag{1}$$

Another important quantity connected with the particle, is its settling velocity $\mathbf{V}_T$ in an infinite quiescent fluid,$\mathbf{V}_T \sim \mathbf{g}\tau$.
It is usually assumed that the Reynolds number for the relative motion between particle and fluid is small with respect to unity. Then the drag is represented by Stokes's law. For particles where a $\leq l$, l being the Kolmogorov dissipation scale, the pertinent equation of motion for the particle is (Maxey & Riley 1983)

$$\frac{d\mathbf{v}}{dt} + \tau^{-1}\mathbf{v} = \tau^{-1}\mathbf{u}\ \{\mathbf{X}(t),t\} + \mathbf{g}. \tag{2}$$

We consider first the influence of the turbulence on the particle motion. When the influence of the initial position can be neglected, (2) has as solution

$$\mathbf{v}(t) = \tau^{-1}\int_{-\infty}^{t} \mathbf{u}\{\mathbf{X}(t'\}\exp\left(\frac{t'-t}{\tau}\right)dt + \mathbf{V}_T . \tag{3}$$

For τ small, small particles or light particles, $\mathbf{v}$ equals $\mathbf{u}\ \{\mathbf{X}\ (t),t\}$, and

$$\langle \mathbf{v} \rangle = \langle \mathbf{u}\{\mathbf{X}(t'),t'\}\rangle + \mathbf{V}_T. \tag{4}$$

Reeks (1977) suggested and Maxey (1987a) proved in great detail that for a completely random velocity field the mean settling speed equals $\mathbf{V}_T$. For a real turbulent $\mathbf{u}$ Maxey (1987a) concludes that the first contribution to the settling velocity $\langle \mathbf{v} \rangle$ comes from third order correlations and involves

$$\left\langle \frac{\partial u_j}{\partial x_l} \frac{\partial u_l}{\partial x_j} u_i(t) \right\rangle. \qquad (5)$$

For a Gaussian velocity field this three point correlation is zero. Nevertheless, for a turbulent flow the influence on the settling speed can be substantial as the numerical simulation by Wang & Maxey (1993) shows. In general the settling speed increases under the influence of the small turbulent scales. Wang & Maxey (1993) offer the explanation that particles are swept outward, when approaching a vortical region from above, toward regions where they obtain an excess settling speed.

For settling particles, the dispersion coefficient $D_p \sim \langle v^2 \rangle \tau$ may differ substantially from that of the fluid D, say, of order $D \sim u_0 L$, where u_0 is a measure for the turbulent velocity fluctuation. This follows both from experiments (Snyder & Lumley 1971), from analytic work (Reeks 1977, Pismen & Nir 1978) and from DNS (Uijttewaal 1995). Heavy particles with a low value of u_0/V_T cross fluid particle trajectories frequently, which causes their autocorrelation to be less than that of fluid particles, leading to $D_p/D<1$. For large τ and times $t >> \tau$, the velocity correlation seen by the particle along its trajectory nears the Eulerian velocity correlation and consequently D_p/D may be larger than one.

At a sufficiently high particle concentration, there is a feedback of the particle motion on the turbulence. With $a \leq l$, turbulence is damped, as shown in the experiments of Tsugi et al (1984) . However, when particles get larger, the relative motion becomes more important and turbulence is enhanced.

3. Bubbles in turbulent flow

Crowe et al (1996), writing on turbulent dispersed flow, exclude bubbly flow because of the totally different response of bubbles to the fluid motion, as compared with solid particles. Yet, some aspects remain, such as the "trajectory crossing " effect, and the way in which the velocity of rise in vertical flow is affected. Bubbles are drawn into eddies and display a bias towards regions with upgoing velocities, just as solid particles display preference for eddy regions with downward velocity.

Otherwise, there are important differences, indeed, as follows from the eqn. of motion (Auton et al 1988)

$$\frac{d\mathbf{v}}{dt} = 3\frac{D\mathbf{u}}{Dt} - (\mathbf{v} - \mathbf{u})\mathrm{x}\omega - 2\mathbf{g} - \tau_v^{-1}(\mathbf{v} - \mathbf{u}). \tag{6}$$

The main differences with (2) are: fluid inertia is much more dominant, through the term 3D**u**/Dt (D/Dt = material derivative in the fluid), the added mass, with associated relaxation time $\tau_v = a^2/18\nu$, and the lift force (second term on r.h.s.). The latter pushes bubbles towards regions where **u** is less, overshadowing the effect of eddy encounter, mentioned above. It appears (Spelt & Biesheuvel 1996) that the effect of turbulence on bubble motion depends on much more crude properties of the spectrum, as compared with solid particles. These authors by numerical simulation and by analysis, obtained results for bubble dispersion as well. As in the case of particles, the bubble dispersion coefficient D_b is less than D for small u_0/V_T, where V_T is now the speed of rise in an infinite liquid.

4. Pseudo turbulence

A rough estimate for the velocity **v** of a bubble placed in a fluid flow with local velocity **u** is (cf 5) $\mathbf{v} = \mathbf{V}_T + 3\mathbf{u}$.

This learns that random motions of bubbles may induce considerable velocity fluctuations, called pseudo-turbulence. Under the assumption of viscous potential flow around a bubble, this can be calculated for zero u_0 (Biesheuvel & Van Wijngaarden 1984), and even for arbitrary u_0. For the excess turbulent energy impaired to the the liquid, this gives as estimate

$$\alpha V_T^2 \left\{ const. + O\left(u_0^2 / V_T^2\right)\right\}. \tag{7}$$

Measurements of pseudo turbulence have been reported by Theofanous & Sullivan (1982), Lance & Bataille (1991), Stewart (1995). All these show significantly larger excess energies.

This, presumably, is due to the flow around largely deformed bubbles, involving a complicated interplay between surface tension, vorticity accumulation, shape instability and wake instability.

5. Miscellaneous

Topics worth mentioning in the context of turbulent two-phase flow are the generation of gravity waves on the surface of a liquid by a turbulent wind flow (Belcher & Hunt 1993) and the enormous effect of bubbles on sound emission by a turbulent flow (Crighton & Ffowcs Williams 1969).

Acknowledgement

I thank Arie Biesheuvel, René Oliemans, Wim Uijttewaal and Peter Spelt for their help, in preparing this paper.

References:

Auton, T.R., Hunt, J.C.R. & Prud'homme (1988) The force exerted on a body in inviscid unsteady non-uniform rotational flow. *J. Fluid Mech.* **197**, 241-257.

Belcher, S.E. & Hunt, J.C.R.(1993) Turbulence shear flow over slowly moving waves, *J. Fluid Mech.* **251**, 109-148.

Besnard, D.C. & Hanlow, F.H. (1988) Turbulence in multiphase flow. *Int. Jnl. Multiph. Flow.* **14**,6 679-699.

Biesheuvel, A. and Wijngaarden, L van (1984) Two-phase flow equations for a dilute dispersion of gas bubbles in liquid. *J. Fluid Mech.*,**148**, 301-318.

Crighton, D.G. and Ffowcs Williams (1969) Sound generation by turbulent two-phase flow. *J. Fluid Mech.*36, 585-603.

Crowe, C.T., Troutt, T.R. and Chung, J.N.(1996) Numerical models for two-phase turbulent flows. *Ann.Rev. Fluid Mech.* **28**, 11-43.

Elghobashi, S.E. and Abou-Arab J.W. (1983). A two equation for two-phase flows. *Phys. Fl.* A26, 931-938.

Lance, M. and Bataille, J. (1991) Turbulence in the liquid phase of a uniform bubbly air-water flow *J. Fluid Mech.* **222**, 95-118.

Launder, B.E. (1992) On the modelling of turbulent industrial flows. *Comp. Meth. Appl. Sc.* 91-102

Maxey, M.R. & Riley, J.J. (1983) Equation of motion for a small rigid sphere in a nonuniform flow. *Phys. Fluids* **26**, 883-889.

Maxey, M.R. (1987a) The gravitational setting of aerosol particles in homogeneous turbulence and random fields. *J. Fluid Mech.* 174, 441-465.

Maxey, M.R. (1987b) The motion of small sperical particles in a cellular flow field. *Phys. Fluids* **30**, 1915-1928.

Pismen, L.M. & Nir, A (1978) On the motion of suspended particles in statisionary homogeneous turbulence. *J. Fluid Mech.* **84**, 193-196.

Reeks, M.W. (1977) On the dispersion of small particles in an isotropic turbulent fluid.. *J. Fluid Mech.* **83**, 3, 529-546.

Snyder, W.H. & Lumley, J.L. (1971) Some measurements of particle velocity autocorreclation functions in a turbulent flow. *J.Fluid Mech.* **48**, 1, 41-71,

Spelt, P.D.M. & Biesheuvel, A. (1996) Numerical simulations of the motion of gas bubbles in homogeneous isotropic turbulence. Under review. with *J. Fluid Mech.*

Stewart, C.W. (1995) Bubble interaction in low-viscosity liquids. *Int. J. Multiphase Flow.***21**, 6, 1037-1046.

Theofanous, T.G. and Sullivan, J. (1982) Turbulence in two-phase dispersed flows, *J. Fluid Mech.* **116**, 343-362.

Tsjugi, Y, Morikawa, Y and Shiomi, H. (1984) LDV measurements of an air-solid two-phase flow in a vertical pipe. *J. Fluid Mech.* **139**, 417-434.

Uijttewaal, W. (1995) Particle motion in turbulent pipe flow. Techn. Univ. Delft. **MEAH-128.**

Wang, L-P and Maxey , M.R. (1993) Settling velocity and concentration distribution of heavy particles in homogeneous isotropic turbulence. *J. Fluid Mech.***256**, 27-68.