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TUTORIAL

THE PROGRESSIVE ATTENUATION OF HIGH-FREQUENCY ENERGY IN SEISMIC REFLECTION DATA*

A. ZIOLKOWSKI and J.T. FOKKEMA**

ABSTRACT

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Seismic reflection data always exhibit a progressive loss of high-frequency energy with time. This effect is partly attributable to irreversible processes such as the conversion of elastic energy into heat (commonly known as absorption), and partly to reversible processes associated with interference between reflected waves arriving at different times. This paper looks only at reversible linear elastic effects at normal incidence and asks the following question: if there were no such absorption, would there still be a progressive loss of high-frequency energy?

Using normal incidence and a layered elastic earth model we prove the following results.

1. The normal incidence response of a sequence of plane parallel elastic layers is non-white.
2. The pressure wave reflected by a layer that is thin compared with a wavelength is differentiated with respect to the incident wave.
3. The transmission response of a thin layer is consequently low-pass and the transmission response of a sequence containing many thin layers is very low-pass.
4. The well-known effect of the transport of acoustic energy by peg-leg multiples within thin layers is identical with this low-pass transmission response.
5. It follows that the high frequency energy is reflected back early in the seismogram.
6. By comparison, very low-frequencies are transmitted through the layered sequence easily and are reflected with difficulty. There is probably a lack of low-frequency energy in the reflection seismogram, by comparison with the spectrum of the incident plane wave.

It follows that any meaningful evaluation of frequency-dependent absorption in seismic data cannot take place unless the frequency-dependent linear elastic effects are taken into account first.

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Introduction

Seismic reflection data always exhibit a progressive attenuation of high-frequency energy with time. This is illustrated in fig. 1, which shows a stacked seismic section of total two-way traveltime. There is clearly more high-frequency energy present in the upper part of the section than in the lower part.

This progressive attenuation of high frequency energy with two-way traveltime is attributable to two causes: anelastic effects and elastic effects. The anelastic effects are the processes responsible for the irreversible conversion of elastic energy into heat and are usually known as 'absorption'. They are often characterized by a quality factor Q, which is usually regarded as being frequency-independent within the seismic bandwidth and defines the attenuation per wavelength: the higher Q, the less the attenuation per wavelength.

There are also reversible elastic effects, in which the energy is conserved, which can cause the same phenomenon. This is particularly evident when the layering is cyclic (O'Doherty and Anstey 1971; Schoenberger and Levin 1974). Interference occurs between reflected waves which arrive at different times, and some frequencies are reflected back at greater amplitude than others. The interference pattern depends on the reflection coefficients, the layer velocities and the layer thicknesses, and it is not immediately obvious that there will be a preferential transmission of low frequencies. This paper is concerned with these reversible elastic effects, and looks for some insight into these propagation phenomena.

Some aspects of the behaviour of elastic waves, such as the determination of the reflection and transmission coefficients at an interface between two media, are better studied using wave theory. Other aspects, such as the behaviour of multiples, can better be understood using ray theory. Both approaches are used in this paper.

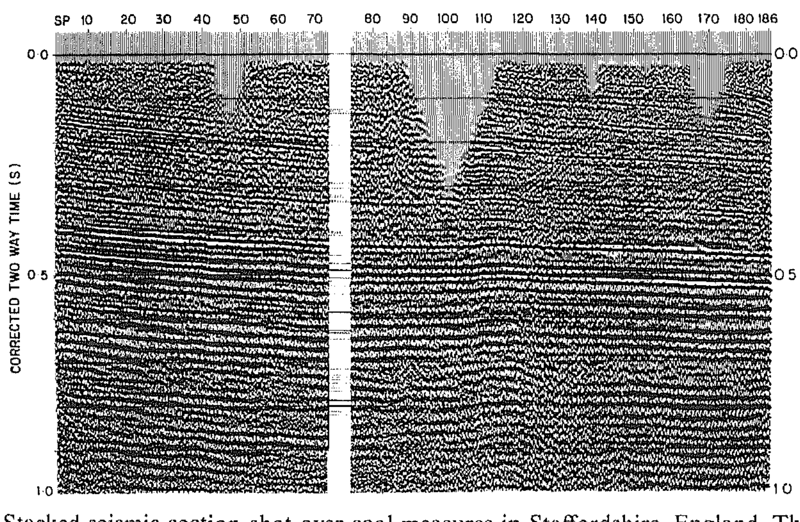

Fig. 1. Stacked seismic section shot over coal measures in Staffordshire, England. The lithological log at shot-point 74 has been converted from depth to two-way traveltime for comparison with the seismic data. (Reproduced from Ziolkowski 1979).

We confine the discussion to normal incidence and begin with a proof that the reflection response of a sequence of plane parallel layers is non-white. We believe that this result is obvious from the work of Treitel and Robinson (1966) but is evidently not well-known, since the assumption of whiteness is prevalent in much of seismic data processing, particularly in the deconvolution and wavelet extraction processes. We then begin the analysis of the wave propagation phenomena with a derivation of the reflection and transmission coefficients at a plane interface. We then derive the normal incidence reflection and transmission response of a single layer. If the layer is thin compared with a wavelength, it is seen that the reflection response is that of a linear high pass filter; the transmission response is correspondingly low-pass. The problem is analyzed with both wave theory and ray theory to show the relation between the filtering effect and the peg-leg multiples within the layer.

A consideration of a sequence containing many thin layers shows that the high frequencies must be filtered out in transmission, both on the way down and on the way back up. The low frequencies are correspondingly transmitted easily but are reflected with small amplitudes. The seismic reflection data *must* exhibit a progressive attenuation of high-frequency energy with time, whether irreversible absorption effects are present or not.

The Non-Whiteness of the Earth Reflection Response

The attenuation of high frequency energy with time is a phenomenon that is well-known in reflection seismology. Very often this observed attenuation of high-frequency energy is attributed entirely to *absorption*. We shall show that this effect *must occur* whether there is absorption or not. There is no need to invoke irreversible effects such as absorption to explain this effect in the data; straightforward reversible linear effects can account for much of the low pass filtering of seismic data, as noted by O'Doherty and Anstey (1971), by Schoenberger and Levin (1974) and by Rüter and Schepers (1978).

It is often assumed in seismic data processing that, in the absence of absorption, the reflection response is white. This assumption is particularly important in deconvolution and in various wavelet extraction methods. We do not understand the rationale for this assumption and wish to show, at the outset, that it is certainly wrong for plane waves normally incident to a sequence of plane layers.

Treitel and Robinson (1966) used ray theory and z-transforms to find the reflection and transmission response of a plane impulsive pressure wave normally incident on a sequence of linearly elastic plane layers. They showed that when the stack of layers has a perfect reflector at its base, the reflection sequence has a dispersive all-pass response; that is, all the energy which goes in, is eventually returned to the surface but with phase delays which are frequency-dependent. Treitel and Robinson (1966) called this the 'all-pass theorem'. An all-pass response has a white spectrum.

The physical reason for the white response is that the layers are elastic and can neither absorb nor create energy. Thus the downward energy flux equals the upward energy flux in each layer. In the case of a rigid reflector at the bottom, the lowest layer must return all downgoing energy to the layer above; its reflection response is therefore white. The upcoming energy transmitted into the layer above exactly equals the downgoing energy transmitted down through to the bottom layer. Therefore the reflection response of the bottom two layers is white. The argument follows identically through every layer in the sequence. Therefore the reflection response of the sequence is white.

This all-pass theorem requires a perfect reflector at the bottom of the sequence. If there is no perfect reflector, the response is not all-pass. This is a corollary of the all-pass theorem. It follows that in real geology where a reflection coefficient of 0.4 is enormous, there is *never* a perfect reflector at the bottom of the sequence, and the normal incidence response is therefore *never* all-pass, and therefore *never* white.

Reflection and Transmission of a Plane Wave at Normal Incidence

A plane pressure wave in the time domain and the frequency domain

A small amplitude plane pressure wave $p(z, t)$ traveling parallel to the z-axis in a homogeneous isotropic elastic medium propagates according to the linear homogeneous one-dimensional wave equation (e.g. Berkhout 1982)

$$\partial_z^2 p(z, t) - \frac{1}{v_p^2} \partial_t^2 p(z, t) = 0, \tag{1}$$

in which ∂_z^2 denotes the second partial derivative with respect to distance z, ∂_t^2 denotes the second partial derivative with respect to time t, and the velocity of propagation v_p is defined as

$$v_p^2 = (\lambda + 2\mu)/\rho, \tag{2}$$

where λ and μ are the Lamé coefficients and ρ is the density. Solutions of (1) are of the form

$$p(z, t) = p^+(t - z/v_p) + p^-(t + z/v_p), \tag{3}$$

in which we denote the waves traveling in the positive z-direction by p^+ and the waves traveling in the negative z-direction by p^-.

It is convenient to look at the propagation phenomena in the frequency domain, using the Fourier transform. Let $p(z, t)$ and $P(z, \omega)$ be a Fourier transform pair:

$$p(z, t) \leftrightarrow P(z, \omega),$$

where ω is the angular frequency and the Fourier transform relations are defined as

$$P(z, \omega) = \int_{-\infty}^{\infty} p(z, t) e^{+j\omega t} \, dt, \tag{4}$$

$$p(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(z, \omega) e^{-j\omega t} \, d\omega, \tag{5}$$

in which we choose the sign convention to be consistent with the convention used for our propagating waves. (We appreciate that this is not the sign convention generally used in geophysics. We find it preferable because, in the frequency domain, waves traveling in the positive z-direction are associated with positive sign in the exponent, while waves traveling in the negative z-direction are associated with the negative sign in the exponent.)

Differentiating (5) with respect to time twice, we have

$$\partial_t p(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} -j\omega P(z, \omega) e^{-j\omega t} \, d\omega, \tag{6}$$

$$\partial_t^2 p(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} -\omega^2 P(z, \omega) e^{-j\omega t} \, d\omega, \tag{7}$$

from which we recognize the Fourier transform pairs

$$\partial_t \leftrightarrow -j\omega \tag{8}$$

and

$$\partial_t^2 \leftrightarrow -\omega^2. \tag{9}$$

Transforming the (1) into the frequency domain, and using (9), we have

$$\partial_z^2 P(z, \omega) + \frac{\omega^2}{v_p^2} P(z, \omega) = 0, \tag{10}$$

which is the frequency-domain expression for the linear one-dimensional homogeneous wave equation.

Relationship between pressure and particle displacement in a plane wave

The propagating wave has a pressure gradient in the z-direction $\partial_z p(z, t)$, which is related to the particle displacement $u_z(z, t)$ by Newton's second law of motion:

$$\partial_z p(z, t) = -\rho \partial_t^2 u_z(z, t), \tag{11}$$

where ρ is the density. Using (9) we may transform (11) into the frequency domain as

$$\partial_z P(z, \omega) = \rho \omega^2 U_z(z, \omega), \tag{12}$$

in which $U_z(z, \omega)$ is the Fourier transform of the particle displacement.

In seismology, we usually measure particle velocity, not displacement. The particle velocity can be obtained from the displacement by differentiation with respect to time. In the frequency domain for a plane wave, this time differentiation corresponds to a simple multiplication by $-j\omega$ as we have shown in (8).

Reflection and transmission for the pressure and displacement

The 'reflection coefficient' and the 'transmission coefficient' at the boundary between two media are not completely meaningful terms unless the disturbance is specified. Pressure and displacement are clearly not the same thing, so it matters what kind of disturbance we are talking about. In this section we derive the normal incidence reflection and transmission coefficients for both the plane pressure wave and the corresponding displacement.

Consider a plane sinusoidal pressure wave of *unit* amplitude in medium I normally incident on a plane interface separating medium I from another medium II, as shown in fig. 2, where the coordinates are chosen such that the interface is the plane $z = 0$ and z is positive into medium II. Let the densities of the two media be ρ_1 and ρ_2, and their compressional wave velocities be v_1 and v_2, respectively.

The incident pressure wave is, in the frequency domain, $\exp\{j\omega(z/v_1 - t)\}$ and propagates in the positive z-direction. Note that the propagation in the positive z-direction is associated with a positive sign in the exponent. This arises from our sign convention for the Fourier transform, (4) and (5).

There is a reflected pressure wave in medium I,

$$R_p \exp\{j\omega(-z/v_1 - t)\},$$

traveling in the negative z-direction with amplitude R_p, and there is a transmitted pressure wave in medium II,

$$T_p \exp\{j\omega(z/v_2 - t)\},$$

which has amplitude $T_p \cdot R_p$ and T_p are the reflection and transmission coefficients of the pressure wave at the interface.

In medium I the pressure field is

$$P^{\mathrm{I}}(z, \omega) = \exp\{j\omega(z/v_1 - t)\} + R_p \exp\{j\omega(-z/v_1 - t)\} \tag{13}$$

and in medium II, it is

$$P^{\mathrm{II}}(z, \omega) = T_p \exp\{j\omega(z/v_2 - t)\}. \tag{14}$$

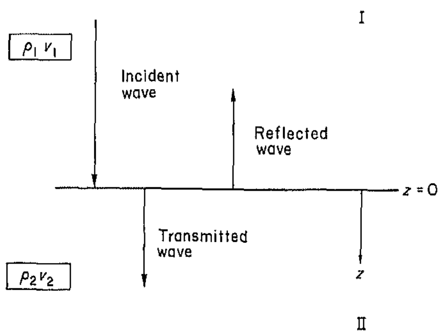

Fig. 2. Reflection and transmission of a plane pressure wave at normal incidence to a plane interface.

Using (12) we can find the displacements in the two media:

$$U_z^{\mathrm{I}}(z,\omega)=\frac{j}{\rho_1\omega v_1}\left[\exp\{j\omega(z/v_1-t)\}-R_p\exp\{j\omega(-z/v_1-t)\}\right], \tag{15}$$

$$U_z^{\mathrm{II}}(z,\omega)=\frac{j}{\rho_2\omega v_2}T_p\exp\{j\omega(z/v_2-t)\}. \tag{16}$$

The pressure and displacement are both continuous at the boundary $z=0$, therefore,

$$P^{\mathrm{I}}(0,\omega)=P^{\mathrm{II}}(0,\omega) \tag{17}$$

and

$$U_z^{\mathrm{I}}(0,\omega)=U_z^{\mathrm{II}}(0,\omega). \tag{18}$$

Imposing (17) on (13) and (14), and (18) on (15) and (16) yields

$$1+R_p=T_p, \tag{19}$$

$$\frac{1-R_p}{\rho_1v_1}=\frac{T_p}{\rho_2v_2}. \tag{20}$$

These conditions hold for all values of t and ω_1 and therefore the common factor $\exp(-j\omega t)$ has been cancelled. From (19) and (20) we find the reflection and transmission coefficients of the pressure wave:

$$R_p=\frac{\rho_2v_2-\rho_1v_1}{\rho_2v_2+\rho_1v_1}, \tag{21}$$

$$T_p=\frac{2\rho_2v_2}{\rho_2v_2+\rho_1v_1}. \tag{22}$$

Equation (15) shows that the amplitude of the incident displacement is

$$\frac{j}{\rho_1\omega v_1},$$

while the amplitude of the reflected displacement is

$$\frac{-R_pj}{\rho_1\omega v_1}.$$

It follows that the reflection coefficient for the displacement is

$$R_u=\left(\frac{-R_pj}{\rho_1\omega v_1}\right)\Big/\left(\frac{+j}{\rho_1\omega v_1}\right)=-R_p=\frac{\rho_1v_1-\rho_2v_2}{\rho_1v_1+\rho_2v_2}, \tag{23}$$

From (15) and (16) we find the transmission coefficient for the displacement,

$$T_u=\frac{\rho_1v_1}{\rho_2v_2}T_p=\frac{2\rho_1v_1}{\rho_2v_2+\rho_1v_1}. \tag{24}$$

Using the extreme case of coal–rock interface, where typical velocities are (van Riel 1965)

v_p coal = 2500 m/s, v_p rock = 3500 m/s,

ρ coal = 1.5 Mg/m^3, ρ rock = 2.5 Mg/m^3,

we find the reflection coefficient of the pressure wave for a rock–coal interface

$$R_p = -0.4,$$

and the transmission coefficient

$$T_p = 0.6.$$

Normal Incidence Reflection and Transmission Response of a Plane Layer

Wave Theory

We now consider the normal incidence reflection and transmission responses of a layer of thickness d, density ρ_2, and compressional wave velocity v_2, sandwiched between two identical half-spaces with density ρ_1 and compressional wave velocity v_1, as shown in fig. 3. The top of the layer is taken to be the plane $z = 0$, and the z-axis is positive downwards into the layer as shown.

Proceeding as above, we have a pressure field in medium I,

$$P^{\mathrm{I}}(z, \omega) = \exp\{j\omega(z/v_1 - t)\} + R \exp\{j\omega(-z/v_1 - t)\}, \tag{25}$$

and a displacement field,

$$U_z^{\mathrm{I}}(z, \omega) = \frac{j}{\rho_1 v_1 \omega} \cdot [\exp\{j\omega(z/v_1 - t)\} - R \exp\{j\omega(-z/v_1 - t)\}], \tag{26}$$

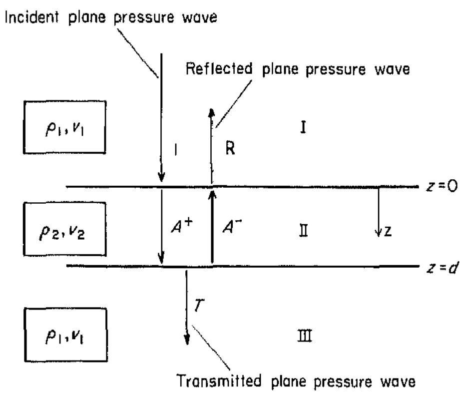

Fig. 3. Reflection and transmission of a plane pressure wave at normal incidence to a layer of thickness d.

in which R is the normal incidence reflection response of the system for a unit amplitude sinusoidal pressure wave incident from medium I. Equation (12) has been used to relate the particle displacement to the pressure.

In medium II, the layer, we have both downgoing and upgoing waves; that is, waves traveling in the positive and negative z-directions. The pressure field in medium II is

$$P^{\text{II}}(z, \omega) = A^{+}\exp\{j\omega(z/v_2 - t)\} + A^{-}\exp\{j\omega(-z/v_2 - t)\}, \tag{27}$$

where A^+ and A^- are complex and $|A^+|$ and $|A^-|$ are the amplitudes of the downgoing and upgoing pressure waves, respectively. Similarly the displacement field in medium II is

$$U_z^{\text{II}}(z, \omega) = \frac{j}{\rho_2 v_2 \omega}[+A^{+}\exp\{j\omega(z/v_2 - t)\} - A^{-}\exp\{j\omega(-z/v_2 - t)\}]. \tag{28}$$

In medium III there are only transmitted waves,

$$P^{\text{III}}(z, \omega) = T\exp\{j\omega(z/v_1 - t)\}, \tag{29}$$

$$U_z^{\text{III}}(z, \omega) = \frac{jT}{\rho_1 v_1 \omega}\exp\{j\omega(z/v_1 - t)\}, \tag{30}$$

where T is the transmission response of the system.

In (25)–(30) there are four unknowns R, T, A^+ and A^-. These are related by the conditions of continuity of pressure and displacement at the two interfaces $z = 0$ and $z = d$.

At $z = 0$, continuity of pressure yields

$$1 + R = A^{+} + A^{-}, \tag{31}$$

while continuity of displacement yields

$$\frac{1}{\rho_1 v_1}(1 - R) = \frac{1}{\rho_2 v_2}(A^{+} - A^{-}). \tag{32}$$

At the boundary $z = d$, the two corresponding continuity conditions yield

$$A^{+}\exp(j\omega d/v_2) + A^{-}\exp(-j\omega d/v_2) = T\exp(j\omega d/v_1), \tag{33}$$

$$\frac{1}{\rho_2 v_2}[A^{+}\exp(j\omega d/v_2) - A^{-}\exp(-j\omega d/v_2)] = \frac{T}{\rho_1 v_1}\exp(j\omega d/v_1). \tag{34}$$

This system of four linear simultaneous equations can be solved to find R and T, the reflection and transmission responses. After a little algebra, we find

$$R = \frac{(1 - \exp(2j\omega d/v_2))R_p}{1 - R_p^2\exp(2j\omega d/v_2)}, \tag{35}$$

$$T = \frac{(1 - R_p^2)}{1 - R_p^2\exp(2j\omega d/v_2)}\exp\{j\omega d(1/v_2 - 1/v_1)\}. \tag{36}$$

It is possible also to find A^+ and A^-, the (complex) amplitudes of the downgoing and upgoing pressure waves in the layer. As far as we are concerned, however, the principal interest is in the reflection and transmission responses R and T. Before making any further analysis of these responses it is worthwhile solving the same problem with ray theory.

If we had wanted simply to derive (35) and (36) one derivation would have been sufficient. It could even have been made mathematically more elegant by the use of matrices. Our motivation in giving two derivations, using simple steps, is to arrive at a better understanding of the problem, and to show clearly the relationship between the wave-theory solution and the ray-theory solution.

In the wave-theory approach, we write down equations for the pressure and displacement in each layer, including both up- and downgoing waves; we then connect the layers by the continuity of pressure and displacement at the boundaries. If this is done correctly the reflection response contains all primaries and all multiples. But to see the relationship between the primaries and the multiples, and to evaluate the importance of the different contributions, it is better to use ray theory.

Ray theory

The situation is illustrated in fig. 4. We consider an impulsive plane pressure wave of unit amplitude normally incident from medium I on the interface with medium II at time $t = 0$. This impulse is partially reflected at the interface with an amplitude from (21),

$$R_{12} = \frac{\rho_2 v_2 - \rho_1 v_1}{\rho_2 v_2 + \rho_1 v_1}.$$

This reflected impulse travels back into medium I, leaving the interface with medium II at time $t = 0$. The transmitted impulsive pressure wave proceeds into medium II with amplitude from (22),

$$T_{12} = \frac{2\rho_2 v_2}{\rho_2 v_2 + \rho_1 v_1}.$$

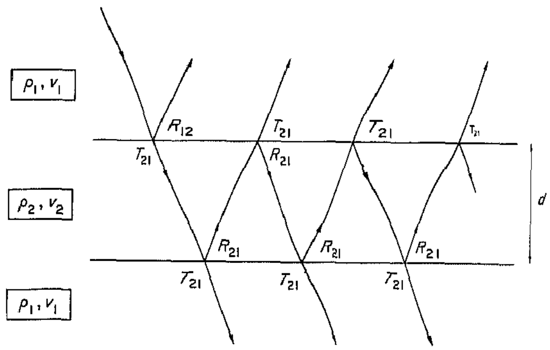

Fig. 4. Reflection and transmission ray paths of an impulsive plane wave at normal incidence to a layer of thickness d.

At the lower boundary this impulse is partially transmitted into medium III with a transmission coefficient,

$$T_{21} = \frac{\rho_1 v_1}{\rho_2 v_2} T_{12}, \tag{37}$$

and partially reflected back into the layer with a reflection coefficient,

$$R_{21} = -R_{12}. \tag{38}$$

This internally reflected impulse gives rise to two infinite series of impulses which are transmitted through the top and bottom of the layer, as shown. The reflection impulse response of the layer is the infinite time series

R_{12}	at	$t = 0$,
$T_{12} \cdot R_{21} \cdot T_{21}$	at	$t = 2\Delta t$,
$T_{12} \cdot R_{21} \cdot R_{21} \cdot R_{21} \cdot T_{21}$	at	$t = 4\Delta t$,
etc.,		etc.,

where

$$\Delta t = d/v_2 \tag{39}$$

is the one-way traveltime through the layer. The transmission impulse response of the layer is the infinite time series

$T_{12} \cdot T_{21}$	at	$t = \Delta t$,
$T_{12} \cdot R_{21} \cdot R_{21} \cdot T_{21}$	at	$t = 3\Delta t$,
$T_{12} \cdot R_{21} \cdot R_{21} \cdot R_{21} \cdot R_{21} \cdot T_{21}$	at	$t = 5\Delta t$,
etc.,		etc.

Using Z-transform notation (one always runs into trouble using z as the depth coordinate as well as the unit delay operator. We aim to avoid any possible confusion between the two different z's) where Z represents a time delay of Δt, Z^2 represents a time delay of $2\Delta t$, and so on (e.g., Robinson 1967), we may write the reflection impulse response as

$$R(Z) = R_{12}\{1 - T_{12} \cdot T_{21} \cdot Z^2(1 + R_{12}^2 Z^2 + R_{12}^4 Z^4 + \cdots)\} \tag{40}$$

and the transmission impulse response as

$$T(Z) = T_{12} \cdot T_{21} Z(1 + R_{12}^2 Z^2 + R_{12}^4 Z^4 + \cdots) \tag{41}$$

To relate $R(Z)$ and $T(Z)$ in these equations to our corresponding frequency domain responses R and T derived in (35) and (36) using wave theory, we note that a time delay Δt corresponds to a phase shift of $\exp(j\omega\Delta t)$ for a wave of angular frequency ω, and a time delay of $2\Delta t$ corresponds to a phase shift of

$$\exp(2j\omega\Delta t) = [\exp(j\omega\Delta t)]^2. \tag{42}$$

That is, if we take Z to be on the unit circle

$$Z = \exp(j\omega\Delta t), \tag{43}$$

the Z-transforms $R(z)$ and $T(z)$ can be related to the frequency domain. In (40) and (41), the infinite series

$$S(Z) = 1 + R_{12}^2 Z^2 + R_{12}^4 Z^4 + \cdots \tag{44}$$

converges because $|Z| = 1$, on the unit circle, and R_{12}^2 is less than 1. Therefore, summing the geometric progression, we find

$$S(Z) = \frac{1}{1 - R_{12}^2 Z^2} \tag{45}$$

and

$$R(Z) = R_{12}\left[1 - \frac{T_{12} T_{21} Z^2}{1 - R_{12}^2 Z^2}\right], \tag{46}$$

$$T(z) = \frac{T_{12} T_{21} Z}{1 - R_{12}^2 Z^2}. \tag{47}$$

Noting that $R_{12}^2 + T_{21} \cdot T_{12} = 1$, we see that (46) and (47) can be written as

$$R(z) = \frac{(1 - Z^2)R_p}{1 - R_p Z^2}, \tag{48}$$

and

$$T(z) = \frac{(1 - R_p^2)Z}{1 - R_p^2 Z^2}. \tag{49}$$

Comparing (48) and (35) we see that R is identical with $R(z)$ when z is defined by (43) to be on the unit circle. Note that there is a small difference between (36) and (49). In (49), there is a factor Z, whereas in (36) there is a corresponding factor $\exp\{j\omega d(1/v_2 - 1/v_1)\}$. The difference is the factor $\exp(-jd/v_1)$, which is simply a phase delay caused by a difference in the reference plane.

Thin-layer approximation

We have seen that wave theory and ray theory give exactly the same reflection response for a plane wave normally incident on an elastic layer embedded in some other elastic material. For the purpose of investigating the frequency dependence it is interesting to study the response for a thin layer. The term 'thin' is defined below.

The complex exponential can be written as

$$\exp(2j\omega d/v_2) = \cos(2\omega d/v_2) + j\sin(2\omega d/v_2). \tag{50}$$

For $2\omega d/v_2 \ll \pi$, we may approximate (50) as

$$\exp(2j\omega d/v_2) \approx 1 + j2\omega d/v_2. \tag{51}$$

Substituting this approximation into (35) we see that the layer reflection response is

$$R \approx \frac{-j\omega R_p 2d/v_2}{1 - R_p^2 - jR_p^2 2\omega d/v_2}. \tag{52}$$

The imaginary part of the denominator of the right-hand side for $R_p < 0.4$ is negligible, thus (52) may be approximated as

$$R(\omega) \approx \frac{-j\omega R_p \cdot 2d}{v_2(1 - R_p^2)}, \tag{53}$$

which is the *thin layer reflection response* to a normally incident plane pressure wave. The amplitude of the full reflection response (35) is shown in fig. 5. The thin layer approximation corresponds to the straight lines sloping downwards to the left.

We see that the layer can be regarded as thin when

$$2\omega d/v_2 \ll \pi,$$

$$\text{or} \quad 4fd \ll v_2, \tag{54}$$

$$\text{or} \quad d \ll \frac{L}{4},$$

where $f = \omega/2\pi$ is the frequency and L is the wavelength. Coal seams up to 1 m thick for example, with velocity 2500 m/s can be regarded as 'thin' for seismic frequencies up to 100 Hz.

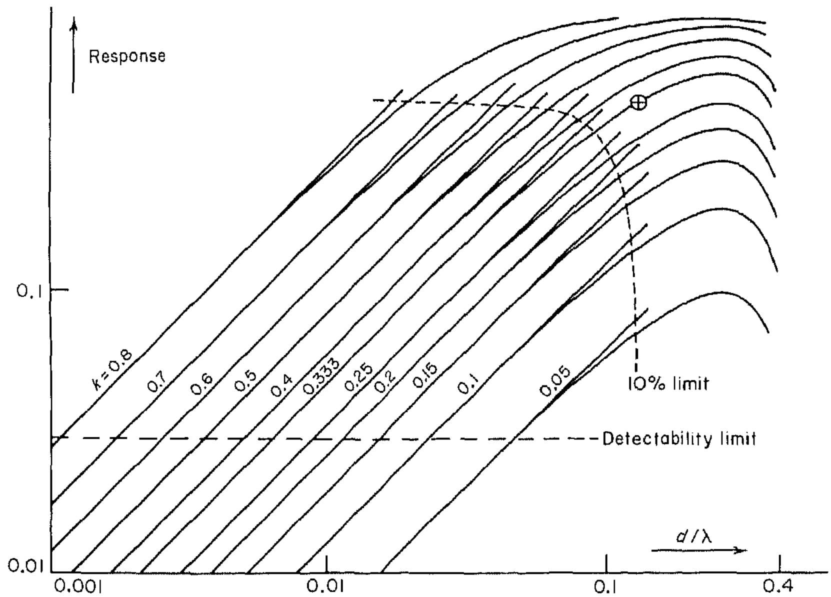

Fig. 5. Reflection response for a sinusoidal wave of an interbedded layer as a function of layer thickness and reflection coefficient. (Reproduced from Koefoed and de Voogd 1980).

In (53) the factor

$$\left|\frac{2R_p}{v_2(1 - R_p^2)}\right|$$

depends only on the acoustic properties of the two media. It increases with increasing acoustic impedance contrast between the media. As long as inequality (54) is satisfied, the amplitude of the response $R(\omega)$ increases linearly with the thickness of the layer d and with the frequency ω. The frequency in this thin layer response enters only as the factor $-j\omega$, which is the frequency domain correspondent of differentiation in the time domain. Thus the incident seismic pressure wave appears to be differentiated on reflection from the thin layer. This was noticed by Widess (1973).

To summarize the results of the thin layer analysis, we note:

1. the amplitude of the thin layer reflection response increases with the acoustic contrast;
2. the amplitude of the thin layer reflection response increases linearly with the layer thickness, therefore any layer of finite thickness gives a response;
3. the amplitude of the thin layer reflection response increases linearly with frequency; the reflected wave is differentiated with respect to the incident wave, and the thin layer reflection response is thus a high-pass filter.

A plot of the amplitude of the response R as a function of the ratio d/L_c, where $L_c = v_2/f$ is the wavelength in the layer has been calculated by Koefoed and de Voogd (1980), and is shown in fig. 5 for various values of the reflection coefficient $k = |R_p|$. The linear behaviour for small d/L_c is obvious.

Normal Incidence Reflection and Transmission Response of a Sequence of Thin Layers

The low-pass transmission effect

The reflection response of a thin layer is identical with that of a linear high-pass filter that acts so as to differentiate the incident pressure wave $p(z, t)$. Since the thin-layer reflection response is high-pass, it follows that the transmission response is low-pass. The response is given by (36). The factor

$$\exp\{j\omega d(1/v_2 - 1/v_1)\}$$

has modulus 1 at all frequencies and therefore contains only phase information. The amplitude spectrum is

$$|T(\omega)| = \frac{|1 - R_p^2|}{|1 - R_p^2 \exp(2j\omega d/v_2)|}, \tag{55}$$

which can also be found from (49) using ray theory. The response $|T(\omega)|$ is plotted in fig. 6. It has a minimum at an angular frequency $\omega = \pi v_2/2d$; that is, when d is equal to a quarter wavelength. This is what we should expect: the reflection

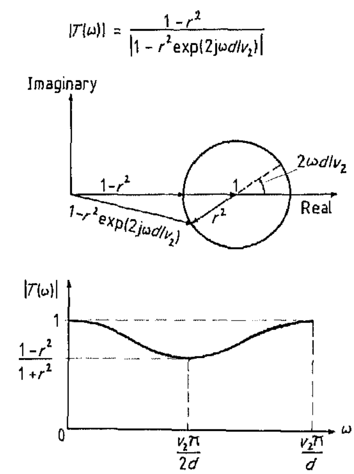

Fig. 6. The layer transmission response. For layers less than a quarter wavelength thick, the response is low-pass.

response is a maximum at the same frequency (Widess 1973). This frequency is sometimes known as the 'tuning' frequency. Below this frequency the transmission response is low-pass. The low-pass filtering effect of a single thin coal seam was noted by Rüter and Schepers (1978).

Consider now a sequence of n such thin seams of thickness d, separated by much thicker layers of varying thicknesses, as shown in fig. 7, and consider the transmitted pressure wave at the bottom of the stack of layers. We could solve the problem

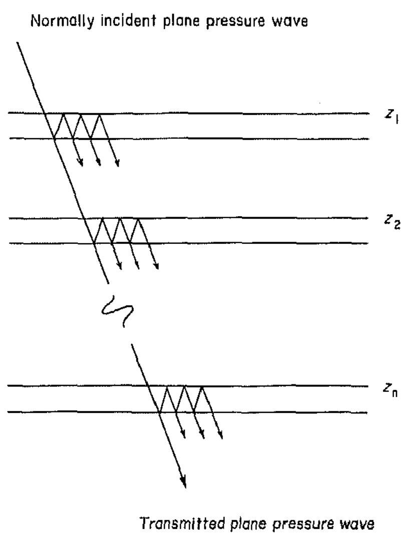

Fig. 7. Plane pressure wave normally incident on a sequence containing n identical thin layers.

using ray theory or wave theory, including all the up and downgoing waves numerically. The ray theoretical approach was used by Rüter and Schepers (1978) and the wave theoretical approach by Hughes and Kennett (1983). Without a computer some insight into the problem can be achieved using a simplifying assumption.

If we assume that the plane wave $p(z, t)$ at the top of our layered sequence is a short pulse in the time domain, we may neglect multiples which occur between seams because these arrive too late to be of interest; but we cannot neglect the very short period internal multiples within the seams. We may therefore say that the transmission response of the stack of layers to an impulsive plane wave $p(z, t)$ at the top of the stack, is the cascaded response of all n thin layers, plus a time delay to account for the traveltime in the thick layers. In the frequency domain this approximate response is computed by multiplying the identical responses:

$$|T_n(\omega)| = |T(\omega)|^n = \frac{|1 - R_p^2|^n}{|1 - R_p^2 \exp(2j\omega d/v_2)|^n}, \tag{56}$$

that is, each thin layer acts like a low-pass filter and the high frequencies in the transmitted wave become progressively attenuated by the sequence of layers: in effect the transmitted wave is passed through the same low-pass filter n times. The response (56) is plotted in fig. 8.

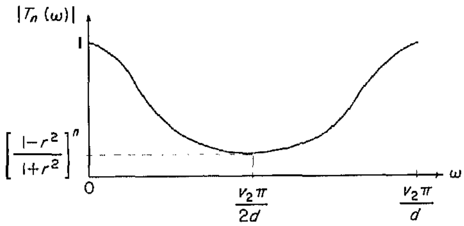

Fig. 8. The n-layer transmission response. When the layers are thin the response is very low pass.

The seismic waves which travel to the bottom of the stack of layers must be low-pass filtered by every thin layer in the sequence. The reflected wave returning from the deepest layer must be filtered again by the same sequence of layers. The low-pass filtering effect for waves reflected back to the surface is therefore squared.

From this simple argument we can see that the reflection response from any sequence containing thin layers is bound to appear as progressively lower frequency with traveltime. Figure 1 is a seismic section over coal measures. It demonstrates this progressive lack of high frequencies. This seismic line was shot over a borehole which had been cored and logged; the lithological log has been displayed at shot point 74 on the same time scale as the seismic section (using the velocity log to convert the depths into two-way traveltime) and shows clearly that there are numerous thin layers, many of them coal seams, which are thin compared with the seismic wavelengths.

The effect of peg-leg multiples on amplitudes

In the ray-theory approach another interesting phenomenon becomes apparent: the way in which the transmitted energy is built up with time by peg-leg multiples. Consider fig. 4 and the argument above. We showed there that the transmission impulse response can be considered as a sequence $T(Z)$ given by (41),

$$T(Z) = T_{12}T_{21}Z(1 + R_{12}^2 Z^2 + R_{12}^4 Z^4 + \cdots),$$

in which the first term is the direct transmitted impulse and the subsequent terms are peg-leg multiple reflections generated within the layer. The contribution of these peg-legs was summed to give the total response of (47),

$$T(Z) = \frac{T_{12}T_{21}Z}{1 - R_{12}^2 Z^2},$$

in which the numerator is the response without peg-legs and the denominator represents the effect of the peg-legs. As we have already seen, the denominator is also responsible for the low-pass filter effect on transmission.

For n such responses, the z-transform is

$$T_n(z) = T^n(Z) = \frac{(T_{12}T_{21})^n Z^n}{(1 - R_{12}^2 Z^2)^n}. \tag{57}$$

We denote this peg-leg response contained in the denominator by

$$\mathrm{PL}_n(z) = \frac{1}{(1 - R_{12}^2 Z^2)^n}; \tag{58}$$

it may be written to first order as

$$\begin{aligned}\mathrm{PL}_n &= (1 - R_{12}^2 Z^2)^{-n}\\ &\approx 1 + nR_{12}^2 Z^2.\end{aligned} \tag{59}$$

Thus, the effect of the peg-legs is to add energy to the transmitted pulse, delayed by the two-way traveltime in the layer. If there are enough such layers the continued effect can be significant, and the amplitude of the peg-leg contributions can be larger than that of the direct pulse. We note that these peg-leg contributions cannot be separated from the low pass transmission response. The importance of such peg-leg multiples was first analyzed in detail by O'Doherty and Anstey (1971) who remarked (p. 444), "the multiply-reflected signal in a series of thin plates bounded by interfaces of opposite polarity is always of the same sign as the direct transmitted signal, and tends to overtake it in amplitude".

Hughes and Kennett (1983) compared synthetic seismograms over coal measures with and without peg-leg multiples, using a full wave-theoretical approach. Their results, shown in figs 9 and 10 clearly show that it is the short-period multiples within the coal seams themselves which are the main mechanism for the transport of energy to the deeper layers.

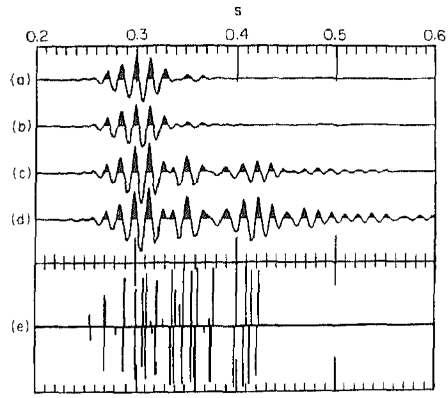

Fig. 9. Comparison of the synthetic seismogram for the acoustic approximation calculated with specific levels of multiples: (a) only primary ray paths for all the layers have been included in this calculation; (b) the primary ray paths in the coal seams have been included along with all possible ray paths (both primary and multiple) for the country rock; (c) in addition to the ray paths considered in (b), the coal seams were also allowed to have first order multiples; (d) the complete synthetic seismogram, which contained all possible ray paths in the coal and the country rock layers; (e) two-way traveltimes and normal incidence reflection coefficients for each interface are illustrated. The scale for the reflection coefficients is =0.5 with the positive axis upwards. (Reproduced from Hughes and Kennett 1983.)

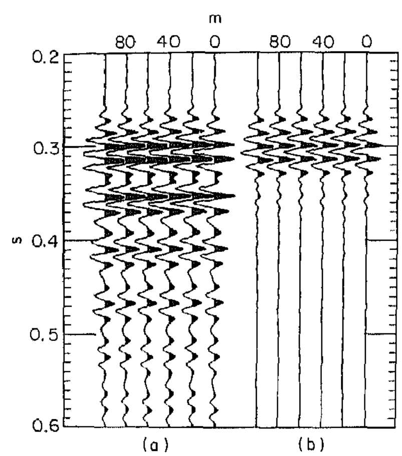

Fig. 10. The effect of changing the origin of the peg-leg multiples: (a) all possible paths through the coal seams and only primary paths through the country rock layers were included in this seismogram; (b) only primary paths in the coal seams and all possible paths in the country rock were considered in this calculation. (Reproduced from Hughes and Kennett 1983.)

Both O'Doherty and Anstey (1971) and Schoenberger and Levin (1974) have remarked on the low-pass filtering effect of *cyclically layered sequences* on the transmitted seismic wave. What we have shown here is that it is not so much *cyclic layering* that is responsible for this effect, but simply the presence of *thin layers*. The distribution of these thin layers through the layered sequence is not important, although they are much more likely to occur in zones where the deposition has been cyclic.

Any single thin layer embedded in a much thicker layer of contrasting acoustic impedance has a low-pass transmission response. If the acoustic impedance contrast is large, the low-pass response is more pronounced than if the contrast is small. A sequence of thin layers, whether in a zone of cyclic deposition or not, has a correspondingly increased low-pass transmission response.

It should be noted that *at low enough frequency any layer can be regarded as thin.* Since the thin-layer reflection response is high pass while the transmission response is low pass, the *very low frequencies* propagate easily through the layered earth, and therefore the normal incidence reflection response does not contain very low frequencies. It is probable, therefore, that the power spectrum of the normal incidence plane wave response lacks very low frequencies (rather than high frequencies) in comparison to the power spectrum of the incident plane wave. To our knowledge, this has never been checked. It is clearly something that needs to be examined.

Discussion and Conclusions

We are aware that our analysis is very simple and is confined to the case of normal incidence which has been studied many times before and often using a more elegant approach than we use here. Indeed, there is nothing new in our theory. In the application of this well-known theory we have discovered things that ought to be obvious (and which, no doubt, *are* obvious to some people), but which do not seem to be widely understood by practising exploration geophysicists. We have felt it worthwhile, therefore, to put our analysis and conclusions in writing.

We have shown by a simple corollary of Treitel and Robinson's (1966) all-pass theorem that the normal incidence reflection response of a sequence of plane parallel elastic layers is not white. It follows that any data processing step, such as spiking deconvolution, that relies on the assumption that the earth reflection response is white, will fail to recover the earth response correctly.

We have also analyzed the normal incidence response of a layer bounded by two identical half-spaces, using both wave theory and ray theory. When the layer is thin compared with a wavelength the reflection response is high pass and the layer acts as a differentiator; the transmission response is correspondingly low pass; these effects increase with the acoustic contrast. Thus, the wave reflected from a thin layer contains a higher proportion of high-frequency energy than the incident wave, while the transmitted wave contains a higher proportion of low-frequency energy.

In a geological sequence containing many thin layers, whether cyclic or not, the low-pass transmission responses of the thin layers are cascaded. Each thin layer preferentially reflects the higher frequencies and the spectrum of the transmitted wave is shifted progressively towards the lower frequencies as it passes through successive thin layers. This progressive shift to low frequencies is caused entirely by the peg-leg multiples that are generated within the thin layers and appear as a tail on the transmitted pulse. Waves reflected from the base of the sequence have to pass through the sequence twice and therefore contain relatively less high-frequency energy than waves reflected from the top of the sequence. Since deeper reflections occur later, it follows that we must see reflection seismograms with high frequency energy concentrated towards the beginning.

At least part of the observed progressive attenuation of high-frequency energy with time in seismic reflection data is caused by this process, in which only reversible elastic effects are at work. That is, this effect occurs whether there are irreversible processes or not. No doubt there are also irreversible processes at work in which energy is absorbed by the rock layers. We have not considered such irreversible processes. The relative importance of thin layers and absorption in the progressive attenuation of high-frequency energy in seismic data cannot be assessed without taking the elastic effects of thin layers into account first—for instance, by some sort of forward modeling procedure.

Finally, since any layer becomes a thin layer if the wavelength is long enough, we conclude that the very lowest frequencies will be reflected with very low amplitudes, due to the high-pass reflection response of thin layers. It follows that reflection seismograms will be deficient in the very lowest frequencies. We do not know whether this is a problem in practice. There is a problem in constructing the low-frequency part of the inverted acoustic impedance log from seismic data, but whether this is caused by a deficiency in low-frequency energy in the source signal or by the earth-filtering effect we have been discussing, is not known. This is something which should be studied.

Acknowledgments

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