TWILIGHT FOR THE ENERGY CONDITIONS?*

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The tension, if not outright inconsistency, between quantum physics and general relativity is one of the great problems facing physics at the turn of the millennium. Most often, the problems arising in merging Einstein gravity and quantum physics are viewed as Planck scale issues (10^{19} GeV, 10^{-34} m, 10^{-45} s), and so safely beyond the reach of experiment. However, over the last few years it has become increasingly obvious that the difficulties are more widespread: There are already serious problems of deep and fundamental principle at the semi-classical level, and worse, certain classical systems (inspired by quantum physics, but in no sense quantum themselves) exhibit seriously pathological behaviour. One manifestation of these pathologies is in the so-called "energy conditions" of general relativity. Patching things up in the gravity sector opens gaping holes elsewhere; and some "fixes" are more radical than the problems they are supposed to cure.

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1. Energy Conditions of General Relativity

Even if you restrict attention to the purely classical regime, Einstein gravity (general relativity) is a tremendously complex theory. In the field equations $G^{\mu\nu} = 8\pi G T^{\mu\nu}$, the left-hand-side, the Einstein tensor $G^{\mu\nu}$, is complicated enough by itself. But it is at least a universal function of the spacetime geometry. In contrast the right-hand-side, the stress-energy tensor $T^{\mu\nu}$, is not universal but instead depends on the particular type of matter and interactions you choose to insert in your model.

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Table 1.   Pointwise energy conditions.

Name	Abbreviation	Definition	Current status
Trace energy condition	TEC	$\rho - 3p \geq 0$	forgotten
Strong energy condition	SEC	$\rho + 3p \geq 0;\ \rho + p \geq 0$	dead
Null energy condition	NEC	$\rho + p \geq 0$	moribund
Weak energy condition	WEC	$\rho \geq 0;\ \rho_p \geq 0$	moribund
Dominant energy condition	DEC	$\rho \geq 0;\ p \in [-\rho, +\rho]$	moribund

Faced with this situation, you must either resign oneself to performing an immense catalog of special-case calculations, one special case for each conceivable matter Lagrangian you can write down, or try to decide on some generic features that "all reasonable" stress-energy tensors should satisfy, and then try to use these generic features to develop general theorems concerning the strong-field behaviour of gravitational fields.

One key generic feature that most matter we run across experimentally seems to share is that energy densities (almost) always seem to be positive. The so-called "energy conditions" of general relativity[1] is a variety of different ways of making this notion of locally positive energy density more precise. The (pointwise) energy conditions take the form of assertions that various linear combinations of the components of the stress-energy tensor should be positive, or at least non-negative (see Table 1). The so-called "averaged energy conditions" are somewhat weaker, they permit localized violations of the energy conditions, as long as "on average" the energy conditions hold when integrated along null or timelike geodesics.

The variety of energy conditions in use in the relativity community is driven largely by reverse engineering based on the technical requirements of how much you have to assume to easily prove the result you want. By assuming some form of energy condition, some notion of positivity of the stress-energy tensor, as an input hypothesis, it has been possible to prove theorems like the singularity theorems (guaranteeing, under certain circumstances, gravitational collapse and/or the existence of a big bang singularity), the positive energy theorem (guaranteeing the mass of a complex gravitating system as seen from infinity is always positive), the topological censorship theorem (guaranteeing the non-existence of traversable wormholes) or the superluminal censorship (limiting the extent to which light cones can "tip over" in strong gravitational fields). Conversely, the violation of some or all of these energy conditions would point towards exotic physical possibilities (see Ref. 2 for some examples).

Over the years, opinions have changed as to how fundamental some of the specific energy conditions are. One particular energy condition (the trace energy condition, TEC) has now been completely abandoned and forgotten. The TEC was the assertion that the trace of the stress-energy tensor should always be negative (or positive depending on metric conventions), and was popular for a while during the 1960's. However, once it was realized that stiff equations of state, such as

those appropriate for neutron stars, violate the TEC this energy condition fell into disfavour. It has now been completely abandoned and is no longer cited in the literature — we mention it here as a concrete example of an energy condition being outright abandoned.

There is also general agreement that the strong energy condition (SEC) is dead:

(1) The most naive scalar field theory you can write down, the minimally coupled scalar field, violates the SEC, and indeed curvature-coupled scalar field theories also violate the SEC. There are fermionic quantum field theories where interactions engender SEC violations, and specific models of point-like particles with two-body interactions that violate the SEC.
(2) If you believe in cosmological inflation, the SEC must be violated during the inflationary epoch, and the need for this SEC violation is why inflationary models are typically driven by scalar inflaton fields.
(3) If you believe the recent observational data regarding the accelerating universe, then the SEC is violated on cosmological scales *right now!*
(4) Even if you are somewhat more conservative, and regard the alleged present-day acceleration of the cosmological expansion as "unproven", the tension between the age of the oldest stars and the measured present-day Hubble parameter makes it very difficult to avoid the conclusion that the SEC must have been violated in the cosmologically recent past, sometime between redshift 10 and the present.[3]

Under these circumstances it would be rather quixotic to take the SEC seriously as fundamental physics.

The null, weak, and dominant energy conditions are on the verge of dying. Specifically: Over the last decade or so it has become increasingly obvious that there are quantum effects that are capable of violating *all* the energy conditions, even the weakest of the standard energy conditions. Despite the fact that they are moribund, for lack of truly successful replacements, the NEC, WEC, and DEC are still extensively used in the general relativity community. The weakest of these is the NEC, and it is in many cases also the easiest to work with and analyze. The standard wisdom for many years was that all reasonable forms of matter should at least satisfy the NEC. After it became clear that the NEC (and even the ANEC) was violated by quantum effects two main lines of retrenchment developed:

(1) Many researchers simply decided to ignore quantum mechanics, relying on the classical NEC to prevent grossly weird physics in the classical regime, and hoping that the long sought for quantum theory of gravity would eventually deal with the quantum problems. This is not really a satisfactory response in that NEC violations already show up in semiclassical quantum gravity (where you quantize the matter fields and keep gravity classical), and show up at first order in $\hbar$. Since semiclassical quantum gravity is certainly a good approximation in our immediate neighbourhood, it is somewhat disturbing to see

widespread (albeit small) violations of the energy conditions in the here and now. Many experimental physicists and observational astrophysicists react quite heatedly when the theoreticians tell them that according to our best calculations there should be "negative energy" (energy densities less than that of the flat-space Minkowski vacuum) out there in the real universe. However, to avoid the conclusion that quantum effects can and do lead to locally negative energy densities, and even violations of the ANEC, requires truly radical surgery to modern physics, and in particular we would have to throw away almost all of quantum field theory.

(2) A more nuanced response is based on the Ford–Roman *Quantum Inequalities*.[4] These inequalities are based on the fact that while quantum-induced violations of the energy conditions are widespread they are also *small*, and on the observation that a negative energy in one place and time always seems to be compensated for (indeed, over-compensated for) by positive energy elsewhere in spacetime. This is the so-called *Quantum Interest Conjecture*. While the positive pay-back is not enough to prevent violation of the ANEC (based on averaging the NEC along a null geodesic) the hope is that it will be possible to prove some improved type of space-time averaged energy condition from first principles, and that such a space-time averaged energy condition might be sufficient to enable us to recover the singularity/positive-mass/censorship theorems under weaker hypotheses than currently employed. (Note that this would not eliminate the possibility of weird geometrical effects in the subatomic realm.)

A fundamental problem for this type of approach that is now becoming acute is the realization that there are also serious *classical* violations of the energy conditions.[5] Recently, it has become clear that there are quite reasonable looking classical systems, field theories that are compatible with all known experimental data, and that are in some sense very natural from a quantum field theory point of view, which violate all the energy conditions. Because these are now classical violations of the energy conditions they can be made arbitrarily large, and seem to lead to rather weird physics. (For instance, it is possible to demonstrate that Lorentzian-signature traversable wormholes arise as [unstable] classical solutions of the field equations.) These classical energy condition violations are due to the behaviour of scalar fields when coupled to gravity, so let us devote the next section to present some background on the usefulness and need for scalar field theories in modern physics.

However, before finishing the section, and for completeness, we would like to point out another area present-day physics in which one is confronted with energy condition violations, namely negative tension braneworlds. If physics is what physicists do, then negative tension branes are physics — they are common ancillary objects in braneworld cosmologies based on variants of the Randall–Sundrum construction. For our present purposes this is important because negative tension branes provide classical violations of all the energy conditions in the

higher-dimensional spacetime,[6] and they do so in a way that is completely independent of your opinions concerning scalar fields. These classical violations of the energy conditions easily engender arbitrarily weird physics.

2. Scalar Fields

Scalar fields play a somewhat ambiguous role in modern theoretical physics: on the one hand they provide great toy models, and are from a theoretician's perspective almost inevitable components of any reasonable model of empirical reality; on the other hand the direct experimental/observational evidence is spotty.

- The only scalar fields for which we have really direct "hands-on" experimental evidence are the scalar mesons (pions π; kaons K; and their "charmed", "truth" and "beauty" relatives, plus a whole slew of resonances such as the η, f_0, η', a_0, . . .). Not a single one of these particles are fundamental, they are all quark–antiquark bound states, and while the description in terms of scalar fields is useful when these systems are probed at low momenta (as measured in their rest frame) we should certainly not continue to use the scalar field description once the system is probed with momenta greater than $\hbar$/(bound state radius). Similarly you should not trust the scalar field description if the energy density in the scalar field exceeds the critical density for the quark-hadron phase transition. Thus scalar mesons are a mixed bag: they definitely exist, and we know quite a bit about their properties, but there are stringent limitations on how far we should trust the scalar field description.
- The next candidate scalar field that is closest to experimental verification is the Higgs particle responsible for electroweak symmetry breaking. While in the standard model the Higgs is fundamental, and while almost everyone is firmly convinced that some Higgs-like scalar field exits, there is a possibility that the physical Higgs (like the scalar mesons) might itself be a bound state of some deeper level of elementary particles (e.g., technicolor and its variants). Despite the tremendous successes of the standard model of particle physics we do not (currently) have direct proof of the existence of a fundamental Higgs scalar field.
- A third candidate scalar field of great phenomenological interest is the axion: it is extremely difficult to see how one could make strong interaction physics compatible with the observed lack of strong CP violation, without something like an axion to solve the so-called "strong CP problem". Still, the axion has not yet been directly observed experimentally.
- A fourth candidate scalar field of phenomenological interest specifically within the astrophysics/cosmology community is the so-called "inflaton". This scalar field is used as a mechanism for driving the anomalously fast expansion of the universe during the inflationary era. While observationally it is a secure bet that something like cosmological inflation (in the sense of anomalously fast cosmological expansion) actually took place, and while scalar fields of some type are the most reasonable way of driving inflation, we must again admit that direct

observational verification of the existence of the inflaton field (and its variants, such as quintessence) is far from being accomplished.

• A fifth candidate scalar field of phenomenological interest specifically within the general relativity community is the so-called "Brans–Dicke scalar". This is perhaps the simplest extension to Einstein gravity that is not ruled out by experiment. (It is certainly greatly constrained by observation and experiment, and there is no positive experimental data guaranteeing its existence, but it is not ruled out.) The relativity community views the Brans–Dicke scalar mainly as an excellent testing ground for alternative ideas and as a useful way of parameterizing possible deviations from Einstein gravity. (And experimentally and observationally, Einstein gravity still wins.)

• Finally, the membrane-inspired field theories (low-energy limits of what used to be called string theory) are literally infested with scalar fields. In membrane theories it is impossible to avoid scalar fields, with the most ubiquitous being the so-called "dilaton". However, the dilaton field is far from unique, in general there is a large class of so-called "moduli" fields, which are scalar fields corresponding to the directions in which the background spacetime geometry is particularly "soft" and easily deformed. So if membrane theory really is the fundamental theory of quantum gravity, then the existence of fundamental scalar fields is automatic, with the field theory description of these fundamental scalars being valid at least up to the Planck scale, and possibly higher.

So overall, while we have excellent theoretical reasons to expect that scalar field theories are an integral part of reality, the direct experimental/observational verification of the existence of fundamental scalar fields is still an open question. Nevertheless, we think it fair to say that there are excellent reasons for taking scalar fields seriously, and excellent reasons for thinking that the gravitational properties of scalar fields are of interest cosmologically, astrophysically, and for providing fundamental probes of general relativity.

3. Problems with Scalar Field Theories

The main problem is that, generically, once you couple them to gravity, they violate all the energy conditions even at a classical level. We say generically because of the key role of the so-called curvature coupling, a term of the form $\xi\phi^2 R$ in the Lagrangian of the system that directly couples the scalar field ϕ with the spacetime curvature scalar R. This term is renormalizable by power counting and so must be included in the curved-space scalar field Lagrangian. Even if this term is not there in the bare Lagrangian it will be generated by quantum effects.

If $\xi = 0$ (so-called "minimal coupling") then the SEC is violated classically, though DEC, WEC, and NEC are satisfied. Unfortunately "minimal coupling" is non-generic and unstable to quantum corrections. For any $\xi \neq 0$ *all* the pointwise energy conditions are violated (including the NEC). There are good reasons to believe that the value $\xi = 1/6$ is preferred. Only for $\xi = 1/6$ (so-called "conformal

coupling") does the flat space limit of the stress-energy tensor for the scalar field yield an expression with good renormalization properties. This expression in flat space was called the "new improved stress-energy tensor" to distinguish it from the naive stress-energy tensor previously used.[7]

Indeed, from the quantum field theory perspective, conformal coupling and the new improved stress-energy tensor are arguably the only sensible choice, and it is rather disturbing that this choice leads to violations of all pointwise energy condition (and so to peculiar physics when coupled to gravity). Even worse, under certain circumstances (typically involving trans-Planckian expectation values for the scalar field) even the averaged null energy condition (ANEC) is violated. [Note that trans-Planckian values for a scalar field are not by themselves objectionable. It is only trans-Planckian energy densities that require a full quantum-gravity treatment. For example, many (not all) models of cosmological inflation use trans-Planckian values for the scalar field.] The fact that the ANEC can be violated by classical scalar fields is significant and important (even with the trans-Planckian caveat). The ANEC is the weakest of the energy conditions in current use, and violating the ANEC short circuits *all* the standard singularity/positive-mass/censorship theorems. This observation piqued our interest and we decided to see just how weird the physics could get once you admit scalar fields into your models.

In particular, it is by now well-known that traversable wormholes are associated with violations of the NEC and ANEC, so we became suspicious that there might be an explicit class of exact traversable wormhole solutions to the coupled gravity-scalar field system. We recently found such a class of [unstable] solutions.[8,9] Now traversable wormholes, while certainly exotic, are by themselves not enough to get the physics community really upset: The big problem with traversable wormholes is that if you manage to acquire even one inter-universe traversable wormhole then it *seems* almost absurdly easy to build a time machine — and this does get the physics community upset. At this point, we will again confront ourselves with quantum physics. It has been conjectured by Hawking, (*Chronology Protection Conjecture*),[10] that quantum physics will save the universe by destabilizing the wormhole just as a time machine is about to form. However, it must certainly be emphasized that there is considerable uncertainty as to how serious these causality problems are.

The violations of the energy conditions induced by non-minimally coupled scalar fields, having a classical character, are not restricted in magnitude or duration by any quantum inequality.[11] Thus, even without reaching trans-Planckian values for the scalar field, one can envisage the creation of long-lasting fluxes of negative energy. It is hard to see how these negative energy fluxes can be made compatible with the second law of thermodynamics.[12] As emphasized by Ford and Roman,[11] the solution to this question is tied up with the manner in which the energy flux interacts with matter. In fact, trying to circumvent this issue by throwing the flux into a black hole they found a miraculous preservation of the generalized second law.

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4. Conclusions

There are several responses to the current state of affairs: either we can learn to live with wormholes, and other strange physics engendered by energy condition violations, or we need to patch up the theory. One particularly simple way of dealing with all these problems is to banish scalar fields from your theories: This makes technicolor partisans very happy, but drives supersymmetry supporters, string theorists, and cosmologists to apoplexy. Alternatively, one could forbid non-minimal couplings, or forbid trans-Planckian field values, each one of these particular possibilities is in conflict with cherished notions of *some* segments of the particle physics/ membrane theory/ relativity/ astrophysics communities. Most physicists would be loathe to give up the notion of a scalar field, and conformal coupling is so natural that it is difficult to believe that banning it would be a viable option. Banishing trans–Planckian field values is more plausible, but this is only a partial remedy and also runs afoul of at least some segments of the cosmological inflationary community.

In summary, the conflict between quantum physics and gravity is now becoming acute. Problems are no longer confined to Planck scale physics but are leaking down to arbitrarily low energies and even into the classical realm. These problems appear to be insensitive to and independent of high energy phenomena and so it is not at all clear that a high energy cutoff (string theory, quantum geometry, lattice gravity, etc. . . .) would do anything to ameliorate them. The situation is both puzzling and exciting.

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