Twisted speckle entities inside wave-front reversal mirrors

A. Yu Okulov*

General Physics Institute of the Russian Academy of Sciences, Vavilova Str. 38, 119991 Moscow, Russia

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The previously unknown property of the optical speckle pattern reported. The interference of a speckle with the counterpropagating phase-conjugated (PC) speckle wave produces a randomly distributed ensemble of a twisted entities (ropes) surrounding optical vortex lines. These entities appear in a wide range of a randomly chosen speckle parameters inside the phase-conjugating mirrors regardless to an internal physical mechanism of the wave-front reversal. These numerically generated interference patterns are relevant to the Brillouin PC mirrors and to a four-wave mixing PC mirrors based upon laser trapped ultracold atomic cloud.

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Phase singularities of the optical beams attracted a substantial interest in recent decades from the point of view of optical information processing [1]. Helical wave fronts had been shown to affect a processes of the second-harmonic generation [1], image processing with a photorefractive mirrors [2], phase-conjugated (PC) reflection via nondegenerate four-wave mixing in a cold atoms cloud [3]. The photons with helical wave function had been shown to possess quantized angular momentum $\hbar$ per photon [4]. The optical nonlinearities are capable to transfer the angular momentum from photons to an ensemble of ultracold atoms [3]. This effect is considered as a possible tool for the light storage in addition to the slow light technique based on electromagnetically induced transparency (EIT) [5,6]. Recently the angular momentum transfer to BEC cloud of sodium atoms via stimulated Raman scheme had been observed [7]. Of special interest is the nondegenerate four-wave mixing a cold atomic cloud and PC reflection of the optical vortices (OVs), i.e., Laguerre-Gaussian (LG) beams with helical wave fronts [3]. The reflection of the phase-conjugated LG photons from 10^7 cesium atoms cooled down to $T \approx 10^{-3}$ K and reversal of the orbital angular momentum (OAM) had been interpreted as a consequence of internal macroscopic rotations inside atomic cloud [8]. Afterwards these experimental results were analyzed from a point of view of the angular momentum conservation for the incident and reflected photons from a Brillouin PC mirror [9]. The goal of the current communication is to study the spiral interference patterns with the period of $\lambda/2$ around the *nodes* of optical speckle pattern. These coarse interference patterns turn in rotation the Brillouin medium [10] producing the acoustical vortices carrying OAM [11].

The traditional experimental and numerical technique for the visualization of the optical phase singularities is based on mixing of a wide quasi-plane-wave reference optical beam with a speckle signal [12]. The dark spots (zeros of amplitudes) appearing in intensity distribution are collocated with the helical ramps in phase distribution. The current communication reports a theoretical description of the alternative visualization technique which uses the interference of the speckle wave with the counterpropagating phase-conjugated one. In fact this technique is automatically implemented in a wavefront reversing mirrors where a phase-conjugated wave produces near the bright spot the Bragg grating of dielectric permittivity of the form $\cos[(k_p+k_s)z]$. In the vicinity of the dark line the modulation of the light intensity is more complex: the Bragg grating is transformed into spirals of the form $\cos[(k_p+k_s)z \pm 2\ell\phi]$, where ϕ is the local azimuthal angle, ℓ is topological charge of the phase singularity, and k_p and k_S are wave numbers of incident (pump) and reflected (Stokes) waves, respectively [9,10]. In the real PC mirrors the contrast of such gratings may be reduced due to the interference with a non-phase-conjugated component of the radiation.

The feature compared to the previous findings [12,13] is that the ideal (or close to an ideal) PC mirror visualizes in the speckle patterns a peculiar optical entities, which might be called the *ropes*. Typically each such a rope is composed of a 2 or 3 optical vortex lines (Fig. 1). In contrast to the well-studied straight optical vortex lines, e.g., LG [1,4], the OV in a speckle field are self-twisted as it seen from Fig. 2. Having in mind the well-known fact [12] that OV appear and annihilate as a pair of whirls with opposite circulations [9,10], it is easy to conclude that at least two adjacent OV with opposite topological charges are needed to produce a rope (Fig. 3). The mean length of each OV in Z direction is the Rayleigh range of a speckle pattern $L_R \approx D^2/\lambda$, where D is an average transverse size [in the plane (X,Y)] of the OV core and the wavelength $\lambda = 2\pi/k_p$. The Fig. 1 shows the numerically generated fragment of a speckle pattern which contains three ropes each composed of a set of a vortex lines. Let us describe the numerical procedure for generating the interference pattern of the two counterpropagating speckle fields.

The standard model of the phase conjugation via stimulated Brillouin scattering (SBS) is the Bragg reflection from the sound grating with period $\lambda/2$ moving with the speed of sound v_{ac} [13]. Due to the conservation of momentum $\vec{p} = \hbar\vec{k}_p = \hbar\vec{k}_s + \hbar\vec{k}_{ac}$ the length of the wave vector of sound k_{ac} is equal to doubled length of wave vector of pump light k_p with an accuracy about 10^{-5} [14]. The Doppler effect defines the small (of the order of 10^{-5}) frequency shift of the reflected (Stokes) wave $\omega_{ac} = 2\omega_p n v_{ac}/c$, where n is refractive index of medium, c is the speed of light in vacuum, and ω_p, ω_S is the frequency of the pump and Stokes light, respectively.

*okulov@kapella.gpi.ru; http://www.gpi.ru/~okulov

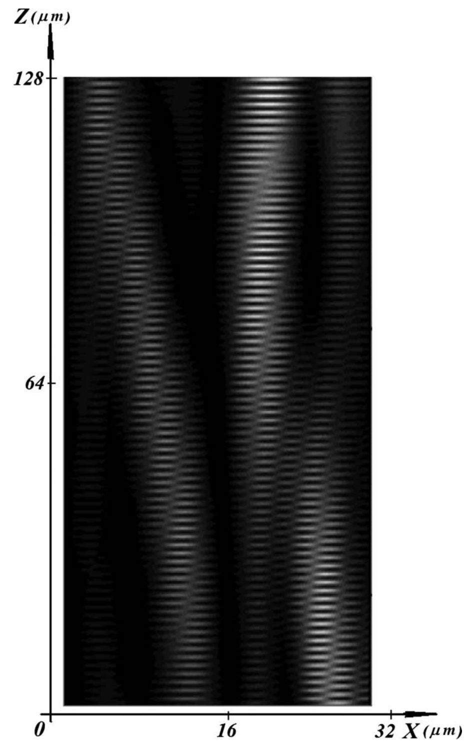

FIG. 1. Grayscale intensity plot for a small volume of a speckle pattern inside PC mirror in (X,Z) plane ($Y=23$ μm). The mean transverse size of a speckle is $D\sim 8$ μm. The mean longitudinal length of a speckle entity is of the order of the Rayleigh range $L_R \approx D^2/\lambda$ for $\lambda \sim 1$ μm. The size of the volume is 32 μm in transverse (X,Y) directions and 128 μm in longitudinal (Z) direction. The step of longitudinal modulation $\cos[(k_p+k_s)z \pm 2\phi]$ induced by the interference of counterpropagating pump and Stokes waves is enlarged here eight times for visualization purposes. The characteristic π phase jump is clearly visible in between adjacent Bragg's cosine-modulated roll patterns.

The evolution in space of the two counterpropagating paraxial laser beams inside Brillouin active medium obeys to Maxwell equations with the cubic nonlinear polarization induced by electrostrictive effect [13]. The linearly polarized pump field E_p moves in a positive direction of Z axis, the reflected Stokes field E_S with the same polarization propagates in opposite direction(Fig. 1). The acoustic field Q is excited via electrostriction [13,15]. The three-wave equations for the interaction of E_p, E_S with adiabatic elimination of the $Q \approx E_p E_S^*$ are

$$\frac{\partial E_p(z,x,y,t)}{\partial z} + \frac{i}{2k_p}\Delta_\perp E_p = -\frac{\gamma^2 \omega_p k_a^2}{32\rho_0 n c \omega_{ac}}|E_S|^2 E_p, \tag{1}$$

$$\frac{\partial E_S(z,x,y,t)}{\partial z} - \frac{i}{2k_S}\Delta_\perp E_S = -\frac{\gamma^2 \omega_S k_a^2}{32\rho_0 n c \omega_{ac}}|E_p|^2 E_S, \tag{2}$$

where $\gamma=\rho(\partial\epsilon/\partial\rho)_S$ is the electrostrictive coupling constant and ρ_0 is the unperturbed density of medium [15]. Consider the phase conjugation with a random-phase plate which produces the chaotic transverse modulation of the phase of the incident E_p with a characteristic size of the transverse inhomogeneity of the order of 5–50 μm [13]. In such a case the complex envelope amplitude of the inhomogeneous speckle field E_p at the entrance to PC mirror is given as a multimode random field [16]:

$$E_p(\vec{r},z=0) \approx E_p^0 \sum_{0<j_x,j_y<N_g} A_{j_x,j_y} \times \exp\left[i2\pi\left\{ \frac{x j_x}{p_x}\kappa_{j_x} + \frac{y j_y}{p_y}\kappa_{j_y} + i\theta_{j_x,j_y} \right\}\right], \tag{3}$$

where random phases θ_{j_x,j_y} are the random numbers from interval $[0,\pi]$, A_{j_x,j_y} are the real amplitudes of the spatial harmonics, p_x, p_y are maximal transverse sizes [in the (X,Y) plane], $\vec{r}=(x,y)=(r,\phi)$, and j_x, j_y are integers corresponding to $N_g=(16,32,64)$ plane waves with random phases $\theta_{jx,jy}$. The random numbers $\kappa_{j_x}=p_x/p'_{jx}$ and $\kappa_{j_y}=p_y/p'_{jy}$ having the uniform distribution in the small vicinity of a 1 are responsible for a random tilt of a plane wave constituting the spatial Fourier spectrum of the light transmitted through a random-phase plate. For the paraxial beam propagation the amplitude and phase structure of the complex field E_p have the following form [17] in an arbitrary Z plane:

$$E_p(\vec{r},z>0) \approx E_p^0 \exp(ik_p z) \sum_{0<j_x,j_y<N_g} \exp[i\theta_{j_x,j_y}] A_{j_x,j_y} \times \exp\left[i2\pi\left\{ \frac{x j_x}{p'_{jx}} + \frac{y j_y}{p'_{jy}} + \frac{z}{2k_p}\left(\frac{j_x^2}{p'^2_{jx}} + \frac{j_y^2}{p'^2_{jy}} \right)\right\}\right]. \tag{4}$$

The interference pattern $I(\vec{r},z)$ is responsible for the sound grating in a Brillouin PC mirror. For the grating produced by a linear superposition of the random plane waves we have

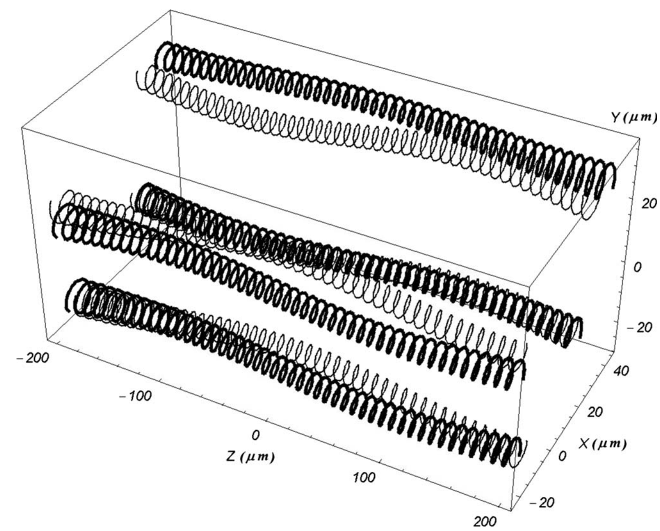

FIG. 2. The plot of the light intensity *maxima* at a given moment t in a small volume inside the PC mirror. The four pairs of spiral interference patterns with opposite handedness are located randomly in space. Their diameters and directions are changed smoothly due to diffraction. The step of longitudinal modulation $\cos[(k_p+k_s)z \pm 2\phi]$ is enlarged here 16 times.

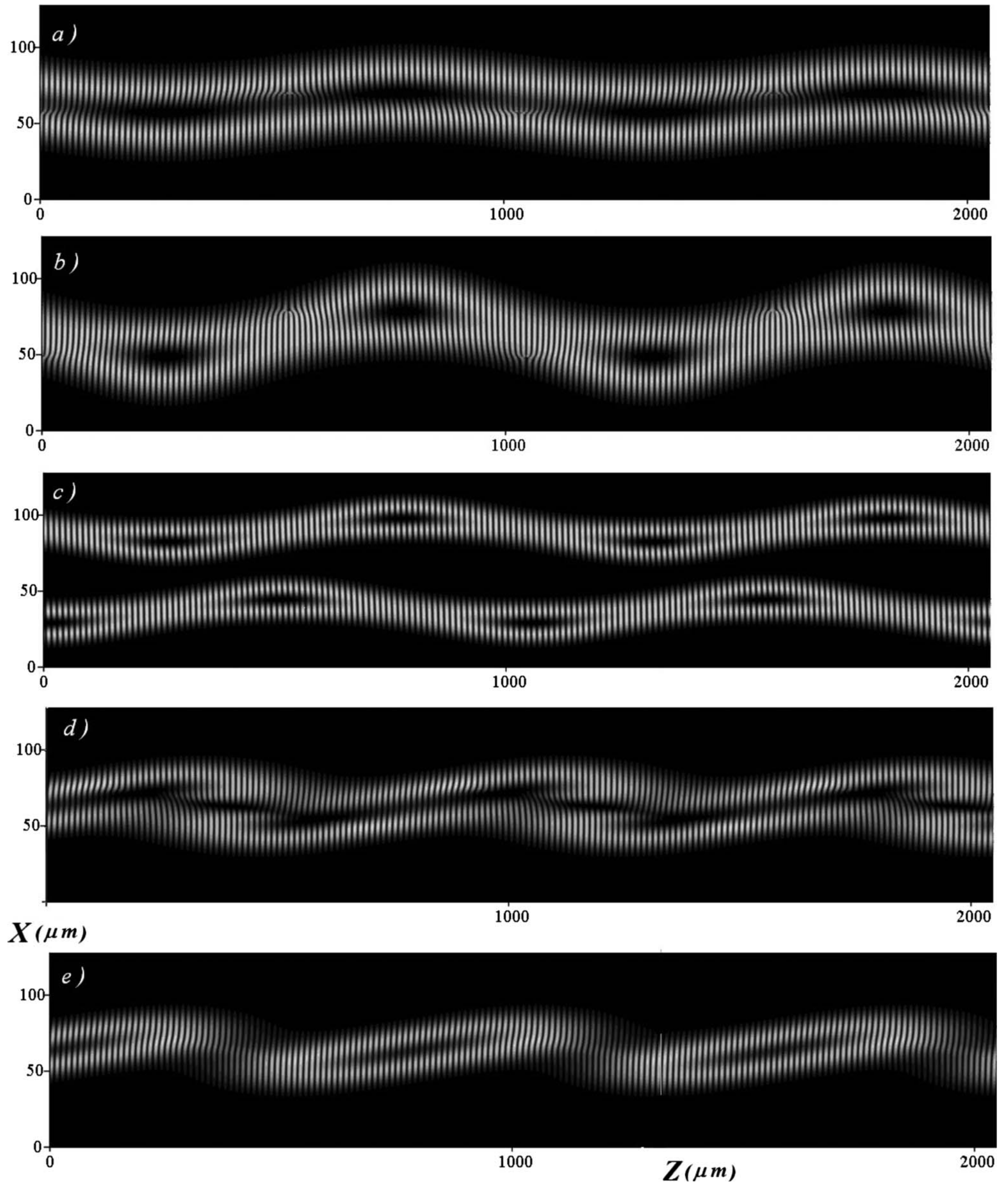

FIG. 3. Grayscale intensity plot for the variational ansatz [Eq. (7)] substituted into Eq. (5). The 128×2048 point arrays are used for the intensity distributions [Eq. (5)] in (X,Z) planes at fixed $Y=64$. The single sinusoidally modulated LG_0^1 vortex line [see Eq. (7)] is shown for (a) $M=0.1D$ and (b) $M=D$. Two LG_0^1 distant vortex lines (c) with parallel, (d) and opposite (e) topological charges $\pm\ell$ constitute the rope when the distance between them is reduced to zero (d,e). The step of longitudinal modulation is enlarged here 32 times.

$$I(\vec{r},z>0)\approx|E_p(\vec{r},z)+E_S(\vec{r},z)|^2, \tag{5}$$

where the phase-conjugated Stokes wave E_s is given by

$$E_s(\vec{r},z>0)\approx E_p^0\exp(-ik_sz)\sum_{0<j_x,j_y<N_g}\exp[-i\theta_{j_x,j_y}]A_{j_x,j_y}\times\exp\left[-i2\pi\left\{\frac{xj_x}{p'_{jx}}+\frac{yj_y}{p'_{jy}}+\frac{z}{2k_s}\left(\frac{j_x^2}{p'^2_{jx}}+\frac{j_y^2}{p'^2_{jy}}\right)\right\}\right], \tag{6}$$

provided that $E_S\approx E_p^*$ (noiseless PC mirror). Such an approach presumes the ideally perfect phase conjugation with ultimate reflection $R=1$ or interference of the noninteracting fields $E_S\sim E_p^*$ when the right hand sizes of Eq. (1) and (2) are equal to zero.

The intensity snapshots given by Eq. (5) were obtained numerically by summation of series (4) and (6) on standard Intel platform with dual-core 1.86 Ghz processor and 1 Gbyte memory using standard educational software. The numerical simulations show that optical vortex lines *intertwine* and form ropes. The average length of a rope is of the order of Rayleigh range D^2/λ. This may be interpreted as a consequence of diffractive divergence within the angle λ/D (Fig. 1). The longitudinal modulation by the Bragg factor $\cos[(k_p+k_S)z\pm 2\phi]$ is accompanied by an additional bending and twisting at characteristic wavelength λ_K. The latter is of the order of a several tenth of optical λ, resembling Kelvin modes of the vortex lines [18,19]. The handedness of helical ropes changes randomly from one entity to another: the clockwise and counterclockwise ropes appear with the equal probability. Despite the statistical nature of a speckle pattern (Fig. 1) the ropes are *structurally stable*: they appear at a different locations and have a variable sizes but their morphology is reproduced from a given statistical realization to another one.

The physical interpretation of these numerically observed patterns is given in Fig. 3 using Eq. (5) with the electrical fields $E_p = E_S^*$ in the form of a two overlapping elementary LG_0^1 optical vortices:

$$E_p \approx \exp[ik_p z - (x_1^2 + y_1^2) + i\ell\phi_1]\sqrt{(x_1^2 + y_1^2)} + \exp[ik_p z - (x_2^2 + y_2^2) \pm i\ell\phi_2]\sqrt{(x_2^2 + y_2^2)}, \quad (7)$$

where the location of the vortex cores $[x_1 D = x + M\cos(2\pi z/\lambda_K)], [y_1 D = y + M\sin(2\pi z/\lambda_K)]$ and $[x_2 D = x + M\cos(2\pi z/\lambda_K + \pi/2)], [y_2 D = y + M\sin(2\pi z/\lambda_K + \pi/2)]$ changes sinusoidally with period λ_K along Z axis, ϕ_1 and ϕ_2 are the local azimuthal angles around vortex cores. Such vortex-vortex [Fig. 3(d)] and vortex-antivortex [Fig. 3(e)] pair interference pattern is used here as a variational ansatz in Eq. (5). Figure 3 reproduces qualitatively a some features of the numerically generated speckle patterns (Fig. 1).

In summary the nontrivial topology of the multiply connected optical speckle patterns was demonstrated numerically. Using the interference with the counterpropagating phase-conjugated speckle field we have shown the hidden twisted geometry of the multimode wave field composed of the randomly tilted plane waves [16]. Noteworthy the ropes exist without phase-conjugated counterpart: the reflection from PC mirror makes these twisted entities visible due to the characteristic π phase jump in between the adjacent Bragg's sinusoidally modulated roll patterns (Fig. 1). Thus it seems probable that a rope entities exist in a wide class of a superpositional physical fields such as an electromagnetic or acoustical ones [11]. The other most evident examples are the blackbody radiation field in a cavity or a monochromatic field in a cavity with a rough mirrors. Taking into account the phenomena of a so-called *nonlinear superposition* which take place for collisions of the optical solitons, vortices [20], and formation of the optical vortex lattices [21] one might expect the finding of a stable ropes in a *nonlinear* fields. The relevant examples are photorefractive PC mirrors [2], ultracold matter, e.g., phase-conjugating PC mirrors based on nondegenerate four-wave mixing in a trapped gases [3,8] and the situations of the slow [5,6,22] and the fast light [23,24] propagation in a resonant medium.

The reported result extends the set of the possible forms of the photon's wave function. In addition to the well-known photon wave functions in the form of the elementary optical vortex lines, e.g., LG beams which possess a helical wave fronts [1,4], the twisted entities each composed of the several vortex lines offer a particular form of the photon's wave function [25,26].

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