Calc. Var. 20, 231–233 (2004) **Calculus of Variations**
DOI: 10.1007/s00526-003-0260-7

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Erratum

Two-bubble solutions in the super-critical Bahri-Coron's problem

Received: 7 November 2003 / Accepted: 18 November 2003 /
Published online: 25 February 2004 – © Springer-Verlag 2004

Professor O. Rey showed us a gap in our paper [1]. More precisely, he pointed us out that estimate (5.5), and consequently (5.11), are valid for dimensions $3 \leq N \leq 6$, if we use the weighted L^∞ norm $\| \cdot \|_*$ introduced in page 122.

We can recover all the results of our paper [1] and for any dimension $N \geq 3$, if we modify the definition of the $*$-norm. The consequent changes inside the body of the paper are relatively minor, but necessary. We will devote this erratum to give the new definition of the $*$-norm and the complete list of the main changes inside the paper.

The definition of $*$-norm, introduced at page 122 in [1] should be

$$\|\psi\|_* = \sup_{x \in \Omega_\varepsilon} |\omega^{-\beta}(x)\psi(x)| + |\omega^{-(\beta + \frac{1}{N-2})}(x)\nabla\psi(x)|,$$

where

$$\omega(x) = \left((1 + |x - \xi_1'|^2)^{-\frac{N-2}{2}} + (1 + |x - \xi_2'|^2)^{-\frac{N-2}{2}}\right)$$

and $\beta = 1$ if $N = 3$ and $\beta = \frac{2}{N-2}$ if $N \geq 4$.

The statement of Lemma 4.1 still holds true. We just have to remark that, since ϕ_ε can be written as in formula (4.10), then $\phi_\varepsilon \in C^1$ and

$$\begin{aligned} &\partial_{x_j}\phi_\varepsilon(x) - (p+\varepsilon)\int_{\Omega_\varepsilon} \partial_{x_j} G_\varepsilon(x,y) V^{p+\varepsilon-1}\phi_\varepsilon dy \\ = &-\int_{\Omega_\varepsilon} \partial_{x_j} G_\varepsilon(x,y) h_\varepsilon \, dy \\ &- \sum c_{ij} \int_{\Omega_\varepsilon} V_i^{p-1} Z_{ij} \partial_{x_j} G_\varepsilon(x,y)\, dy \quad x \in \Omega_\varepsilon, \end{aligned}$$

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where G_ε denotes the Green's function of Ω_ε. Hence, we get the following estimates

$$\begin{aligned}
&\int_{\Omega_\varepsilon} \partial_{x_j} G_\varepsilon(x,y)|h_\varepsilon|\,dy \\
&\le \|h_\varepsilon\|_{**} C \int_{\mathbb{R}^N} \frac{1}{|x-y|^{N-1}} \left((1+|y-\xi_1'|^2)^{-2} + (1+|y-\xi_2'|^2)^{-2}\right)\,dy \\
&\le C\|h_\varepsilon\|_{**}|\log\varepsilon|^m \omega^{\beta+\frac{1}{N-2}}(x); \\
&\left|\sum c_{ij}\int_{\Omega_\varepsilon} V_i^{p-1} Z_{ij}\partial_{x_j} G_\varepsilon(x,y)\,dy\right| \\
&\le C\left(\|\phi_\varepsilon\|_\rho + \|h_\varepsilon\|_{**}\right)\sum\int_{\mathbb{R}^N}\frac{1}{|x-y|^{N-1}}\left((1+|y-\xi_i'|^2)^{-\frac{N+3}{2}}\right) \\
&\le C(\|\phi_\varepsilon\|_\rho + \|h_\varepsilon\|_{**})\omega^{\beta+\frac{1}{N-2}}(x);
\end{aligned}$$

$$\int_{\Omega_\varepsilon} \partial_{x_j} G_\varepsilon(x,y) V^{p+\varepsilon-1}|\phi_\varepsilon|dy \le C\|\phi_\varepsilon\|_\rho \omega^{\beta+\frac{1}{N-2}}(x).$$

So that we have

$$|\partial_{x_j}\phi_\varepsilon(x)| \le C(\|\phi_\varepsilon\|_\rho + \|h_\varepsilon\|_{**}|\log\varepsilon|^m)\omega^{\beta+\frac{1}{N-2}}(x).$$

Formula (4.11) should be

$$|\phi_\varepsilon(x)| \le C(\|\phi_\varepsilon\|_\rho + \|h_\varepsilon\|_{**}|\log\varepsilon|^m)\omega^{\beta}(x).$$

Finally, the last formula from above at page 125 should read as

$$\|\phi_\varepsilon\|_* \le C(\|\phi_\varepsilon\|_\rho + \|h_\varepsilon\|_{**}|\log\varepsilon|^m),$$

from which the conclusion of the lemma follows.

The estimates given at the middle of page 129 should be the following.

Assume that $N > 6$. In the region where $dist(y,\partial\Omega_\varepsilon) \ge \delta\varepsilon^{-\frac{1}{N-2}}$, for some $\delta > 0$, then $V(y) \ge \alpha_\delta \bar{V}(y)$ for some $\alpha_\delta > 0$; hence in this region, we have

$$|\bar{V}^{-\frac{4}{N-2}} N_\varepsilon(\eta)| \le C\bar{V}^{2\beta-1}\|\eta\|_*^2 \le C\varepsilon^{2\beta-1}\|\eta\|_*^2.$$

On the other hand, when $dist(y,\partial\Omega_\varepsilon) \le \delta\varepsilon^{-\frac{1}{N-2}}$, we have

$$V(y) = C(y)\varepsilon^{\frac{N-1}{N-2}} dist(y,\partial\Omega_\varepsilon) + O(\delta^2\varepsilon + \varepsilon^{\frac{N+1}{N-2}})$$

with $C(y) \ge C_0 > 0$; hence,

$$V(y) \ge \frac{C_0}{2}\varepsilon^{\frac{N-1}{N-2}} dist(y,\partial\Omega_\varepsilon)$$

as $dist(y,\partial\Omega_\varepsilon) \le \delta\varepsilon^{-\frac{1}{N-2}}$, provided that $\delta > 0$ is chosen small enough. These estimates are a consequence of the fact that the Green function of the domain Ω vanishes linearly with respect to $dist(x,\partial\Omega)$ ad $x \to \partial\Omega$.

Hence we can conclude that, if $dist(y, \partial\Omega_\varepsilon) \le \delta\varepsilon^{-\frac{1}{N-2}}$, then

$$|\bar{V}^{-\frac{4}{N-2}} N_\varepsilon(\eta)| \le \bar{V}^{-\frac{4}{N-2}} V^{p-2} \|\eta\|^2$$

$$\le C\bar{V}^{-\frac{4}{N-2}} \left(\varepsilon^{\frac{N-1}{N-2}} dist(y, \partial\Omega_\varepsilon)\right)^{p-2} dist(y, \partial\Omega_\varepsilon)^2 \|\nabla\eta(y)\|^2$$

$$\le C\varepsilon^{-\frac{4}{N-2}+\frac{N-1}{N-2}(p-2)-\frac{p}{N-2}+2\beta+\frac{2}{N-2}} \|\eta\|_*^2 \le C\varepsilon^{2\beta-1}\|\eta\|_*^2.$$

Formula (5.5), and hence (5.11), still hold true.

The first formula from above in page 131 should be

$$\|N_\varepsilon(\psi+\phi_1) - N_\varepsilon(\psi+\phi_2)\|_{**} \le$$

$$\begin{cases} \bar{V}^{2\beta-1}(\|\phi_1\|_* + \|\phi_2\|_* + \|\psi\|_*)\|\phi_1-\phi_2\|_* & \text{if } N \le 6 \\ \varepsilon^{2\beta-1}(\|\phi_1\|_* + \|\phi_2\|_* + \|\psi\|_*)\|\phi_1-\phi_2\|_* & \text{if } N > 6 \end{cases}$$

$$\le \varepsilon^{\min\{2\beta,1\}}\|\phi_1-\phi_2\|_*.$$

The second formula from below at page 131 should be

$$\bar{V}^{-\frac{4}{N-2}+\beta}|D_{\bar{\phi}} N_\varepsilon(\phi+\psi))|$$

$$\le C \begin{cases} \bar{V}^{2\beta-1}\|\phi+\psi\|_* & \text{if } N \le 6 \\ \varepsilon^{2\beta-1}\|\phi+\psi\|_* & \text{if } N > 6 \end{cases} \le C\varepsilon^{\min\{2\beta,1\}}.$$

The forth formula from below at page 132 should be

$$|(D_{\xi'}N)(\xi', \Lambda, \bar{\phi})| \le C\bar{V}^{\frac{N-1}{N-2}}|(V+\bar{\phi})_+^{p+\varepsilon-1} - V^{p+\varepsilon-1} - (p+\varepsilon-1)V^{p+\varepsilon-2}\bar{\phi}|$$

$$\le C \begin{cases} \bar{V}^{\frac{5}{N-2}+\varepsilon+\beta}\|\bar{\phi}\|_* & \text{if } N \le 6 \\ \varepsilon^{\frac{5}{N-2}+\varepsilon+\beta}\|\bar{\phi}\|_* & \text{if } N > 6. \end{cases}$$

Reference

1. del Pino, M., Felmer, P., Musso, M.: Two-bubble solutions in the super-critical Bahri-Coron's problem. Calc. Var. **16**, 113–145 (2003)