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Two-channel Linear Phase FIR QMF Bank Minimax Design via Global Nonconvex Optimization Programming

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ABSTRACT

In this correspondence, a two-channel linear phase finite impulse response (FIR) quadrature mirror filter (QMF) bank minimax design problem is formulated as a nonconvex optimization problem so that a weighted sum of the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter is minimized subject to specifications on these performances. A modified filled function method is proposed for finding the global minimum of the nonconvex optimization problem. Computer numerical simulations show that our proposed design method is efficient and effective.

Index Terms—Two-channel linear phase FIR QMF bank minimax design, nonconvex optimization problem, filled function, global optimization.

I. INTRODUCTION

Since transition bandwidths of the filters in two-channel filter banks are usually larger than those in multi-channel filter banks, lengths of the filters in two-channel filter banks are usually shorter than those in multi-channel filter banks. Moreover, as only a single prototype filter is required for the design of a QMF bank and all other filters are derived from the prototype filter, the total number of filter coefficients required for the design of a QMF bank is usually smaller than those in general filter banks. Furthermore, as the linear phase property of the filters guarantees no phase distortion of the filter bank and the FIR property of the filters guarantees the bounded input bounded output stability of the filter bank, two-channel linear phase FIR QMF banks find many applications in image and video signal processing [1].

Unlike a multi-channel QMF bank [2], [3], a two-channel QMF bank could not achieve the exact perfect reconstruction with the prototype filter having very good frequency selectivity [4]. Hence, it is useful to design a two-channel QMF bank so that a weighted sum of the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter is minimized subject to specifications on these performances. Nevertheless, this QMF bank minimax design problem is a nonconvex optimization problem. As nonconvex optimization problems usually consist of many local minima [14], it is usually stuck at these local minima and very difficult to find the global minimum if conventional gradient based approaches are employed for finding the global minimum.

When nonconvex optimization problems consist of finite numbers of local minima, it is possible to find the global minimum of these nonconvex optimization problems. There are mainly two different approaches for finding the global minimum of these nonconvex optimization problems. The first type of the approaches is nongradient based approaches, such as evolutionary algorithm based approaches [5], [6]. These approaches keep generating evaluation points randomly. Those evaluation points with better performances are kept, while those evaluation points with poor performances are ignored. However, computational complexities of these nongradient based approaches are very high because most of the evaluation points are ignored. The second type of the approaches is filled function approaches [7]-[12]. The definition of filled functions and the working principle of filled function methods are discussed in Section II. Nevertheless, it is very challenging to find a filled function that satisfies the required properties. To tackle this difficulty, filled functions with several parameters are defined [7]-[12]. However, there is no general rule for the selection of these parameters. In this correspondence, extra constraints are imposed on the optimization problems so that the required properties of the filled function are guaranteed to be satisfied.

In this correspondence, a modified filled function method is proposed for finding the global minimum of a two-channel linear phase FIR QMF bank minimax design problem. The outline of this correspondence is as follows. In Section II, the definition of filled functions and the working principle of filled function methods are reviewed. In Section III, a two-channel linear phase FIR QMF bank minimax design problem is formulated as a nonconvex optimization problem and a modified filled function method is proposed for finding the global minimum of the nonconvex optimization problem. In Section IV, computer numerical simulations are illustrated. Finally, conclusions are drawn in Section V.

II. REVIEW ON DEFINITION OF FILLED FUNCTIONS AND WORKING PRINCIPLE OF FILLED FUNCTION METHODS

A filled function [7]-[12] is a function satisfying the following properties: (a) the current local minimum of the original cost function is the current local maximum of the filled function; (b) the whole current basin of the original cost function is a part of the current hill of the filled function; (c) the filled function has no stationary point in any higher basins of the original cost function; and (d) there exists a local minimum of the filled function which is in a lower basin of the original cost function.

Some terminologies related to filled functions have been used above. Notably, a basin of a function is defined as the subset of the domain of the optimization variables such that any points in this subset will give the same local minimum of the function via conventional gradient based optimization methods. A hill of a function is defined as the subset of the domain of the optimization variables such that any points in this subset will give the same local maximum of the function via conventional gradient based optimization methods. A higher basin of a function is a basin of the function with the cost value of the local minimum of the basin being higher than that of the current basin of the function. A lower basin of a function is a basin of the function with the cost value of the local minimum of the basin being lower than that of the current basin of the function.

Due to property (a), by evaluating the filled function at a point slightly deviated from the current local minimum of the original cost function, a lower filled function value can be obtained. Hence, the filled function could kick away from the current local minimum of the original cost function. Due to properties (b)-(d), the current local minimum of the filled function is neither in the current basin nor any higher basins of the original cost function. Hence, the current local minimum of the filled function is in a lower basin of the original cost function. As a result, by finding the next local minimum of the original cost function, i.e., searching the neighborhood around the current local minimum of the filled function, a better local minimum of the original cost function can be obtained. Following these procedures, if the original cost function contains a finite number of local minima [14], then the global minimum of the original cost function will be eventually reached.

III. PROBLEM FORMULATION AND MODIFIED FILLED FUNCTION METHOD

A. *Problem formulation*

Denote the transpose operator, the conjugate operator and the conjugate transpose operator as the superscripts ^{T}, * and $^+$, respectively, and the modulus operator as $|\cdot|$. Let the transfer functions of the lowpass and the highpass analysis filters of a two-channel linear phase FIR QMF bank be $H_0(z)$ and $H_1(z)$, respectively, and those of the synthesis filters of the filter bank be $F_0(z)$ and $F_1(z)$, respectively. Here, $H_0(z)$ is the transfer function of the prototype filter. Denote the impulse response of the prototype filter as $h(n)$, the passband and the stopband of the prototype filter as B_p and B_s, respectively, the length of the prototype filter as N, the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter as δ_p and δ_s, respectively, the specifications on the acceptable bounds on the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter as ε_p and ε_s, respectively, and the desired magnitude response of the prototype filter as $D(\omega)$. In this correspondence, it is assumed that the prototype filter is even length and symmetric. Let the polyphase components of $H_0(z)$ be $E_0(z^2)$ and $E_1(z^2)$, that is

$$H_0(z) \equiv E_0(z^2) + z^{-1}E_1(z^2). \tag{1}$$

Denote the transfer function of the filter bank as $T(z)$, the maximum amplitude distortion of the filter bank as δ_a, and the specification on the acceptable bound on the maximum amplitude distortion of the filter bank as ε_a. Let the vector containing these distortions and the even-time index filter coefficients be $\mathbf{x}$, that is

$$\mathbf{x} \equiv [\delta_a, \quad \delta_p, \quad \delta_s, \quad h(0), \quad h(2), \quad \cdots, \quad h(N-2)]^T. \tag{2}$$

In order to achieve both the aliasing free condition and the QMF pairs condition, the relationships among the analysis filters and the synthesis filters are governed by

$$H_1(z) = H_0(-z), \tag{3a}$$

$$F_0(z) = 2H_0(z) \tag{3b}$$

and

$$F_1(z) = -2H_0(-z). \tag{3c}$$

As the prototype filter is even length and symmetric, we have

$$H_0(z) = \sum_{n=0}^{\frac{N}{2}-1} h(2n)z^{-2n} + z^{-1}\sum_{n=0}^{\frac{N}{2}-1} h(2n)z^{-(N-2-2n)}, \tag{4}$$

$$E_0(z) = \sum_{n=0}^{\frac{N}{2}-1} h(2n)z^{-n}, \tag{5}$$

$$E_1(z) = \sum_{n=0}^{\frac{N}{2}-1} h(2n)z^{-\left(\frac{N}{2}-1-n\right)} = z^{-\left(\frac{N}{2}-1\right)}E_0(z^{-1}) \tag{6}$$

and

$$T(z) = 4z^{-1}E_0(z^2)E_1(z^2) = 4z^{-(N-1)}E_0(z^2)E_0(z^{-2}). \tag{7}$$

Denote

$$\boldsymbol{\eta}(\omega) \equiv \begin{bmatrix} 0, & 0, & 0, & 1, & e^{-j\omega}, & \cdots, & e^{-j\left(\frac{N}{2}-1\right)\omega} \end{bmatrix}^T, \tag{8}$$

then

$$T(\omega) = 4e^{-j\omega(N-1)}\mathbf{x}^T(\boldsymbol{\eta}(2\omega))^*(\boldsymbol{\eta}(2\omega))^T\mathbf{x}. \tag{9}$$

Obviously, the filter bank does not suffer from the phase distortion and the amplitude distortion of the filter bank can be expressed as $\left|4\mathbf{x}^T(\boldsymbol{\eta}(2\omega))^*(\boldsymbol{\eta}(2\omega))^T\mathbf{x}-1\right|$. Denote

$$\mathbf{Q}(\omega) \equiv 8(\boldsymbol{\eta}(2\omega))^*(\boldsymbol{\eta}(2\omega))^T, \tag{10}$$

then the amplitude distortion of the filter bank can be further expressed as $\left|\frac{1}{2}\mathbf{x}^T\mathbf{Q}(\omega)\mathbf{x}-1\right|$. Denote

$$\boldsymbol{\iota}_a \equiv [1, \quad 0, \quad \cdots, \quad 0]^T, \tag{11}$$

then the constraint on the maximum amplitude distortion of the filter bank can be expressed as

$$\frac{1}{2}\mathbf{x}^T\mathbf{Q}(\omega)\mathbf{x} - \boldsymbol{\iota}_a^T\mathbf{x} - 1 \le 0 \tag{12}$$

and

$$-\frac{1}{2}\mathbf{x}^T\mathbf{Q}(\omega)\mathbf{x}-\boldsymbol{\iota}_a^T\mathbf{x}+1\le 0 \quad \forall\,\omega\in[-\pi,\pi]. \tag{13}$$

Denote

$$\boldsymbol{\kappa}(\omega)\equiv 2\left[0,\quad 0,\quad 0,\quad \cos\left(\left(\frac{N-1}{2}\right)\omega\right),\quad \cos\left(\left(\frac{N-5}{2}\right)\omega\right),\quad \cdots,\quad \cos\left(\left(\frac{3-N}{2}\right)\omega\right)\right]^T, \tag{14}$$

then

$$\begin{aligned} H_0(\omega)&=(\boldsymbol{\eta}(2\omega))^T\mathbf{x}+e^{-j\omega(N-1)}(\boldsymbol{\eta}(2\omega))^+\mathbf{x} \\ &=e^{-j\omega\left(\frac{N-1}{2}\right)}\left(\left[0,\quad 0,\quad 0,\quad e^{j\left(\frac{N-1}{2}\right)\omega},\quad e^{j\left(\frac{N-5}{2}\right)\omega},\quad \cdots,\quad e^{-j\left(\frac{N-3}{2}\right)\omega}\right]\mathbf{x}+\left[0,\quad 0,\quad 0,\quad e^{-j\left(\frac{N-1}{2}\right)\omega},\quad e^{-j\left(\frac{N-5}{2}\right)\omega},\quad \cdots,\quad e^{j\left(\frac{N-3}{2}\right)\omega}\right]\mathbf{x}\right) \\ &=e^{-j\omega\left(\frac{N-1}{2}\right)}(\boldsymbol{\kappa}(\omega))^T\mathbf{x} \end{aligned}, \tag{15}$$

and the passband ripple magnitude of the prototype filter can be expressed as $\left|(\boldsymbol{\kappa}(\omega))^T\mathbf{x}-D(\omega)\right| \quad \forall\,\omega\in B_p$.

Define

$$\boldsymbol{\iota}_p\equiv[0,\quad 1,\quad 0,\quad \cdots,\quad 0]^T, \tag{16}$$

then the constraint on the maximum passband ripple magnitude of the prototype filter can be expressed as

$$\left|(\boldsymbol{\kappa}(\omega))^T\mathbf{x}-D(\omega)\right|\le\boldsymbol{\iota}_p^T\mathbf{x} \quad \forall\,\omega\in B_p. \tag{17}$$

Define

$$\mathbf{A}_p(\omega)\equiv[\boldsymbol{\kappa}(\omega)-\boldsymbol{\iota}_p,\quad -\boldsymbol{\kappa}(\omega)-\boldsymbol{\iota}_p]^T \tag{18}$$

and

$$\mathbf{c}_p(\omega)\equiv[D(\omega),\quad -D(\omega)]^T, \tag{19}$$

then the constraint on the maximum passband ripple magnitude of the prototype filter can be further expressed as

$$\mathbf{A}_p(\omega)\mathbf{x}-\mathbf{c}_p(\omega)\le\mathbf{0} \quad \forall\,\omega\in B_p. \tag{20}$$

Similarly, define

$$\boldsymbol{\iota}_s\equiv[0,\quad 0,\quad 1,\quad 0,\quad \cdots,\quad 0]^T, \tag{21}$$

$$\mathbf{A}_s(\omega)\equiv[\boldsymbol{\kappa}(\omega)-\boldsymbol{\iota}_s,\quad -\boldsymbol{\kappa}(\omega)-\boldsymbol{\iota}_s]^T \tag{22}$$

and

$$\mathbf{c}_s(\omega)\equiv[D(\omega),\quad -D(\omega)]^T, \tag{23}$$

then the constraint on the maximum stopband ripple magnitude of the prototype filter can be expressed as

$$\mathbf{A}_s(\omega)\mathbf{x}-\mathbf{c}_s(\omega)\le\mathbf{0} \quad \forall\,\omega\in B_s. \tag{24}$$

Define

$$\mathbf{A}_b\equiv[\mathbf{I},\quad \mathbf{0}] \tag{25}$$

and

$$\mathbf{c}_b\equiv[\varepsilon_a,\quad \varepsilon_p,\quad \varepsilon_s]^T, \tag{26}$$

in which $\mathbf{I}$ is the 3×3 identity matrix, then the specifications on the acceptable bounds on the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter can be expressed as

$$\mathbf{A}_b\mathbf{x}-\mathbf{c}_b\le\mathbf{0}. \tag{27}$$

In order to minimize a weighted sum of the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter subject to the specifications on these performances, the filter bank design problem is formulated as the following optimization problem:

Problem $(\mathbf{P})$

$$\min_{\mathbf{x}} \quad f(\mathbf{x})\equiv(\alpha\,\boldsymbol{\iota}_a+\beta\,\boldsymbol{\iota}_p+\gamma\,\boldsymbol{\iota}_s)^T\mathbf{x}, \tag{28a}$$

$$\text{subject to} \quad g_1(\mathbf{x},\omega)\equiv\frac{1}{2}\mathbf{x}^T\mathbf{Q}(\omega)\mathbf{x}-\boldsymbol{\iota}_a^T\mathbf{x}-1\le 0 \quad \forall\,\omega\in[-\pi,\pi], \tag{28b}$$

$$g_2(\mathbf{x},\omega)\equiv-\frac{1}{2}\mathbf{x}^T\mathbf{Q}(\omega)\mathbf{x}-\boldsymbol{\iota}_a^T\mathbf{x}+1\le 0 \quad \forall\,\omega\in[-\pi,\pi], \tag{28c}$$

$$g_3(\mathbf{x},\omega)\equiv\mathbf{A}_p(\omega)\mathbf{x}-\mathbf{c}_p(\omega)\le\mathbf{0} \quad \forall\,\omega\in B_p, \tag{28d}$$

$$g_4(\mathbf{x},\omega)\equiv \mathbf{A}_s(\omega)\mathbf{x}-\mathbf{c}_s(\omega)\le \mathbf{0} \quad \forall\omega\in B_s \tag{28e}$$

and
$$g_5(\mathbf{x})\equiv \mathbf{A}_b\mathbf{x}-\mathbf{c}_b\le \mathbf{0}, \tag{28f}$$

where α, β and γ are the weights of different criteria for formulating the cost function, $f(\mathbf{x})$ is the cost function, and $g_1(\mathbf{x},\omega)$, $g_2(\mathbf{x},\omega)$, $g_3(\mathbf{x},\omega)$, $g_4(\mathbf{x},\omega)$ and $g_5(\mathbf{x})$ are the constraint functions of the optimization problem.

As the set of the filter coefficients satisfying the constraints (28b) and (28c) is nonconvex, the optimization problem is a nonconvex optimization problem. In general, it is difficult to find the global minimum of a nonconvex optimization problem.

B. Modified filled function method

To find the global minimum of a nonconvex optimization problem, the following algorithm is proposed.

Algorithm

Step 1: Initialize a minimum improvement factor ε, an accepted error ε', an initial search point $\tilde{\mathbf{x}}_1$, a positive definite matrix $\mathbf{R}$ and an iteration index $k=1$.

Step 2: Find a local minimum of the following optimization Problem ($\mathbf{P}_f$) using our previous proposed integration approach with the initial search point $\tilde{\mathbf{x}}_k$ [13].

Problem ($\mathbf{P}_f$)

$$\min_{\mathbf{x}} \quad \text{(28a)},$$

subject to (28b)-(28f),

$$g_6(\mathbf{x})\equiv \boldsymbol{\iota}_a^T(\mathbf{x}-(1-\varepsilon)\tilde{\mathbf{x}}_k)\le 0, \tag{29a}$$

$$g_7(\mathbf{x})\equiv \boldsymbol{\iota}_p^T(\mathbf{x}-(1-\varepsilon)\tilde{\mathbf{x}}_k)\le 0 \tag{29b}$$

and
$$g_8(\mathbf{x})\equiv \boldsymbol{\iota}_s^T(\mathbf{x}-(1-\varepsilon)\tilde{\mathbf{x}}_k)\le 0, \tag{29c}$$

where $g_6(\mathbf{x})$, $g_7(\mathbf{x})$ and $g_8(\mathbf{x})$ are the constraint functions we imposed. Denote the obtained local minimum as $\mathbf{x}_k^*$.

Step 3: Find a local minimum of the following optimization Problem ($\mathbf{P}_H$) using our previous proposed integration approach with the initial search point $\mathbf{x}_k^*$ [13].

Problem ($\mathbf{P}_H$)

$$\min_{\mathbf{x}} \quad H(\mathbf{x})\equiv(\alpha\boldsymbol{\iota}_a+\beta\boldsymbol{\iota}_p+\gamma\boldsymbol{\iota}_s)^T\mathbf{x}+\frac{1}{(\mathbf{x}-\mathbf{x}_k^*)^T\mathbf{R}(\mathbf{x}-\mathbf{x}_k^*)}, \tag{30a}$$

subject to (28b)-(28f),

$$g_6'(\mathbf{x})\equiv \boldsymbol{\iota}_a^T(\mathbf{x}-(1-\varepsilon)\mathbf{x}_k^*)\le 0, \tag{30b}$$

$$g_7'(\mathbf{x})\equiv \boldsymbol{\iota}_p^T(\mathbf{x}-(1-\varepsilon)\mathbf{x}_k^*)\le 0 \tag{30c}$$

and
$$g_8'(\mathbf{x})\equiv \boldsymbol{\iota}_s^T(\mathbf{x}-(1-\varepsilon)\mathbf{x}_k^*)\le 0, \tag{30d}$$

where $H(\mathbf{x})$ is the filled function we defined, and $g_6'(\mathbf{x})$, $g_7'(\mathbf{x})$ and $g_8'(\mathbf{x})$ are the constraint functions we imposed. Denote the obtained local minimum as $\tilde{\mathbf{x}}_{k+1}$. Increment the value of k.

Step 4: Iterate Step 2 and Step 3 until

$$\left\|(\alpha\boldsymbol{\iota}_a+\beta\boldsymbol{\iota}_p+\gamma\boldsymbol{\iota}_s)^T(\mathbf{x}_k^*-\mathbf{x}_{k-1}^*)\right\|\le\varepsilon'. \tag{31}$$

Take the final vector of $\mathbf{x}_k^*$ as the global minimum of the original optimization problem.

Step 1 is an initialization of the proposed algorithm. In order not to terminate the algorithm when the convergence of the algorithm is slow and to have a high accuracy of the solution, both ε and ε' should be chosen as small values. Also, as $\tilde{\mathbf{x}}_1$ is an initial search point of the optimization algorithm, this initial search point should be in the feasible set. However, in general it is difficult to guarantee that $\tilde{\mathbf{x}}_1$ is in the feasible set, it should be chosen in such a way that most of the constraints are satisfied. Moreover, as $\mathbf{R}$ is a positive definite matrix, it controls the spread of the hill of $H(\mathbf{x})$ at $\mathbf{x}_k^*$. If $\mathbf{R}$ is a diagonal matrix with all diagonal elements being the same and positive, then large values of these diagonal elements will result to a wide spread of the hill of $H(\mathbf{x})$ at $\mathbf{x}_k^*$ and vice versa. Since the local minima of nonconvex optimization problems could be located very close together [14], the spread of the hill of $H(\mathbf{x})$ at $\mathbf{x}_k^*$ should be small and the diagonal elements of $\mathbf{R}$ should be chosen as small positive numbers. Step 2 is to find a local minimum of $f(\mathbf{x})$. As the constraints (29a)-(29c) are imposed on the Problem ($\mathbf{P}_f$), the maximum amplitude distortion of the filter bank, the maximum ripple magnitude and the maximum stopband ripple magnitude of the prototype filter corresponding to the new obtained local minimum are guaranteed to be lower than that corresponding to $\tilde{\mathbf{x}}_k$. Similarly, Step 3 is to find a local minimum of $H(\mathbf{x})$. As the constraints (30b)-(30d) are imposed on the Problem ($\mathbf{P}_H$), the maximum amplitude distortion of the filter bank, the maximum ripple magnitude and the maximum stopband ripple magnitude of the prototype filter are guaranteed to be lower than that corresponding to $\mathbf{x}_k^*$. Step 4 is a termination test procedure. If the difference of the weighted performance between two consecutive iterations is smaller than a certain bound ε', then the algorithm is terminated.

It has been discussed in Section I that conventional filled function methods require that (a) the current local minimum of the original cost function is the current local maximum of the filled function; (b) the whole current basin of the original cost function is a part of the current hill of the filled function; (c) the filled function has no stationary point in any higher basins of the original cost function; and (d) there exists a local minimum of the filled function which is in a lower basin of the original cost function. As $\mathbf{R}$ is a positive definite matrix and $\mathbf{x}_k^*$ is in the denominator of $H(\mathbf{x})$, $H(\mathbf{x}) \to +\infty$ as $\mathbf{x} \to \mathbf{x}_k^*$. Hence, $\mathbf{x}_k^*$ is the global maximum of $H(\mathbf{x})$ and property (a) is guaranteed to be satisfied. As the constraints (30b)-(30d) are imposed on the Problem ($\mathbf{P}_H$), when a new local minimum of $H(\mathbf{x})$ is found, this new local minimum of $H(\mathbf{x})$ will not be located at $\mathbf{x}_k^*$ and the original cost value evaluated at $\tilde{\mathbf{x}}_{k+1}$ will guarantee to be lower than that at $\mathbf{x}_k^*$. Hence, properties (b)-(d) are guaranteed to be satisfied. As a result, the proposed algorithm guarantees to reach the global minimum of the nonconvex optimization problem.

As the efficiency of general nonconvex optimization algorithms would depend on the initial search points, the total number of local minima of the optimization problems and the stopping criteria of the optimization algorithms, there is always a tradeoff between the accuracy of the obtained solutions and the efficiency of the optimization algorithms. For nongradient based approaches, as most of the evaluation points are ignored, the effectiveness of these algorithms is low. On the other hand, our proposed method guarantees to obtain the local minimum in each iteration, the effectiveness of our proposed algorithm is high. Hence, for the same period of time, our proposed method would obtain a better solution than that of nongradient based approaches.

IV. NUMERICAL COMPUTER SIMULATIONS

In order to have a fair comparison, the performance of the QMF banks designed via our proposed method is compared to that designed via the minimax approach discussed in [4]. We choose the same passband, stopband, filter length, maximum passband ripple magnitude, maximum stopband ripple magnitude and desirable magnitude response of the prototype filter as that in [4], that is

$$B_p = [-0.4\pi, 0.4\pi], \tag{32}$$

$$B_s = [0.6\pi, \pi] \cup [-\pi, -0.6\pi], \tag{33}$$

$$N = 36, \tag{34}$$

$$\varepsilon_p = -50\ \text{dB}, \tag{35}$$

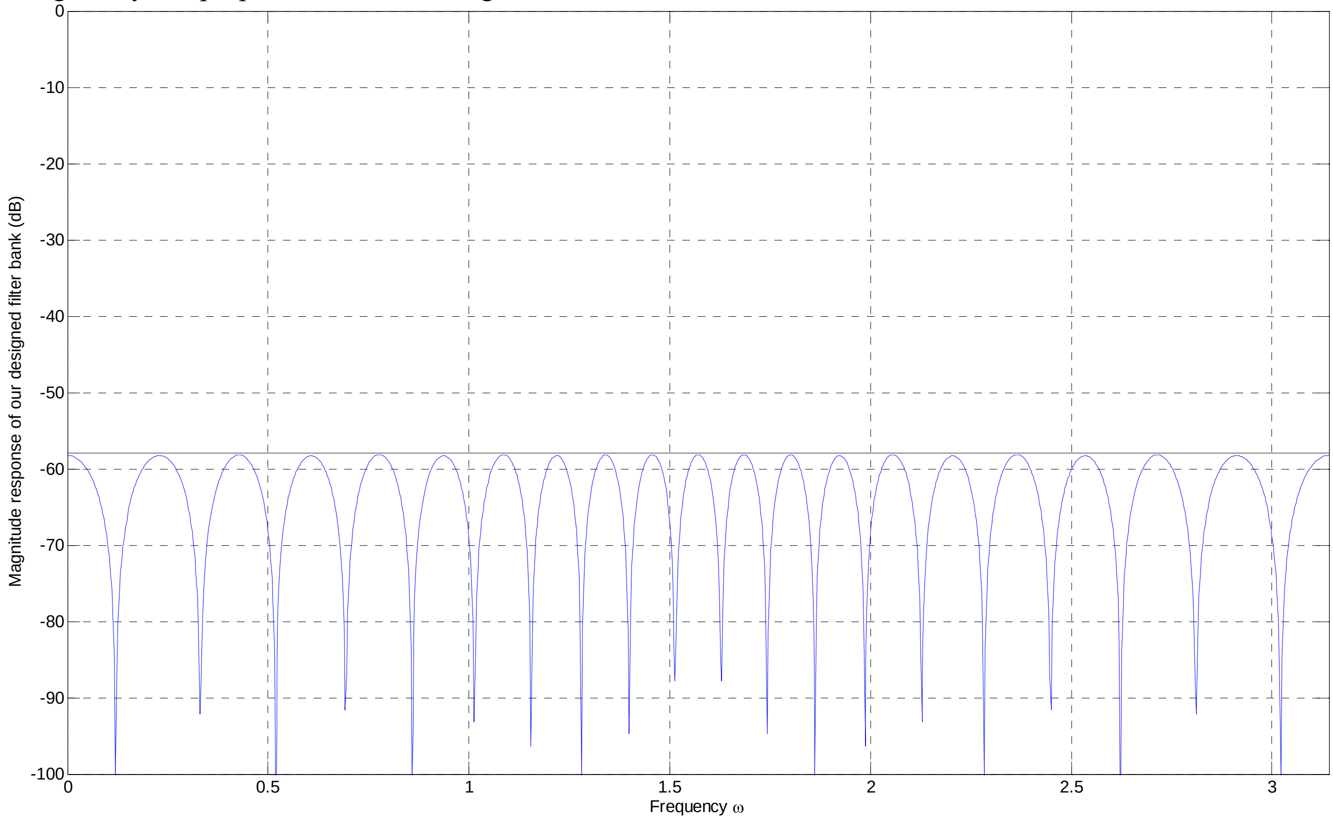
$$\varepsilon_s = -50\ \text{dB} \tag{36}$$

and

$$D(\omega) = \begin{cases} 1 & \omega \in B_p \\ 0 & \omega \in B_s \end{cases}. \tag{37}$$

In order to guarantee that the performance of the QMF bank designed via our proposed method is better than that in [4], the specification on the maximum amplitude distortion of the filter bank is chosen as $\varepsilon_a = -58$ dB, which is better than that in [4] ($\varepsilon_a = 0.003 = -50.4576$ dB). In order not to have any bias among the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter, all the weights in the cost function are chosen to be the same, that is $\alpha = \beta = \gamma = 1$. In this correspondence, $\varepsilon = \varepsilon' = 10^{-6}$ are chosen which is small enough for most applications. $\tilde{\mathbf{x}}_1$ is chosen as the filter coefficients obtained via the Remez exchange algorithm, which guarantee to satisfy the specifications on the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter. $\mathbf{R}$ is chosen as the diagonal matrix with all diagonal elements equal to 10^{-3}, which is small enough for most applications.

To compare the efficiency of the designed method, our proposed method only takes three iterations to converge and the total time required for the computer numerical simulations is 1.6 seconds. On the other hand, the method discussed in [4] takes 68 iterations to converge and the total time required for the computer numerical simulations is 80 seconds. Hence, it can be concluded that the method discussed in [4] requires more computational efforts than our proposed method and our proposed method is more efficient than that discussed in [4]. The magnitude responses of the filter banks as well as the magnitude responses of the prototype filters in both the passband and the stopband designed via our proposed method are shown in Figure 1. It can be seen from Figure 1 that the prototype filter designed by our proposed method could achieve $\delta_p = -64.2416$dB and $\delta_s = -50.3625$dB, and the QMF bank could achieve $\delta_a = -58.1557$dB. It can be checked easily that the QMF bank designed via our proposed method achieves better performances on the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude and the maximum stopband ripple magnitude ripple of the prototype filter than that designed by the method discussed in [4]. This is because the QMF bank designed by the method discussed in [4] is not the global minimum, while that designed by our proposed method is the global minimum.



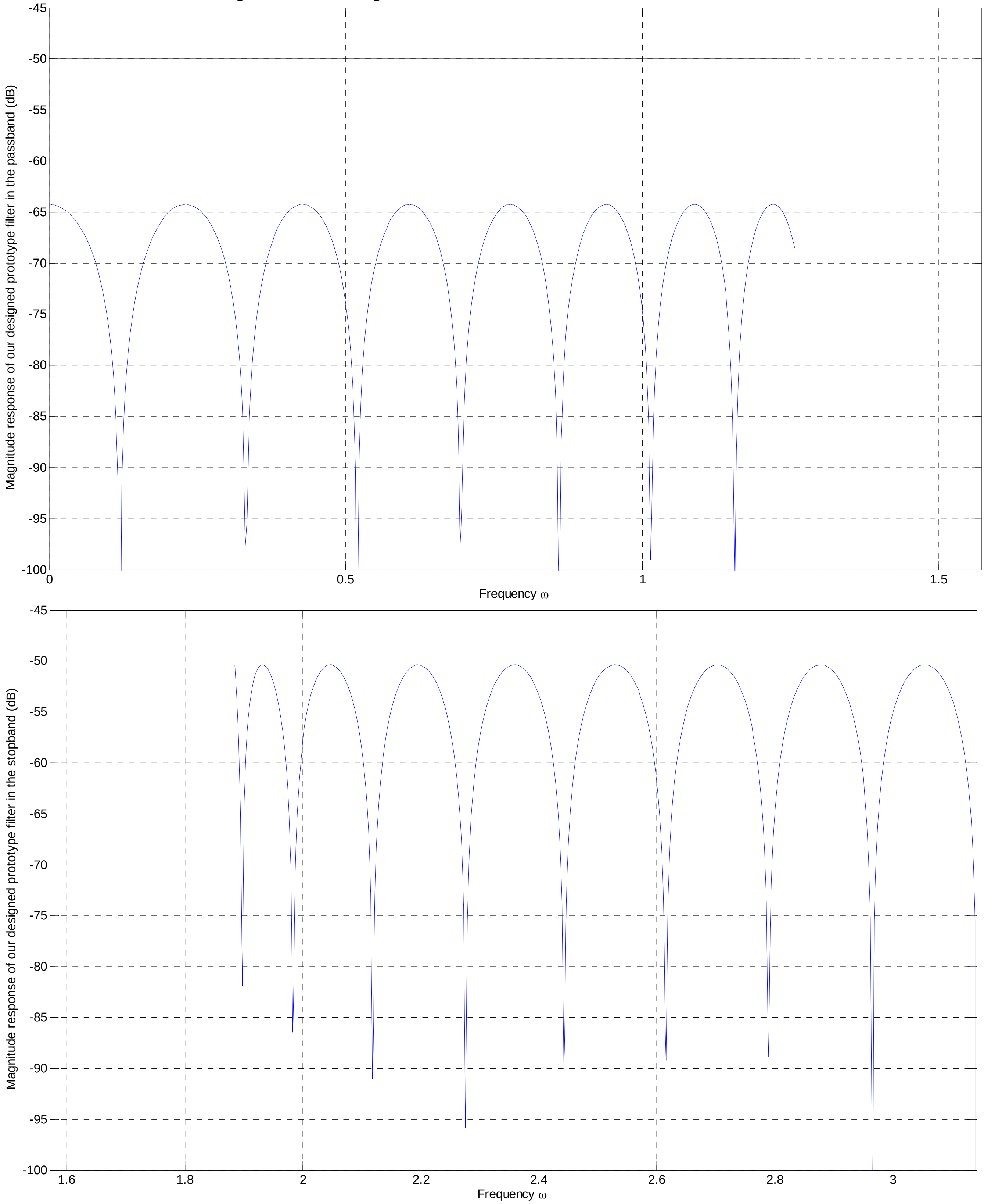


Figure 1. (a) Magnitude response of the filter bank. (b) Magnitude response of the prototype filter in the passband. (c) Magnitude response of the prototype filter in the stopband.

V. CONCLUSIONS

This correspondence proposes a modified filled function method for the design of a two-channel linear phase FIR QMF bank so that a weighted sum of the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter is minimized. The proposed method could find the global minimum of the nonconvex optimization problem efficiently.

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