Two Conjectures or how to win £1100

BRYAN THWAITES

Much as I would have liked to have accepted your Editor's invitation to reminisce over the last fifty years of personal educational activity – perhaps with special reference to the singularity of the early 1960s – it struck me that whatever I might write would be almost incomprehensibly irrelevant to the teacher in today's classroom. He, or she, seems to me to have a task infinitely more daunting in almost every respect than that which I faced when I was a schoolmaster in the 1950s, or when I was heavily engaged in the next decade with curriculum reform.

So instead, it occurred to me that at least some of the younger readers of the *Gazette* may not have come across a couple of conjectures which have fascinated and frustrated me – the first going back to 1951; the second much more recent. Both are easily understood and handled by the average ten-year-old, and so there will be some who will attribute my continuing interest in such apparently elementary mathematics to well-advanced senility. Be that as it may, I back up my interest with the offer of real money which, even in the days of the National Lottery, may encourage some worthwhile investigations in the classroom.

As to the provenance of the first, it may best be explained by the opening paragraphs of my survey published in the twenty-first anniversary issue of the *Bulletin of the Institute of Mathematics and its Applications*, of April 1985:

> ' At 4.00 p.m. on Monday, July 21st, 1952, seventeen clever, but not particularly mathematical, boys were probably wondering how their mathma don – as a mathematics teacher was called at the school in question – was going to amuse them during their last few periods with him. Those were happy days when, at least at that place, teaching continued until the last minute of the last hour of the last day of the last term of the year: examinations, such as they were, came and went without disturbing the steady rhythm of education and a boy could opt to do some serious mathematics even he had already obtained an Oxbridge scholarship in same other subject.
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> The don whom they were awaiting had already foreseen, in his aeronautical researches at the National Physical Laboratory and Imperial College, the potential of electronic computers, having used Hollerith punched-card machines in a pseudo-programmable sense rather than as pure tabulators of numerical values of formulae. He thought as he thinks now: that in the context of computers the dreams of the present are the reality of a decade or so later.

Naturally, he tried to pass on such thoughts to his pupils, and so in that July he wondered how to exemplify the ideas of iteration and binary trees. The tools then available were, of course, only pencil and paper (and log tables); so restriction to positive integers seemed sensible. He thus devised the following iterative algorithm: if a member of an iterative sequence of positive integers is even, then it is halved, otherwise it is multiplied first by an odd constant to which is added a second odd constant.

As it happened, this proved to be more than enough to keep the seventeen interestedly occupied for the week; for it was soon discovered that one particular pair of odd constants, 3 and 1, respectively, seemed to possess a unique property. Thus was *Thwaites Conjecture* postulated – it is believed for the first time, since it was only in the 1960s that it began to crop up apparently independently elsewhere and to attract titles such as *Collatz*, *Syracuse*, *Kakatuni*, *Ulam* and others.'

The unique property appeared to be that, no matter what integer was chosen to start with, the iteration always led to unity. And that is the challenge – to prove, or disprove – that that particular iteration reduces to unity for all positive integers. And the first person to succeed in that challenge receives one thousand pounds. Which being interpreted means that, in my view and after well over forty years of unsuccessful work, I have come to believe that it is unprovable.

The second conjecture is even simpler as far as numerical skill is concerned. I have been unable to trace the origin of it, though I have generalised what I first saw. It goes as follows:

'Take any set of N rational numbers. Form another set by taking the positive differences of successive members of the first set, the last such difference being formed from the last and first members of the original set. Iterate. Then in due course the set so formed will consist entirely of zeros if and only if N is a power of two.'

Although neither I, nor others who have been equally intrigued, have yet proved this, one's instinct is that here is a provable conjecture; and so the prize for the first successful proof, or disproof, is a mere hundred pounds.

Good luck!

(N.B. Solutions should be sent to Professor Thwaites.)

SIR BRYAN THWAITES

Milnthorpe, Winchester SO22 4NF

One, two, many

'The market for those aged over 19, unlike the 16-19 year olds, is almost infinite.' Val Sunderland, Assistant Principal, Calderdale College.

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