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Two-Dimensional Massless Quantum Electrodynamics in the Landau-Gauge Formalism and the Higgs Mechanism

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The Schwinger model is considered in the Landau-gauge formalism of quantum electrodynamics. This model can be solved exactly on the assumption of no radiative corrections to the anomaly. It is found that the photon obtains a non-zero mass through the Higgs mechanism. In this case, the would-be Nambu-Goldstone boson is an associated boson which is constructed from a pair of two-component massless fermions. This would-be Nambu-Goldstone boson appears as a result of the spontaneous breaking of the gauge invariance of the first kind, and it becomes unphysical through the Higgs mechanism. However, as all the fermions themselves decouple from photons, they cannot appear as real particles in our world.

§ 1. Introduction

Recently several ideas are proposed to explain why the constituent particles do not appear as real particles. The most fantastic idea is infrared shielding, i.e.; if there were an extremely long range force, it would bind the constituents and they could appear not as free particles but only as bound states. In this case, we can hope that the Yang-Mills fields play a role of a long-range force and also that the symmetry breaking occurs because such a dynamical system is very unstable owing to the existence of long range correlations. Then we expect that the gauge fields get a mass through the Higgs mechanism and the constituent particles cannot appear since the Nambu-Goldstone boson becomes unphysical through the Higgs mechanism.

In order to analyze the conjecture, we consider the two-dimensional massless QED (the Schwinger model). We can solve this model exactly and show explicitly that the gauge invariance of the first kind is broken spontaneously, and the Nambu-Goldstone boson appears as a bound state of the massless free fermions.

This paper is organized as follows: In § 2, we consider the Schwinger model in the Landau gauge. In § 3, we solve the Dirac equation for the electron in the gauge field. In § 4, we reconstruct the electromagnetic current from the fermion wave functions obtained in § 3. This technique is due to Lowenstein. In § 5, we construct the associated boson from the massless free fermions. This is just the Nambu-Goldstone boson and appears as a bound state of the fermions. In § 6, we discuss the construction of the Hilbert space and the representation of the field operators. We prove that the charge Q cannot annihilate the vacuum, that is, spontaneous symmetry breaking occurs. Conclusions are given in § 7.

§ 2. Two-dimensional QED

In this section, we deal with the two-dimensional quantum electrodynamics (QED) in the Landau gauge. The Lagrangian is

$$L=-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}+B\partial_\mu A^\mu+\bar{\Psi}(i\gamma^\mu\partial_\mu)\Psi-eJ^\mu A_\mu\,, \qquad (2\cdot1)$$

where Ψ is a massless fermion field operator, A_μ the photon field operator and B the Lagrange multiplier field operator.[1] The canonical variables are obtained in the usual manner, and we see B is a zero norm field operator. We must consider the theory in the indefinite-metric Hilbert space. The S-matrix has the usual properties when restricted to the physical subspace $\mathrm{H_{phys}}$, which is defined as follows:

$$|\mathrm{phys}\rangle \text{ is in } \mathrm{H_{phys}} \text{ if and only if } B^{(+)}(x)|\mathrm{phys}\rangle=0\,. \qquad (2\cdot2)$$

This is always possible because B satisfies a free field equation.

The commutation relations which we will use are as follows: (These relations can be derived from the canonical commutation relations. Details are shown in Ref. 1).)

$$[A_\mu(x),\ B(y)]=-i\partial_\mu{}^x D(x-y)\,, \qquad (2\cdot3)$$

$$[B(x),\ B(y)]=0, \qquad (2\cdot4)$$

$$[B(x),\ J_\mu(y)]=0\,, \qquad (2\cdot5)$$

where $D(x)$ is formally given by

$$D(x)=-i\int\frac{d^2p}{(2\pi)}\delta(p^2)\,\varepsilon(p^0)\exp(-ipx)$$

$$=-\frac{1}{2}\varepsilon(x^0)\,\theta(x^2). \qquad (2\cdot6)$$

The equations of motion are

$$\Box_x A_\mu(x)-\partial_\mu{}^x B(x)=eJ_\mu(x)\,, \qquad (2\cdot7)$$

$$(i\gamma^\mu\partial_\mu-e\gamma^\mu A_\mu(x))\Psi(x)=0\,. \qquad (2\cdot8)$$

The variation with respect to B leads to the following equation which ensures the Landau gauge for the photon;

$$\partial^\mu A_\mu=0\,. \qquad (2\cdot9)$$

We also have the conservation of the current from the gauge invariance

$$\partial^\mu J_\mu=0\,, \qquad (2\cdot10)$$

then, (2·7) together with (2·9) and (2·10) implies

$$\Box_x B(x) = 0 . \tag{2·11}$$

Now, the reason why the two-dimensional massless QED can be solved exactly is that there are no radiative corrections to the vertex part. This fact is easily confirmed since

$$\gamma^\mu \text{ (product of the odd number of } \gamma \text{ matrices) } \gamma_\mu = 0 .$$

Then the one-particle-irreducible part of the photon self-energy $\pi_{\mu\nu}(q)$ is given by the only one-fermion-loop diagram. As a result, we obtain

$$\pi_{\mu\nu}(q) = -(g_{\mu\nu}q^2 - q_\mu q_\nu)\left(\frac{e}{\sqrt{\pi}}\right)^2 \frac{1}{q^2 + i\varepsilon} . \tag{2·12}$$

This contribution is just that of the massless boson. This massless spectrum is to give the gauge field a mass. As we will prove later, the gauge invariance of the first kind is broken spontaneously; therefore this contribution is to be considered as that of the Nambu-Goldstone boson.

There are, however, no other contributions to the photon spectral function in this model, whence we can obtain the complete Green's function of the photon as follows:

$$\langle T(A_\mu(x) A_\nu(y))\rangle_0$$
$$= i\int \frac{d^2p}{(2\pi)^2} \exp(-ip(x-y))\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2 + i\varepsilon}\right)\frac{1}{p^2 - m^2 + i\varepsilon} , \tag{2·13}$$

where

$$m = e/\sqrt{\pi} . \tag{2·14}$$

Constructing the theory on the Landau gauge, we obtain the Green's function as that of the R-gauge massive vector particle.[2] The most important assumption is that A_μ is a free field operator because the two-point Green's function is that of free field operator. In general, this is not necessarily true since the indefinite metric is used.[3] If there is no indefinite metric, we can easily prove that such a field operator must be free, using the Federbush-Johnson theorem. In order to check the assumption, we must investigate all the photon Green's functions, and we will see the following probable counterexample (Fig. 1).

There may be other diagrams to cancel this contribution. Actually a very formal calculation shows that A_μ and J_μ satisfy the free field equations

$$\Box_x(\Box_x + m^2) A_\mu(x) = 0 ,$$
$$\Box_x(\Box_x + m^2) J_\mu(x) = 0 . \tag{2·15}$$

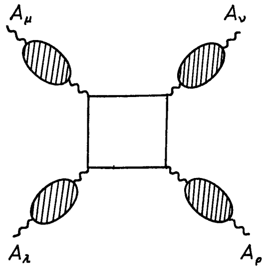

Fig. 1. The Feynman diagram which represents the probable counterexample in the four-point Green's function of the photon field operators.

Indeed if there are no radiative corrections to the Adler anomaly,[4] we have the following equation between the Heisenberg operators A_μ and J_μ:

$$\varepsilon^{\mu\nu}\partial_\mu(eJ_\nu(x)) = -\tfrac{1}{2}m^2\varepsilon^{\mu\nu}F_{\mu\nu}(x), \tag{2·16}$$

which together with (2·7), (2·9), (2·10) and (2·11) leads to Eq. (2·15). Even if A_μ is not free, the conclusions are still valid when we restrict ourselves to the Fock space of the massive gauge field. Actually if we define the asymptotic field $A_\mu^{\text{in(out)}}=$ weak-lim A_μ, the commutation relations between the A_μ's and B are equal to those between the $A_\mu^{\text{in(out)}}$'s and B. Following the discussion of Strocchi, we see that the difference between A_μ and $A_\mu^{\text{in(out)}}$ is zero at least in the charge zero sector. In the following, we assume A_μ is free, and A_μ and J_μ satisfies (2·15).

In this case, on introducing another auxiliary field $\tilde{\chi}$, we can write the following equations for A_μ.[2]

$$(\Box_x+m^2)A_\mu(x)-\partial_\mu\tilde{\chi}(x)=0\,, \tag{2·17}$$

$$\partial^\mu A_\mu(x)=0\,, \tag{2·18}$$

$$\Box_x\tilde{\chi}(x)=0\,, \tag{2·19}$$

where the commutation relations are

$$[\tilde{\chi}(x),\ \tilde{\chi}(y)]=-im^2D(x-y), \tag{2·20}$$

$$[A_\mu(x),\ \tilde{\chi}(y)]=-i\partial_\mu{}^xD(x-y). \tag{2·21}$$

A_μ can be divided into the Proca field U_μ and the massless ghost field $\tilde{\chi}$[6],[7]

$$A_\mu(x)=\frac{1}{m^2}\partial_\mu{}^x\tilde{\chi}(x)+U_\mu(x), \tag{2·22}$$

where the commutation relation of the Proca field is given by

$$[U_\mu(x),\ U_\nu(y)]=-i\left(g_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{m^2}\right)\Delta(x-y;\ m^2). \tag{2·23}$$

§ 3. Fermion wave function

We consider the Dirac equation (2·8) in this section. We first introduce the following notations:

$$x=(x^0,\ x^1),\quad u=x^0+x^1,\quad v=x^0-x^1\,,$$

$$p=(p^0,\ p^1),\quad A=(A_0,\ A_1),\quad A_\pm=A_0\pm A_1\,,$$

and γ-matrices are

$$\gamma^0=\gamma_0=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \gamma^1=-\gamma_1=\begin{pmatrix}0&1\\-1&0\end{pmatrix},\quad \gamma^5=\gamma^0\gamma^1=\begin{pmatrix}-1&0\\0&1\end{pmatrix}. \tag{3·1}$$

However, we must remark that there is an ambiguity in the definition of the massless vector field in two dimensions. Indeed on introducing the pseudo-scalar field $\tilde{\varphi}$, we can rewrite A_μ as follows:

$$A_\mu = U_\mu + \frac{1}{m^2}[\kappa\partial_\mu\tilde{\chi} + (1-\kappa)\,\varepsilon_{\mu\nu}\partial^\nu\tilde{\varphi}], \tag{3·2}$$

where $\tilde{\varphi}$ is defined by the requirement

$$\partial_\mu\tilde{\chi} = \varepsilon_{\mu\nu}\partial^\nu\tilde{\varphi}\,, \tag{3·3}$$

then the commutation relations are

$$\begin{aligned}[]
[\tilde{\varphi}(x),\ \tilde{\varphi}(y)] &= -im^2 D(x-y), \\
[\tilde{\varphi}(x),\ \tilde{\chi}(y)] &= im^2\tilde{D}(x-y),
\end{aligned} \tag{3·4}$$

where

$$\tilde{D}(x) = \tfrac{1}{2}\varepsilon(x^0)\,\theta(-x^2). \tag{3·5}$$

Of course, this ambiguity does not have influence on the representation of the operators A_μ, B and f, where f is a free fermion field operator.

Now, since the Dirac equation (2·8) becomes

$$\begin{pmatrix} 0 & 2i\partial_u - eA_+(u,\ v) \\ 2i\partial_v - eA_-(u,\ v) & 0 \end{pmatrix}\begin{pmatrix} \Psi_1(u,\ v) \\ \Psi_2(u,\ v)\end{pmatrix} = 0\,, \tag{3·6}$$

these equations can be easily solved on introducing the following new field operators:

$$\begin{aligned}
\phi_+ &= -\frac{2}{m^2}[\kappa\tilde{\chi} + (1-\kappa)\tilde{\varphi}] - \int U_+(u',\ v)\,du'\,, \\
\phi_- &= -\frac{2}{m^2}[\kappa\tilde{\chi} - (1-\kappa)\tilde{\varphi}] - \int U_-(u,\ v')\,dv'\,.
\end{aligned} \tag{3·7}$$

We define

$$\int U_{+(-)}du'(v') \equiv \tilde{U}_{+(-)}\,. \tag{3·8}$$

Because the Proca field obeys the equations

$$\begin{aligned}
\left(4\frac{\partial^2}{\partial u\partial v} + m^2\right)U_\mu &= 0\,, \\
\frac{\partial}{\partial u}U_- + \frac{\partial}{\partial v}U_+ &= 0\,,
\end{aligned} \tag{3·9}$$

we see*)

*) This solution was pointed out by N. Nakanishi.

$$\tilde{U}_{-} = -\tilde{U}_{+} = \frac{2}{m^2}\varepsilon^{\mu\nu}\partial_\mu U_\nu \equiv \frac{2}{m^2}\varphi \tag{3·10}$$

and

$$[\varphi(x),\ \varphi(y)] = im^2\Delta(x-y;\ m^2). \tag{3·11}$$

Thus Ψ is now parametrized by κ and given by

$$\Psi = \exp\left\{-\frac{i\sqrt{\pi}}{m}[\kappa\tilde{\chi} + \gamma^5[(1-\kappa)\tilde{\varphi} - \varphi]]\right\} f. \tag{3·12}$$

The parameter κ should de determined by the canonical anti-commutation relations and the canonical commutation relations, i.e., we require for the space-like two points x and y

$$\{\Psi(x),\ \Psi^*(y)\}_+ = 0, \tag{3·13}$$

$$[\Psi(x),\ B(y)] = 0, \tag{3·14}$$

$$[\Psi(x),\ A_\mu(y)] = 0. \tag{3·15}$$

Using the commutation relations cited above and (4·5), (4·6) and (6·2) which will be proved later, we see that (3·13) implies $\kappa=0$ or 1, and (3·14) and (3·15) imply $\kappa=0$ uniquely. Therefore we obtain

$$\Psi = \exp\left\{-\frac{i\sqrt{\pi}}{m}\gamma^5[\tilde{\varphi}^- - \varphi^-]\right\} f \exp\left\{-\frac{i\sqrt{\pi}}{m}\gamma^5[\tilde{\varphi}^+ - \varphi^+]\right\}, \tag{3·16}$$

where the indices $(-)$, $(+)$ denote the creation and annihilation parts of the operators respectively.*)

§ 4. Construction of the electromagnetic current

In this section, we construct the electromagnetic current using the solution obtained in the previous section. As we have mentioned there, we assume A_μ is a free field. Then we can construct the current J_μ following the method of Lowenstein.[5),9)] (At this point, we must remark that f and $\phi_\pm$ commute with each other. This is possible because $[B(x), f(y)]$ and $[\chi(x), f(y)]$ have the opposite signs to each other.)

We define $J^\mu(x) = N(\bar{\Psi}\gamma^\mu\Psi)(x)$. Then, as will be shown in the Appendix, we obtain

$$eJ^\mu(x) = e:\bar{f}\gamma^\mu f:(x) - \partial^\mu\tilde{\chi}(x) - m^2U^\mu(x). \tag{4·1}$$

*) This solution is consistent with the solution of Schwinger and Casher, Kogut and Susskind.[9)] Then as was pointed out by them, the mass singularity of fermion is absent from the vacuum expectation value of the time ordered scalar currents. In § 4, we will prove that the ghost field is a linear combination of the would-be Nambu-Goldstone boson χ and B. Then we can say that since the Nambu-Goldstone boson becomes unphysical through the Higgs mechanism, the resultant ghost field cancels out the mass singularity of fermion.

On the other hand, from (2·6) and (2·13), we have

$$eJ^\mu(x) = -\partial^\mu B(x) - m^2 U^\mu(x). \tag{4·2}$$

Therefore we must conclude

$$\tilde{\chi}(x) = \chi(x) + B(x), \tag{4·3}$$

$$e:\bar{f}\gamma^\mu f:(x) = \partial^\mu \chi(x). \tag{4·4}$$

The commutation relations (2·3), (2·4), (2·5) and (4·4) lead to the following relation:

$$[\chi(x),\ \chi(y)] = +im^2 D(x-y), \tag{4·5}$$

$$[\chi(x),\ B(y)] = -im^2 D(x-y). \tag{4·6}$$

These relations are of course consistent with (2·20). (4·6) implies that χ becomes an unphysical operator,[6),7)] and (4·4) implies that χ is essentially the associated boson, then we see that the associated boson becomes unphysical through the Higgs mechanism.

Finally we consider the electromagnetic current (4·2). We see the following relations using (2·4), (4·5) and (4·6):

$$[eJ_0(x),\ eJ_1(y)]_{\text{etc}} = +im^2 \partial_1{}^x \delta(x^1 - y^1), \tag{4·7}$$

$$[A_\mu(x),\ J_\nu(y)]_{\text{etc}} = 0\,. \tag{4·8}$$

The first equation is just the Schwinger term and equal to the result obtained by him. The second equation shows a more delicate problem. If the canonical quantization is done in QED, this commutation relation must be zero because J_μ does not explicitly include A_μ and its derivatives. Therefore this relation means that the canonical quantization is done successfully in the model.

§ 5. The associated boson and the Nambu-Goldstone boson

In order to analyze the Higgs mechanism, we construct the associated boson. Let f be the free fermion field. As is easily seen,

$$J^\mu_{\text{free}}(x) = :\bar{f}\gamma^\mu f:(x), \quad J^{\mu:5}_{\text{free}}(x) = :\bar{f}\gamma^5\gamma^\mu f:(x) = \varepsilon^\mu{}_\nu J^\nu_{\text{free}}(x),$$

$$\partial_\mu J^\mu_{\text{free}}(x) = \partial_\mu J^{\mu:5}_{\text{free}}(x) = 0\,,$$

then we see $\Box J^\mu_{\text{free}} = 0$.

These relations suggest that there is a scalar field operator $J(x)$ which satisfies $J^\mu_{\text{free}}(x) = \partial^\mu J(x)/\sqrt{\pi}$. We call J the associated boson. In fact as Kleiber pointed out,[11)] we can construct J from f and $\bar{f}$. This fact is essentially due to the special properties of the two-dimensional massless fermion.

The free massless fermion field at time zero is expanded as follows:

$$f(x^0 = 0,\ x^1) = \int \frac{dp^1}{\sqrt{2\pi p^0}} \exp(ip^1 x^1)\,(u(p^1)\,a(p^1) + v(p^1)\,b^*(-p^1)),$$

where

$$u(p^1)=\begin{pmatrix}p^1\theta(p^1)\\ p^1\theta(-p^1)\end{pmatrix},\quad v(p^1)=\begin{pmatrix}p^1\theta(-p^1)\\ p^1\theta(p^1)\end{pmatrix}, \tag{5·1}$$

and the commutation relations are

$$\{a(p^1),\ a^*(p'^1)\}_+=\delta(p^1-p'^1),$$
$$\{b(p^1),\ b^*(p'^1)\}_+=\delta(p^1-p'^1),$$
$$\text{others}=0\,. \tag{5·2}$$

The boson creation and annihilation operators $d^-(p^1)$ and $d^+(p^1)$ are now constructed by

$$d^+(p^1)=\int\frac{dq^1}{2\pi p^0\sqrt{|p^1+q^1||q^1|}}\{\theta(p^1):\Psi_1{}^*(q^1)\Psi_1(p^1+q^1):$$
$$+\theta(-p^1):\Psi_2{}^*(q^1)\Psi_2(q^1+p^1):\}, \tag{5·3}$$
$$d^-(p^1)=d^+(p^1)^*,$$

where

$$\Psi(p^1)=u(p^1)a(p^1)+v(p^1)b^*(-p^1). \tag{5·4}$$

$d^-(p^1)$ and $d^+(p^1)$ satisfy the usual commutation relations between the boson creation and annihilation operators

$$[d^-(p^1),\ d^-(p'^1)]=0\,,$$
$$[d^+(p^1),\ d^+(p'^1)]=0\,,$$
$$[d^+(p^1),\ d^-(p'^1)]=\delta(p^1-p'^1). \tag{5·5}$$

The associated boson J is expanded by using d^- and d^+ as follows:

$$J(x)=-i\int\frac{dp^1}{\sqrt{2\pi p^0}}(\exp(ipx)d^-(p^1)-\exp(-ipx)d^+(p^1)) \tag{5·6}$$

with $p=(p^0,p^1)=(|p^1|,p^1)$. On the other hand, the unphysical boson χ is given by

$$\chi(x)=\int\frac{mdp^1}{\sqrt{2\pi p^0}}(\exp(ipx)\beta^-(p^1)+\exp(-ipx)\beta^+(p^1)), \tag{5·7}$$

with $p=(p^0,p^1)=(|p^1|,p^1)$. Then $mJ(x)=\chi(x)$ implies

$$d^-(p^1)=-i\beta^-(p^1),\ d^+(p^1)=i\beta^+(p^1). \tag{5·8}$$

Since the symmetry is broken as will be proved in § 6, χ must be regarded as the Nambu-Goldstone boson. Then we can see explicitly that the associated boson becomes the Nambu-Goldstone boson and the unphysical particle through the Higgs mechanism.

The important comment is the meaning of the Wick product. Strictly speaking, we cannot define the Wick product without defining the vacuum. The Wick product $:\bar{f}\gamma^{\mu}f:$ is defined only on the fermion Fock space. Our analysis implies that the Fock space of the associated boson which can be embedded in the fermion Fock space becomes the subspace included in $\mathrm{H}_{\mathrm{unphys}}$. The details are shown in the following section.

We consider the associated boson as a bound state of the fermion, and the Nambu-Goldstone boson. The above relations show the detailed structure of the Higgs mechanism.

§ 6. Construction of the Hilbert space and the representation of the field operator

The fermion field operator Ψ must be defined on the Hilbert space $\mathrm{H}=\mathrm{H}_{\mathrm{phys}}\oplus\mathrm{H}_{\mathrm{unphys}}$. Then we consider the free fermion field f which appears in Ψ as the operator on H, not on the fermion Fock space. As we have shown, the associated boson can be constructed from f and $\bar{f}$, and does not commute with B. Then f is an unphysical operator on H. We can explicitly show the following relations:

$$\left[\frac{1}{m}\chi(x),\ f(y)\right]=+\sqrt{\pi}\,\{D(x-y)+\gamma^5\tilde{D}(x-y)\}f(y),$$

$$\left[\frac{1}{m}\chi(x),\ \bar{f}(y)\right]=-\sqrt{\pi}\,\bar{f}(y)\,\{D(x-y)-\gamma^5\tilde{D}(x-y)\}. \tag{6·1}$$

In addition to these relations, using (2·11), (4·6) and the fact that f is a free massless fermion field, we see that the following relations must hold:

$$\left[\frac{1}{m}B(x),\ f(y)\right]=-\sqrt{\pi}\,\{D(x-y)+\gamma^5\tilde{D}(x-y)\}f(y),$$

$$\left[\frac{1}{m}B(x),\ \bar{f}(y)\right]=+\sqrt{\pi}\,\bar{f}(y)\,\{D(x-y)-\gamma^5\tilde{D}(x-y)\}. \tag{6·2}$$

Then we can confirm that $[\chi(x)+B(x),\ f(y)]=0$, while f is an unphysical operator.

In the following, we discuss the Hilbert space H. As we assume A is free, A can be represented on the Fock-type space. In other words, $\mathrm{H}_{\mathrm{phys}}$ is constructed only by the Fock space of the Proca field and the B field operators.[3),11)] On the other hand, the unphysical subspace is more complicated; for example this contains the fermion Fock space but we cannot consider it as the Fock space in the usual sense. Actually, f does not commute with B. B and f can be represented on H, but the explicit form cannot be determined at this step. (See for example, Ref. 11).)

If the symmetry breaking does not occur, the charge Q can be represented on the Hilbert space which can be decomposed into the charge sectors H_q;

$$\mathrm{H}=\sum_q \oplus \mathrm{H}_q. \tag{6·3}$$

On H_q, Q is only the scalar multiplication operator. The vacuum must be contained in H_0, in other words, the charge must annihilate the vacuum. In our case, this is impossible. The charge is formally given by

$$Q=\lim_{L\to\infty}\int_{-L}^{L}(\Box A_0-\partial_0 B)\,dx^1. \tag{6·4}$$

The electromagnetic current has the zero mass spectrum, then Q cannot be well defined in general. Actually we can prove the following equation from (4·6):

$$\langle[Q,\ \chi(x)]\rangle_0=-im^2. \tag{6·5}$$

This implies that the gauge invariance of the first kind is spontaneously broken; thus we can regard χ as the Nambu-Goldstone boson. In this case, the Nambu-Goldstone boson appears as the associated boson.

§ 7. Conclusions

As we have shown in the preceding sections, the symmetry breaking occurs in two-dimensional massless QED. This is because the associated boson can be constructed from the fermions and directly becomes the Nambu-Goldstone boson. The photon gets a mass through the Higgs mechanism, and the Nambu-Goldstone boson becomes unphysical, while the free massless fermions f and $\bar{f}$ themselves become unphysical because the Nambu-Goldstone boson is a fusion of them rather than a bound state of them.

There is an ambiguity, however, in the definition of the massless vector operators in two dimensions. This ambiguity is determined by the canonical commutation relations between the fermion field operator Ψ and the other boson field operators B and A_μ. The resultant Ψ is consistent with the solution of Schwinger. Thus we can confirm the result of Casher, Kogut and Susskind that the massless spectrum of a fermion is absent from the vacuum expectation value of the time ordered scalar currents.

Finally, we would like to point out the following facts concerning the symmetry breakings in two dimensions. As was pointed out by Coleman,[13] there are untamable infrared singularities associated with the massless bosons in two dimensions. Then a sensible field theory in two dimensions cannot contain massless bosons. However, in the case of discrete symmetries, since the Nambu-Goldstone bosons do not appear (for example, as in the case of the two-dimensional Ising model),[14] symmetry breakings can take place. In our case, the Nambu-Goldstone boson becomes unphysical through the Higgs mechanism, and the physical subspace is constructed only by the Fock space of the massive

Proca field and the B field operators. Thus our theory does not contradict Coleman's criterion.

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Appendix

—Construction of the electromagnetic current—

In this section, we construct the electromagnetic current in a gauge invariant way. Now we have the solution of the Dirac equation as follows:

$$\Psi(x)=\mathscr{D}(x)f(x),$$

where

$$\mathscr{D}(x)=\begin{pmatrix}\exp\left(\frac{ie}{2}\phi_-(x)\right) & 0 \\ 0 & \exp\left(\frac{ie}{2}\phi_+(x)\right)\end{pmatrix}$$

and $f(x)$ is a free fermion field operator. This expression is very formal; we must consider the redefinition of the field operator. One way is given by Jaffe, and we must consider the new field operator $\Psi(x)\equiv:\Psi(x):$, where the Wick product : : is taken on the vacuum of the Hilbert space and not of the Fock space. However, in this paper, following Lowenstein, we define Ψ by (3·16). This is always possible because $\phi_\pm$ and f are free and commutable with each other.

We must construct a current which is free from the light-cone singularity and satisfies locality, covariance and gauge invariance. Following the method of Lowenstein, we can define the electromagnetic current $J^\mu(x)=\mathrm{N}(\overline{\Psi}\gamma^\mu\Psi)(x)$ by

$$\begin{aligned}\mathrm{N}(\overline{\Psi}\gamma^\mu\Psi)(x)=\lim \mathrm{N}(\varepsilon)\{&\overline{\Psi}(x+\varepsilon)\gamma^\mu\Psi(x)\\ &-\langle\overline{\Psi}(x+\varepsilon)\gamma^\mu\Psi(x)\rangle_0(1-ie\varepsilon_\nu A^\nu(x))\},\end{aligned}$$

where N is a normalization factor and $(1-ie\varepsilon_\nu A^\nu(x))$ is inserted to ensure the gauge invariance of the electromagnetic current. The calculation is straightforward because $\phi_\pm$ do commute with f, contrary to the case of the Thirring model, and the result is just that of § 4.

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