Two-dimensional periodic structures in a nonlinear resonator

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Received October 5, 1989; accepted January 16, 1990

The formation of a periodic transverse structure in the light field in a resonator consisting of mirrors and a thin layer of a nonlinear medium is considered. It is shown that, when the resonator length is a multiple of one half of the Talbot length, there exist periodic structures that are self-imaged under propagation through both the empty part of the resonator and the nonlinear medium. The influence of aperture losses is estimated. It is suggested that this type of a resonator may be useful for diode-array-pumped solid-state lasers.

1. INTRODUCTION

Transverse effects in nonlinear-optical devices are now being widely investigated. Recently experiments have demonstrated synchronization of a laser array by means of the Talbot effect,[1] spatial turbulence and self-organization in a nonlinear-optical system with two-dimensional feedback,[2] and transverse mode locking in a CO_2 laser.[3] The most interesting theoretically, in my opinion, is the development of analytical approaches; their importance can not be overestimated because they provide ways to analyze experimental situations and to test numerical codes.

A wide spectrum of approaches is being discussed. One approach considers the nonlinear coupling of the different transverse modes of an empty resonator (Gauss–Hermite or Gauss–Laguerre modes) and considers the patterns that arise as a result of transverse mode locking.[4,5] Another approach relies on the possibility of solving exactly some envelope equations, by means of inverse scattering transformation within a sufficiently long nonlinear medium, and adjusting their solutions at the boundaries and reflecting surfaces. This method describes the transverse effects in a passive bistable nonlinear interferometer as the formation of the solitary waves of the nonlinear Schrödinger equation.[6]

A third approach reduces the nonlinear diffusion equation that describes a semiconductor bistable interferometer[7] to a kicked dynamics problem. The input laser field is approximated by a set of equidistant δ spikes, so, for one spatial dimension, the integration becomes straightforward and the values of a bistable parameter in nearest-neighbor pixels are connected by a two-dimensional area-preserving map. A fourth approach lies in the approximation of the nonlinear medium as a thin layer, within which the diffraction is negligible. The other part of the resonator is considered to be empty, and the wave field there is transformed by a Fresnel–Kirchhoff integral. As a result, the dynamics of the transverse (and temporal) structure is computed by successively iterating a nonlinear local map (one- or two-dimensional) and a linear nonlocal map (generally speaking, of an infinite number of dimensions).[8] Historically, this method was developed in microwave radiophysics for a traveling-wave tube generator with delayed feedback (see Ref. 9 and references therein). Recently this method has been rediscovered[10] and has been applied to the passive bistable nonlinear interferometer, for which the occurrence of transverse period doublings and switching waves was predicted. A nonlinear resonator with a phase-conjugate mirror also has been modeled by this method.[11]

In Ref. 12 it was assumed that a wide-aperture (large Fresnel number), active optical resonator with a thin active element and nonlinear losses (due to generation of optical harmonics or stimulated light scattering) could be approximately described by a one-dimensional map. This provides a possibility of observing period-doubling behavior, which results in the formation of regular and chaotic transverse structures. It was assumed that such a description is valid only if the effective Fresnel number $ka^2/2\pi n L_R$ is sufficiently high [k is the wave number, L_R is the resonator length, n is the number of passes through the resonator (iterates of the map), a is the effective size of the transverse inhomogeneity of the wave field]. Fortunately, this restriction is not necessary for periodic transverse structures. Our goal is to show that when the resonator length is a multiple of one half of the Talbot length, the phenomenon of self-imaging of the periodic fields will still take place in the presence of a thin layer of an arbitrary nonlinear medium. The paper is organized as follows: In Section 2 the essential features of self-imaging are described, an exact expression for Talbot beam diffraction at a finite aperture is obtained, and its propagation through the thin nonlinear slice is considered. In Section 3 the problem of the efficiency of diode-array-pumped lasers is briefly discussed, and afterward a Talbot cavity with a thin nonlinear amplifying slice inside is proposed. In Section 4 the results are summarized.

2. TALBOT RESONATORS WITH A THIN NONLINEAR LAYER

In 1836 Talbot[13] viewed with a magnifying lens white light passing through an equidistant grating made by Fraunhofer. Talbot was intrigued by the fact that, when the focus of the lens was shifted gradually away from the grating, distinct color bands arose parallel to the lines of the grating. For a given position of the focus, these bands were of two colors that were always complementary, i.e., green and red or blue and yellow. When the lens was gradually shifted, each member of these pairs of complementary colors replaced the other periodically. He observed also that these bands became more distinct when the source size was decreased (white light became more spatially coherent). Sometime later Rayleigh[13] repeated Talbot's experiments and gave the correct explanation of this effect. His arguments will be illustrated below from a modern viewpoint.

Consider a two-dimensional grating at $z = 0$ that is formed by a set of a mutually coherent Gaussian beams emitted normally from the nodes of the rectangular grid (Fig. 1). After passing the so-called Fresnel length $L_{\mathrm{fr}} = ka^2/4\pi$ in the positive z direction, the transverse structure of such a wave pattern is highly distorted (a, introduced above, is now the effective width of the single Gaussian beam). At $z = L_{\mathrm{fr}}$ the overlapping of the beams becomes significant (because the period of the grating b is of the order of a), and strong interference occurs. The transverse structure in this plane is certainly regular and periodic, but it has no similarity to the field in the plane $z = 0$ (destructive interference). It seems obvious that the interference will become constructive when the optical path difference between different Huygens waves emitted by the nodes of a grating in the plane $z = 0$ to a given node in the plane z is a multiple of a wavelength λ. Really, this gives the distance $z = kb^2/4\pi$, where all partial waves are in phase. In this plane a perfect rectangular grid is formed by diffraction, but its period is one half that of the initial field.[14] The ideal reproduction of the initial field is obtained in the planes $z = L_t = l_t kb^2/\pi$, where l_t is an integer (see Fig. 1), although the optical-path difference is not a multiple of λ for any pair of Huygens waves. This formula obtained by Rayleigh for monochromatic light explains the periodicity and colors of the Talbot bands, because the period of reproduction L_t is inversely proportional to the wavelength.

Such behavior of a Talbot beam is quite different from the behavior of a high-order Hermite–Gaussian beam (TEM_{mn} mode). Although the transverse structure of a TEM_{mn} mode also takes the form of a rectangular grid, it is nonperiodic, and its flow in the z direction is monotonic rather than oscillatory.[15] Another free-space propagation mode is a Bessel beam, whose nondiffracting behavior resembles that of a Talbot beam. A Bessel beam also exhibits oscillations of its transverse structure along the z axis and requires an infinite aperture for unpertubed propagation.[16] The Talbot and Bessel beams are both examples of a unique phenomenon of self-imaging, which is discussed in Ref. 17.

Now Rayleigh's formula will be supported by evaluation of an exact Kirchhoff–Fresnel integral that includes finite-aperture effects. In the paraxial approximation, two-dimensional diffraction of the field $E(x, y, z)$ is described by

$$E(x, y, z) = \frac{ik}{2\pi z}\exp(ikz) \times \iint_{-\infty}^{+\infty} \exp\left\{\frac{ik}{2z}[(x'-x)^2 + (y'-y)^2]\right\} \times D(x', y')E(x', y', z=0)\mathrm{d}x'\mathrm{d}y', \quad (1)$$

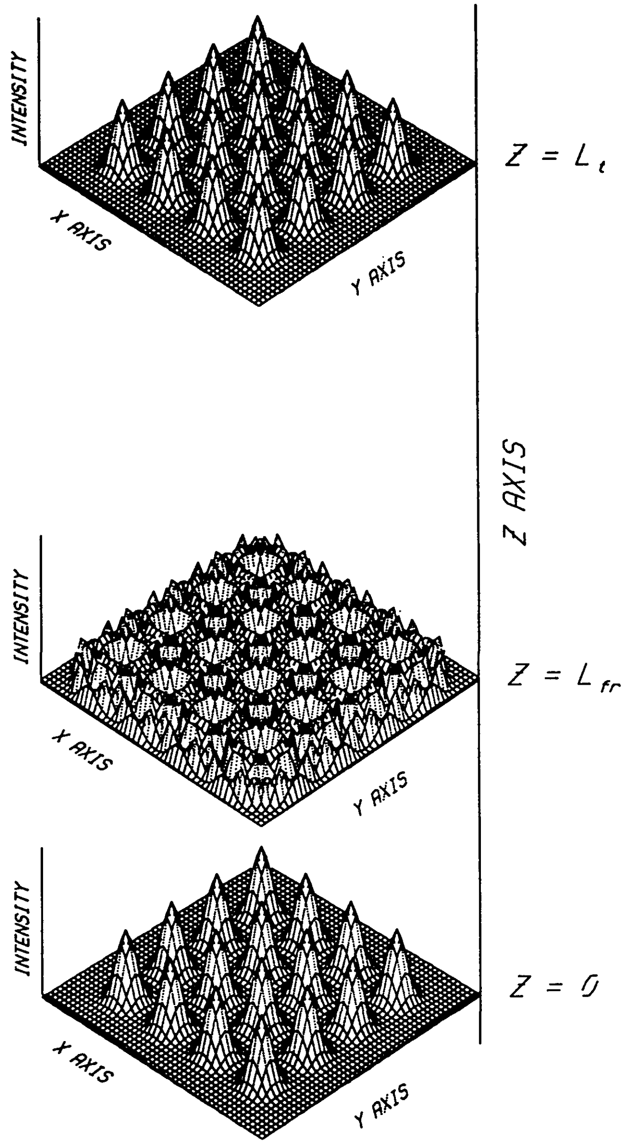

Fig. 1. Transverse profile destruction at $z = L_{\mathrm{fr}}$ and its reconstruction at $z = L_t$.

where z is the longitudinal distance, $D(x, y)$ describes the cutoff of $E(x, y, z = 0)$ by the aperture, which could be a steplike function (for a pinhole) or a smooth one (as in the present paper), and $E(x, y, z = 0)$ is the input laser field in the plane $z = 0$, which is assumed to be spatially periodic. The last-named expression could be expanded in a Fourier series of the form

$$E(x, y, z = 0) = \sum A_{mn} \exp[i2\pi(nx/b_x + my/b_y)], \quad (2)$$

where b_x and b_y are the periods on the x and y axes, respectively. The aperture function is assumed to have a Gaussian form: $D(x, y) = \exp(-x^2/2d_x^2 - y^2/2d_y^2)$. Therefore integral (1), when Eq. (2) is substituted, is easily calculated (because all of the components of the sum are of the Gaussian type):

$$E(x, y, z) = \exp(ikz) \sum A_{mn} \exp\left[i2\pi\left(\frac{nx}{b_x} + \frac{my}{b_y}\right)\right] \times \exp\left[-\frac{(2b_x n - x)^2}{(1 + iz/kd_x^2)2d_x^2} - \frac{(2b_y m - y)^2}{(1 + iz/kd_y^2)2d_y^2} + i2\pi \frac{\pi z}{k} \times \left(\frac{n^2}{b_x^2} + \frac{m^2}{b_y^2}\right)\right]\left[\left(1 + \frac{iz}{kd_x^2}\right)\left(1 + \frac{iz}{kd_y^2}\right)\right]^{-1/2}. \quad (3)$$

To understand this expression, consider first the infinite aperture limit $d_x = d_y = \infty$:

$$E(x, y, z) = \exp(ikz) \sum A_{mn} \exp\left[i2\pi\left(\frac{nx}{b_x} + \frac{my}{b_y}\right)\right] \times \exp\left[i2\pi \frac{\pi z}{k}\left(\frac{n^2}{b_x^2} + \frac{m^2}{b_y^2}\right)\right]. \quad (4)$$

This equation shows that if $b_x = lb_y = b$ and $\pi z/kb^2 = l_t$ (where l and l_t are integers), then the argument of the second exponent under the sum in Eq. (4) is a multiple of $i2\pi$. Therefore the field in the planes $z = L_t = l_t kb^2/\pi = 2l_t b^2/\lambda$ reproduces exactly the field in the plane $z = 0$ (Talbot effect; see Fig. 1). Note that this effect is mathematically due, first, to the infinite limits of integration in Eq. (1) and, second, to $D(x, y) =$ constant. Hence it is physically due to the infinite aperture. The finite aperture perturbations of the self-imaging are described by Eq. (3), which for a circular aperture ($d_x = d_y = d$) takes the form

$$E(x, y, z) = \exp(ikz) \sum A_{mn} \exp\left[i2\pi\left(\frac{nx}{b} + \frac{mly}{b}\right)\right] \times \exp\left[-\frac{(2bl_t n - x)^2 + (2bll_t m - y)^2}{2d^2(1 + il_t b^2/\pi d^2)}\right] \times [1 + il_t b^2/\pi d^2]^{-1}. \quad (5)$$

Here the denominator and the second exponent under the sum are responsible for perturbations. The denominators of Eq. (3) contain an expression kd^2/z that is equal to the Fresnel number of the whole aperture rather than of a single inhomogeneity of the transverse structure. When this number increases to a value close to unity, the self-imaged Talbot beam crashes. Similar behavior was observed with Bessel beams,[16] in which the unique multiple $(1 + z/kd^2)$ determines the decay of the transverse structure. In this paper consideration is limited to the case of large Fresnel numbers when $b/d = 0.1$. Thus the denominator has the value $1 + i \times 0.01$, and the aperture losses are relatively small.

In this approximation it is possible to neglect the finite-aperture effects and to consider the transformation of the periodic wave field in a thin nonlinear medium. The wave propagation through this medium is described by iterates of the local nonlinear map[8]:

$$E_{n+1}(x, y, z) = f[x, y, E_n(x, y, z)]. \quad (6)$$

An arbitrary nonlinear function $f = GE[1 - \tanh(\kappa LGE)]$ (Ref. 12) or $f = E_n \exp[ikL(n_0 + n_2|E_n|^2)]$ (Ref. 10) can be expanded in a power series of the form

$$f(E) = \alpha E + \beta E^2 + \epsilon E^3 + \ldots.$$

Thus the periodic field, transformed within the nonlinear layer, will be

$$f(E) = \alpha \sum A_{mn} \exp\left[i2\pi\left(\frac{nx}{b_x} + \frac{my}{b_y}\right)\right] + \beta \sum A_{mn}A_{ts} \exp\left[i2\pi\left(\frac{nx}{b_x} + \frac{my}{b_y} + \frac{tx}{b_x} + \frac{sy}{b_y}\right)\right] + \ldots. \quad (7)$$

We see that the local nonlinear transformation increases the number of spatial harmonics, whereas the periodicity of $E(x, y)$ under the above transformation is maintained. This is true if the coefficients α, β are x, y independent or if they are spatially inhomogeneous with the same period as $E(x, y)$. (In Section 3 a practical, interesting example of such an inhomogeneous nonlinear medium is considered.) Hence the field $f[E(x, y)]$ propagates once more through the empty part of the resonator and is self-imaged after one round trip. This fact was not mentioned in earlier publications[18] in which a set of nonlinear amplifying CO_2 tubes was analyzed with an assumption of constant distribution of a field in each tube. In the present paper it is shown that this restriction can be removed, at least for a thin, nonlinear layer.

3. SPATIALLY PERIODIC END-PUMPED SOLID-STATE LASERS

Diode-pumped solid-state lasers have demonstrated favorable performance characteristics. For example, an end-pumped Nd:YAG microlaser exhibits excellent optical and electrical efficiency that is due to perfect coincidence of the population-inversion distribution with the TEM_{00} mode.[19] Side pumping by diode arrays increases the total emitted power,[20] but it decreases the overall efficiency, and, what is particularly important, it tends to destroy TEM_{00} output. The reason lies in a significant mismatch between the population inversion and the resonator field. In my opinion, a possible way to match the profile of the inversion to the resonator mode shape would be to use a spatially periodic diode-array and end pumping (Fig. 2). The thin, nonlinear-medium layer (Nd:YAG, tetraphosphate lithium neodymium[16]) should have a spatially periodic distribution of inversion with a period, for example, $b = 200\ \mu$m, $L_t = 8$ cm, $\lambda = 1\ \mu$m, an effective size of inhomogeneity $a = 50\ \mu$m, and a size of aperture $d = 2000\ \mu$m. Then $b^2/d^2 = 0.01$ [see Eq. (5)], and diffraction losses are relatively small. Hence it is adequate to use a rough approximation, as if diffraction were absent[12] at the initial stage of the transverse structure formation. In Eq. (6) f corresponds to saturable gain that has the transverse profile of superimposed Gaussian spikes (Fig. 2).

An expression for f is deduced in the following way. Consider the thin layer of an amplifying medium near $z = 0$. Wave $E_-(x, y, z)$ penetrates the Nd layer through the antireflection-coated right-hand surface and is almost totally reflected at the left-hand surface, creating the wave $E_+(x, y, z)$ (Fig. 3a). The nonresonant absorption γ does not affect the propagation because of its low value in solid-state amplifi-

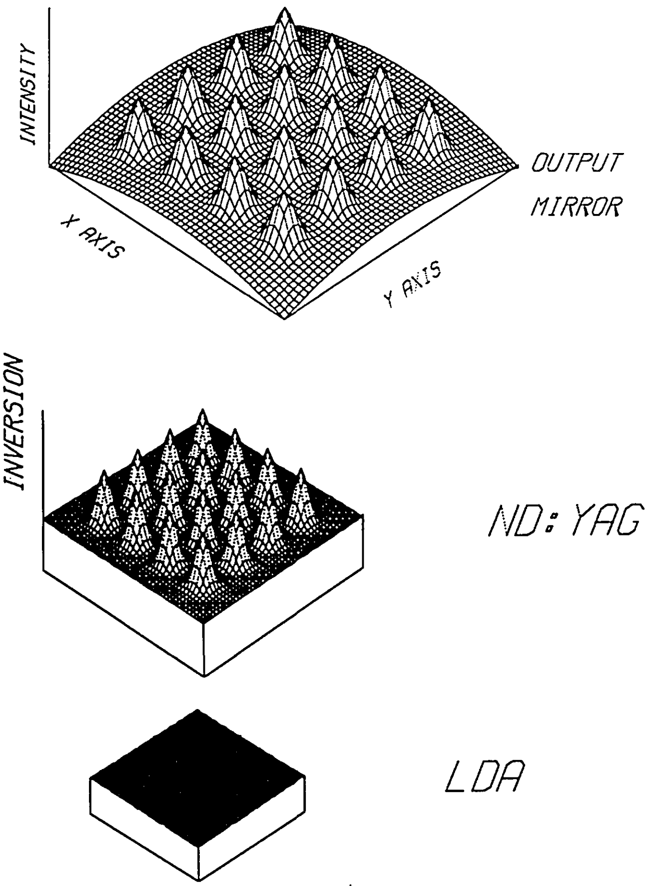

Fig. 2. Configuration of a resonator for a diode-pumped solid-state laser: LDA, laser diode array; Nd:YAG, active medium.

ers: $\exp(-\gamma z) = \exp(-0.001z)$. Since diffraction is assumed to be negligible in the active layer, the following equations for intensities $F(z) = E_+^2$ and $B(z) = E_-^2$ are valid:

$$\frac{dF}{dz} = \frac{\sigma NF}{1 + \sigma T_1(F + B)},$$

$$\frac{dB}{dz} = -\frac{\sigma NB}{1 + \sigma T_1(F + B)}, \tag{8}$$

where σ is the cross section for stimulated emission, N is initial inversion, and T_1 is the inversion lifetime.[21] The boundary conditions are (see Fig. 3a) $F(0) = R_b B(0)$, $F(L) = F$, $B(L) = B$. The first integral of this system is $BF = C =$ constant. After substitution, the variables are easily separated. The result is

$$F \exp[\sigma T_1(F - C/F)] = F(0)\exp\{\sigma T_1[F(0) - C/F(0)] + \sigma NL\},$$

$$B \exp[\sigma T_1(B - C/B)] = B(0)\exp\{\sigma T_1[B(0) - C/B(0)] - \sigma NL\},$$

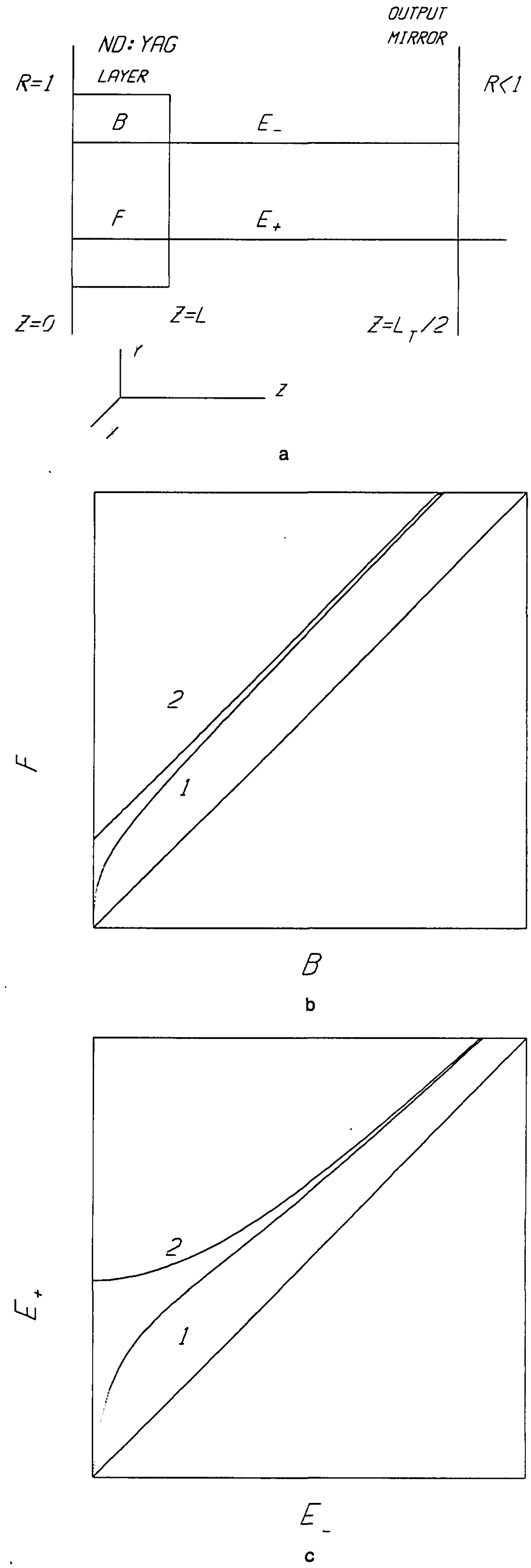

Fig. 3. a, Boundary conditions for calculation of counterpropagating fields $E_+(z), E_-(z)$. R_b and R_0 are the reflectances of the left and the output mirrors, respectively. b, Output intensity F versus 1, input intensity B; 2, its asymptote $F = B + NL/T_1$. c, Output field E_+ versus 1, input field E_-; 2, its asymptote $E_+ = E_-(1 + NL/T_1E_-^2)^{1/2}$.

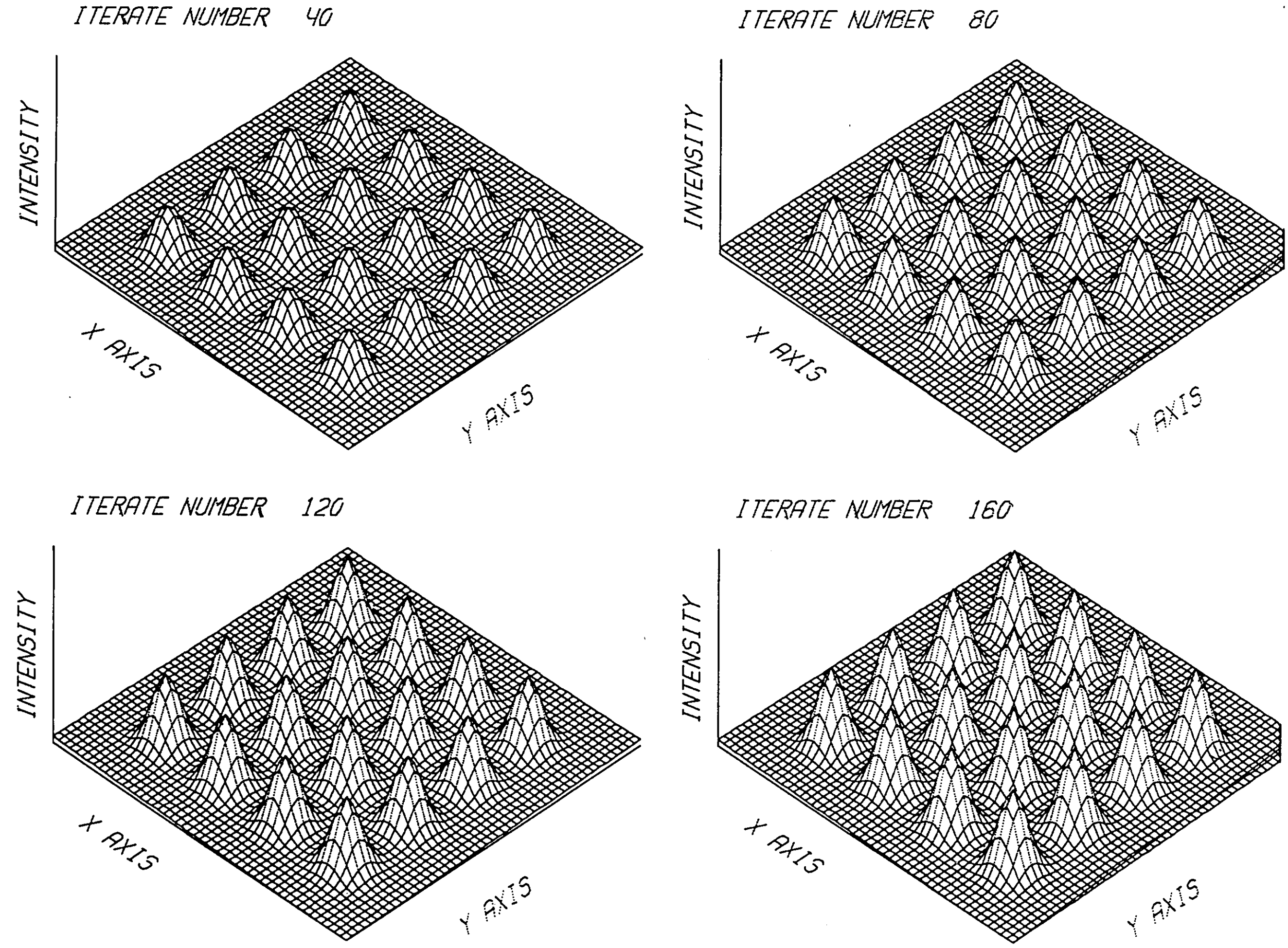

Fig. 4. Evolution of transverse structure in the resonator of microlaser (Fig. 2) calculated by successive iterations of the map [Eq. (10)] with a spatially periodic inversion profile (central 4 × 4 spikes).

where N is assumed to be z independent [otherwise σNL should be replaced on $\int_0^L \sigma N(z)\mathrm{d}z$]. The simplest solution obtained with the help of boundary conditions (when $R_b = 1$) is equivalent to Eq. (7) of Ref. 12:

$$F\exp(2\sigma T_1 F) = B\exp(2\sigma T_1 B + 2\sigma NL). \tag{9}$$

The solution of Eq. (9) is plotted in Fig. 3b. In contrast to the traveling-wave solution, there is a multiple 2 in the exponents (owing to simultaneous action of E_+^2 and E_-^2 on the inversion). This equation connects the intensities of the amplified (F) and incident (B) waves. The equation for E_+ and E_- is trivially obtained from Eq. (9), and its solution is plotted in Fig. 3c (this curve was obtained for an absorbing medium in Ref. 22). Now take into account that the forward E_+ and backward E_- waves are connected at the output mirror by the boundary condition $E_- = R_0^{1/2}E_+$.

The final step of this consideration is based on the following assumptions: (i) $E_+(x, y)$ is spatially periodic, (ii) R_0 is x, y independent, and (iii) the optical length of the resonator is one half of the Talbot length. Consequently, free-space propagation leads to the imaging of the amplified field E_+ $(z = L, x, y)$ into the incident field $E_-(z = L, x, y)R_0^{1/2}$. Denoting $E_n(x, y) = E_-(z = L, x, y)$, it is easy to obtain the following map, which connects amplitudes $E_n(x, y)$ from the nth pass to the $(n + 1)$th pass:

$$(E_{n+1}/R_0^{1/2})\exp(\sigma T_1 E_{n+1}^2/R_0) = E_n\exp(\sigma T_1 E_n^2 + \sigma NL). \tag{10}$$

Nonlinear propagation of the light field $E(x, y)$ inside the nonlinear resonator (Figs. 2 and 3a) had been modeled by iterates of the map [Eq. (10)] with a spatially periodic distribution of the inversion $N = N(x, y)$ (in the form of a rectangular grid; see Fig. 2). The initial conditions for E_n (x, y) were chosen in the form of a weak plane wave and a weak coherent ripple. The result of the iterates is presented in Fig. 4. The small-signal gain at the maxima of inversion was equal to 2, the highly saturated gain was equal to 1.2, and the amount of the total nonresonant losses per pass (mainly output mirror reflectance R_0) was 0.8. Although the model [Eq. (10)] is local, i.e., "two infinitesimally close points on the transverse structure may undergo uncorrelated temporal motions,"[23] it describes a physically reasonable distribution of the wave field. The latter is smooth owing to the smooth distribution of gain.

4. CONCLUSION

In the present paper a simple theoretical model, describing the formation of the periodic transverse modes, has being developed [Eq. (10)]. The main result is that in the infinite-aperture limit the nonlinear evolution of the transverse structure of a periodic diffracting field could be computed by iterates of a one-dimensional map. As was shown, there exist certain periodic transverse modes with a period determined by the distribution of the inversion. These modes in some region of the parameters are independent of the initial conditions. The above model gives an example of a system in which spatially coherent, temporally chaotic solutions are possible (for another example see Ref. 6).

The possible application discussed above is, in fact, a combination of earlier research on solid-state lasers with spatially periodic output mirror reflectance,[24] a radiating mirror-type semiconductor laser,[25] and synchronization of a laser array by means of the Talbot effect.[1] The idea of using spatially periodic end pumping could help to improve the efficiency of diode-array pumping. Recent experiments on phase locking of diode arrays[14] have revealed some pecularities of Talbot mode discrimination. An analysis of these effects, along with computer simulations, will be published elsewhere.

ACKNOWLEDGMENTS

The author is grateful to A. N. Oraevsky and V. A. Alekseev for useful debate and to M. S. Kurdoglyan and A. G. Molchanov for helpful discussions.

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