Two-dimensional tunable photonic crystals

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We call a photonic crystal tunable if its spectrum can be altered by an external electric or magnetic field. One of the two constitutive components of the proposed periodic composite structure has either electric permittivity or magnetic permeability dependent on the external electric or magnetic field. Consequently, the electromagnetic spectrum of the photonic crystal can be altered over a wide range by the external quasistationary uniform field. The tunable photonic crystal exhibits some useful features reminiscent of those accompanying the well-known electronic topological phase transitions in metals. Thorough theoretical analysis of a two-dimensional tetragonal periodical structure is undertaken. This specific periodic structure exhibits the most important features of a tunable photonic crystal. [S0163-1829(98)06305-X]

I. INTRODUCTION

The propagation of electromagnetic waves in periodic and disordered dielectrics has attracted much attention in recent years.[1–7] One of the remarkable features of periodic dielectric structures, often referred to as photonic crystals, is that they can have gaps (stop bands) in the frequency spectrum. That phenomenon is of great theoretical and practical importance and it can be employed in a variety of new optical devices (see, for instance, Refs. 1, 5, and 7). Until recently, the overwhelming majority of experimental and theoretical investigations on the electromagnetic properties of photonic crystals have dealt primarily with geometric aspects of the problem. More precisely, photonic crystals have been considered as composite structures made up of two lossless isotropic media with different refractive indices n_1 and n_2. Further specificity has been of purely geometrical nature: different shapes and dimensions of different fragments of the entire structure, different space symmetry, different kinds of local defects or partial disorder, etc. We demonstrate in this paper that dielectric materials with somewhat more complicated physical properties than simply losslessness can give more flexibility in the design of photonic crystals. Our focus will be on those photonic crystals whose characteristics can be controlled by a moderate external magnetic or electric field.

Let us consider a spatially periodic composite structure with at least one component displaying a nonlinearity in the electric or the magnetic susceptibility. If the amplitudes $E(t)$ and $H(t)$ of the electric and magnetic fields of the propagating electromagnetic (EM) wave are sufficiently small then the wave can be treated within linear approximation. In addition to that, if external uniform field $\mathbf{H}_0$ or $\mathbf{E}_0$ is strong enough, it may substantially alter the material tensors

$$\varepsilon=\varepsilon(\mathbf{E}_0,\mathbf{H}_0) \quad \text{or} \quad \mu=\mu(\mathbf{E}_0,\mathbf{H}_0) \tag{1}$$

and thereby alter the entire spectrum of the medium. We will use the subscript ''0'' to refer to external (controlling) field which is assumed to be strong enough to cause a nonlinear response according to Eq. (1).

With few exceptions, the magnitude of an applied external field must be much greater compared to the amplitude of the corresponding components of the propagating electromagnetic wave, i.e.,

$$E_0 \gg E(t) \quad \text{or} \quad H_0 \gg H(t). \tag{2}$$

In most cases the condition (2) must be imposed for EM wave propagation to be a linear problem whereas the material tensors ε and/or μ vary with the external field. In some cases though, the condition (2) is not required. For instance, for nearly static external field may cause substantial nonlinear response. At the same time the propagating EM wave with sufficiently high frequency can be treated within linear approximation even if its amplitude is comparable with the amplitude of the external field. This is especially likely if the main effect caused by an external quasistationary field $\mathbf{E}_0$ (or $\mathbf{H}_0$) reduces to a rearrangement of the domain structure in a thermodynamic equilibrium state. A similar effect may occur if the external field $\mathbf{E}_0$ (or $\mathbf{H}_0$) alternates thereby causing a resonant response of the medium. Indeed, in the resonant case there can be a pronounced nonlinear behavior even for a relatively small amplitude of the controlling field. If the frequency of the propagating wave is not resonant, it still can be treated within the linear theory.

It seems unlikely that one of the material tensors can be altered by an external field while the material remains essentially isotropic. Hence, the material tensors can be substantially anisotropic. In general, the controlling field may be time- and space-dependent and must be treated as an inseparable part of the electrodynamic problem. In view of the previous discussion on the relationships between the external fields and the propagating EM wave, one may consider the much simpler problem where $\mathbf{E}_0$ (or $\mathbf{H}_0$) is just a stationary or quasistationary parameter that alters the material tensors.

In the latter case evidently the physical nature of the controlling parameter does not play any significant role.

A controllable alteration of the photonic band structure may have numerous physical and practical aspects. We will focus primarily on a single and basic question: how does the external field affect the propagation of electromagnetic waves for a given fixed frequency Ω? At first glance, the only remarkable effect of tunability is the possibility of switching between transparent and opaque states, depending on whether the frequency Ω falls in a transmittance band or a photonic band gap. This appears to be true for the case of one-dimensional periodical structures. But for two- and three-dimensional periodicity more careful consideration shows that in addition to that there are other interesting phenomena. For instance, if the fixed frequency Ω was originally situated within a photonic band gap, then the gradual alteration of the photonic band structure caused by the external controlling field will result in at least two distinctive transitions accompanied by a dramatic modification of the character of electromagnetic wave propagation through the medium. To a certain degree, the corresponding transitions are similar to those well known in the theory of electronic topological phase transitions (see Ref. 8 and references therein). Since there are only few qualitatively different anomaly types in electromagnetic wave propagation, we can find a practical example that enables us to demonstrate all these interesting features altogether. In Sec. III such an example will be studied in great detail.

If the amplitude of the propagating electromagnetic wave is also too strong to be treated within linear approximation, some qualitatively new interesting effects can occur even in the case of one-dimensional periodical structures, see, for example, Refs. 9–12. Those questions are beyond the scope of our consideration.

The rest of the paper is organized as follows. The next section is devoted to a brief illustrative discussion of the materials that can can be used as constitutive components for tunable photonic crystals and their expected properties. Then we undertake an extensive theoretical analysis of the two-dimensional (2D) tetragonal structure.

II. MATERIALS FOR TUNABLE PHOTONIC CRYSTALS

There exist many dielectric materials with pronounced nonlinearity in the electric or magnetic properties. In particular, most of ferroelectrics display a substantial dependence $\varepsilon=\varepsilon(\mathbf{E}_0)$. On the other hand, magnetically ordered crystals, especially ferromagnets and ferrimagnets, are likely to manifest a magnetic nonlinearity $\mu=\mu(\mathbf{H}_0)$ even in a relatively low external magnetic field $\mathbf{H}_0$. Unless otherwise specified, we restrict ourselves to the case of the electric-field-dependent tensor $\varepsilon=\varepsilon(\mathbf{E}_0)$, having in mind that the entire consideration holds for the case of the controlling magnetic field as well. In spite of the formal mathematical similarity, the electric and magnetic cases may differ significantly. For the known lossless dielectric materials with strong dependence $\varepsilon(\mathbf{E}_0)$, the electromagnetic properties are substantially different from those of the media with ''magnetic-type'' nonlinearity like $\mu(\mathbf{H}_0)$, especially so if the appropriate frequency range is concerned. We study here primarily the general features of the tunable photonic crystals as described in the Introduction. Thus we do not focus on a particular material since there is a variety of different situations, and for a concrete problem with specified frequency range suitable materials can be selected to meet the requirements.

As we pointed out in the Introduction, one of the two constitutive components of the tunable photonic crystal must be made of a material with substantial nonlinearity of the electric permittivity $\varepsilon=\varepsilon(\mathbf{E}_0)$. Note that the overwhelming majority of ferroelectric materials also do meet this requirement. Hence, the main problem is how to find those dielectrics which, first, would be practically lossless in the given frequency range and, second, would manifest sufficiently high electric permittivity at that frequency range. For some materials the above restrictions may be critical for infrared or optical frequencies, but in the microwave range up to 10^{11} sec^{-1} there exist hundreds of dielectrics with satisfactory physical characteristics. Particularly, the most attractive would be a situation when a small shift in the impressed controlling field would lead to a significant alteration of ε in one of the two constitutive components. There are two conspicuous situations where such behavior should be expected. First, when the frequency ω of the propagating EM wave lies in a vicinity of a resonance frequency of the medium. The second situation occurs in a vicinity of a ferroelectric phase transition accompanied by a strong anomaly in the electric susceptibility (see, for example, Ref. 13 and references therein). On the other hand, in the vicinity of a phase transition or an electro-dipole resonance, the absorption effects may increase dramatically and this would be highly undesirable. In a sense, the above two situations are extreme and may not be of interest if only a relatively moderate band structure rearrangement is required.

Before we proceed further, let us briefly touch upon possible applications of magnetic materials as the active elements of tunable photonic devices. Most of the so-called soft ferromagnets and ferrimagnets display high magnetic permeability with tensor $\mu=\mu(\mathbf{H}_0)$ being strongly dependent on $\mathbf{H}_0$, and from this point of view they would be ideal materials for tunable photonic crystals. The problem is that magnetic susceptibility $\chi(\omega)$ of the common ferromagnets and ferrimagnets at high frequencies becomes very small. The frequency at which $\mu(\omega)$ drops significantly is usually much lower than that of $\varepsilon(\omega)$ and lies somewhere within the radio-frequency range. Another problem is that in the presence of the magnetic field $\mathbf{H}_0$, the temporal dispersion of magnetic susceptibility tensor may involve a substantial increase of the imaginary antisymmetric components like those responsible for Faraday rotation. This fact may significantly complicate the entire electromagnetic band structure. Nevertheless, the possible advantages of creating tunable photonic devices based on magnetically ordered materials are so attractive that it would certainly make sense to investigate thoroughly this issue. With these exceptions, most of the principal results of the following sections equally hold for both electric and magnetic-field controlled photonic crystals.

III. TWO DIMENSIONAL TETRAGONAL PHOTONIC CRYSTAL

Let us consider a photonic crystal with tetragonal symmetry and 2D periodicity as shown in Fig. 1. Suppose that the

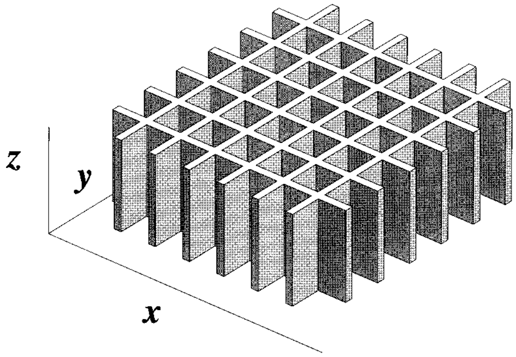

FIG. 1. A slab of tetragonal 2D photonic crystal composed of lossless dielectric material embedded in air background. Only the waves propagating in the xy plane are considered.

magnetic permeability μ of both constitutive components at the frequency range of interest is just the identity tensor $\mathbf{I}$, i.e.,

$$\mu=\mathbf{I}.$$

The electric permittivity tensor $\varepsilon=\varepsilon(\mathbf{r})$ is assumed to be real and position-dependent, it takes on two different values ε_1 and ε_2, since there are two constitutive components. In the absence of external field $\mathbf{E}_0$, both tensors ε_1 and ε_2 are assumed to be isotropic,

$$\text{For } \mathbf{E}_0=0: \quad \varepsilon_1=\varepsilon\mathbf{I}, \quad \varepsilon_2=\mathbf{I}. \tag{3}$$

For simplicity, the second constitutive component of the tetragonal structure in Fig. 1 is assumed to be void, therefore $\varepsilon_2=\mathbf{I}$ regardless of the external field. Space symmetry of this 2D periodic structure belongs to the tetragonal point group $4/mmm$.

The uniform electric field $\mathbf{E}_0$ applied along the z direction affects the tensor $\varepsilon_1=\varepsilon_1(\mathbf{E}_0)$ as follows:

$$\varepsilon_1=\begin{bmatrix}\varepsilon_\perp & 0 & 0\\ 0 & \varepsilon_\perp & 0\\ 0 & 0 & \varepsilon_\parallel\end{bmatrix}; \quad \varepsilon_2=\mathbf{I}, \tag{4}$$

where

$$\varepsilon_\perp=\varepsilon_\perp(\mathbf{E}_0), \quad \varepsilon_\parallel=\varepsilon_\parallel(\mathbf{E}_0); \quad \mathbf{E}_0\|\mathbf{z}. \tag{5}$$

Hence, $\varepsilon_1(\mathbf{E}_0)$ is not isotropic any more. Formally, the electric field $\mathbf{E}_0\|\mathbf{z}$ lowers the tetragonal space symmetry of the system down to $4mm$. But the actual effective symmetry of the macroscopic Maxwell equations remains $4/mmm$. The reason is that the only physical characteristic of the medium entering the macroscopic Maxwell equations is tensor $\varepsilon_1=\varepsilon_1(\mathbf{E}_0)$, which is always invariant under the space inversion operation—irrespective of the uniform applied field.[14] Therefore, the external field $\mathbf{E}_0\|\mathbf{z}$ will not further complicate the procedure of spectrum calculations. In particular, since in the case of the symmetry group $4/mmm$ (but not $4mm$!) the xy plane coincides with the mirror plane of the photonic crystal, EM modes with two different polarizations (E mode and H mode) will be independent and can be analyzed separately.

There exist a variety of dielectric materials in which dielectric constant can be altered substantially by an external

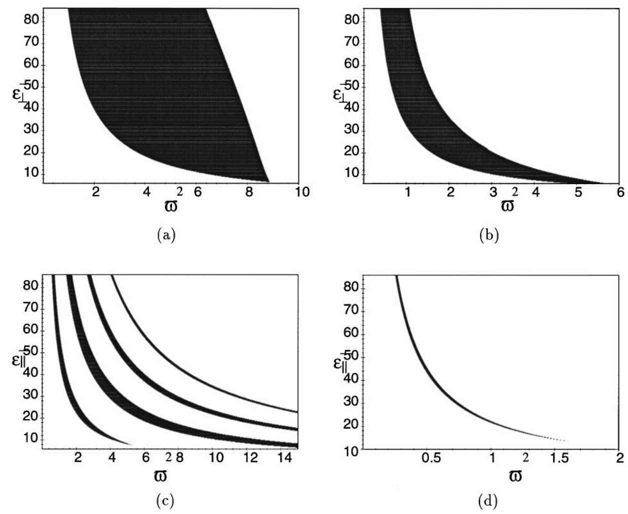

FIG. 2. Dependence of photonic gaps on the dielectric constant: (a) for H mode, $\delta=0.1$; (b) $\delta=0.3$; (c) for E mode $\delta=0.1$; (d) $\delta=0.3$.

electric field. The character of the corresponding dependence $\varepsilon_1(\mathbf{E}_0)$ may be qualitatively different for different types of dielectric media, as ferroelectric crystals vs centrosymmetric ones. Since the medium depends on the external field $\mathbf{E}_0$ parametrically, for the problems considered there is no need to present the explicit dependence of $\varepsilon_\perp$ and $\varepsilon_\parallel$ on $\mathbf{E}_0$. Instead, in further considerations we deal only with the dependence of different spectral characteristic on the quantities $\varepsilon_\perp$ and $\varepsilon_\parallel$.

Our treatment of lossless magnetic media is based on the classical Maxwell's equations

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \quad \nabla\cdot\mathbf{D}=0, \quad \mathbf{D}=\varepsilon\mathbf{E}, \tag{6}$$

$$\nabla\times\mathbf{H}=\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}, \quad \nabla\cdot\mathbf{B}=0, \quad \mathbf{B}=\mathbf{H}, \tag{7}$$

where $\mathbf{E}$, $\mathbf{D}$, $\mathbf{H}$, and $\mathbf{B}$ are the electric field and induction and the magnetic field and induction, respectively, and c is the velocity of light. In the two component 2D periodic medium we suppose μ and ε to be dependent only on x and y. The periodicity of the medium then is described by

$$\varepsilon(x+Lg_1,y+Lg_2)=\varepsilon(x,y), \tag{8}$$

where g_j are integers and L is the linear dimension of the square primitive cell of the tetragonal lattice $\mathbb{L}$. The axis z we will call the principal axis of the photonic crystal. The imposed external electric field $\mathbf{E}_0$ aligned along the principal axis z of the photonic crystal

$$\mathbf{E}_0=\begin{bmatrix}0\\0\\h\end{bmatrix} \tag{9}$$

alters the tensor $\varepsilon=\varepsilon(x,y;h)$, i.e., $\varepsilon_\perp=\varepsilon_\perp(x,y;h)$ and $\varepsilon_\parallel=\varepsilon_\parallel(x,y;h)$. In particular, the Maxwell equations (6) and (7) will depend on z component h of the external electric field $\mathbf{E}_0$.

We consider only the EM waves propagating perpendicularly to the principal axis z, which is equivalent to the assumption that the EM field depends only on x and y, i.e.,

$$\mathbf{H}=\mathbf{H}(x,y), \quad \nabla\cdot\mathbf{H}=0; \quad \mathbf{E}=\mathbf{E}(x,y), \quad \nabla\cdot\varepsilon\mathbf{E}=0. \tag{10}$$

Denoting

$$(x,y)=\mathbf{r}$$

and proceeding in the standard fashion, we introduce the harmonic in time fields

$$\mathbf{H}(\mathbf{r},t)=\mathbf{H}(x,y)e^{-i\omega t}, \quad \mathbf{E}(\mathbf{r},t)=E(x,y)e^{-i\omega t}, \tag{11}$$

$$\mathbf{H}=\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}, \quad \mathbf{E}=\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}, \tag{12}$$

and arrive at the following eigenvalue problem

$$\nabla\times\mathbf{E}(\mathbf{r})=\frac{i\omega}{c}\,\mathbf{H}(\mathbf{r}), \quad \nabla\times\mathbf{H}(\mathbf{r})=-\frac{i\omega}{c}\,\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}), \tag{13}$$

$$\nabla\cdot\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r})=0, \quad \nabla\cdot\mathbf{H}(\mathbf{r})=0. \tag{14}$$

The above equations are evidently reduced to

$$\nabla\times\nabla\times\mathbf{E}(\mathbf{r})=\frac{\omega^2}{c^2}\,\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}) \tag{15}$$

or

$$\nabla\times\varepsilon^{-1}(\mathbf{r})\nabla\times\mathbf{H}(\mathbf{r})=\frac{\omega^2}{c^2}\,\mathbf{H}(\mathbf{r}). \tag{16}$$

It is sufficient to analyze the spectrum of any one of the problems (15) or (16) for either **E** or **H** and then, the remaining one can be found by means of Eq. (13). Using standard symmetry arguments one can verify that the spectral problems (15) or (16) can be reduced to the analysis of two kind of modes: (i) E-polarized fields (or TM modes) when H_z

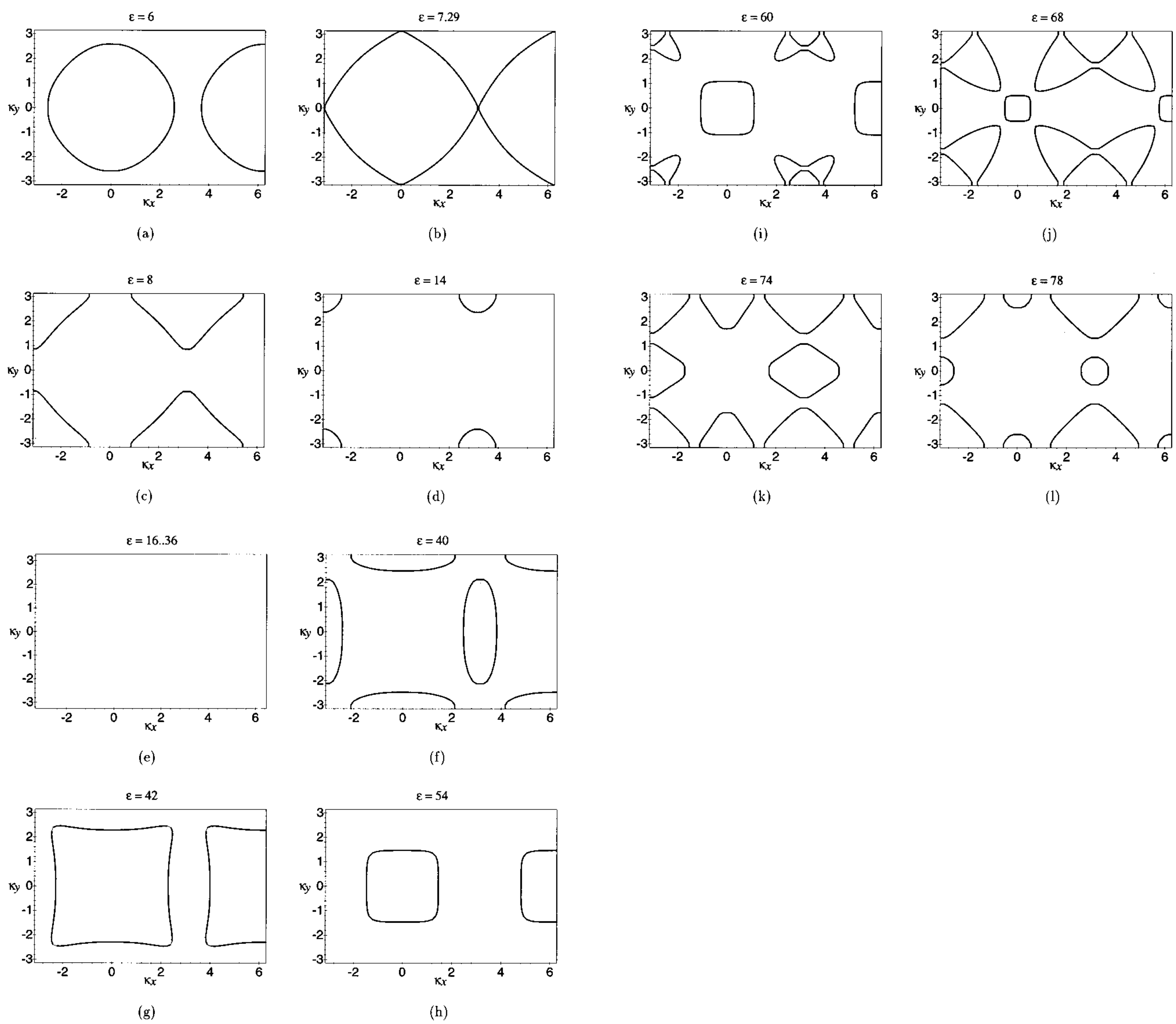

FIG. 3. Successive cross-sections of the H-mode spectrum arranged in ascending order of ε: the case (e) corresponds to the gap location of the frequency $\Omega=1.41$.

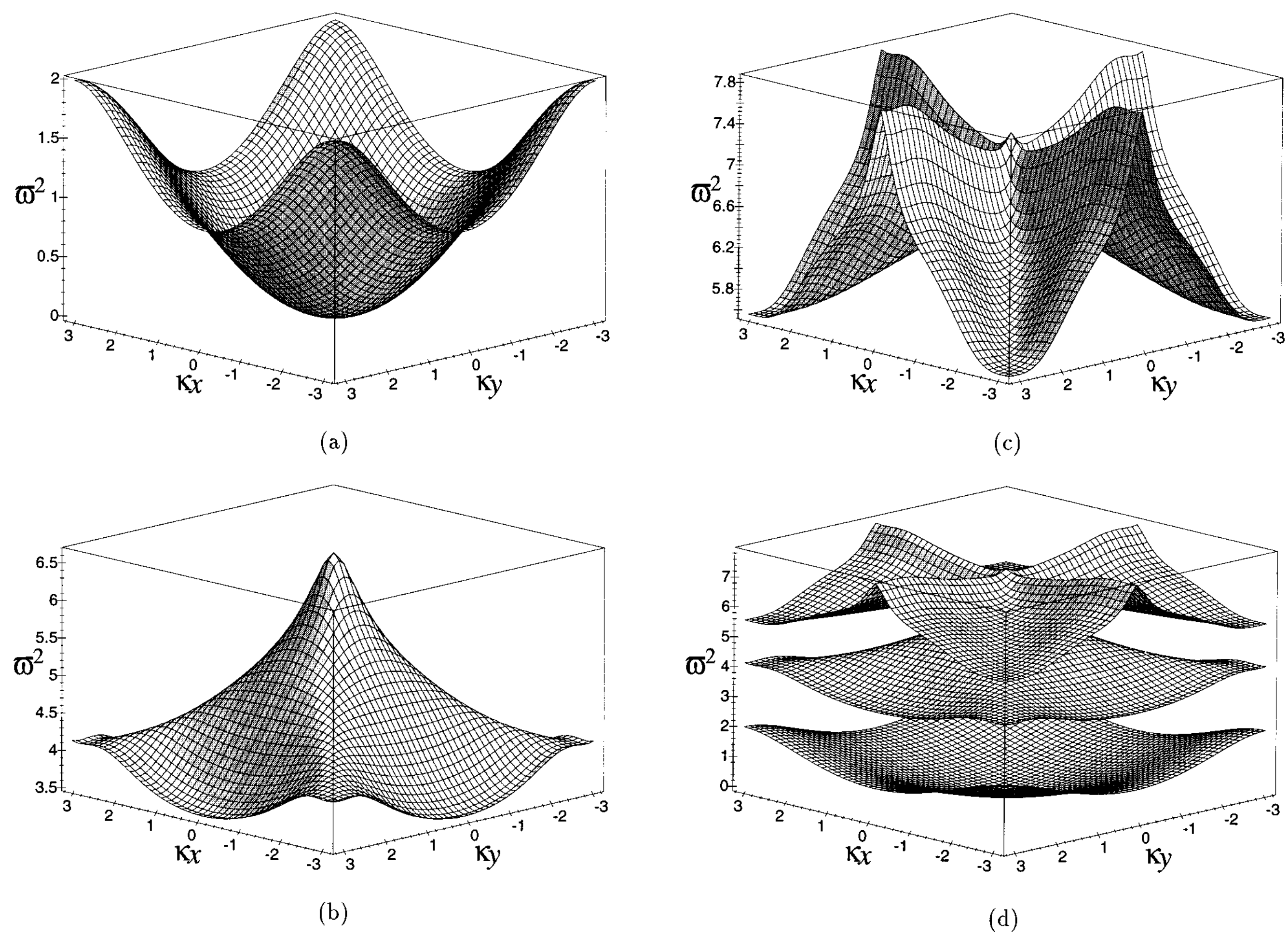

FIG. 4. Three lowest bands of the H-mode spectrum for $\varepsilon=16$: (a) $n=1$; (b) $n=2$; (c) $n=3$; (d) all three bands together. In this case the frequency $\Omega=1.41$ falls in the gap between the first and the second bands [see Fig. 3(e)].

$=0$ and $E_x=E_y=0$ and (ii) H-polarized fields (or TE modes) when $E_z=0$ and $H_x=H_y=0$. Namely, for E-polarized fields the following equation holds:

$$\mathbf{E}=\begin{bmatrix}0\\0\\E_z(\mathbf{r})\end{bmatrix},\quad \mathbf{H}=-\frac{ic}{\omega}\nabla\times\mathbf{E}=-\frac{ic}{\omega}\begin{bmatrix}\partial_y E_z(\mathbf{r})\\-\partial_x E_z(\mathbf{r})\\0\end{bmatrix},\tag{17}$$

whereas for H-polarized fields we have

$$\mathbf{H}=\begin{bmatrix}0\\0\\H_z(\mathbf{r})\end{bmatrix},\quad \mathbf{E}=\frac{ic}{\omega\varepsilon_\perp}\nabla\times\mathbf{H}=\frac{ic}{\omega\varepsilon_\perp}\begin{bmatrix}\partial_y H_z(\mathbf{r})\\-\partial_x H_z(\mathbf{r})\\0\end{bmatrix}.\tag{18}$$

Hence, each of the vector problems (15) and (16) is reduced to the following set of two scalar eigenvalue problems

$$-\Delta E_z(\mathbf{r})=\omega^2c^{-2}\varepsilon_\parallel(\mathbf{r})E_z(\mathbf{r}),\tag{19}$$

$$-\nabla\varepsilon_\perp^{-1}(\mathbf{r})\nabla H_z(\mathbf{r})=\omega^2c^{-2}H_z(\mathbf{r}).\tag{20}$$

Since $\varepsilon_\perp(\mathbf{r})$ and $\varepsilon_\parallel(\mathbf{r})$ are $\mathbb{L}$-periodic functions, we seek the eigenfunctions of the spectral problems (19) and (20) in the Bloch form

$$E_z(\mathbf{r})=e^{i\mathbf{kr}}E_\mathbf{k}(\mathbf{r}),\quad H_z(\mathbf{r})=e^{i\mathbf{kr}}H_\mathbf{k}(\mathbf{r});\quad \mathbf{k}=(k_x,k_y),\tag{21}$$

where $E_\mathbf{k}(\mathbf{r})$ and $H_\mathbf{k}(\mathbf{r})$ are $\mathbb{L}$-periodic functions. Then plugging them in Eqs. (19) and (20) we obtain

$$-[\partial_{x,k_x}^2+\partial_{y,k_y}^2]E_\mathbf{k}(\mathbf{r})=\omega^2c^{-2}\varepsilon_\parallel(\mathbf{r})E_\mathbf{k}(\mathbf{r}),\tag{22}$$

$$\partial_{j,k_j}=\partial_j-ik_j,\quad j=x,y;\tag{23}$$

$$-[\partial_{x,k_x}\varepsilon_\perp^{-1}(\mathbf{r})\partial_{x,k_x}+\partial_{y,k_y}\varepsilon_\perp^{-1}(\mathbf{r})\partial_{y,k_y}]H_\mathbf{k}(\mathbf{r})$$
$$=\omega^2c^{-2}H_\mathbf{k}(\mathbf{r}),\tag{24}$$

where $\mathbf{r}$ is in the primitive cell of the two-dimensional lattice $\mathbb{L}$ and $\mathbf{k}$ runs the primitive cell of the lattice $\mathbb{L}'$ dual to $\mathbb{L}$.

The mathematical properties of the eigenvalue problems (22) and (24) for square periodic geometries were thoroughly analyzed in Refs. 15 and 16 by analytical methods and then in Ref. 17 numerically. Based on those methods we carry out the computation for photonic crystals in the next section.

IV. COMPUTATION OF BANDS AND GAPS

In this section we carry out the computation of the spectral attributes of a tunable dielectric photonic crystal of square geometry as on Fig. 1. In this case, the lattice $\mathbb{L}$

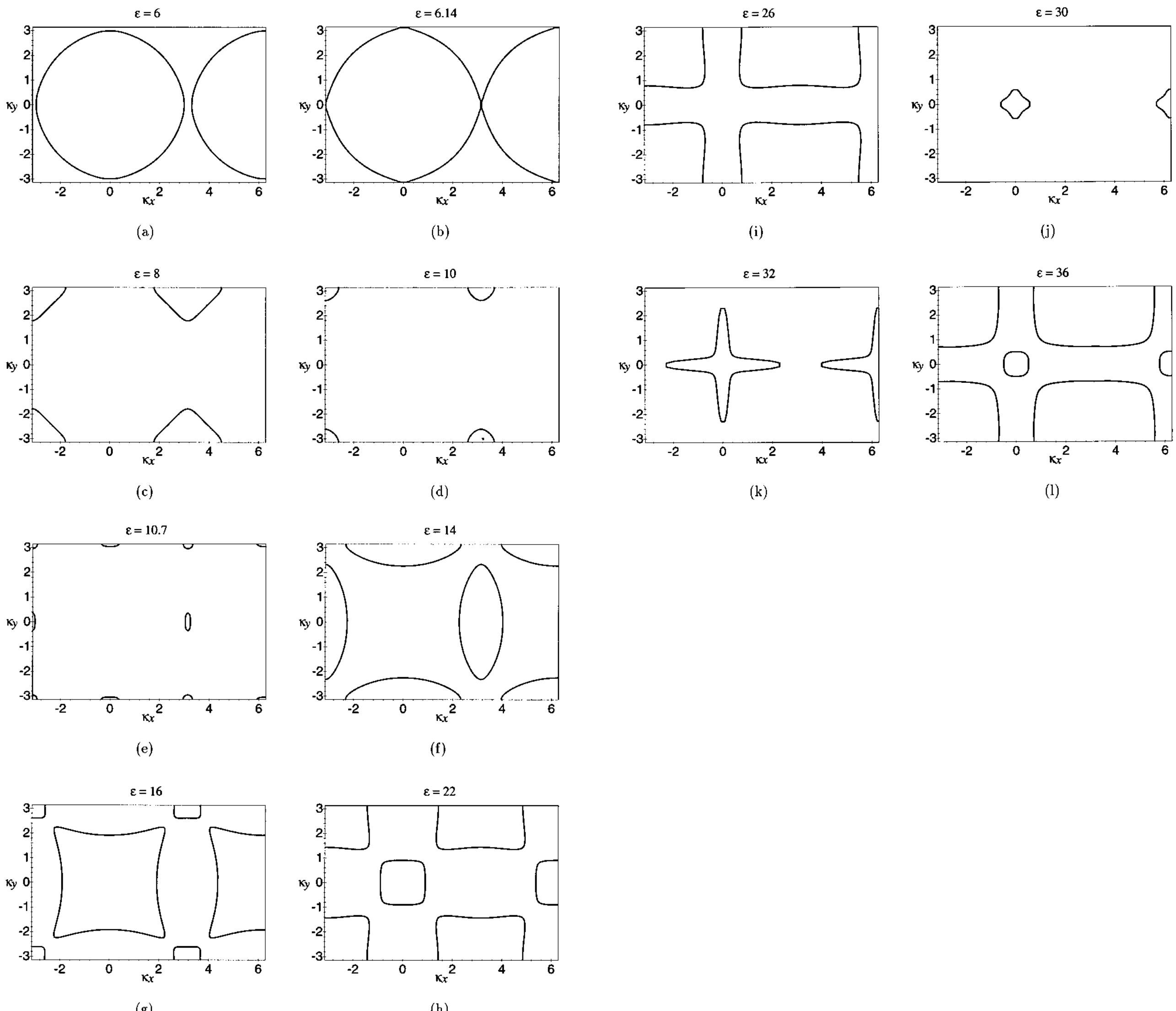

FIG. 5. Successive cross sections of the E-mode spectrum arranged in ascending order of ε.

$=L\mathbb{Z}^2$, where $\mathbb{Z}^2$ is the square lattice with the unit square primitive cell $[0,1]^2$ and L is the linear dimension of the square primitive cell $[0,L]^2$ of the photonic crystal. Then the problems (22) and (24) can be rewritten in the form

$$-(\partial^2_{x',m_{x'}}+\partial^2_{y,m_{y'}})E_{\mathbf{m}}(\boldsymbol{\rho})=\varpi^2\overline{\varepsilon_{\parallel}}(\boldsymbol{\rho})E_{\mathbf{m}}(\boldsymbol{\rho}), \tag{25}$$

$$-\left[\partial_{x',m_{x'}}\frac{1}{\overline{\varepsilon_{\perp}(\boldsymbol{\rho})}}\partial_{x',m_{x'}}+\partial_{y',m_{y'}}\frac{1}{\overline{\varepsilon_{\perp}(\boldsymbol{\rho})}}\partial_{y',m_{y'}}\right]H_{\mathbf{m}}(\boldsymbol{\rho})$$
$$=\varpi^2 H_{\mathbf{m}}(\boldsymbol{\rho}), \tag{26}$$

where $\boldsymbol{\rho}$, $\mathbf{m}$, ϖ, $\overline{\varepsilon_{\perp}}(\boldsymbol{\rho})$ and $\overline{\varepsilon_{\parallel}}(\boldsymbol{\rho})$ are dimensionless quantities defined by

$$\varpi^2=\omega^2\frac{\varepsilon_{\perp}(0)L^2}{c^2},\quad \boldsymbol{\rho}=(x',y')=\frac{\mathbf{r}}{L},$$

$$\mathbf{m}=(m_{x'},m_{y'})=L\mathbf{k}; \tag{27}$$

$$\overline{\varepsilon_{\perp}}(\boldsymbol{\rho})=\frac{\varepsilon_{\perp}(\boldsymbol{\rho})}{\varepsilon_{\perp}(0)},\quad \overline{\varepsilon_{\parallel}}(\boldsymbol{\rho})=\frac{\varepsilon_{\parallel}(\boldsymbol{\rho})}{\varepsilon_{\perp}(0)}. \tag{28}$$

Now in the problems (25) and (26) $\boldsymbol{\rho}$ runs the unit cell $[0,1]^2$ and $\mathbf{m}$ runs the cell $[-\pi,\pi]^2$. Solving the eigenvalue problems (25) and (26) we find the band dispersion relationships $\omega_n(\mathbf{k})$ where $n=1,2,\ldots$ is the band index. Dependence of photonic gaps on the dielectric constant is shown in Fig. 2.

V. GROUP VELOCITY ANOMALIES

Most of the qualitative results concerning different aspects of tunability equally apply to E and H modes. So, wherever it is appropriate, we will not specify the EM wave polarization.

Let us pick the frequency Ω so that in the absence of external field it falls in a gap of the electromagnetic spectrum $\omega_n(\mathbf{k})$. This implies that equation $\omega_n(\mathbf{k})=\Omega$ has no solutions for any $\mathbf{k}$ lying in the xy plane, i.e.,

$$\Omega\neq\omega_n(\mathbf{k})\quad\text{for any } n \text{ and } \mathbf{k}. \tag{29}$$

The external field can alter the entire electromagnetic spectrum including the location and the very existence of a particular spectral gap. As a consequence, the fixed frequency Ω can find itself within a neighboring *transmittance band* in which, by definition, the equation

$$\Omega = \omega_n(\mathbf{k}) \tag{30}$$

has a solution. The band number n may take on one or several values, depending on whether the equation $\Omega = \omega_n(\mathbf{k})$ has solutions for a single spectral branch, or for several of them simultaneously. Equation (30) defines the equifrequency curves in $\mathbf{k}$ space.

For a particular value of Ω and ε, the equifrequency curve represents the plane cross section of the appropriate band $\omega = \omega_n(\mathbf{k})$. In the case of overlapping bands the equifrequency curve is a superposition of individual contributions from each separate band n. This curve may comprise connected or disconnected pieces originated from the same or different bands. In the course of the spectral structure modification caused by alteration of the controllable parameter ε, the shape and connectivity of the equifrequency curve will change dramatically. In this section we find out what kind of modifications the equifrequency curve undergoes, and how these changes affect the conditions of EM wave propagation.

The best way to elucidate the whole picture is to start with a specific example. This example contains the characteristic features most of which persist in any kind of tunable photonic crystals. All those features develop even for the simplest situation, when in a transmittance band the equation (30) has a solution for a single spectral band. The more complicated cases of overlapping bands will not bring about essentially new features, rather we will simply have the superposition of the curves associated with different bands $\omega_n(\mathbf{k})$.

In our example we consider the situation when, in the course of band-structure modification caused by altering the controllable parameter ε, the fixed frequency Ω finds itself, first, in the lowest band. Then, the frequency Ω falls in the lowest gap situated between the first and the second bands. Finally, the gap moves down further and Ω falls in the second band.

In Fig. 3 the successive cross sections of $\omega_n(\mathbf{k})$ are arranged in ascending order of ε. The equifrequency curves presented in Figs. 3(a)–3(d) are defined by the equation $\Omega = \omega_1(\mathbf{k})$, and those in Figs. 3(f)–3(h) are defined by the equation $\Omega = \omega_2(\mathbf{k})$. The further cross sections involve additional bands.

In Fig. 4 we depicted the three lowest bands $\omega_n(\mathbf{k})$, $n = 1,2,3$ of the H-mode spectrum for $\varepsilon = 16$. In this case the frequency Ω falls in the gap located between the first and the second bands [see Fig. 3(e)].

For the case of E mode the corresponding set of cross sections of the first and the second bands is presented in Fig. 5.

Let start our discussion with the case of the H mode.

To understand the features of the EM wave propagation under the band-structure modification we study the group velocity

$$\mathbf{v}_n(\mathbf{k}) = \nabla_{\mathbf{k}} \omega_n(\mathbf{k}) \tag{31}$$

that characterizes speed and direction of wave propagation.

Let us start with a gap situation represented in Fig. 4 and Fig. 3(e). When controllable parameter ε decreases, the fixed frequency Ω finds itself in the first band. Under that transition from the opacity to the transmittance, EM properties of the photonic crystal will experience a dramatic transformation that can be traced in Figs. 3(e)–3(d). Let us consider those transformation in more detail.

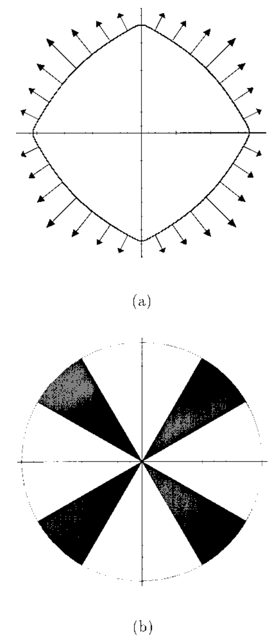

FIG. 6. The second critical point corresponding to that of Fig. 3(b): (a) the equifrequency curve; the arrows point in the directions of the group velocity; (b) the hatched sectors indicate the allowed directions of the EM wave propagation.

(1) Right after the transition from the opacity [Fig. 3(e)] to the transmittance [Fig. 3(d)] the group velocity $\mathbf{v}_1(\mathbf{k})$ is isotropic in the xy plane and has small magnitude. In general, a transition between opaque and transparent states occurs when Ω coincides with an absolute extreme value of the corresponding band $\omega_n(\mathbf{k})$. In our case *the first critical value* of ε is defined by the equation $\max_{\mathbf{k}} \omega_1(\mathbf{k};\varepsilon) = \Omega$.

(2) Further decrease of ε leads to the development of a strong anisotropy of the EM wave propagation that culminates at *the second critical point* [see Fig. 3(b) and Fig. 6]. This critical point occurs when Ω coincides with a saddle point of the band $\omega_1(\mathbf{k};\varepsilon)$. At this point the EM waves can only propagate within two narrow angles along the directions [110] and [110]. Indeed, as illustrated in Fig. 6, for all $\mathbf{k}$ on the equifrequency curve $\omega_1(\mathbf{k};\varepsilon) = \Omega$ the direction of the group velocity $\mathbf{v}_1(\mathbf{k})$ takes on only the values close to *just two directions*, namely, the directions of the square diagonals. An extreme anisotropy in the EM wave propagation exists not only in a vicinity of the second critical point but also at some other values of ε [see, for example, Fig. 3(g)].

(3) Further decrease of ε makes the condition of the EM wave propagation nearly isotropic [Fig. 3(a)].

(4) In addition to the two mentioned critical points associated with the two topological modifications of the equifrequency curve, there must exist another distinctive anomaly in the light propagation. Indeed, at some point of the spectrum transformation from Figs. 3(d)–3(a) a closed piece of the equifrequency curve must lose its convexity [as is the case with Fig. 3(c)] and then must gain it back. As a consequence, for some directions of the EM wave propagation there will be several waves propagating with different speed. In other words, there will be several vectors $\mathbf{v}_1(\mathbf{k})$ of the same direc-

tion but different magnitudes. And there will be at least two critical points at which this multivalence appears and vanishes.

Clearly, the total number of critical points associated with local extremes or saddle points of $\omega_n(\mathbf{k})$, as well as those associated with zero curvature points on the equifrequency curve $\omega_n(\mathbf{k})=\Omega$, may differ for different bands. But the described above critical points are always present.

Consider now the case of increasing parameter ε which is presented in Figs. 3(f)–3(l). The most significant difference compared to the case of decreasing ε is that right after the transition point between the gap and the second band, the EM wave propagation will be essentially anisotropic, and for each direction $\mathbf{v}_2(\mathbf{k})/|\mathbf{v}_2(\mathbf{k})|$ of the group velocity there will be two waves propagating with different speed $|\mathbf{v}_2(\mathbf{k})|$.

In the cases Figs. 3(i)–3(f) we have curves originated from both the second and the third bands.

Let us turn now to the case of the E mode. The set of the spectral cross sections presented in Fig. 5 looks very similar to that of the H mode in Fig. 3. The only important difference is that for the chosen values of Ω and δ, in the course of increasing ε the frequency Ω moves from the first band directly to the second one without being in the gap. Indeed, in Fig. 5(e) one can see the instant of a slight overlapping of the two adjacent bands. The gap opens at $\varepsilon > 15$ when the fixed frequency Ω is already far away in the second band. Note that the rest of the anomalies in the EM wave propagation remains in place.

VI. CONCLUSIONS

We have studied the basic properties of 2D tunable photonic crystals. It has been shown that the transition between opaque and transparent states of the system inevitably involves some distinctive anomalies, including extreme anisotropy of EM properties. Since the analysis of the tunability effects has been based on the precise band-structure calculations, the predicted anomalies can be unambiguously identified experimentally. Most of the qualitative aspects of the tunability effects can be extended, we believe, to the case of 3D photonic crystals.

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