Two-element free-electron lasers

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The interaction between the electrons and the radiation in a free-electron laser leads to a shift and a spread of the electron velocity distribution. The electron dynamics of a two-element system are studied in the small signal region. It is found that the efficiency and gain can be increased through introduction of an adjustable drift distance between two identical wigglers.

Theoretical studies of free-electron lasers indicate that the efficiency of such devices is limited by the small fraction of energy loss of the electrons that can take place in a single pass. The first operation of the free-electron laser as an oscillator achieved an efficiency of less than 0.01%.[1] It has been proposed to recycle the electron beam in order to increase the overall efficiency.[2] However, the analysis of the electron dynamics in a free-electron laser, using coupled Maxwell–Boltzmann equations,[3] showed that the velocity spread of an electron beam after the interaction is much larger than its velocity shift. This broadening leads to a considerable reduction of the gain and thus the usefulness of the reused beam.

In this Letter we point out that it is possible to improve the device efficiency by using multiple interaction regions that are properly spaced. For a quantitative analysis we use the single-electron model, which has been proved very useful in describing the transverse[4] and longitudinal[5] free-electron lasers. In this model, the trace of each electron is uniquely determined by its initial velocity v and entry phase ϕ. The output velocity of v' of an electron is found to second order in the field amplitude:

$$v' = v + \Delta v$$
$$= v + \frac{c^2}{\omega L}\left\{\eta\left(\frac{1-\cos\zeta}{\zeta}\sin\phi + \frac{\sin\zeta}{\zeta}\cos\phi\right) - \eta^2\left[\frac{1-\cos\zeta-\frac{\zeta}{2}\sin\zeta}{\zeta^3} + \frac{\sin\zeta-\zeta}{2\zeta^3}\sin(2\phi-\zeta)\right]\right\}, \quad (1)$$

where L is the length of interaction region, $\zeta = [(1/v_p) - (1/v)]\omega L$ is the phase slippage in one transit of an electron with velocity v relative to the wave whose phase velocity is v_p. η is the field expansion constant, and its explicit form depends on the type of free-electron laser. For example, $\eta = eE\omega L^2/mc^3\gamma^3$ in the longitudinal free-electron laser,[5] while $\eta = 2e^2BEL^2/m^2c^4\gamma^2$ in the Stanford free-electron laser.[4] In general, the velocity change of a single electron passing through an arbitrary interaction region can be written as

$$\Delta v = \Delta + \sqrt{2}\,\sigma\cos\theta, \quad (2)$$

where Δ is the average velocity shift, σ represents the velocity spread, and $\theta = \phi + \delta(\zeta)$ still serves as a label of individual electrons. δ is a function depending on ζ only.

If the initial electron beam is monoenergetic and if the electron distribution in ϕ space is uniform each time the electron beam re-enters the interaction region, we find that the velocity shift is proportional to the number of circulations N, while the velocity spreads to $\sqrt{N}$. As the velocity spread becomes larger after each circulation, the maximum number of circulations is thus determined by the maximum allowable velocity spread σ_{max}. The total velocity shift then becomes

$$\Delta_{max} = (\Delta/\sigma^2)\sigma_{max}^2. \quad (3)$$

Since the extractable energy from the electron beam is proportional to Δ_{max}, the overall efficiency of the device depends on the value of the factor $R \equiv (\Delta/\sigma^2)$. Consequently, an increase in the efficiency can be achieved by either enhancing the single-pass shift Δ or reducing the single-pass spread σ.

The velocity spread is due to the different entry phases of electrons. If we can "invert" the interaction between wave and electrons in a second interaction region, then we may expect a reduction in the velocity spread. Such inversion can be achieved if the entry phase of each electron is shifted by π radians with respect to that of the first region. Thus two-stage devices have been proposed[6,7] that consist of two identical interaction regions separated by a drift distance between them. Because of the velocity difference between wave and electrons, the π shift of the entry phase can be obtained by adjusting the value of L_D (Fig. 1).[8]

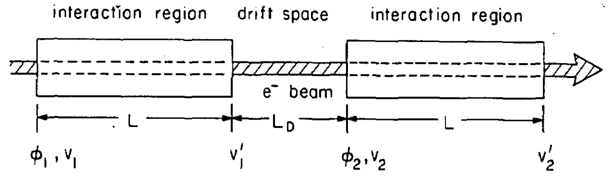

Fig. 1. A two-element free-electron laser.

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As a separate, but related, issue we consider the problem of maximizing the single-pass velocity shift and thus the single-pass gain by using two interaction regions. The basic reasoning for this approach is derived from the operation of the klystron.[9] The first interaction region acts as a buncher, giving rise to a strong velocity modulation. In the drift space between the two interaction regions, the velocity modulation gives rises to bunches (current modulation). These bunches are then made to enter the second interaction region at the optimum phase for deceleration and energy extraction.

Using the single-electron formalism[5] in the small-gain regime, we are able to formulate the interaction process in both interaction regions. The final velocity that is to be expressed as a function of initial entry phase is obtained by relating physical quantities in the second region to those in the first region. The expression of final velocity is then used to obtain the velocity spread and shift.

In terms of the general expression (2), the velocity shift and spread of a two-element free-electron laser are found to be

$$\sigma = \frac{\sqrt{2}\, c^2\eta}{\omega L \zeta}\,[(1-\cos\zeta)(1+\cos\alpha)]^{-1/2},$$

$$\Delta = -\frac{c^2\eta^2}{\omega L}\left[f(\zeta)\left(2+\frac{L_D}{L}\right)\sin\alpha + g(\zeta)(1+\cos\alpha)\right], \tag{4}$$

where

$$\alpha = \omega L_D\left(\frac{1}{c}-\frac{1}{v}\right)+\zeta,$$

$$f(\zeta) = (1-\cos\zeta)/\zeta^2,$$

$$g(\zeta) = (2-2\cos\zeta-\zeta\sin\zeta)/\zeta^3.$$

Since the device efficiency depends on Δ/σ^2, we try to maximize Δ and temporarily neglect the problem of velocity spread. Two terms with different dependencies on ΩT are involved in Δ. The maximum of $g(\zeta)$ is 0.135 at $\zeta = 2.6$, while $f(\zeta) = 0.5$ at $\zeta = 0$. Near resonance ($\zeta = 0$), the first term dominates over the second. Furthermore, the presence of L_D in the first term makes it possible to increase the shift by using longer drift distances. Choosing $\zeta = 0$ and $\alpha = \pi/2$, we have

$$\Delta = -\frac{c^2\eta^2}{\omega L}\left(1+\frac{L_D}{2L}\right) \text{ and } \sigma = \frac{c^2\eta}{\omega L} \tag{5}$$

for the shift and spread of the two-element device.

We next compare Eqs. (5) with the shift and spread of a single-element device of length $2L$, which is the total interaction length in Eqs. (5). The result is shown in Table 1, where we have arbitrarily chosen L_D to be $2L$.

Equations (5) show that for a given total interaction distance the velocity shift can be enhanced by a two-element system, especially when the electron-drift distance is much larger than the length of each interaction region. Furthermore, it is also shown that the increase in the velocity shift is not accompanied by an increase in the spread of electrons.

The parameter α is determined from the drift distance. In the high relativistic limit ($\gamma \gg 1$),

Table 1. Gain Enhancement of a Two-Element Device[a]

	Conditions: Single-Element $\zeta = 2.6$	Conditions: Two-Element $\zeta = 0;\ \alpha = \pi/2$
Δ	$-0.54\,\frac{c^2\eta^2}{\omega L}$	$-\frac{2c^2\eta^2}{\omega L}$
σ	$1.05\,\frac{c^2\eta}{\omega L}$	$\frac{c^2\eta}{\omega L}$

[a] The comparison of the velocity shift and spread is given for a single-element device of length $2L$, which is the total interaction length of the two-element device. The drift distance is arbitrarily chosen to be $2L$, which results in a four times larger velocity shift.

$$\alpha = -\frac{\delta L_D}{\lambda\gamma^2}. \tag{6}$$

The optimum operation can be achieved within a change of 2π in α, which means an adjustment of the drift distance within ΔL_D,

$$\Delta L_D = 2\lambda\gamma^2. \tag{7}$$

For an estimate of a typical value of ΔL_D we consider the Stanford device and find that ΔL_D is about 5 cm. This value is very reasonable for a practical experimental setup.

Next we consider the problem of velocity spread. It is found that the first-order contribution to the spread σ can be rendered zero by choosing $\zeta = 2\pi$ or $\alpha = \pi$. Unfortunately, we also find that the second-order shift becomes zero under either of these two conditions. Qualitatively, the spread is now of second order and the shift is fourth order in the field amplitude.

This result follows directly from Madey's theorem,[10] which can be written in the relativistic approximation as

$$\langle \Delta v\rangle_\phi = \frac{1}{2}\frac{\partial}{\partial v}[\langle \Delta v^2\rangle_\phi], \tag{8}$$

where triangular brackets represent the ensemble average over ϕ. Using the expression (4) for Δv, we have

$$\Delta \equiv \langle \Delta v\rangle_\phi = \sigma\frac{\partial\sigma}{\partial v}. \tag{9}$$

It is obvious that Δ is identically zero if σ vanishes. We conclude that, up to second order, it is impossible to eliminate the first-order spread without sacrificing the second-order gain.

We have considered the problem of increasing the shift and eliminating the spread separately. However, the final purpose is to improve the value of R that can be obtained from Eqs. (5),

$$R = \frac{\omega L}{2c^2}\left[\left(2+\frac{L_e}{L}\right)\tan\frac{\alpha}{2} + \left(\frac{2}{\zeta}-\cot\frac{\zeta}{2}\right)\right]. \tag{10}$$

The optimum condition for an efficient operation of the device is then found by optimizing Eq. (10) with changing α and ζ. Although the shift depends on α and ζ in a complicated manner, the expression for R contains two terms that depend exclusively on α and ζ. So the optimum value for α and ζ can be found independently. Consider the first term. It becomes infinitely large when $\alpha \rightarrow \pi$. The second term, which depends only on ζ, approaches infinity as $\zeta \rightarrow 2\pi$. As we have shown before, at these two values the shift is actually zero. However, σ^2 approaches zero at a faster rate than Δ, which results in an increasing value of R.[11]

In conclusion, we have shown that the efficiency of a free-electron laser in beam circulation and the gain of a single-pass device can be highly improved by using a two-element system. For the gain enhancement, the system is operated at the resonance and α is equal to $\pi/2$. The gain is found linearly related to the drift distance. For the efficiency improvement we choose ζ as close as possible to 2π and α as close as possible to π. However, the choice of ζ and α must be such that single-pass gain is higher than the threshold condition.

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8. We have considered using a magnet system in the drift space. However, it is found that the velocity dependence of the magnet is a second-order effect that is too small to affect the first-order bunching effect.
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11. If σ_{max} is defined as the maximum allowable width of electron distribution, where the gain drops by a certain factor, it is found that σ_{max} does not depend on parameters strongly near zeros of Δ.

Efficient higher-Stokes-order Raman conversion in molecular gases: errata

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The following references were inadvertently omitted during the editorial process:

Reference 6 should also have included works by V. Wilke and W. Schmidt, "Tunable coherent radiation source covering a spectral range from 185 to 880 nm," Appl. Phys. 18, 177–181 (1979); J. A. Paisner and R. S. Hargrove, "Tunable, efficient VUV and IR generation using high order stimulated Raman scattering in H_2," presented at OSA/IEEE Conference on Laser Engineering and Applications, May 30–June 1, 1979, paper II-4.

Reference 8 should also have included a paper by W. R. Trutna, Jr., Y. K. Park, and R. L. Byer, "The dependence of Raman gain on pump laser bandwidth," IEEE J. Quantum Electron. **QE-15**, 648–655 (1979).