Two Flaws In Business Cycle Accounting*

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October 18, 2006

Abstract

Using 'business cycle accounting' (BCA), Chari, Kehoe and McGrattan (2006) (CKM) conclude that models of financial frictions which create a wedge in the intertemporal Euler equation are not promising avenues for modeling business cycle dynamics. There are two reasons that this conclusion is not warranted. First, small changes in the implementation of BCA overturn CKM's conclusions. Second, one way that shocks to the intertemporal wedge impact on the economy is by their spillover effects onto other wedges. This potentially important mechanism for the transmission of intertemporal wedge shocks is not identified under BCA. CKM potentially understate the importance of these shocks by adopting the extreme position that spillover effects are zero.

*The first author is grateful for the financial support of a National Science Foundation grant. We are grateful for several key discussions with Mark Gertler, as well as for discussions with Martin Eichenbaum.

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1. Introduction

Chari, Kehoe and McGrattan (2005) (CKM) argue that a procedure they call Business Cycle Accounting (BCA) is useful for identifying promising directions for model development.[1] The key substantive finding of CKM is that financial frictions like those analyzed by Carlstrom and Fuerst (1997) (CF) and Bernanke, Gertler and Gilchrist (1999) (BGG) are not promising avenues for studying business cycles. Based on our analysis of business cycle data for the US in the 1930s and for the US and 14 other OECD countries in the postwar period, we find that the CKM conclusion is not warranted.

The BCA strategy begins with the standard real business cycle (RBC) model, augmented by introducing four shocks, or 'wedges'. A vector autoregressive representation (VAR) for the wedges is estimated using macroeconomic data on output, consumption, investment and government consumption.[2] The macroeconomic data are assumed to be observed with a small measurement error whose variance is fixed a priori. The fitted wedges have the property that when they are fed simultaneously to the augmented RBC model, the model reproduces the four macroeconomic data series up to the small measurement error. The importance of a particular wedge is determined by feeding it to the model, holding all the other wedges constant, and comparing the resulting model predictions with the data. One of the wedges, the intertemporal wedge, is the shock that enters between the intertemporal marginal rate of substitution in consumption and the rate of return on capital. CKM argue that this wedge contributes very little to business cycle fluctuations, for the following two reasons: (i) the wedge accounts for only a small part of the movement in macroeconomic variables during recessions and (ii) the wedge drives consumption and investment in opposite directions, while these two variables display substantial positive comovement over the business cycle. CKM assert that their conclusions are robust to various model perturbations, including the introduction of adjustment costs in investment.

There are two reasons that BCA does not warrant being pessimistic about the usefulness of models of financial frictions such as those proposed in CF or BGG. First, CKM's conclusions are not robust to small changes in the way they implement BCA. For example, when we redo CKM's calculations for the 1982 recession, we reproduce their finding that the intertemporal wedge accounts for essentially no part of the decline in output below trend at the trough of the recession. When we introduce a modest amount of investment adjustment costs, the intertemporal wedge accounts for a substantial 34 percent of the drop in output at the trough of the recession.[3] We then consider an alternative specification of the intertemporal wedge which is at least as plausible as the one CKM work with. CKM define the intertemporal wedge as an ad valorem tax on the price of investment goods. We argue that the CF and BGG models motivate considering an alternative formulation in which the

[1] This strategy is closely related to that advocated in Parkin (1988), Ingram, Kocherlakota and Savin (1994), Hall (1997), and Mulligan (2002).

[2] The last variable includes government consumption and net exports.

[3] Our adjustment costs are 'modest' in two senses. First, they imply a steady state elasticity of the investment-capital ratio to the price of capital equal to unity. This lies in the middle of the range of empirical estimates reported in the literature. Second, the adjustment cost function has the property that the quantity of resources lost due to investment adjustment costs is small, even in the wake of the enormous decline in investment in the early 1930s (see section 4 below for a detailed discussion).

wedge is modeled as a tax on the gross rate of return on capital. When we work with this alternative formulation, the intertemporal wedge accounts for 26 percent of the drop in output at the trough of the 1982 recession. But, when we also drop CKM's model of measurement error, that quantity jumps to 52 percent. Notably, the CKM model of measurement error is overwhelmingly rejected in the post war US data. So, BCA actually places a range of 0 to 52 percent on the fraction of the drop in output accounted for by the intertemporal wedge in the 1982 recession. This range is sufficiently wide to comfortably include most views about the importance of the intertemporal wedge.

We show that, at a qualitative level, economic theory predicts the lack of robustness in BCA that we find. The intertemporal wedge associated with different perturbations of the RBC model represent different ways of bundling the fundamental economic shocks to the economy. As a result, the BCA experiment of feeding measured wedges to an RBC model represents fundamentally different economic experiments under alternative specifications of the RBC model. Since the experiments are different, the outcomes are expected to be different too. Our results show that these expected differences are quantitatively large enough to overturn CKM's conclusions.

Second, CKM's analysis ignores that the financial shocks which drive the intertemporal wedge may have spillover effects onto other wedges.[4] It is not possible to determine the magnitude of these effects with BCA, because BCA leaves the fundamental shocks to the economy unidentified. In fact, the VAR for the wedges estimated under BCA is consistent with a wide range of possible spillover patterns. In terms of CKM's conclusion (i) above, we show that the financial shocks which drive the intertemporal wedge could account for as much as 70-100 percent of reductions in output in US recessions, including the Great Depression. We obtain the same finding for several other countries in the OECD. Regarding CKM's conclusion (ii), we show that once spillover effects are taken into account, financial shocks which drive the intertemporal wedge can drive consumption and investment in the same direction.

CKM understand that the fundamental economic shocks are not identified under BCA. However, the implications they draw from this observation are very different from the ones we draw. They say, 'Our method is not intended to identify the primitive sources of shocks. Rather, it is intended to help understand the mechanisms through which such shocks lead to economic fluctuations.'[5] We find that, without the ability to identify the economic shocks, a potentially important part of the mechanism by which these shocks affect the economy - the spillover effects - is also not identified. In effect, BCA offers a menu of observationally equivalent assessments about the importance of shocks to the intertemporal wedge. By focusing exclusively on the extreme case of zero spillovers, CKM select the element in the menu which minimizes the role of intertemporal shocks. We show that there are other elements in that menu which assign a very large role to intertemporal shocks.

[4] Recent developments in economic modeling suggest a variety of mechanisms by which these spillover effects can occur. For example, it is known that in models with Calvo-style wage-setting frictions (see, e.g., Erceg, Henderson and Levin (2000)), a shock outside the labor market can trigger what looks like a preference shock for labor, or a 'labor wedge'. Similarly, variable capital utilization can have the effect that a non-technology shock triggers a move in measured TFP, or the 'efficiency wedge'.

[5] The quote is taken from the CKM introduction. It summarizes CKM's comments in section 3 of their paper.

Following is an outline of the paper. In the following section, we describe the model used in the analysis. In section 3, we elaborate on the observational equivalence results discussed above. In section 4, we discuss our model solution and estimation strategy. In section 5 we discuss the lack of identification of spillover effects in BCA. In section 6 we discuss the wedge decomposition under BCA and our modification to take into account spillovers. Section 7 displays the results of implementing BCA on various data sets. Concluding remarks appear in section 8. Additional technical details appear in three Appendices.

2. The Model and the Wedges

This section describes the model used in the analysis. In addition, we discuss the wedges and, in particular, our two specifications of the intertemporal wedge.

According CKM's version of the RBC model, households maximize:

$$E\sum_{t=0}^{\infty}\left(\beta\left(1+g_n\right)\right)^t\left[\log c_t+\psi\log\left(1-l_t\right)\right],\ 0<\beta<1,$$

where c_t and l_t denote per capita consumption and employment, respectively. Also, g_n is the population growth rate and $\psi>0$ is a parameter. The household budget constraint is

$$c_t+\left(1+\tau_{x,t}\right)x_t\leq r_tk_t+\left(1-\tau_{l,t}\right)w_tl_t+T_t,$$

where T_t denotes lump sum taxes, x_t denotes investment and $\tau_{l,t}$ denotes the labor wedge. Here, k_t denotes the beginning-of-period t stock of capital divided by the period t population. The variable, $\tau_{x,t}$, is CKM's specification of the intertemporal wedge. The technology for capital accumulation is given by:

$$\left(1+g_n\right)k_{t+1}=\left(1-\delta\right)k_t+x_t-\Phi\left(\frac{x_t}{k_t}\right)k_t, \tag{2.1}$$

where $\Phi\left(\zeta\right)$ is symmetric about $\zeta=b$, where b is the steady state investment-capital ratio. In addition, to ensure that Φ has no impact on steady state, we suppose that $\Phi\left(b\right)=\Phi'\left(b\right)=0$.

The household maximizes utility by choice of $\{c_t,k_{t+1},l_t,x_t\}$, subject to its budget constraint, the capital evolution equation, the laws of motion of the wedges and the usual inequality constraints and no-Ponzi scheme condition.

The resource constraint is:

$$c_t+g_t+x_t=y\left(k_t,l_t,Z_t\right)=k_t^{\alpha}\left(Z_tl_t\right)^{1-\alpha}, \tag{2.2}$$

where

$$Z_t=\tilde{Z}_t\left(1+g_z\right)^t,$$

and $\tilde{Z}_t$, the 'efficiency' wedge, is an exogenous stationary stochastic process. In the resource constraint, g_t denotes government purchases of goods and services plus net exports, which is assumed to have the following trend property:

$$g_t=\tilde{g}_t\left(1+g_z\right)^t,$$

where $\tilde{g}_t$ is a stationary, exogenous stochastic process and $g_z \geq 0$.

Combining firm and household first order necessary conditions for optimization in the case $\Phi = 0$,

$$\frac{-u_{l,t}}{u_{c,t}} = (1 - \tau_{l,t}) y_{l,t} \tag{2.3}$$

$$u_{c,t} = \beta E_t u_{c,t+1} \frac{y_{k,t+1} + (1 + \tau_{x,t+1}) P_{k',t+1} \left[1 - \delta - \Phi\left(\frac{x_{t+1}}{k_{t+1}}\right) + \Phi'\left(\frac{x_{t+1}}{k_{t+1}}\right)\frac{x_{t+1}}{k_{t+1}}\right]}{(1 + \tau_{x,t}) P_{k',t}} \tag{2.4}$$

where $u_{c,t}$ and $-u_{l,t}$ are the derivatives of period utility with respect to consumption and leisure, respectively. In addition, $y_{l,t}$ and $y_{k,t}$ are the marginal products of labor and capital, respectively. Also, the price of capital, $P_{k',t}$, is

$$P_{k',t} = \frac{1}{1 - \Phi'\left(\frac{x_{t+1}}{k_{t+1}}\right)}. \tag{2.5}$$

The equilibrium values of $\{c_t, k_{t+1}, l_t, x_t\}$ are computed by solving (2.1), (2.2), (2.3), (2.4), subject to the transversality condition and the following law of motion for the exogenous shocks:

$$s_t = [I - P] P_0 + P s_{t-1} + u_t,\ s_t = \begin{pmatrix} \log \tilde{Z}_t \\ \tau_{l,t} \\ \tau_{x,t} \\ \log \tilde{g}_t \end{pmatrix},\ E u_t u_t' = QQ' = V, \tag{2.6}$$

where P_0 is the 4×1 vector of unconditional means for s_t and

$$P = \begin{bmatrix} \bar{P} & 0 \\ 0 & p_{44} \end{bmatrix},\ Q = \begin{bmatrix} \bar{Q} & 0 \\ 0 & q_{44} \end{bmatrix}. \tag{2.7}$$

Here, P is stationary and $\bar{P}$ is not otherwise restricted. The symmetric matrix, V, in (2.6) must satisfy the zero restrictions implicit in $QQ' = V$, and the zeros in the lower diagonal part of Q in (2.7). We follow CKM in implementing the zero restrictions in our analysis of the US Great Depression. We do this in our analysis of OECD countries as well. In our analysis of postwar US data, we allow all elements of P and all elements in the lower triangular part of Q to be non-zero. The parameters of (2.6) are P_0, P, and V, possibly with the indicated zero restrictions on V and the zero and stationarity restrictions on P.

We consider an alternative specification of the intertemporal wedge. Our specification is motivated by our analysis of the version of the CF model with adjustment costs and by our analysis of BGG. In Appendix A, we derive equilibrium conditions for a version of the CF model with $\Phi \neq 0$. We establish a proposition displaying a set of wedges which, if added to the RBC economy, ensure that the equilibrium allocations of the RBC economy coincide with those of our version of the CF economy with investment adjustment costs. We show that the intertemporal wedge has the following form:

$$u_{c,t} = \beta E_t u_{c,t+1} \left(1 - \tau_{t+1}^k\right) R_{t+1}^k, \tag{2.8}$$

where,

$$R_t^k = \frac{y_{k,t} + P_{k',t}\left[1 - \delta - \Phi\left(\frac{x_t}{k_t}\right) + \Phi'\left(\frac{x_t}{k_t}\right)\frac{x_t}{k_t}\right]}{P_{k',t-1}} \tag{2.9}$$

Note that in the alternative formulation, the wedge is a tax on the gross return to capital, in contrast to CKM's value-added tax on investment purchases, $\tau_{x,t}$. In Appendix A we show that the CF model with adjustment costs implies τ_{t+1}^k is a function of uncertainty realized at date t, but not at date $t+1$.[6] We follow CKM in presuming that all wedges implied by the CF financial frictions apart from the intertemporal wedge, $1 - \tau_{t+1}^k$, are quantitatively small and can be ignored.

In Appendix B we derive the intertemporal wedge associated with the BGG model. That model also implies that the intertemporal wedge enters as $1-\tau_{t+1}^k$ in (2.8). The only difference is that under BGG, τ_{t+1}^k is a function of the period $t+1$ realization of uncertainty.[7]

In our alternative specification of the intertemporal wedge, we allow τ_t^k to respond to current and past information. This assumption encompasses both the CF and BGG financial friction models, since the econometric estimation is free to produce a τ_t^k whose response to current information is very small.

3. General Observations on the Robustness of BCA to Modeling Details

In later sections, we show that the conclusions of BCA for the importance of the intertemporal wedge are not robust to alternative specifications of the intertemporal wedge, and to alternative specifications of investment adjustment costs. This finding may at first seem puzzling in light of a type of observational equivalence result emphasized in CKM. An example of this type of result which occurs when BCA is done with a linearly approximated RBC model is the following. Consider an RBC economy with, say, no investment adjustment costs (i.e., $\Phi = 0$) and a particular time series representation for the wedges. After introducing adjustment costs (i.e., $\Phi \neq 0$), one can find a new representation of the intertemporal wedge which ensures that the equilibria of the economies with and without adjustment costs coincide.[8] This is an observational equivalence result because it implies that the likelihood of a

[6] That appendix provides a careful derivation of our result, because our finding for the way the intertemporal wedge enters (2.8) differs from CKM's finding. CKM consider the case, $\Phi = 0$, in deriving the wedge representation of the CF model. The results for the $\Phi = 0$ and $\Phi \neq 0$ cases are qualitatively different. When $\Phi = 0$ capital producers simply produce increments to the capital stock, which capital owners add to the existing undepreciated capital by themselves. When $\Phi \neq 0$, old capital is a fundamental input in the production of new capital. In this case, we assume that the capital producers must purchase the economy's entire stock of capital in order to produce new capital, so that their financing requirements and the associated frictions are different. There are perhaps other ways of arranging the production of new installed capital when $\Phi \neq 0$. We find our way convenient because it results in an intertemporal wedge that virtually coincides with the one we derive for BGG

[7] CKM derive the intertemporal wedge for a version of the BGG model in which banks have access to complete state-contingent markets. Our wedge formula applies to the model analyzed in BGG, which does not permit complete markets.

[8] Here, we make use of our asumption that analysis is done using log-linear approximation. In this case, the only effect of the change in Φ is to change the rate of return on capital. For example, in the linear

set of allocations is invariant to the presence of adjustment costs. This case of adjustment costs is just example of the type of observational equivalence result we have in mind. For example, consider an RBC economy in which the intertemporal wedge is of the $\tau_{x,t}$ type emphasized by CKM. Given a specification of the joint time series representation of $\tau_{x,t}$ and the other wedges, the $\tau_{x,t}$ RBC model implies a set of equilibrium allocations. Now consider an alternative RBC economy in which the intertemporal wedge is of the τ_t^k type. There exists a specification of the joint stochastic process for τ_t^k and the other wedges having the property that the equilibrium allocations in the τ_t^k RBC model coincide with those in the $\tau_{x,t}$ RBC model. Again, this stochastic process is identified from the requirement that the after tax rates of return in the two economies coincide. In both of the above examples, it is clear that the observational equivalence result depends on the assumption that the time series representations used for the shocks are sufficiently flexible to accommodate any specification for the stochastic process of the wedges.[9]

We wish to stress here that the equilibrium observational equivalence result does not imply a 'BCA robustness result'. In particular, the outcome of BCA (i.e., the outcome of feeding fitted wedges, one at a time, to a model) is not expected to be robust to the specification of investment adjustment costs, or to whether the intertemporal wedge is modeled as $\tau_{x,t}$ or τ_t^k. There are two reasons for this lack of robustness. One is practical and reflects that the analyst must confine him/herself to a specific parametric time series representation of the wedges, thus potentially ruling out one of the conditions of the observational equivalence result. The other, deeper, reason is the one mentioned in the introduction. Even if the analyst uses a completely flexible time series representation of the wedges, the intertemporal wedge represents a different bundle of fundamental shocks under alternative perturbations of the model. Feeding the measured intertemporal wedge to an RBC model under alternative model perturbations represents a different experiment and so is expected to produce a different outcome.

To illustrate these observations, suppose the data are generated by an RBC model in which intertemporal wedge is the τ_t^k type, with a certain specification of the adjustment cost function, Φ. The joint time series representation of the wedges is given by (2.6), in which P and Q are diagonal. Thus, each wedge is uncorrelated with all other wedges, at all leads and lags. In this case, BCA has a clear interpretation: when the estimated intertemporal wedge is fed to the baseline RBC model, the simulations display the model's response to a particular history of past innovations to that wedge alone. Suppose the econometrician is provided with an infinite amount of data, but misspecifies the adjustment cost function, Φ. As in BCA, the econometrician only estimates the joint time series representation of the wedges, and holds the misspecified Φ and other nonstochastic parts of the economy fixed. We assume that the econometrician's time series representation for the wedges is sufficiently flexible to encompass the quasi-true time series of the wedges that is implied by the observational equivalence result. We obtain insight into BCA by deriving that time series representation. The requirement that the after tax rates of return in the econometrician's model coincide

approximation the law of motion for the capital stock, (2.1), is always linear and invariant to a.

[9] This is a special case of a well-known result that econometric identification often hinges on having sufficient restrictions on the unobserved shocks.

with the true after tax rate of return implies, using (2.9):

$$1-\tau_{t+1}^{k}=\left(1-\bar{\tau}_{t+1}^{k}\right)\frac{y_{k,t}+\bar{P}_{k',t}\left[1-\delta-\bar{\Phi}\left(\frac{x_t}{k_t}\right)+\bar{\Phi}'\left(\frac{x_t}{k_t}\right)\frac{x_t}{k_t}\right]}{y_{k,t}+P_{k',t}\left[1-\delta-\Phi\left(\frac{x_t}{k_t}\right)+\Phi'\left(\frac{x_t}{k_t}\right)\frac{x_t}{k_t}\right]}. \tag{3.1}$$

Here, a $-$ over a variable indicates the value of the variable in the true model and absence of a $-$ indicates the value estimated by the econometrician who misspecifies Φ. The endogenous variables on the right side of the equality in (3.1) are specific functions of the history of the innovations driving the wedges in the actual economy. Then, according to (3.1), the adjusted time series representation of τ_t^k is the convolution of these functions with the function on the right of the equality in (3.1). We derive this map from the fundamental innovations in the economy to τ_t^k using linearization.

Consider the true specification of Φ and the true joint time series representation of the wedges, s_t, given in (2.6). Let z_t denote the list of endogenous variables in the model, i.e., $z_t=(c_t,x_t,k_{t+1},l_t,\tau_t^k)$, where the quantity variables are measured in log deviations from steady state and τ_t^k is in deviation from steady state. The equilibrium conditions of z_t may be written in the form:

$$E_t\left[\alpha_0 z_{t+1}+\alpha_1 z_t+\alpha_2 z_{t-1}+\beta_0 s_{t+1}+\beta_1 s_t\right]=0,\ \text{with } s_t=Ps_{t-1}+Q\varepsilon_t.$$

Here, $s_t=\left(\log\tilde{Z}_t,\tau_{l,t},\bar{\tau}_t^k,\log\tilde{g}_t\right)'$. The expectational difference equation is composed of the intertemporal first order condition (2.8), the intratemporal first order condition (2.3), the law of motion for capital (2.1), the resource constraint, (2.2), and the mapping from $\bar{\tau}_t^k$ to τ_t^k, (3.1), all after suitable log-linearization. The solution to this system is written $z_t=Az_{t-1}+Bs_t$, or, when expressed in moving average form[10]:

$$z_t=\left[I-AL\right]^{-1}B\left[I-PL\right]^{-1}Q\varepsilon_t.$$

Let τ denote the 5-dimensional column vector with all zeros, except a 1 in the 5th location. Then, the time series representation for τ_t^k is

$$\tau_t^k=\tau\left[I-AL\right]^{-1}B\left[I-PL\right]^{-1}Q\varepsilon_t.$$

This is the convolution of (3.1) with the time series representation of the (linearized) variables in (3.1). Let ν denote the 3 by 4 matrix constructed by deleting the third row of the 4-dimensional identity matrix and let S_t denote the 3 dimensional vector obtained by deleting $\bar{\tau}_t^k$ from s_t. We conclude that the econometrician who misspecifies Φ will estimate the following joint time series representation for the wedges in his misspecified model:

$$\begin{pmatrix}\tau_t^k\\ S_t\end{pmatrix}=\begin{bmatrix}\tau\left[I-AL\right]^{-1}B\\ \nu\end{bmatrix}\left[I-PL\right]^{-1}Q\varepsilon_t.$$

By inspection, it is clear that in general, the new joint series representation of $\left(\tau_t^k,S_t\right)$ has a moving average component. To see this, it is useful to examine the *iid* case, $P=0$ and

[10] For further discussion, see Christiano (2002).

$Q = I$. Note first that $\tau\left[I-A\right]^{-1}B$ has the following form:

$$\begin{aligned}
\tau\left[I-A\right]^{-1}B &= \tau\begin{bmatrix} 1 & 0 & -a_{13}L & 0 & 0 & 0 \\ 0 & 1 & -a_{23}L & 0 & 0 & 0 \\ 0 & 0 & 1-a_{33}L & 0 & 0 & 0 \\ 0 & 0 & -a_{43}L & 1 & 0 & 0 \\ 0 & 0 & -a_{53}L & 0 & 1 & 0 \\ 0 & 0 & -a_{63}L & 0 & 0 & 1 \end{bmatrix}^{-1} B = \tau\begin{bmatrix} 1 & 0 & -L\frac{a_{13}}{La_{33}-1} & 0 & 0 & 0 \\ 0 & 1 & -L\frac{a_{23}}{La_{33}-1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{La_{33}-1} & 0 & 0 & 0 \\ 0 & 0 & -L\frac{a_{43}}{La_{33}-1} & 1 & 0 & 0 \\ 0 & 0 & -L\frac{a_{53}}{La_{33}-1} & 0 & 1 & 0 \\ 0 & 0 & -L\frac{a_{63}}{La_{33}-1} & 0 & 0 & 1 \end{bmatrix} B \\
&= \begin{bmatrix} 0 & 0 & -a_{63}L & 0 & 0 & 1 \end{bmatrix} B \\
&= \begin{bmatrix} B_{51}-a_{63}B_{31}L & B_{52}-a_{63}B_{32}L & B_{53}-a_{63}B_{33}L & B_{54}-a_{63}B_{34}L \end{bmatrix},
\end{aligned}$$

where B_{ij} denotes the ij^{th} element of B. We conclude that the new joint representation of the wedges is:

$$\begin{pmatrix} \tau_t^k \\ S_t \end{pmatrix} = \begin{bmatrix} \begin{pmatrix} B_{51}-a_{63}B_{31}L & B_{52}-a_{63}B_{32}L & B_{53}-a_{63}B_{33}L & B_{54}-a_{63}B_{34}L \end{pmatrix} \\ \nu \end{bmatrix} \varepsilon_t.$$

Note that the intertemporal wedge has a pure, first order moving average representation, even though τ_t^k in the correctly specified economy is *iid* and a function only of the third element of ε_t. Evidently, the wedges in the misspecified economy do not obey the same first order VAR(1) representation that s_t does. Thus, the analyst who is restricted VAR(1) (or, VAR(q), $q < \infty$) representations for the wedges misrepresents the reduced form of the data. Under these circumstances, it is not surprising that the conclusions of BCA will be different, across different specifications of Φ.

Now, suppose that the analyst adopts a sufficiently flexible time series representation of the wedges, so that the specification error described in the previous paragraph does not occur. The intertemporal wedge, $\bar{\tau}_t^k$, computed by the econometrician working with the correct specification of Φ is a function of just the current realization of the third element of ε_t. In the alternative specification, τ_t^k is a function of the entire history of all elements of ε_t. Clearly, feeding the estimated intertemporal wedge to the model is a different experiment across the two different specifications of Φ. This is why we do not expect the results of BCA to be robust to perturbations in the RBC model.

4. Model Solution and Estimation

Here, we describe how we assigned values to the model parameters. A subset of the parameters were not estimated. These were set as in CKM:

$$\begin{aligned} \beta &= 1/1.03,\ \alpha = 0.35,\ \delta = 0.0464,\ \psi = 2.24, \\ g_n &= 0.015,\ g_z = 0.016. \end{aligned} \tag{4.1}$$

Here, β, δ, g_n, and g_z are expressed at annual rates. These are suitably adjusted when we analyze quarterly data. The first subsection below discusses the estimation of the parameters of the exogenous shocks, P_0, P, and V, using data on output, consumption, investment and government consumption plus net exports. Estimation is carried out conditional on a parameterization of the adjustment cost function. The parameterization of the adjustment cost function is discussed in the second subsection. The third subsection rebuts some criticisms of the investment adjustment cost function expressed in CKM. Their criticisms suggest that investment adjustment costs are, in effect, a 'nonstarter'. Since they are not empirically interesting, they therefore do not constitute a compelling basis for criticizing BCA. We explain why we disagree with this assessment.

4.1. Estimating the Parameters of the Time Series Representation of the Wedges

For the US Great Depression, we used annual data covering the period, 1901-1940.[11] Quarterly data covering the period 1959Q1-2004Q3 were used for the US and quarterly data over various periods were used on 14 other OECD countries.[12] Following CKM, the elements of the matrices, P and V are estimated subject to the zero restrictions described in section 2, and to the restriction that the maximal eigenvalue of P not exceed 0.995.

The first step of estimation is to set up the model's solution in state space - observer form:

$$\begin{aligned} Y_t &= H\left(\xi_t;\gamma\right) + \upsilon_t && (4.2) \\ \xi_t &= F\left(\xi_{t-1};\gamma\right) + \eta u_t && (4.3) \\ \gamma &= \left(P, P_0, V\right),\ \eta = \begin{pmatrix} \tilde{0} \\ I \end{pmatrix},\ E\upsilon_t\upsilon_t' = R,\ Eu_tu_t' = V, \end{aligned}$$

where $\tilde{0}$ is a 1×4 vector of zeros and ξ_t is the state of the system:

$$\xi_t = \begin{pmatrix} \log \tilde{k}_t \\ s_t \end{pmatrix}, \tag{4.4}$$

where $\tilde{k}_t = k_t/\left(1+g_z\right)^t$. Also, Y_t is the observation vector:

$$Y_t = \begin{pmatrix} \log \tilde{y}_t \\ \log \tilde{x}_t \\ \log l_t \\ \log \tilde{g}_t \end{pmatrix}, \tag{4.5}$$

where $\tilde{x}_t = x_t/\left(1+g_z\right)^t$. Finally, υ_t is a 4×1 vector of measurement errors, with

$$R = 0.0001 \times I_4, \tag{4.6}$$

where I_4 is the four-dimensional identity matrix and CKM set the scale factor exogenously (see CKM (technical appendix, page 16)). We refer to this specification of R as the 'CKM

[11] These data were taken from CKM, as supplied on Ellen McGrattan's web site.

[12] US data are the data associated with the CKM project, and were taken from Ellen McGrattan's web page. With two exceptions, data for other OECD countries were taken from Chari, Kehoe and McGrattan (2002), also on Ellen McGrattan's web site. Data on hours worked were taken from the OECD productivity database. These data are annual and were converted to quarterly by log-linear interpolation. Population data were taken from the OECD national databases and log-linearly intertpolated to quarterly.

measurement error assumption'. We repeat the analysis under CKM measurement error, as well as with $R = 0$.

As noted in the introduction, the CKM specification of measurement error has an impact on the analysis. CKM do not explain why they include measurement error, nor do they discuss the a priori evidence which leads them to the specific values they choose for the measurement error variance.[13] We do have reason to believe the data are measured with error. However, we know of no reason to take seriously the notion that CKM's specification even approximately captures actual data measurement error.[14]

We implement BCA using first and second-order approximations to the model's equilibrium conditions. Consider the first order approximation. In this case, the representation of the policy rule is:

$$\log \tilde{k}_{t+1} = (1-\lambda)\lambda_0 + \lambda \log \tilde{k}_t + \psi s_t, \tag{4.7}$$

where λ_0 and λ are scalars and ψ is a 1×4 row vector. Then, (4.2)-(4.3) can be written:

$$\begin{aligned} \xi_t &= F_0 + F_1 \xi_{t-1} + \eta \varepsilon_t, \\ F_0 &= \left[\begin{array}{c} (1-\lambda)\lambda_0 \\ (I-P) P_0 \end{array} \right], \; F_1 = \left[\begin{array}{cc} \lambda & \psi \\ 0 & P \end{array} \right], \end{aligned}$$

where F_0 is a 5×1 column vector, and F_1 is a 5×5 matrix. Also,

$$Y_t = H_0 + H_1 \xi_t + \upsilon_t, \tag{4.8}$$

where H_0 is a 4×1 column vector and H_1 is a 4×4 matrix. The Gaussian likelihood is constructed using F_0, F_1, H_0, H_1, V, R, and $Y = (Y_1, ..., Y_T)$ (see Hamilton (1994)). These in turn can be constructed using γ, R. Thus, the likelihood can be expressed as $L(Y|\gamma; R)$.

For the nonlinear case, we use the algorithm in Schmitt-Grohe and Uribe (2004) to obtain second order approximations to the functions, F and H in (4.2) and (4.3). It is easy to see that even if u_t is Normally distributed, Y_t will not be Normal in this nonlinear system. We nevertheless proceed to form the Gaussian density function using the unscented filter described in Wan and van der Merwe (2001). It is known that under certain conditions, Gaussian maximum likelihood estimation has the usual desirable properties, even when the data are not Gaussian.

4.2. Investment Adjustment Costs

To analyze the version of the model with adjustment costs, we must parameterize the investment adjustment cost function, Φ. Our calibration is based on our interpretation of the variable, $P_{k',t}$. On this dimension, the CF and BGG models differ slightly (for details, see Appendices A and B). Both agree that $P_{k',t}$ is the marginal cost, in units of consumption

[13] As already noted, other parameter values are also fixed in the analysis, such as production function parameters. Dogmatic priors like this can perhaps be justified by appealing to analyses based on other data, such as observations on income shares. We are not aware of any such argument, however, that can be used as a basis for adopting the dogmatic priors in (4.6).

[14] Based on what we know about the way data are collected, there is strong a priori reason to question the CKM model of measurement error. For a careful discussion, see Sargent (1989).

goods, of producing new capital when only (2.1) is considered.[15] However, in the CF model, financial frictions introduce a wedge between the market price of capital and $P_{k',t}$. Still, in practice the discrepancy between $P_{k',t}$ and the market price of new capital in the CF model with adjustment costs may be quantitatively small. To see this, it is instructive to consider the response of the variables in the CF model (where $P_{k',t} = 1$ always) to a technology shock. According to CF (see Figure 2 in CF), the contemporaneous response of the market price of capital is only one-tenth the contemporaneous response of investment. That simulation suggests that the distinction between $P_{k',t}$ and the market price of capital may not be large in the CF model.

In the BGG model, financial frictions arise inside the relationship between the managers of capital and banks, and so the frictions do not open wedge between the marginal cost of capital and $P_{k',t}$. As a consequence, $P_{k',t}$ corresponds to the market price of capital in the BGG model.

Under the interpretation of $P_{k',t}$ as the market price of capital, we can calibrate Φ based on empirical estimates of the elasticity of investment with respect to the price of capital (i.e., Tobin's q). From (2.5), this is

$$\frac{d\log\left(x_t/k_t\right)}{d\log P_{k',t}} = \frac{1}{\Phi''\left(b\right)b}. \tag{4.9}$$

According to estimates reported in Abel (1980) and Eberly (1997), Tobin's q lies in a range of 0.6 to 1.4. Interestingly, if we just consider the period of largest fall in the Dow Jones Industrial average during the Great Depression, 1929Q4 to 1932Q4, the ratio of the percent change in investment to the percent change in the Dow is 0.68.[16] This is an estimate of Tobin's q under the assumption that the movement in the Dow reflects primarily the price of capital, and not its quantity.[17] This estimate lies in the middle of the Abel-Eberly range of estimates. A unit Tobin's q elasticity implies $\Phi''\left(b\right) = 1/b$.

Another factor impacting on our choice of $\Phi''\left(b\right)$ is the model's implication for the rate of return on capital, R^k. Figure 1A shows the results corresponding to Tobin's q elasticities 1/2, 1, 3 and ∞ (the latter corresponds to $\Phi''\left(b\right) = 0$). For each elasticity, the model was estimated using the linearization strategy and using quarterly US data covering the period 1959QIV-2003QI. For these calculations, the only feature of Φ that is required is the value of $\Phi''\left(b\right)$. The model-based estimate of R_t^k, (2.9), was computed using the two-sided Kalman smoother.[18] The US data on R_t^k were constructed using Robert Shiller's data on real dividends and real stock prices for the $S\&P$ composite index. In the case of both model-based and actual R_t^k, we report centered, equally weighted, 5 quarter moving averages. Note that without adjustment costs, the model drastically understates the volatility in R_t^k. With

[15] It is easy to verify that $P_{k',t}$ in (2.1) corresponds to the price of investment goods (i.e., unity) divided by the marginal product of investment goods in producing end of period capital.

[16] This is the ratio of the log difference in investment to the log difference in the Dow, over the period indicated. Both variables were in nominal terms.

[17] By associating the model's capital stock with what is priced in the Dow, we are implicitly taking the position that capital in the model corresponds to both tangible and intangible capital.

[18] See Hamilton (1994) for a discussion. The two-sided smoother is required because we do not use empirical data on the capital stock, which is an input in (2.9). Presumably, the smoother estimates the capital stock by combining the investment data with the capital accumulation equation.

a Tobin's q elasticity of 3 (i.e., $\Phi''(b) = 1/(3b)$) the model still substantially understates that volatility. With an elasticity around unity, the model begins to reproduce the volatility of R^k, though it is still somewhat low. Only with an elasticity around 1/2 does the model nearly replicate the volatility of R^k. These results reinforce our impression that the data suggest a Tobin's q elasticity of unity or less. To be conservative, we work with an elasticity of unity.

4.3. Responding to CKM's Criticisms About Adjustment Costs

CKM criticize the use of adjustment costs with a unit Tobin's q elasticity for two reasons. According to their first critique, adjustment costs with a unit Tobin's q elasticity imply that an unreasonably large amount of resources are absorbed by adjustment costs during collapse of investment in the Great Depression. This conclusion is based on the arbitrary assumption that the adjustment cost function, Φ, is globally quadratic. But, we show that other functional forms for Φ can be found with the property, $\Phi''(b) = 1/b$, whose global properties do not imply that an inordinate amount of resources were used up in investment adjustment costs in the Great Depression. Second, CKM assert that an adjustment cost formulation which implies a static relationship between the investment-capital ratio and Tobin's q is empirically implausible. But, we show that BCA lacks robustness even with the specification of adjustment costs proposed in Christiano, Eichenbaum and Evans (2004), which does not imply a static relationship the investment-capital ratio and Tobin's q. This adjustment cost function, in which adjustment costs are a function of the change in the flow of investment, also does not imply that an inordinate amount of resources were used up in adjustment costs during the collapse of investment in the 1930s.[19]

The globally quadratic adjustment cost formulation adopted by CKM is:

$$\Phi\left(\frac{x_t}{k_t}\right) = \frac{a}{2}\left(\frac{x_t}{k_t} - b\right)^2,$$

so that $\Phi''(b) = a$. Imposing that Tobin's q elasticity is unity, the resources lost to adjustment costs, as a fraction of output, is given by:

$$\Phi\left(\frac{x_t}{k_t}\right) = \frac{1}{2}\left(\lambda_t - 1\right)^2 \frac{x}{y\mu_t}, \tag{4.10}$$

according to (2.1). Here, x/y is the steady state investment to output ratio. In (4.10), we have used $x = bk$ in the steady state. Here, λ_t is the time t investment-capital ratio, expressed as a ratio to its steady state value, b. Also, μ_t is the output-capital ratio, expressed as a ratio to its steady state value, y/k. Figure 6 indicates that output was 10 percent below trend in 1930, and then fell another 10 percent in each of 1931 and 1932. In 1933, the trough of the Depression, it fell yet another 5 percent, so that by 1933 output was a full 35 percent

[19] This adjustment cost function has the additional advantage that it receives empirical support from the analysis of housing investment (see Rosen and Topel (1988)) and aggregate Tobin's q data (see Matsuyama (1984)), in addition to the empirical evidence in Christiano, Eichenbaum and Evans (2006). Also, this adjustment cost formulation has economically interesting microfoundations, as shown in Lucca (2006) and Matsuyama (1984).

below trend. The drop in investment was even more dramatic. In 1930, 1931, 1932 and 1933 it was about 30, 50, 70 and 70 percent below trend, respectively. Using our capital accumulation equation, we infer that the stock of capital was 10 percent below trend in 1933.

Since investment was 70 percent below its trend in 1933 and the capital stock was 10 percent its trend then, we infer that the investment to capital ratio is 60 percent below steady state, i.e., $\lambda_{1933} = 0.40$. Output was 35 percent below steady state in 1933, and we infer that the output-capital ratio was 25 percent below trend, so that $\mu_{1933} = 0.75$. Substituting these into (4.10),

$$\Phi\left(\frac{x_t}{k_t}\right) = \frac{1}{2}(0.40 - 1)^2 (0.23)/0.75 = 0.055,$$

or 5.5 percent. Given that output was 35 percent below trend in 1933, the implication is that 16 percent of the drop in output reflected resources lost to adjustment costs associated with the low level of investment. To see how sensitive this conclusion is to the choice of functional form for Φ, consider Figure 1B, which graphs (4.10) for $100\lambda_t$ ranging from 40 percent to 160 percent, holding $x/(y\mu_t)$ fixed at 0.31. Note how the quadratic curve hits the vertical axis at 5.5 percent. The other curve in Figure 1B coincides with the quadratic function for λ_t roughly in its range for postwar business cycles. Outside this range, the alternative function is flatter than the quadratic, and it hits the vertical axis at 2.5 percent. The alternative adjustment cost function has a much more modest implication for the amount of resources lost to adjustment costs as investment collapsed in the Great Depression. Yet, the implications of the model with the alternative adjustment cost function for postwar business cycles coincides with the implications of the model with the quadratic adjustment cost function.[20]

To address CKM's second concern about adjustment costs, we also considered the following formulation:

$$(1 + g_n) k_{t+1} = (1 - \delta) k_t + \left[1 - \frac{a}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2\right] x_t.$$

With this formulation of adjustment costs, investment responds differently to permanent and temporary changes in the price of capital. This addresses one of CKM's concerns about investment adjustment costs. To address the other concern, we needed to assign a value to a. For this, we estimated the parameters of the joint time series representation of the wedges for various values of a, using postwar US data. We found that with $a = 3.75$ the model's implications for the volatility of the rate of return on capital virtually coincides with the implications of our baseline model with a unit Tobin's q elasticity. We then used the Balke and Gordon quarterly data on investment and output in the 1930s to compute the fraction of output lost due to adjustment as investment plunged at the start of the Great Depression. We found that the largest fraction of output lost due to adjustment costs in the period 1929Q1-1933Q1 was 1.46 percent. According to the Balke and Gordon data, investment

[20] The alternative adjustment cost function is a 10^{th} degree polynomial, and so it has a continuous derivative of every order. It was constructed as follows. We constructed a 'target' function by splicing the quadratic function in the range, $\lambda \in (0.85, 1.15)$, with straight lines on either end. The straight lines have slope equal to that of the quadratic function at the point where they meet. The 10^{th} degree polynomial was fit by standard Chebyshev interpolation.

rose sharply starting in 1933Q2. Adjustment costs were larger then, but adjustment costs in expansions are less of a concern to CKM.[21] We conclude that with the alternative adjustment costs, neither of CKM's two objections apply.

Significantly, our finding that BCA is sensitive to the presence of adjustment costs is also true when the adjustment costs are in terms of the change in investment. Ignoring the spillover effects between wedges, as CKM do, we calculated the percent of the fall in output due to the intertemporal wedge at the trough of five postwar US recessions. For the 1970, 1974, 1980, 1990 and 2000 recessions, the percentages are 17, 30, 14, 26, and 43, respectively. All these are substantial amounts and certainly do not warrant the CKM conclusion that financial frictions which manifest themselves primarily in the intertemporal wedge are not worth pursuing.

5. Identification, the Importance of Financial Frictions and BCA

In the introduction we discussed the sense in which the importance of financial frictions is not identified under BCA. We explain this here. We describe a statistic which we use to characterize the importance of financial frictions. We show that a range of values for this statistic is consistent with the same value of the likelihood function.

Until now, the basic shocks driving the system have been u_t in (2.6). The interactions among these shocks are left almost completely unrestricted under BCA. In part, this is because the u_t's are found to be highly correlated in practice. This correlation is assumed to reflect that the elements of u_t are overlapping combinations of different fundamental economic shocks. Because fundamental economic shocks are assumed to be primitive and to have separate origins, they are often assumed to be uncorrelated. We make this uncorrelatedness assumption here. Denote the 5×1 vector of fundamental economic shocks by e_t. We normalize their variances to unity, so that $Ee_te_t' = I$. We assume that the fundamental shocks are related to the u_t's by the following invertible relationship:

$$u_t = Ce_t, \ Ee_te_t' = I, \ CC' = V, \tag{5.1}$$

where C has the structure of Q in (2.7).[22] It is well known that even with a particular estimate of V in hand, there are many C's that satisfy $CC' = V$. Alternative specifications of C that preserve the property, $CC' = V$, are observationally equivalent with respect to a set of observations, $Y = (Y_1, ..., Y_T)$. Because this property plays a key role in our analysis, it is useful to state it as a proposition:

[21] According to Balke and Gordon's data, per capita real investment, including durable goods, (1929 dollars), was 44, 65, 119, and 83 in the first to fourth quarters of 1933. Our estimate of the percent of aggregate output lost to adjustment costs is 0.77, 3.09, 17.04, and 1.69 for each of the four quarters in 1933. The number for 1933Q3 is very large. However, we note that it is generated by a rise in investment, not a fall. In addition, we are suspicious that investment rose 83 percent in 1933Q3 and then fell about 30 percent in 1933Q4. This sharp volatility is consistent with the possibility that measurement error overstated the level of invesment in 1933Q3.

[22] We are assuming that the fundamental economic shocks can be recovered from the space of current and past shocks. Lippi and Reichlin (1993) challenge this assumption and discuss some of the implications of its failure. See also Sims and Zha (1996) and Fernandez-Villaverde, Rubio-Ramirez and Sargent (2006).

Proposition 1. Consider a set of model parameter values, $\gamma = (P, P_0, V)$, with likelihood value, $L(Y|\gamma; R)$. Perturbations of C such that $CC' = V$ have no impact on the likelihood, L.

Obviously, Proposition 1 applies for both the linear and the nonlinear strategies we use to approximate the likelihood. Although BCA makes many detailed economic assumptions (e.g., details about utility and technology), it does not make the assumptions needed to identify the fundamental economic disturbances, e_t, to the economy.

We suppose, for the purpose of our discussion, that the third element in e_t corresponds to the financial frictions shock which originates in the intertemporal wedge, which is the third element of s_t.[23] To discuss the difficulty of pinning down the importance of financial frictions, it is useful to develop a constructive characterization of the family of C's that satisfy (5.1).[24] Write

$$C = \bar{C}W, \tag{5.2}$$

where W is any orthonormal matrix and $\bar{C}$ is the unique lower diagonal matrix with non-negative diagonal elements having the property that $\bar{C}\bar{C}' = V$. Although each C in (5.2) is observationally equivalent by Proposition 1, each C implies a different e_t. To see this, note that for any sequence of fitted disturbances, u_t, one can recover a time series of e_t using

$$e_t = C^{-1}u_t = W'\bar{C}^{-1}u_t. \tag{5.3}$$

To see how many e_t's there are, for given V and sequence u_t, let

$$W = \frac{1}{2a}\begin{bmatrix} a & b & c & d \\ -b & a & e & f \\ -c & -e & a & g \\ -d & -f & -g & a \end{bmatrix},$$

where $g = (cf - de)/b$. It is easy to verify that W is orthonormal for each $\theta = (a, b, c, d, e, f)$. For a fixed set of observed u_t, $t = 1, ..., T$, there is a different sequence, e_t, $t = 1, ..., T$, associated with almost all $\theta \in R^6$. According to Proposition 1, the likelihood of the data based on the linear approximation is constant with respect to variations in θ.

We are now in a position to describe our measure of the importance of financial frictions. This measure combines the two mechanisms by which financial frictions can matter. The first is that financial frictions represent a source of shocks (see Figure 2). For us, the stand-in for these shocks is e_{3t}. These operate on the economy by driving the intertemporal wedge, s_{3t}, (see (i) in Figure 2) and through spillover effects onto other wedges ((iii) in Figure 2). The second mechanism reflects that financial frictions modify the way non-financial friction shocks, e_{1t}, e_{2t}, e_{4t}, affect the economy. They do so by inducing movements in the intertemporal wedge (see (ii) in Figure 2). Our measure of the importance of financial frictions is the ratio of what the variance of HP-filtered output would be if only the financial frictions were operative, to

[23] In an agency cost model, these shocks could be perturbations to the variance of idiosyncratic disturbances affecting entrepreneurs, or to the survival rate of entrepreneurs. See Christiano, Motto and Rostagno (2004, 2006) for examples.

[24] Here, we follow the strategy pursued in Uhlig (2002).

the total variance of HP-filtered output. We construct this formally as follows. The wedges, s_t, have the following moving average representation (here, we ignore constant terms):

$$s_t = [I - PL]^{-1} Q\varepsilon_t = F(L)\varepsilon_t,$$

say. Define

$$\tilde{s}_t = \tilde{F}(L)\varepsilon_t.$$

Here, $\tilde{F}(L)$ denotes the version of $F(L)$ in which all elements have been set to zero, except those in the third column and the third row (i.e., $\tilde{F}(L)$ is $F(L)$ with the (i,j) elements set to zero, for $i,j = 1,2,4$.) The dynamics of $\tilde{s}_t$ reflect the mechanisms by which the financial frictions affect the wedges. The fact that the $3,3$ element of $F(L)$ is kept in $\tilde{F}(L)$ means that the financial friction shock is permitted to exert its effect on the intertemporal wedge, s_{3t}. The fact that we keep the other elements of the third column of $F(L)$ means that we include in $\tilde{F}(L)$ the spillover effects from the financial friction shock to the other wedges. Regarding the other elements of ε_t, $\tilde{F}(L)$ only includes their spillover effects onto the intertemporal wedge. This is our way of capturing the notion that financial frictions modify the transmission of non-financial shocks. Although $\tilde{s}_t$ represents the component of s_t corresponding to financial frictions, it is important to bear in mind that it is not an orthogonal decomposition of s_t.[25] For example, it is possible for the variance of $\tilde{s}_t$ to exceed that of s_t.

Write (4.7) in lag operator form:

$$\log \tilde{k}_t = \frac{\gamma L}{1-\lambda L} s_t,$$

and express the linearized observer equation, (4.8), as follows:

$$Y_t = h_0 s_t + h_1 \log \tilde{k}_t + v_t$$

where h_0 is a 4×4 matrix and h_1 is a 4×1 column vector. Then,

$$Y_t = H(L)F(L)\varepsilon_t + v_t,$$

where

$$H(L) = h_0 + h_1 \frac{\gamma L}{1-\lambda L}.$$

The representation of Y_t that reflects only the financial frictions is denoted $\tilde{Y}_t$, and is as follows:

$$\tilde{Y}_t = H(L)\tilde{F}(L)\varepsilon_t + v_t. \tag{5.4}$$

The spectral densities of $\tilde{Y}_t$ and Y_t are, respectively,

$$S_{\tilde{Y}}(\omega) = H(e^{-i\omega})\tilde{F}\left(e^{-i\omega}\right)\tilde{F}\left(e^{i\omega}\right)' H\left(e^{i\omega}\right)' + R$$

$$S_Y(\omega) = H(e^{-i\omega})F\left(e^{-i\omega}\right)F\left(e^{i\omega}\right)' H\left(e^{i\omega}\right)' + R.$$

[25] That is, $\tilde{s}_t$ and $s_t - \tilde{s}_t$ are correlated. Since $var(s_t) = var(\tilde{s}_t) + var(s_t - \tilde{s}_t) + 2cov(\tilde{s}, s_t - \tilde{s}_t)$, it is possible for $var(\tilde{s}_t) > var(s_t)$ if the covariance term is sufficiently negative.

The variance of Y_t, denoted C_0, can be computed by solving the following expression for large N :

$$C_0 = \frac{1}{N} S_Y(\omega_0) + \frac{2}{N} \sum_{k=1}^{\frac{N}{2}-1} re\left(S_Y(\omega_k)\right) + \frac{1}{N} S_Y(\omega_{N/2}),\ \omega_j = \frac{2\pi j}{N}.$$

The variance of $\tilde{Y}_t$, $\tilde{C}_0$, can be computed in an analogous way.

Our measure of the importance of financial frictions, f, is the 1,1 element of $\tilde{C}_0$, which we denote $\tilde{C}_0^{11}$. Our measure of financial frictions scales this by the 1,1 element of C_0 :

$$f = \frac{\tilde{C}_0^{11}}{C_0^{11}}. \tag{5.5}$$

Since it is a ratio of variances, f must be positive. However, because (5.5) is not based on an orthogonal decomposition, f may be larger than unity. The importance of financial frictions is not identified, because almost all perturbations in θ imply different values of f, but the same value of the likelihood, by Proposition 1.

6. Wedge Decompositions

We describe decompositions of the data during a recession which begins in period $t = t_1$ and ends in period $t = t_2$. CKM's strategy, which we call the 'baseline decomposition', is as follows. CKM ask how the recession would have unfolded if only the wedge, s_{3t}, evolved as it did and the other wedges remained constant at their values at the start of the recession. We find the sequence, ε_t, $t = t_1, ..., t_2$ which has the property that when this is input into (2.6), the third element of the simulated s_t, $t = t_1, ..., t_2$, coincides with its estimated values and the other elements of s_t are fixed at their value at $t = t_1$.

We investigate an alternative strategy for assessing the role of financial frictions, which recognizes the roles played by these frictions discussed in the introduction and in section 5. Such a strategy would choose a value for the rotation parameter, θ and use the implied sequence of e_t's to simulate (5.4). However, because $\tilde{F}$ is an infinite-ordered moving average representation, we decided this strategy is impractical and we devised a closely related one instead. The strategy we implemented ('rotation decomposition') recognizes that financial shocks drive both the intertemporal wedge and have spillover effects on other wedges. But, it does not capture the spillover effects from other shocks onto the intertemporal wedge. In this sense our rotation decomposition understates the role of financial frictions. However, we mitigate the latter effect by working with the rotation, θ, which maximizes the role of financial frictions, f.

The rotation decomposition is constructed as follows. We compute u_t, $t = t_1, ..., t_2$, and the value, θ^*, of $\theta \in R^6$ which maximizes f in (5.5). Then, we fix W and compute the implied sequence, e_t, for $t = t_1, ..., t_2$ using (5.3) and the value of C implied by (5.2). Next, set to zero all but the third element in e_t. After that, we compute the implied sequence of disturbances, $u_t^{\theta^*}$, $t = t_1, ..., t_2$ using (5.1). Here, the superscript θ^* highlights the dependence on the rotation parameter, θ^*. For input into our state space - observer system, (4.3)-(4.2), we require ε_t. We compute a sequence, $\varepsilon_t^{\theta^*}$, $t = t_1, ..., t_2$ using $\varepsilon_t^{\theta^*} = C^{-1} u_t^{\theta^*}$.

7. Empirical Results

This section documents two problems with BCA: conclusions are sensitive to modeling details and to the position one takes on spillover effects. In the first subsection we discuss the results for US postwar recessions. We then consider postwar recessions in the remaining OECD countries. Finally, we consider the US in the Great Depression.

7.1. US Postwar Recessions

7.1.1. Sensitivity of Baseline Decomposition to Modeling Details

In our analysis of the post-war US data, we examine five recessions. The 1982 recession, which is emphasized in CKM, is highlighted in the text. Details about the other post war US recessions are provided in Appendix C. Consider Table 1, which presents summary results for the 1982 US recession. The statistic reported in Table 1 is the fraction of the decline in output at the recession trough which is accounted for by the intertemporal wedge. The trough of the recession is defined as the quarter when detrended output achieves its minimum value. Panel 1a displays results based on the CKM specification of the intertemporal wedge (i.e., τ_{xt}) and Panel 1b displays results for the alternative specification (τ_t^k). In addition, results based on the baseline and rotation decompositions and with and without investment adjustment costs are reported. Finally, the table shows the impact of including CKM measurement error at the estimation stage of computing the wedges.

Turning to the CKM version of the wedge in Panel 1a we see that, regardless of whether measurement error is included in the analysis, adjustment costs make a substantial difference. Without investment adjustment costs, the intertemporal wedge contributes essentially nothing to the decline in output (or investment) in the 1982 recession. With adjustment costs, the intertemporal wedge accounts for roughly 30 percent of the decline in output at the trough of that recession. Evidently, adjustment costs have a very large impact on inference. At the same time, the impact of measurement error is nil, when we work with the CKM version of the intertemporal wedge.

Turning to the alternative specification of the intertemporal wedge, in Panel 1b we see that measurement error now matters a great deal. For example, with no measurement error and with adjustment costs, the intertemporal wedge accounts for over half the decline in output at the trough of the 1982 recession. With measurement error, that number falls to a much smaller (though still substantial!) 22 percent. The first column in the table shows that the CKM measurement error specification is strongly rejected by a likelihood ratio test whether or not adjustment costs are included in the analysis. So, the likelihood directs us to pay attention to the results without measurement error.

The results in Panel 1b show how much the specification of the intertemporal wedge matters. When CKM measurement error is used and there are no adjustment costs, the alternative formulation of the intertemporal wedge accounts for a substantial 24 percent of the drop in output at the trough of the 1982 recession. This stands in sharp contrast with the nearly zero percent drop implied by the CKM measure of that wedge. Interestingly, with the alternative measure of the wedge and with CKM measurement error, adjustment costs matter very little. When we set measurement error to zero (inducing a very large jump in the likelihood!) then adjustment costs matter a great deal, even with the alternative specification of the intertemporal wedge.

A more complete representation of our findings is reported in Figure 3, which displays results for the baseline decomposition of US data in the 1982 recession. To save space, Figure 3 reports results only for the alternative specification of the intertemporal wedge. The alternative version of the wedge is of special interest because of its conformity with the model in BBG.

In Figure 3, the circles indicate the zero line. The line with diamonds indicates the evolution of the data in response to all the wedges. By construction, the line with diamonds corresponds to the actual (detrended) data. The line marked with stars indicates the baseline decomposition when we estimated the model with the CKM specification of measurement error. The left column of graphs indicates results based on setting adjustment costs in investment to zero (i.e., $\Phi = 0$). The right column of graphs indicates results based on setting adjustment costs in investment to a level which implies a Tobin's q elasticity of unity. Note that for results based on estimation using the CKM measurement error specification, the intertemporal wedge accounts for relatively little of the movement in output, investment, hours worked and consumption. This conclusion is not sensitive to the introduction of adjustment costs in investment.

The line in Figure 3 indicated by pluses displays results based on estimation with measurement error set to zero. In the left column, we see that if the only wedge that had been active in the 1982 recession had been the intertemporal wedge, the US economy would have experienced a substantial boom (this can also be seen in Table 1). Investment would have been massively above trend, and consumption would have been massively below trend. These results show how sensitive BCA can be to seemingly minor details. Measurement error is very small under the CKM measurement error specification, yet it has a large impact on the outcome of BCA.

Measurement error also has a big impact on the assessment of the importance of adjustment costs. Comparing results in the left and right columns of Figure 3, we see that when measurement error is set to zero in estimation, then adjustment costs make a big difference to the assessment of the importance of the intertemporal wedge. The boom in output produced by the intertemporal wedge in the absence of adjustment costs becomes a recession when adjustment costs are turned on. As noted above, with adjustment costs the intertemporal wedge accounts for a very substantial 52 percent of the drop in output at the trough of the 1982 recession.

Results for four other US postwar recessions are presented in the appendix, and they generally support our findings for the 1982 recession: BCA results sensitive to the position taken on measurement error, the specification of the intertemporal wedge and on adjustment costs in investment.

7.1.2. The Potential Importance of Spillovers

The evidence for the 1982 recession in Figure 3 and for the other recession episodes is that the intertemporal wedge, when it has any impact at all, drives consumption and investment in opposite directions. At first, this may seem damaging to the proposition that shocks which drive the intertemporal wedge are important in business cycles, because consumption and investment are both procyclical in the data. This section shows that the opposite-signed response of consumption and investment is simply an artifact of ignoring spillover effects. Once spillover effects are taken into account, the evidence from BCA is consistent with consumption and investment responding with the same sign to an intertemporal wedge shock.

We quantify the potential importance of spillover effects by considering our rotation decomposition, discussed in section 6. Table 1 indicates that the intertemporal wedge accounts for almost the whole of the 1982 recession under the rotation decomposition, under almost all model perturbations. The one exception occurs in the case of no measurement error, no adjustment costs and τ_t^k intertemporal wedge.

We can see the results more completely for the alternative representation of the wedge, in Figure 4 (from here on, only results for the alternative representation of the wedge are presented). The left column of that figure reproduces the results of CKM's baseline decomposition from Figure 3. The right column displays the results based on the rotation decomposition. All results in Figure 4 are based on setting measurement error to zero. This is consistent with our remarks above, according to which CKM's measurement error specification has no a priori appeal, and it is overwhelmingly rejected in the post war data.

What we see in the right column of Figure 4 is that the estimated financial shock accounts for nearly the whole of the 1982 recession. Also, the financial shock drives consumption and investment in the same directions. This reflects the operation of spillover effects. We stress that the likelihood of the model on which the results in the left and right columns are based is the same. BCA provides no way to select between the two.

7.2. OECD Postwar Recessions

The results for postwar recessions in OECD countries for which we have data are summarized in Table 2, panel A (no adjustment costs) and Table 2, panel B (adjustment costs). For each country the entry represents the average of a statistic over all the recessions for which we have data. The statistic is the fraction of the decline in output in the trough of a recession, due to the intertemporal wedge. This is measured, as indicated in the table, according to the baseline or rotation decomposition.[26] In each panel, the bottom row is the weighted mean of the corresponding column entries. The weight for a given country is proportional to the number of recessions in that country's data.[27]

Consider first the case where the BCA methodology is closest to CKM, i.e., the case with measurement error, no investment adjustment costs and the baseline wedge decomposition.[28]

[26] The numbers for the United States are different from what is reported in Table 1, because all results in Table 2 are based on P and Q matrices with the zero restrictions indicated in (2.7). In addition, the numbers in Table 2 reflect an average over all recessions in the sample for each country, while Table 1 only pertains to the 1982 recession.

[27] For Belgium, we only have data for the 1990 recession; for Canada, the 1980 and 1990 recessions; for Denmark, the 1990 recession; for Finland, the 1974 and 1990 recessions; for France, the 1980 and 1990 recessions; for Germany, the 1990 recession; for Italy, the 1980 and 1990 recessions; for Japan, the 1990 recession; for Mexico, the 1990 recession; for Holland, the 1980 and 1990 recessions; for Norway, the 1990 recession; for Spain, the 1974 and 1990 recessions; for Switzerland, the 1990 recession; for the UK, the 1974, 1980, and 1990 recessions.

[28] However, recall that we now consider the alternative type of wedge, the τ_t^k wedge motivated by the CF

Note that there are numerous countries with fractions that are well above zero. Some are even above unity, which means that when the intertemporal wedge is fed to the RBC model, the model on average predicts bigger recessions than actually occurred. Overall, the average contribution of the intertemporal wedge to the fall in output in a trough is a substantial 22 percent.

As we found for the United States in the 1982 recession, when we then drop measurement error we find that the intertemporal wedge on average predicts an output boom in the OECD recessions for which we have data (Panel A, right portion). Although the measurement error used in the analysis is quite small, the outcome of BCA is evidently very sensitive to it.

Now consider what happens when we introduce adjustment costs, in Panel B. When we include measurement error in the analysis, there are several countries in which the intertemporal wedge plays a substantial role in recessions. However, there are several where the intertemporal wedge actually predicts a significant boom. As a result, the average contribution of the intertemporal wedge to business cycles across all countries is now about zero. When we now drop measurement error, the importance of the wedge jumps substantially for several countries. For example, it jumps from 15 percent to 46 percent in the United States and 33 percent to 75 percent in Canada. Some, however, such as Switzerland, go from 31 percent to -14 percent when measurement error is dropped. As a result, the overall average is a more modest jump of 16 percent.

Turning to the rotation wedge, we see that under that decomposition, the intertemporal wedge assumes a very large role in most countries. It is logically possible that the entire effect of this substantial importance assigned to financial shocks is due to spillover effects. In this case, one might be tempted to conclude that these are not actually shocks to the intertemporal wedge itself, and are better thought of as shocks to other wedges. To investigate this, we computed the ratio of the variance in HP filtered output due only to the spillover effects of financial shocks, to the total variance in HP filtered output due to financial frictions. This ratio is reported in the column, 'ratio', in Table 2. Note that in the case of no measurement error and investment adjustment costs, the ratio is only 30 percent for the US. Evidently, in US business cycles, the great importance assigned to financial shocks is not coming primarily from spillover effects. In other countries, the ratio is greater than unity, suggesting that spillovers are substantial (see Belgium, Germany and the UK). However, on average the ratio is only 60 percent, suggesting that the financial shocks identified in our rotation decomposition operate on the economy primarily by their direct impact on the intertemporal wedge.

We conclude that our findings for the postwar US also hold up on average across the other countries in the OECD.

7.3. US Great Depression

We now consider results for the US Great Depression. In this episode, the data exhibit substantial fluctuations and so it is perhaps not surprising that there is evidence of inaccuracy in the linear approximation of our model's solution. To quantify the degree approximation error we first estimate the capital stock for each date in the sample, by a two-sided Kalman pro-

and BGG models.

jection using the state-space representation of our model.[29] This, together with the realized wedges for each date, provided us with an estimate of the model's state for each date in the sample. Then, for each t we used the approximate policy rule to compute $(c_t, k_{t+1}, l_t, x_t, y_t)$ as a function of the date t state. We then computed the percent change in each of these 5 variables required for the four equilibrium conditions, (2.1), (2.2), (2.3), (2.8), plus the production function to be satisfied as a strict equality at t. For each t these calculations were done under the assumption that the period $t+1$ decisions are made using the approximate solution. Figure 5 shows that outside the 1930s, the approximation error associated with the linearized policy rule is for the most part fairly small. In the period of the 1930s, however, the approximation error becomes large, briefly reaching 65 percent for investment. We report the same measure of approximation error for the second order approximation to the model solution. In this case, the approximation errors are considerably smaller. Because of this evidence that the first order approximation has substantial approximation error, and because the second order approximation appears to be noticeably more accurate, we only display results for the Great Depression based on the second order approximation.

Consider the results in Figure 6. The left column displays the baseline decomposition and shows that the intertemporal wedge accounts for a substantial 21 percent of the fall in output in the Great Depression. In addition, that wedge drives consumption and investment in opposite directions. When we allow for spillovers using the rotation decomposition, we find that financial shocks may account for as much as 92 percent of the fall in output at the trough of the Great Depression.[30] Moreover, shocks to the intertemporal wedge drive consumption and investment in the same direction. We also did the calculations using the CKM measurement error and the results appear in Figure A9 in Appendix C. The results reported there are qualitatively similar to what emerges from Figure 6.

We conclude that results for the Great Depression are consistent with the findings for the postwar period. Taken as a whole, the evidence from BCA is consistent with the proposition that shocks to the intertemporal wedge play a significant role in business fluctuations.

8. Conclusion

Chari, Kehoe and McGrattan (2006) advocate the use of business cycle accounting to identify directions for improvement in equilibrium models. As a demonstration of the power of the approach, they argue that BCA can be used to rule out a prominent class of financial friction models. In particular, they conclude that models of financial frictions which create wedges in the intertemporal Euler equation are not promising avenues for understanding business cycle dynamics.

We have described two flaws in BCA which undermine its usefulness. First, consistent with economic theory, the results of BCA are not robust to small changes in the modeling environment. Second, BCA necessarily misses key mechanisms by which financial shocks which drive the intertemporal wedge affect the economy. The empirical correlations among

[29] Essentially, this involves using measured investment to compute the capital stock using the capital accumulation equation.

[30] Given the nonlinearity of the model, we could not compute the rotation decomposition as we did for postwar data. Instead, we computed the rotation that minimized the sum of squared deviations between the actual data and the predicted data using the estimated wedges.

wedges are consistent with the possibility that the financial shocks which drive the intertemporal wedge have important spillover effects on other wedges. These spillover effects are not identified under BCA. However, spillover effects are potentially so important that the evidence is consistent with the proposition that financial shocks are the major driving force in postwar recessions in the US and many OECD countries, as well as in the US Great Depression.

Fortunately, there are alternative ways to investigate whether given model features are useful in business cycle analysis. An approach which does not involve so many of the detailed model assumptions used by BCA, but which does incorporate the sort of assumptions needed to identify spillover effects, uses vector autoregressions.[31] An alternative approach works with fully specified, structural models. With the recent advances in computational technology and in economic theory, exploration of alternative models is relatively costless. A full set of references to the literature that explores the sort of financial frictions which are the object of interest in CKM would be too lengthy to include here. See Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2004, 2006) and Queijo (2005), and the references they cite.

Another approach uses a natural way to confront one of the identification problems with BCA. Absent direct observations, it is difficult to identify the intertemporal wedge and the rate of return of capital separately. However, as stressed in Cochrane and Hansen (1992), rates of return are the one type of economic variable on which we have excellent observations. For example, rates of return do not have the problems of interpretation associated with wages and they do not have the measurement error problems associated with observations on quantities like consumption and investment. The recent work of Primiceri, Schaumburg and Tambalotti (2005) carries out an analysis that is similar to business cycle accounting, except that they make use of direct measures of rates of return. They find that the estimates of τ_t^k (which they call 'preference shocks') assign that variable an important role in business cycle fluctuations.[32] A related approach is taken recently in Christiano, Motto and Rostagno (2006), who also include rates of return in the analysis. In addition, they integrate an explicit model of financial frictions and so are able to relate τ_t^k directly to primitive, uncorrelated financial shocks. When they feed the individual shocks to the model, holding other shocks fixed, they find that the financial shocks are an important driving force in business cycles.

[31] For a recent review, see Christiano, Eichenbaum and Vigfusson (2006).

[32] This approach is related to that of Hansen and Singleton (1982, 1983).

A. Appendix A: The Carlstrom-Fuerst Financial Friction Wedge

This section considers a version of the CF model, modified to include the adjustment costs in capital studied in CKM. We identify the version of the RBC model with wedges whose equilibrium coincides with that of the CF model with adjustment costs. We state the result as proposition A.3. For the RBC wedge economy to have the same equilibrium as the CF economy with adjustment costs requires several wedges and other adjustments. We then describe the parameter settings required in the original CF model to ensure that the adjustments primarily take the form of a wedge in the intertemporal Euler equation, and nowhere else. In this respect we follow the approach taken in CKM. To simplify the notation, we set the population growth rate to zero throughout the discussion in the appendix.

A.1. RBC Model With Adjustment Costs

To establish a baseline, we describe the version of the RBC model with adjustment costs. Preferences are:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t\right).$$

The resource constraint and the capital accumulation technology are, respectively,

$$c_t + x_t \leq k_t^{\alpha} \left(Z_t l_t\right)^{1-\alpha} \tag{A.1}$$

and

$$k_{t+1} = (1-\delta) k_t + x_t - \Phi\left(\frac{x_t}{k_t}\right) k_t. \tag{A.2}$$

The first order necessary conditions for optimization are:

$$\frac{-u_{l,t}}{u_{c,t}} = (1-\alpha)\left(\frac{k_t}{l_t}\right)^{\alpha} Z_t^{1-\alpha} \tag{A.3}$$

$$1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}}\left(1+R_{t+1}^k\right), \tag{A.4}$$

where the gross rate of return on capital is:

$$1+R_{t+1}^k = \frac{\alpha\left(\frac{k_{t+1}}{Z_{t+1} l_{t+1}}\right)^{\alpha-1} + P_{k,t+1}}{P_{k' t}}.$$

where $P_{k',t}$ is defined in (2.5) and

$$P_{k,t+1} = P_{k',t+1}\left[1-\delta-\Phi\left(\frac{x_{t+1}}{k_{t+1}}\right)+\Phi'\left(\frac{x_{t+1}}{k_{t+1}}\right)\frac{x_{t+1}}{k_{t+1}}\right]. \tag{A.5}$$

In the following two subsections, we argue that the CF financial frictions act like a tax on the gross return on capital, $1+R_{t+1}^k$, in (A.4). In particular, $1+R_{t+1}^k$ is replaced by

$$\left(1+R_{t+1}^k\right)\left(1-\tau_t^k\right).$$

This statement is actually only true as an approximation. Below we state, as a proposition, what the exact RBC model with wedges is, which corresponds to the CF model. We then explain the sense in which the wedge equilibrium just described is an approximation.

A.2. The CF Model With Adjustment Costs

Here, we develop the version of the CF model in which there are adjustment costs in the production of new capital. The economy is composed of firms, an η mass of entrepreneurs and a mass, $1-\eta$, of households. The sequence of events through the period proceeds as follows. First, the period t shocks are observed. Then, households and entrepreneurs supply labor and capital to competitive factor markets. Because firm production functions are homogeneous, all output is distributed in the form of factor income. Households and entrepreneurs then sell their used capital on a capital market. The total net worth of households and entrepreneurs at this point consists of their earnings of factor incomes, plus the proceeds of the sale of capital. Households divide this net worth into a part allocated to current consumption, and a part that is deposited in the bank. Entrepreneurs apply their entire net worth to a technology for producing new capital. They produce an amount of capital that requires more resources than they can afford with only their own net worth. They borrow the rest from banks. At this point the entrepreneur experiences an idiosyncratic shock which is observed to him, while the bank can only see it by paying a monitoring cost. This creates a conflict between the entrepreneur and the bank which is mitigated by the bank extending the entrepreneur a standard debt contract. After capital production occurs, entrepreneurs sell the new capital, and pay off their bank loan. Households receive a return on their deposits at the bank, and use the proceeds to purchase new capital. Entrepreneurs use their income after paying off the banks to buy consumption goods and new capital. All the newly produced capital is purchased by households and entrepreneurs, and all the economy's consumption goods are consumed. The next period, everything starts all over.

We now provide a formal description of the economy. The household problem is

$$\max_{\{c_{c,t},k_{ct+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t\right),$$

subject to:

$$c_t + q_t k_{c,t+1} \leq w_t^c l_t + \left[r_t + P_{k,t}\right] k_{c,t} \tag{A.6}$$

where c_t and $k_{c,t}$ denote household consumption and the household stock of capital, respectively. In addition, and l_t denotes household employment, w_t^c denotes the household's competitive wage rate, $P_{k,t}$ denotes the price of used capital and q_t denotes the price of capital available for production in the next period (the reason for not denoting this price by $P_{k',t}$ will be clear momentarily). After receiving their period t income, households allocate their net worth (the right side of (A.6)) to c_t and the rest, $w_t^c l_t + \left[r_t + P_{k,t}\right] k_{c,t} - c_t$, is deposited in a bank. These deposits earn a rate of return of zero. This is because markets are competitive and the opportunity cost to the household of the output they lend to the bank is zero. Later in the period, when the deposit matures, the households use the principal to purchase $k_{c,t+1}$ units of capital. The first order conditions of the household are (A.6) with the equality strict and:

$$1 = E_t \beta \frac{u_{c,t+1}}{u_{c,t}} \left[\frac{r_{t+1} + P_{k,t+1}}{q_t}\right] \tag{A.7}$$

$$\frac{-u_{l,t}}{u_{c,t}} = w_t^e, \tag{A.8}$$

where $u_{c,t}$ and $-u_{l,t}$ denote the time t marginal utilities of consumption and leisure, respectively.

The η entrepreneurs' present discounted value of utility is:

$$E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t c_{et}.$$

After the period t shocks are realized, the net worth of entrepreneurs, a_t, is:

$$a_t = w_t^e + [r_t + P_{k,t}] k_{et},$$

where w_t^e is the wage rate earned by the entrepreneur, who inelastically supplies his one unit of labor. The entrepreneur uses the a_t consumption goods, together with a loan from the bank to purchase the inputs into the production of capital goods. Entrepreneurs have access to the technology for producing capital, (2.1). The technology proceeds in two stages. In the first stage, the entrepreneur produces an intermediate input, i_t. In the second stage, that input results in ωi_t units of capital, which has a price, in consumption goods, q_t. The random variable, ω, is independently distributed across entrepreneurs, has mean unity, and cumulative density function,

$$\Psi(z) \equiv prob\,[\omega \leq z].$$

The entrepreneur who wishes to produce i_t units of the capital input faces the following cost function:

$$C(i_t; P_{k,t}) = \min_{\varphi_{k,t}, \varphi_{x,t}} P_{k,t}\varphi_{k,t} + \varphi_{x,t} + \lambda_t \left[i_t - (1-\delta)\varphi_{k,t} - \varphi_{x,t} + \Phi\left(\frac{\varphi_{x,t}}{\varphi_{k,t}}\right)\varphi_{k,t} \right],$$

where the constraint is that the object in square brackets is no less than zero. In addition, $\varphi_{k,t}$ and $\varphi_{x,t}$ denote the quantity of old capital and investment goods, respectively, purchased by the entrepreneur. The first order conditions for $\varphi_{k,t}$ and $\varphi_{x,t}$ are:

$$P_{k,t} = \lambda_t \left[1 - \delta - \Phi\left(\frac{\varphi_{x,t}}{\varphi_{k,t}}\right) + \Phi'\left(\frac{\varphi_{x,t}}{\varphi_{k,t}}\right)\frac{\varphi_{x,t}}{\varphi_{k,t}} \right] \quad \text{(A.9)}$$

$$1 = \lambda_t \left[1 - \Phi'\left(\frac{\varphi_{x,t}}{\varphi_{k,t}}\right) \right], \quad \text{(A.10)}$$

respectively. The reason for denoting the time t price of old capital by $P_{k,t}$ is now apparent. Substituting out for λ in (A.9) from (A.10), we see that the formula for $P_{k,t}$ here coincides with with the one implied by (A.5). The reason for not denoting the price of new capital by $P_{k',t}$ is also apparent. Comparing (A.10) with (2.5) we see that the formula for λ_t coincides with the formula for $P_{k',t}$ in the standard RBC model with adjustment costs. However, the equilibrium value of q_t will not coincide with λ_t here. This is because λ_t does not capture all the costs of producing new capital. It measures the marginal costs implied by the production technology. However, as discussed in detail in CF, it is missing the marginal cost that arises from the conflict between entrepreneurs and banks. This has the consequence that the production of capital necessarily involves some monitoring, and therefore also involves some destruction of capital.

Solving (A.10) for x_t/k_t in terms of λ_t, and using the result to substitute out for $\varphi_{x,t}/\varphi_{k,t}$ in (A.9):

$$P_{k,t}=\lambda_t\left[(1-\delta)-\frac{a}{2}\left(\frac{\frac{1}{\lambda_t}-1}{a}\right)^2+a\left(\frac{\frac{1}{\lambda_t}-1}{a}\right)\left(\frac{\frac{1}{\lambda_t}-1}{a}+b\right)\right]$$

Solving this for λ_t, provides the marginal cost function for producing i_t :

$$\lambda_t=\lambda\left(P_{k,t}\right). \tag{A.11}$$

Because all entrepreneurs face the same $P_{k,t}$, they will all choose the same ratio, $\varphi_{x,t}/\varphi_{k,t}$, regardless of the scale of production, i_t. Moreover, that ratio must be equal to the ratio of aggregate investment to the aggregate stock of capital.

The constant returns to scale feature of the production function implies that the total cost of producing i_t is:

$$C\left(i_t;P_{k,t}\right)=\begin{cases}\lambda\left(P_{k,t}\right)i_t & a>0\\ i_t & a=0\end{cases}$$

Consider an entrepreneur who has a_t units of goods and wishes to produce $i_t\geq a_t$, so that the entrepreneur must borrow $\lambda\left(P_{k,t}\right)i_t-a_t$ from the bank. Following CF, we suppose that the entrepreneur receives a standard debt contract. This specifies a loan amount and an interest rate, R_t^a, in consumption units. If the revenues of the entrepreneur turn out to be too low for him to repay the loan, then the entrepreneur is 'bankrupt' and he simply provides everything he has to the bank. In this case, the bank pays a monitoring cost which is proportional to the scale of the entprepeneur's project, μi_t^a. We now work out the equilibrium value of the parameters of the standard debt contract.

The value of ω such that entrepreneurs with smaller values of ω are bankrupt, is $\bar{\omega}_t^a$, where

$$\left[\lambda\left(P_{k,t}\right)i_t-a_t\right]R_t^a=\bar{\omega}_t^a i_t q_t.$$

Using this we find that the average, across all entrepreneurs with asset level a_t, of revenues is:

$$\begin{aligned}&i_tq_t\int_0^\infty\omega dF\left(\omega\right)-\int_{\bar{\omega}_t^a}^\infty R_t^a\left(\lambda\left(P_{k,t}\right)i_t^a-a_t\right)dF\left(\omega\right)-i_tq_t\int_0^{\bar{\omega}_t^a}\omega dF\left(\omega\right)\\ =\;&i_tq_tf\left(\bar{\omega}_t^a\right),\end{aligned}$$

where

$$f\left(\bar{\omega}_t^a\right)=\int_{\bar{\omega}_t^a}^\infty\omega d\Phi\left(\omega\right)-\bar{\omega}_t^a\left(1-\Phi\left(\bar{\omega}_t^a\right)\right).$$

The average receipts to banks, net of monitoring costs, across loans to all entprepreneurs with assets a_t is:

$$\begin{aligned}&q_ti_t^a\left[\int_0^{\bar{\omega}_t^a}\omega d\Phi\left(\omega\right)-\mu\Phi\left(\bar{\omega}_t^a\right)\right]+\left[\lambda\left(P_{k,t}\right)i_t-a_t\right]R_t^a\left[1-\Phi\left(\bar{\omega}_t^a\right)\right]\\ =\;&q_ti_t^ag\left(\bar{\omega}_t^a\right),\end{aligned}$$

where

$$g\left(\bar{\omega}_t^a\right) = \int_0^{\bar{\omega}_t^a} \omega d\Phi\left(\omega\right) - \mu\Phi\left(\bar{\omega}_t^a\right) + \bar{\omega}_t^a\left[1 - \Phi\left(\bar{\omega}_t^a\right)\right].$$

The contract with entrepreneurs with asset levels, a_t, that is assumed to occur in equilibrium is the one that maximizes the expected state of the entrepreneur at the end of the contract, subject to the requirement that the bank be able to pay the household a gross rate of interest of unity. The interval during which the entrepreneur produces capital is one in which there is no alternative use for the output good. So, the condition that must be satisfied for the bank to participate in the loan contract is:

$$q_t i_t g\left(\bar{\omega}_t^a\right) \geq \lambda\left(P_{k,t}\right) i_t - a_t.$$

The contract solves the following Lagrangian problem:

$$\max_{\bar{\omega}_t^a, i_t} i_t q_t f\left(\bar{\omega}_t^a\right) + \mu\left[q_t i_t g\left(\bar{\omega}_t^a\right) - \lambda\left(P_{k,t}\right) i_t + a_t\right].$$

The first order conditions for $\bar{\omega}_t^a$ and i_t are, after solving out for μ and rearranging:

$$q_t f\left(\bar{\omega}_t^a\right) = \frac{f'\left(\bar{\omega}_t^a\right)}{g'\left(\bar{\omega}_t^a\right)}\left[q_t g\left(\bar{\omega}_t^a\right) - \lambda\left(P_{k,t}\right)\right] \quad \text{(A.12)}$$

$$i_t = \frac{a_t}{\lambda\left(P_{k,t}\right) - q_t g\left(\bar{\omega}_t^a\right)} \quad \text{(A.13)}$$

From (A.12), we see that $\bar{\omega}_t^a = \bar{\omega}_t$ for all a_t, so that $R_t^a = R_t$ for all a_t. It then follows from (A.13) that the loan amount is proportional to a_t. As in the no adjustment cost case in CF, these two properties imply that in studying aggregates, we can ignore the distribution of assets across entrepreneurs.

The expected net revenues of the entrepreneurs, expressed in terms of a_t, are, after making use of (A.13):

$$i_t q_t f\left(\bar{\omega}_t\right) = \frac{q_t f\left(\bar{\omega}_t\right)}{\lambda\left(P_{k,t}\right) - q_t g\left(\bar{\omega}_t\right)} a_t. \quad \text{(A.14)}$$

At the end of the period, after the debt contract with the bank is paid off, the entrepreneurs who do not go bankrupt in the process of producing capital have income that can be used to buy consumption goods and new capital goods:

$$c_{et} + q_t k_{et+1} \leq \begin{cases} i_t q_t \omega - R_t\left(\lambda\left(P_{k,t}\right) i_t - a_t\right) & \omega \geq \bar{\omega}_t \\ 0 & \omega < \bar{\omega}_t \end{cases}. \quad \text{(A.15)}$$

An entrepreneur who is bankrupted in period t must set $c_{et} = 0$ and $k_{et+1} = 0$. In period $t+1$, these entrepreneurs start with net worth $a_{t+1} = w_{t+1}^e$. Entrepreneurs who are not bankrupted in period t can purchase positive amounts of c_{et} and $k_{e,t+1}$ (except in the non-generic case, $\omega = \bar{\omega}_t$). For these entrepreneurs, the marginal cost of purchasing $k_{e,t+1}$ is q_t units of consumption. The time t expected marginal payoff from $k_{e,t+1}$ at the beginning of period $t+1$ is $E_t\left[r_{t+1} + P_{k,t+1}\right]$. In each aggregate state in period $t+1$, the entrepreneur expands his net worth by the value of $\left[r_{t+1} + P_{k,t+1}\right]$ in that state. This extra net worth can be leveraged into additional bank loans, which in turn permit an expansion in the entrepreneur's payoff by investing in the capital production technology. The expected value of this additional payoff (relative to date $t+1$ idiosyncratic uncertainty) corresponds to the coefficient on a_t in (A.14). So, the expected rate of return available to entrepreneurs who are not bankrupt in period t is:

$$E_t\left[\frac{r_{t+1}+P_{k,t+1}}{q_t}\times\zeta_{t+1}\right], \tag{A.16}$$

which they equate to $1/(\beta\gamma)$. Here,

$$\zeta_{t+1}=\max\left[\frac{q_{t+1}f\left(\bar{\omega}_{t+1}\right)}{\lambda\left(P_{k,t+1}\right)-q_{t+1}g\left(\bar{\omega}_{t+1}\right)},1\right].$$

The expression to the left of '$\times$' in (A.16) is the rate of return enjoyed by ordinary households. The reason that ζ_{t+1} cannot be less than unity is that and entprepeneur can always obtain unity, simply by consuming his net worth in the following period and not producing any capital. Averaging over all budget constraints in (A.15):

$$c_{et}+q_tk_{et+1}=\frac{q_tf\left(\bar{\omega}_t\right)}{\lambda\left(P_{k,t}\right)-q_tg\left(\bar{\omega}_t\right)}a_t.$$

Here, c_{et} and $k_{e,t+1}$ refer to averages across all entrepreneurs.

Output is produced by goods-producers using a linear homogeneous technology,

$$y\left(k_t,l_t,\eta,Z_t\right)=k_t^{\alpha}\left(\left(1-\eta\right)Z_tl_t\right)^{1-\alpha-\zeta}\eta^{\zeta}, \tag{A.17}$$

where k_t is the sum of the capital owned by households and the average capital held by entrepreneurs:

$$k_t=\left(1-\eta\right)k_{ct}+\eta k_{e,t}.$$

The argument, η, in y is understood to apply to the second occurrence of η. The arguments in the production function reflect our assumption that the entrepreneur supplies one unit of labor, and households supply l_t units of labor. Profit maximization implies:

$$y_{k,t}=r_t,\ y_{l,t}=w_t^c,\ y_{3,t}=w_t^e. \tag{A.18}$$

We now collect the equilibrium conditions for the economy. The production of new capital goods by the average entrepreneur is:

$$\begin{aligned}&i_t\int_0^{\infty}\omega dF\left(\omega\right)-\mu i_t\int_0^{\bar{\omega}_t}dF\left(\omega\right)\\=\;&i_t\left[1-\mu F\left(\bar{\omega}_t\right)\right].\end{aligned}$$

Since there are η entprepreneurs, the total new capital produced is $k_{t+1}=\eta i_t\left[1-\mu F\left(\bar{\omega}_t\right)\right]$, so that

$$k_{t+1}=\left[\left(1-\delta\right)k_t+x_t-\Phi\left(\frac{x_t}{k_t}\right)k_t\right]\left[1-\mu F\left(\bar{\omega}_t\right)\right]. \tag{A.19}$$

The resource constraint is:

$$\left(1-\eta\right)c_t+\eta c_t^e+x_t=k_t^{\alpha}\left(\left(1-\eta\right)Z_tl_t\right)^{1-\alpha-\zeta}\eta^{\zeta}. \tag{A.20}$$

Substituting (A.18) into (A.7) and (A.8):

$$1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} \frac{y_{k,t+1} + P_{k,t+1}}{q_t} \quad (A.21)$$

$$\frac{-u_{l,t}}{u_{c,t}} = y_{l,t}. \quad (A.22)$$

The budget constraint of the entrepreneur is:

$$c_{et} + q_t k_{et+1} = \lambda \left(P_{k,t}\right) \frac{k_{t+1}}{\eta \left[1 - \mu F\left(\bar{\omega}_t\right)\right]} q_t f\left(\bar{\omega}_t\right) \quad (A.23)$$

The efficiency conditions associated with the contract are:

$$q_t f\left(\bar{\omega}_t\right) = \frac{f'\left(\bar{\omega}_t\right)}{g'\left(\bar{\omega}_t\right)} \left[q_t g\left(\bar{\omega}_t\right) - \lambda\left(P_{k,t}\right)\right] \quad (A.24)$$

$$\frac{k_{t+1}}{\eta \left[1 - \mu F\left(\bar{\omega}_t\right)\right]} = \frac{a_t}{\lambda\left(P_{k,t}\right) - q_t g\left(\bar{\omega}_t\right)} \quad (A.25)$$

$$a_t = y_{3,t} + y_{k,t} k_{et} + P_{k,t} k_{et} \quad (A.26)$$

The intertemporal efficiency condition for the entrepreneur is (assuming the condition, $\zeta_{t+1} \geq 1$, is not binding):

$$E_t \left[\frac{F_{k,t+1} + P_{k,t+1}}{q_t} \times \frac{q_{t+1} f\left(\bar{\omega}_{t+1}\right)}{\lambda\left(P_{k,t+1}\right) - q_{t+1} g\left(\bar{\omega}_{t+1}\right)}\right] = \frac{1}{\gamma\beta} \quad (A.27)$$

Taking the ratio of (A.9) to (A.10), we obtain:

$$P_{k,t} = \frac{1 - \delta - \Phi\left(\frac{x_t}{k_t}\right) + \Phi'\left(\frac{x_t}{k_t}\right)\frac{x_t}{k_t}}{1 - \Phi'\left(\frac{x_t}{k_t}\right)} \quad (A.28)$$

The 10 variables to be determined with the 10 equations, (A.19)-(A.28) are: c_t, $c_{e,t}$, x_t, k_t, $k_{e,t}$, l_t, $P_{k,t}$, q_t, $\bar{\omega}_t$, a_t.

It is convenient to define a sequence of markets equilibrium formally. Let s^t denote a history of realizations of shocks. Then,

Definition A.1. *An equilibrium of the CF economy with adjustment costs is a sequence of prices,* $\{P_k(s^t), q(s^t), w^e(s^t), w^c(s^t), r(s^t)\}$ *, quantities,* $\{c(s^t), c_e(s^t), x(s^t), k(s^t), k_e(s^t), l(s^t), a(s^t)\}$ *, and* $\{\bar{\omega}(s^t)\}$ *such that:*

(i) Households optimize (see (A.21), (A.22))

(ii) Entrepreneurs optimize (see (A.23), (A.26), (A.27), (A.28))

(iii) Firms optimize (see (A.18))

(iv) Conditions related to the standard debt contract are satisfied (see (A.24), (A.25))

(v) The resource constraint and capital accumulations equations are satisfied (see (A.19), (A.20))

A.3. The CF Model as an RBC Model with Wedges

We now construct a set of wedges for the RBC economy in section A.1, such that the equilibrium for the distorted version of that economy coincides with the equilibrium for the CF economy. We begin by constructing the following state-contingent sequences:

$$
\begin{aligned}
\psi\left(s^t\right) &= 1-\mu F\left(\tilde{\bar{\omega}}\left(s^t\right)\right), \\
\tau_x\left(s^t\right) &= \frac{\psi\left(s^t\right)\tilde{q}\left(s^t\right)}{\lambda\left(\tilde{P}_{k,t}\left(s^t\right)\right)}-1, \\
\theta\left(s^t\right) &= \frac{\tilde{P}_k\left(s^t\right)\tau_x\left(s^t\right)}{\tilde{r}\left(s^t\right)}, \\
G\left(s^t\right) &= \eta\left(\tilde{c}^e\left(s^t\right)-\tilde{c}\left(s^t\right)\right), \\
T\left(s^t\right) &= G\left(s^t\right)-\tau_x\left(s^t\right)\tilde{x}\left(s^t\right)-\theta\left(s^t\right)\tilde{r}\left(s^t\right)\tilde{k}\left(s^{t-1}\right), \\
D\left(s^t\right) &= \tilde{w}^e\left(s^t\right)\eta,
\end{aligned}
\tag{A.29}
$$

where $\tilde{q}$, $\tilde{c}^e$, $\tilde{c}$, $\tilde{w}^e$, $\tilde{r}$, $\tilde{k}$, $\tilde{x}$, $\tilde{\bar{\omega}}$ and $\tilde{P}_k$ correspond to the objects without '$\tilde{\ }$' in a CF equilibrium. Also, λ is the function defined in (A.11). In this subsection, we treat D, ψ, θ, τ_x, G and T as given exogenous stochastic processes, outside the control of agents. Here, D, G, and T represent exogenous sequences of profits, government spending and lump sum taxes. Also, θ and τ_x are tax rates on capital rental income and investment good purchases. Finally, ψ is a technology shock in the production of physical capital.

Consider the following budget constraint for the household:

$$
\begin{aligned}
& c\left(s^t\right)+\left(1+\tau_x\left(s^t\right)\right)x\left(s^t\right) \\
\leq\ & \left(1-\theta\left(s^t\right)\right)r\left(s^t\right)k\left(s^{t-1}\right)+w\left(s^t\right)l\left(s^t\right)-T\left(s^t\right)+D\left(s^t\right).
\end{aligned}
\tag{A.30}
$$

Here, r is the rental rate on capital, w is the wage rate, and l measures the work effort of the household. Each of these variables is a function of s^t and is determined in an RBC wedge economy. At time 0 the household takes prices, taxes and $k\left(s^{-1}\right)$ as given and chooses c, k and l to maximize utility:

$$
\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)u\left(c\left(s^t\right),l\left(s^t\right)\right)
$$

subject to the budget constraint, no-Ponzi game and non-negativity constraints. Here, $\pi\left(s^t\right)$ is the probability of history, s^t.

Households operate the following backyard technology to produce new capital:

$$
k\left(s^t\right)=\left[\left(1-\delta\right)k\left(s^t\right)+x\left(s^t\right)-\Phi\left(\frac{x\left(s^t\right)}{k\left(s^{t-1}\right)}\right)k\left(s^{t-1}\right)\right]\psi\left(s^t\right). \tag{A.31}
$$

The first order necessary conditions for household optimization are:

$$u_l\left(s^t\right)+u_c\left(s^t\right)w\left(s^t\right)=0, \tag{A.32}$$

$$\frac{1+\tau_x\left(s^t\right)}{\psi\left(s^t\right)}P_{k'}\left(s^t\right) \tag{A.33}$$

$$= \sum_{s^{t+1}|s^t}\beta\pi\left(s^{t+1}|s^t\right)\frac{u_c\left(s^{t+1}\right)}{u_c\left(s^t\right)}\left[r\left(s^{t+1}\right)\left(1-\theta\left(s^{t+1}\right)\right)+\left(1+\tau_x\left(s^{t+1}\right)\right)P_k\left(s^{t+1}\right)\right],$$

where

$$P_k\left(s^t\right)\equiv\frac{1-\delta-\Phi\left(\frac{x\left(s^t\right)}{k\left(s^t\right)}\right)+\Phi'\left(\frac{x\left(s^t\right)}{k\left(s^t\right)}\right)\frac{x\left(s^t\right)}{k\left(s^t\right)}}{1-\Phi'\left(\frac{x\left(s^t\right)}{k\left(s^{t-1}\right)}\right)}. \tag{A.34}$$

Equation (A.32) is the first order condition associated with the optimal choice of $l\left(s^t\right)$. Equation (A.33) combines the first order order conditions associated with the optimal choice of $x\left(s^t\right)$ and $k\left(s^t\right)$. Also,

$$P_{k'}\left(s^t\right)\equiv\frac{1}{1-\Phi'\left(\frac{x\left(s^t\right)}{k\left(s^{t-1}\right)}\right)}, \tag{A.35}$$

is the pre-tax marginal cost of producing new capital, in units of the consumption good. In addition, $\pi\left(s^{t+1}|s^t\right)\equiv\pi\left(s^{t+1}\right)/\pi\left(s^t\right)$ is the conditional probability of history s^{t+1} given s^t.

The technology for firms is taken from (A.17):

$$y\left(k,l,\eta,Z\right)=k^{\alpha}\left(\left(1-\eta\right)Zl\right)^{1-\alpha-\zeta}\eta^{\zeta},$$

where, as before, the third argument in y refers only to the second occurrence of η. There are three inputs: physical capital, household labor and another factor whose aggregate supply is fixed at η. Profit maximization leads to:

$$r\left(s^t\right)=y_k\left(s^t\right),\ w\left(s^t\right)=y_l\left(s^t\right),\ w^e\left(s^t\right)=y_\eta\left(s^t\right). \tag{A.36}$$

The household is assumed to own the representative firm, and it receives the earnings of η in the form of lump-sum profits, $D\left(s^t\right)$. We do allow allow trade in claims on firms, a restriction that is non-binding on allocations because the households are identical.

We now state the equilibrium for the RBC wedge economy:

Definition A.2. *An RBC wedge equilibrium is a set of quantities,* $\left\{c\left(s^t\right),l\left(s^t\right),k\left(s^t\right),x\left(s^t\right)\right\}$, *and prices* $\left\{P_k\left(s^t\right),P_{k'}\left(s^t\right),r\left(s^t\right),w\left(s^t\right)\right\}$, *and a set of taxes, profits and government spending,* $\left\{G\left(s^t\right),\tau_x\left(s^t\right),\theta\left(s^t\right),T\left(s^t\right)\right\}$, *technology shocks,* $\left\{Z\left(s^t\right),\psi\left(s^t\right)\right\}$, *such that*

(i) The quantities solve the household problem given the prices, taxes, profits, government spending and the shock to the backyard investment technology

(ii) Firm optimization is satisfied

(iii) Relations (A.29) is satisfied, for given state-contingent sequences, $\tilde{q}$, $\tilde{c}^e$, $\tilde{c}$, $\tilde{w}^e$, $\tilde{r}$, $\tilde{k}$, $\tilde{x}$, $\tilde{\tilde{\omega}}$ *and* $\tilde{P}_k$.

The variables to be determined in an RBC wedge equilibrium are c, l, k, x, P_k, $P_{k'}$, r and w. The 8 equations that can be used to determine these are (A.30)-(A.36). It is easily verified that c, l, k, x, P_k, r and w coincide with the corresponding objects in a CF equilibrium. In addition, $P_{k'}$ coincides with $\lambda(P_k)$ in a CF equilibrium. To see this, one verifies that the equilibrium conditions in the RBC wedge economy coincide with the equilibrium conditions in the CF economy. First, (A.31) coincides with (A.19). After using (A.36), we see that (A.32) coincides with (A.22). Consider the household budget equation evaluated at equality. Substituting out for lump sum transfers:

$$\begin{aligned} & c\left(s^t\right)+\left(1+\tau_x\left(s^t\right)\right)x\left(s^t\right) \\ = & \left(1-\theta\left(s^t\right)\right)r\left(s^t\right)k\left(s^{t-1}\right)+w\left(s^t\right)l\left(s^t\right) \\ & \tau_x\left(s^t\right)x\left(s^t\right)+\theta\left(s^t\right)r\left(s^t\right)k\left(s^{t-1}\right)+w^e\left(s^t\right)\eta+G\left(s^t\right), \end{aligned}$$

or,

$$\begin{aligned} & (1-\eta)\,c\left(s^t\right)+x\left(s^t\right)+\eta c^e\left(s^t\right) \\ = & r\left(s^t\right)k\left(s^{t-1}\right)+w\left(s^t\right)l\left(s^t\right)+w^e\left(s^t\right)\eta \\ = & y\left(k\left(s^{t-1}\right),l\left(s^t\right),\eta,Z\left(s^t\right)\right), \end{aligned} \tag{A.37}$$

by linear homogeneity. Here, $Z(s^t)=Z(s_t)$, where s_t is the realization of period t uncertainty. Equation (A.37) coincides with (A.20). Substitute out for θ and τ_x from (A.29) into (A.33), and rearranging, we obtain:

$$1=E_t\frac{u_c\left(s^{t+1}\right)}{u_c\left(s^t\right)}\left[\frac{r\left(s^{t+1}\right)+P_k\left(s^{t+1}\right)}{\frac{1+\tau_x(s^t)}{\psi(s^t)}P_{k'}\left(s^t\right)}\right].$$

Note that by definition of $1+\tau_x(s^t)$ in (A.29),

$$\frac{1+\tau_x\left(s^t\right)}{\psi\left(s^t\right)}P_{k'}\left(s^t\right)=\frac{P_{k'}\left(s^t\right)q\left(s^t\right)}{\lambda\left(P_{k,t}\left(s^t\right)\right)}.$$

Combining (A.11) and (A.10), we find that $\lambda(P_{k,t}(s^t))=P_{k'}(s^t)$, so that the household's intertemporal Euler equation reduces to (A.21), or (after making use of (A.36)):

$$E_t\frac{u_c\left(s^{t+1}\right)}{u_c\left(s^t\right)}\left[\frac{y_k\left(s^{t+1}\right)+P_k\left(s^{t+1}\right)}{P_{k'}\left(s^t\right)}\left(1-\tau^k\left(s^t\right)\right)\right]=1, \tag{A.38}$$

where

$$1-\tau^k\left(s^t\right)=\frac{\psi\left(s^t\right)}{1+\tau_x\left(s^t\right)}.$$

We conclude that conditions (A.19)-(A.22) in the CF economy are satisfied. The remaining equilibrium conditions are satisfied, given (A.29). We state this result as a proposition:

Proposition A.3. *Consider a CF equilibrium, and a set of taxes, technology shocks and transfers computed in (A.29). The objects,* $\{c(s^t), l(s^t), k(s^t), x(s^t)\}, \{P_k(s^t), r(s^t), w(s^t)\}$ *and* $P_{k'}(s^t)=\lambda(P_k(s^t))$ *in the CF equilibrium correspond to an RBC wedge equilibrium.*

For η and ζ close to zero and ψ close to unity, the RBC wedge equilibrium converges to the equilibrium conditions of the RBC model with adjustment costs in section A.1 with a wedge, $1-\tau^k$, in the intertemporal Euler equation, (A.4).

B. Appendix B: The Bernanke-Gertler-Gilchrist Financial Friction Wedge

In this section we briefly review the BGG model and derive the RBC wedge model to which it corresponds. In the model there are households, capital producers, entrepreneurs and banks. At the beginning of the period, households supply labor to factor markets, and entrepreneurs supply capital. Output is then produced and an equal amount of income is distributed among households and entrepreneurs. Households then make a deposit with banks, who lend the funds on to entrepreneurs. Entrepreneurs have a special expertise in the ownership and management of capital. They have their own net worth with which to acquire capital. However, it is profitable for them to leverage this net worth into loans from banks, and acquire more capital than they can afford with their own resources. The source of friction is a particular conflict between the entrepreneur and the bank. In the management of capital, idiosyncratic things happen, which either make the management process more profitable than expected, or less so. The problem is that the things that happen in this process are observed only by the entrepreneur. The bank can observe what happens inside the management of capital, but only at a cost. As a result, the entrepreneur has an incentive to underreport the results to the bank, and thereby attempt to keep a greater share of the proceeds for himself. To mitigate this conflict, it is assumed that entrepreneurs receive a standard debt contract from the bank.

The capital that entrepreneurs purchase at the end of the period is sold to them by capital producers. The latter use the old capital used within the period, as well as investment goods, to produce the new capital that is sold to the entrepreneurs. Capital producers have no external financing need. They finance the purchase of used capital and investment goods using the revenues earned from the sale of new capital.

The budget constraint of households is:

$$c_t + B_{t+1} \leq (1 + R_t) B_t + w_t l_t + T_t,$$

where R_t denotes the interest earned on deposits with the bank, b_t denotes the beginning-of-period t stock of those deposits, w_t denotes the wage rate, l_t denotes employment and T_t denotes lump sum transfers. Subject to this budget constraint and a no-Ponzi condition, households seek to maximize utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t).$$

Households' first order conditions, in addition to the transversality condition, are:

$$\begin{aligned} u_{c,t} &= \beta E_t u_{c,t} (1 + R_{t+1}) \\ \frac{-u_{l,t}}{u_{c,t}} &= w_t. \end{aligned}$$

Firms have the following production function:

$$y_t = k_t^{\alpha} (Z_t l_t)^{1-\alpha} = y(k_t, l_t, Z_t).$$

They rent capital and hire labor in perfectly competitive markets at rental rate, r_t, and wage rate, w_t, respectively. Optimization implies:

$$y_{k,t} = r_t, \ y_{l,t} = w_t.$$

Capital producers purchase investment goods, x_t, and old capital, k_t, to produce new capital, k_{t+1}, using the following linear homogeneous technology:

$$k_{t+1} = (1-\delta) k_t + x_t - \Phi\left(\frac{x_t}{k_t}\right) k_t.$$

The competitive market prices of k_t and k_{t+1} are $P_{k,t}$ and $P_{k',t}$, respectively. Capital producer optimization leads to the following conditions:

$$\begin{aligned} P_{k,t} &= \frac{1}{1-\Phi'\left(\frac{x_t}{k_t}\right)}\left[1-\delta-\Phi\left(\frac{x_t}{k_t}\right)+\Phi'\left(\frac{x_t}{k_t}\right)\frac{x_t}{k_t}\right] \\ P_{k',t} &= \frac{1+g_n}{1-\Phi'\left(\frac{x_t}{k_t}\right)}. \end{aligned}$$

At the end of period t, entrepreneurs have net worth, N_{t+1}, and it is assumed that $N_{t+1} < P_{k',t}k_{t+1}$. As a result, in an equilibrium in which the entire stock of capital is to be owned and operated, entprepreneurs must borrow:

$$b_{t+1} = P_{k',t}k_{t+1} - N_{t+1}. \tag{B.1}$$

As soon as an individual entrepreneur purchases k_{t+1}, he experiences a shock, and k_{t+1} becomes $k_{t+1}\omega$. Here, ω is a random variable that is iid across entrepreneurs and has mean unity. The realization of ω is unknown before the loan is made and it is known only to the entrepreneur after it is realized. The bank which extends the loan to the entrepreneur must pay a monitoring cost in order to observe the realization of ω. The cumulative distribution function of ω is F, where

$$\Pr ob\left[\omega < x\right] = F(x).$$

Entrepreneurs receive a standard debt contract from their bank, which specifies a loan amount, b_{t+1}, and a gross rate of return, Z_{t+1}, in the event that it is feasible for the entrepreneur to repay. The lowest realization of ω for which it is feasible to repay is $\bar{\omega}_{t+1}$, where

$$\bar{\omega}_{t+1}\left(1+R_{t+1}^k\right) P_{k',t}k_{t+1} = Z_{t+1}b_{t+1}. \tag{B.2}$$

For $\omega < \bar{\omega}_{t+1}$ the entrepreneur simply pays all its revenues to the bank:

$$\left(1+R_{t+1}^k\right)\omega P_{k',t}k_{t+1}.$$

In this case, the bank monitors the entrepreneur. Following BGG, we assume that monitoring costs are a fraction, μ, of the total earnings of the entrepreneur:

$$\mu\left(1+R_{t+1}^k\right)\omega P_{k',t}k_{t+1}.$$

At time t the bank borrows b_{t+1} from households. In each state of $t+1$ the bank pays households

$$\left(1+R_{t+1}\right)b_{t+1} \tag{B.3}$$

units of currency. The bank's source of funds in each state of period $t+1$ is the earnings from non-bankrupt entrepreneurs plus the earnings of bankrupt entrepreneurs, net of monitoring costs:

$$\left[1-F\left(\bar{\omega}_{t+1}\right)\right]Z_{t+1}b_{t+1}+(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega dF\left(\omega\right)\left(1+R_{t+1}^k\right)P_{k',t}k_{t+1}. \tag{B.4}$$

We follow BGG, who implicitly suppose that at date t there are no state-contingent markets for currency in date $t+1$. As a consequence, (B.3) must not exceed (B.4) in any date $t+1$ state. This condition, together with competition among banks, leads to:

$$\left[1-F\left(\bar{\omega}_{t+1}\right)\right]Z_{t+1}b_{t+1}+(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega dF\left(\omega\right)\left(1+R_{t+1}^k\right)P_{k',t}k_{t+1}=\left(1+R_{t+1}\right)b_{t+1}.$$

Substituting from (B.2) for $Z_{t+1}b_{t+1}$ and dividing by $\left(1+R_{t+1}^k\right)P_{k',t}k_{t+1}$:

$$\left[1-F\left(\bar{\omega}_{t+1}\right)\right]\bar{\omega}_{t+1}+(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega dF\left(\omega\right)=\left(\frac{1+R_{t+1}}{1+R_{t+1}^k}\right)\frac{b_{t+1}}{P_{k',t}k_{t+1}}.$$

We conclude that the gross return on capital can be expressed:

$$1+R_{t+1}=\left(1-\tau_{t+1}^k\right)\left(1+R_{t+1}^k\right),$$

where the 'wedge', $1-\tau_{t+1}^k$, satisfies:

$$1-\tau_{t+1}^k=\frac{P_{k',t}k_{t+1}}{P_{k',t}k_{t+1}-N_{t+1}}\left(\left[1-F\left(\bar{\omega}_{t+1}\right)\right]\bar{\omega}_{t+1}+(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega dF\left(\omega\right)\right).$$

The wedge, τ_t^k, contains two additional endogenous variables, N_{t+1} and $\bar{\omega}_{t+1}$. These are determined in general equilibrium by the introduction of two additional equations: the condition associated with the fact that the standard debt contract maximizes the utility of the entrepreneur, as well as the law of motion for entrepreneurial net worth.

The resource constraint for this economy is:

$$c_t+G_t+x_t=k_t^{\alpha}\left(Z_tl_t\right)^{1-\alpha},$$

where G_t includes any consumption of entrepreneurs, as well as monitoring costs incurred by banks. As long as these latter can be ignored, then the BGG financial friction is to, in effect, introduce a tax on the rate of return on capital in, $1+R_{t+1}^k$, in (A.4). In particular, $1+R_{t+1}^k$ is replaced by

$$\left(1+R_{t+1}^k\right)\left(1-\tau_{t+1}^k\right).$$

Note there is a slight difference with CF financial frictions in that the latter imply the tax rate is not a function of period $t+1$ uncertainty, while the BGG frictions imply that in general it is a function of this uncertainty.

C. Appendix C: Other Empirical Results

Figures A1 - A9 display additional results for US recessions. Figures A1-A8 pertain to four postwar US recessions. Figures A1 - A4 are the analog of Figure 3 for the 1982 recession. Figures A5 - A8 are the analog of Figure 4. Figure A9 is the analog of Figure 6 for the US Great Depression, except that it is based on BCA, when measurement error is set to zero in estimation.

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Table 1: Summary of Results for the 1982 US Recession

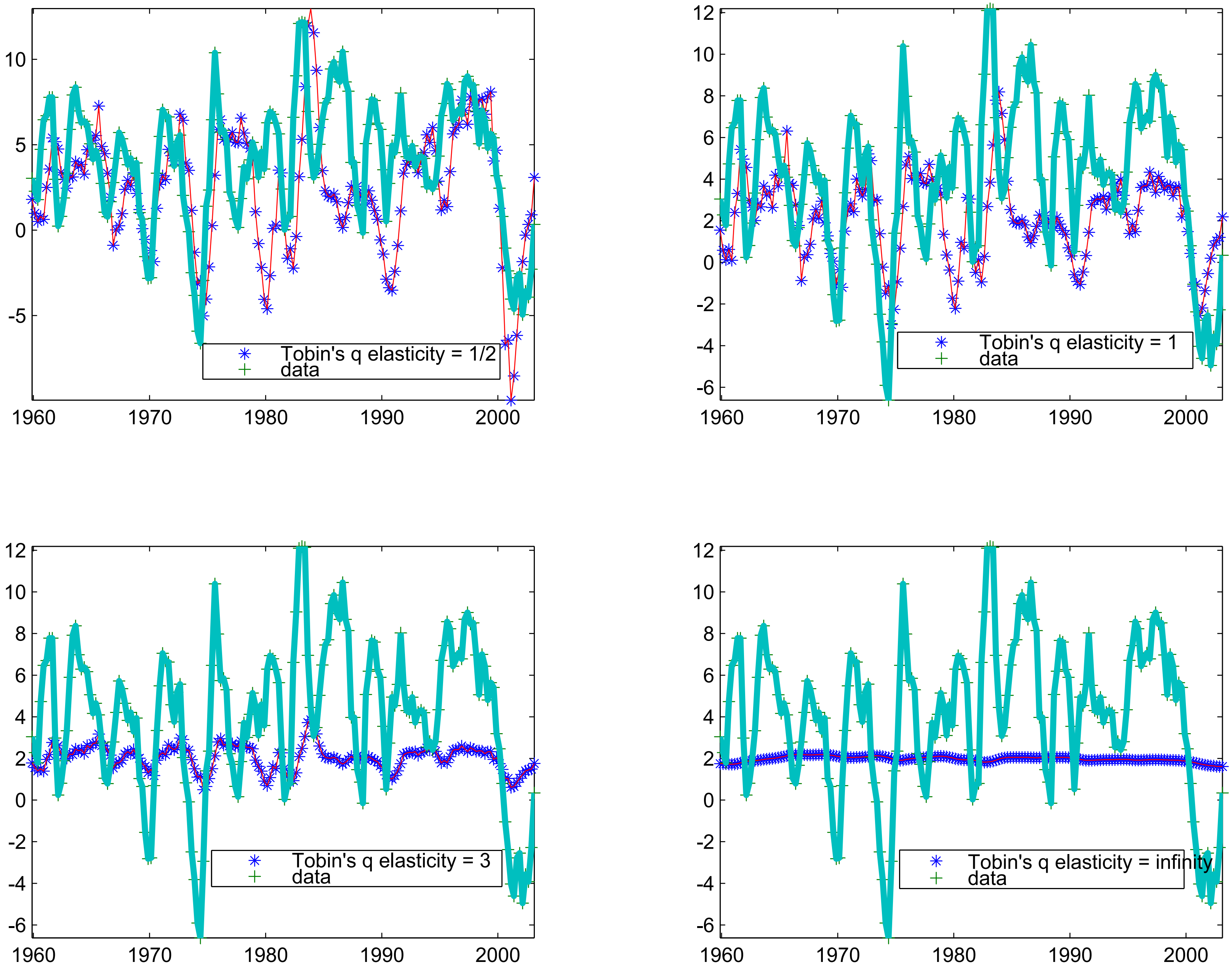
Panel 1a: CKM Version of Intertemporal Wedge ($\tau_{x,t}$)				
	CKM measurement error		No measurement error	
	baseline	rotation	baseline	rotation
	Tobin's q elasticity $= \infty$			
	-0.03	0.92	0.047	0.96
	Tobin's q elasticity $= 1$			
	0.31	1.02	0.28	1.05
Panel 1b: Alternative Version of the Wedge (BGG, τ_t^k)				
Likelihood ratio test of	CKM measurement error		No measurement error	
CKM measurement error	baseline	rotation	baseline	rotation
	Tobin's q elasticity $= \infty$			
481	0.24	1.32	-1.31	-0.01
	Tobin's q elasticity $= 1$			
541	0.22	1.05	0.53	0.93

Notes: (1) Likelihood ratio statistic - twice difference between the log likelihood under estimation with CKM measurement error specification, and under estimation with measurement error set to zero; (2) CKM measurement error: results based on estimation with CKM specification of R in (3.6), (3) No measurement error: based on estimation subject to $R = 0$, (4) baseline decomposition - see text, (5) rotation - rotation of shocks which maximizes importance of financial frictions, as defined in text.

Table 2: Alternative Version of the Wedge (BGG, τ_t^k)						
Panel A: **Tobin's q Elasticity = ∞**						
	CKM Measurement Error			**No Measurement Error**		
	Baseline	**Rotation**	**Ratio**	**Baseline**	**Rotation**	**Ratio**
Country						
United States	0.24	0.90	0.49	-1.11	-0.15	1.29
Belgium	0.61	0.83	0.28	-1.12	0.24	0.31
Canada	0.20	1.11	0.69	-0.51	0.95	1.25
Denmark	0.39	1.15	0.00	0.18	1.11	0.92
Finland	1.18	1.58	0.13	0.24	1.31	0.44
France	0.13	1.63	0.96	-3.10	1.59	0.41
Germany	0.44	1.10	0.77	-1.87	-3.29	0.00
Italy	-4.83	1.85	0.00	-0.19	1.38	0.45
Japan	-0.00	1.02	1.01	0.27	1.67	1.41
Mexico	0.00	1.06	1.00	-0.07	1.05	1.00
Netherlands	2.50	3.02	0.04	-0.01	1.25	0.68
Norway	1.27	-0.23	0.00	-0.56	0.79	0.90
Spain	1.49	1.51	0.00	-0.00	1.66	0.97
Switzerland	-0.14	0.95	1.03	-0.24	0.89	1.01
England	0.25	1.10	0.60	0.04	1.19	0.89
Weighted Mean	0.22	1.30	0.44	-0.59	0.80	0.85

Panel B: **Tobin's q Elasticity = 1**						
	CKM Measurement Error			**No Measurement Error**		
	Baseline	**Rotation**	**Ratio**	**Baseline**	**Rotation**	**Ratio**
Country						
United States	0.15	0.89	0.73	0.46	0.85	0.30
Belgium	-0.09	0.85	1.01	-0.69	1.25	1.39
Canada	0.33	1.26	0.55	0.75	1.86	0.39
Denmark	0.14	1.05	0.99	-0.17	0.95	1.12
Finland	-6.25	-5.72	0.02	-0.60	0.49	0.33
France	1.45	1.43	0.02	1.13	0.99	0.02
Germany	0.33	1.08	1.32	-0.08	0.90	1.22
Italy	0.66	1.29	0.41	-0.51	3.72	0.11
Japan	0.71	1.05	1.12	0.23	2.06	1.01
Mexico	-0.04	0.92	1.06	0.40	1.04	0.41
Netherlands	0.76	1.25	0.43	0.04	1.51	0.65
Norway	-0.06	1.10	0.91	-0.29	1.53	0.78
Spain	1.93	1.28	0.03	0.67	1.29	0.96
Switzerland	0.31	1.11	0.67	-0.14	1.38	0.24
England	-0.01	1.07	0.91	-0.08	1.26	1.24
Weighted Mean	-0.01	0.61	0.61	0.16	1.37	0.60

Figure 1A: Actual and Model Rates of Return, Different Tobin's q Elasticities



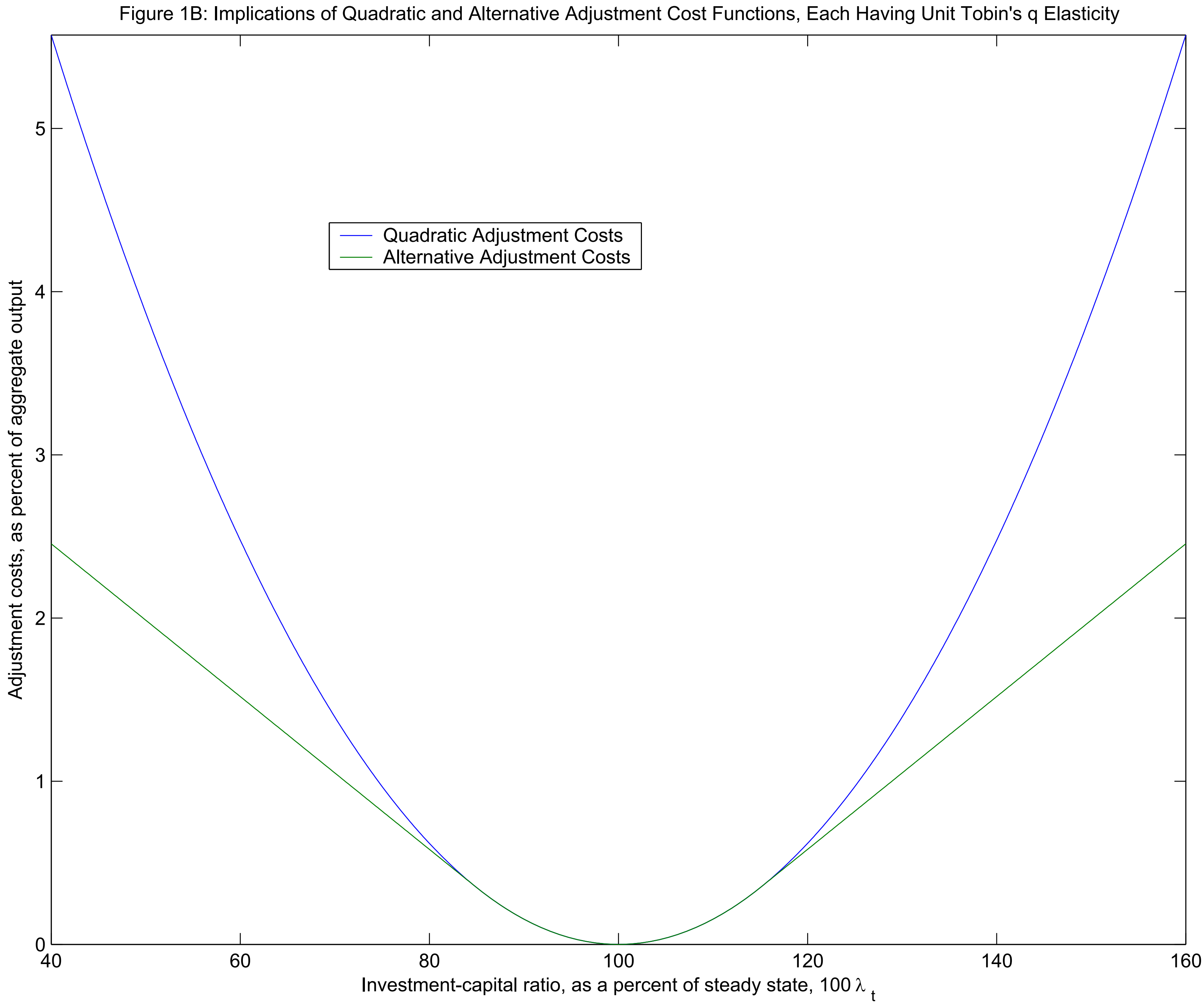
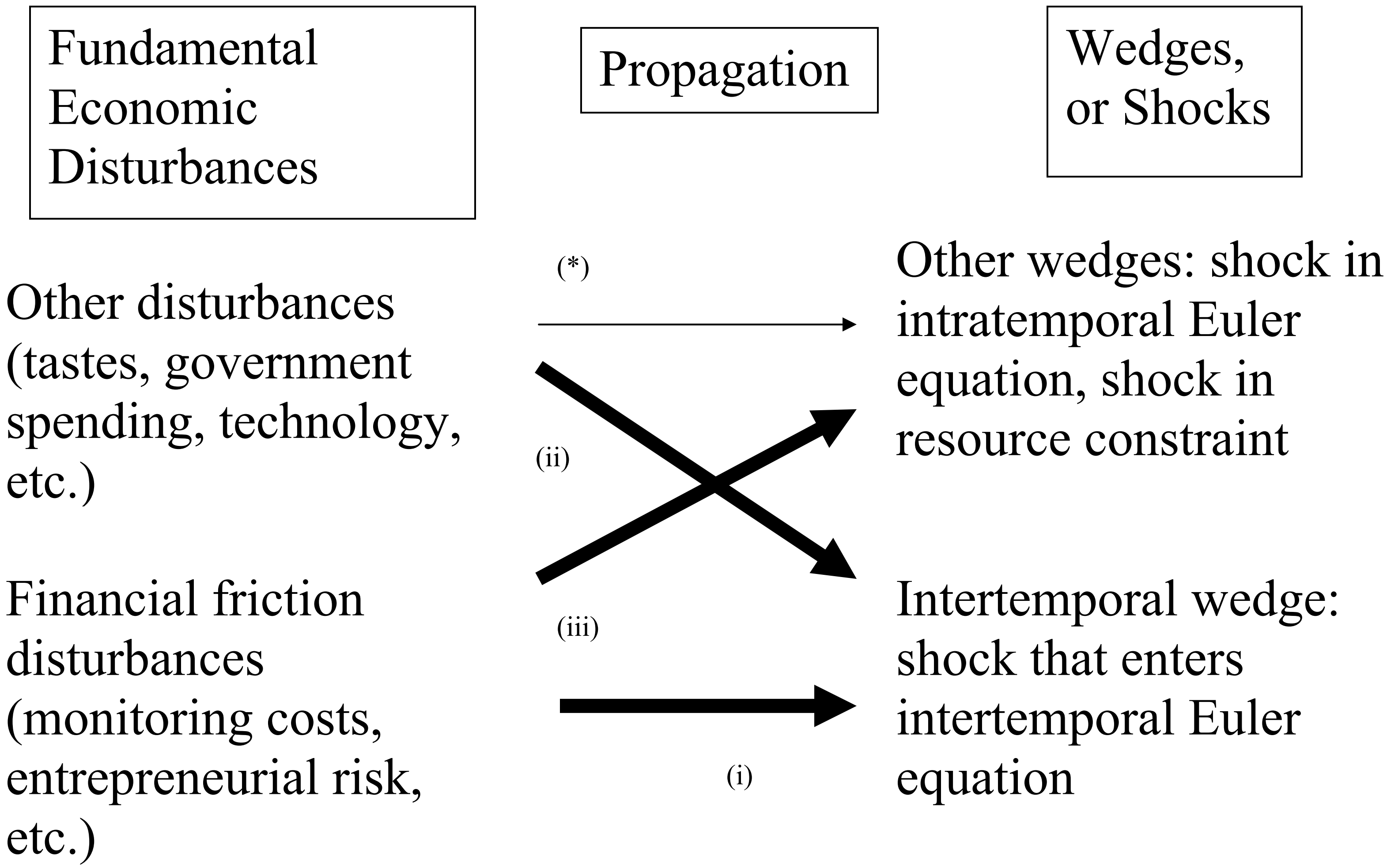


Figure 1B: Implications of Quadratic and Alternative Adjustment Cost Functions, Each Having Unit Tobin's q Elasticity
Quadratic Adjustment Costs
Alternative Adjustment Costs
Adjustment costs, as percent of aggregate output
Investment-capital ratio, as a percent of steady state, 100 λ t
0
1
2
3
4
5
40
60
80
100
120
140
160

Figure 2: The propagation of economic disturbances through wedges



(*) movements in other wedges due to other disturbances
(i) movements in the intertemporal wedge due to financial disturbances
(ii) movements in the intertemporal wedge due to spillover effects from standard disturbances
(iii) movements in other wedges due to spillovers from financial disturbances

Figure 3: Raw Data ('All Wedges') and Various Counterfactual Simulations

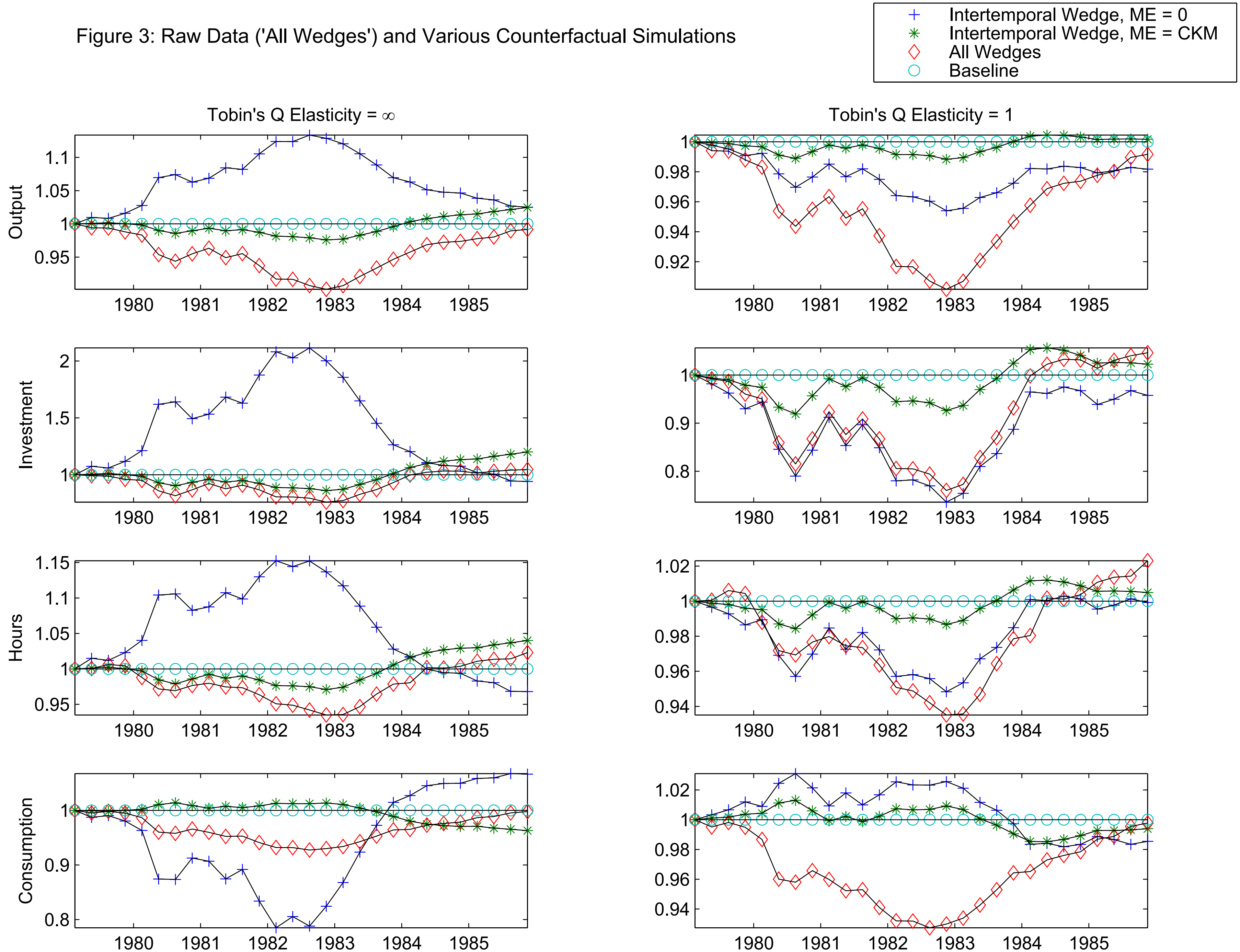
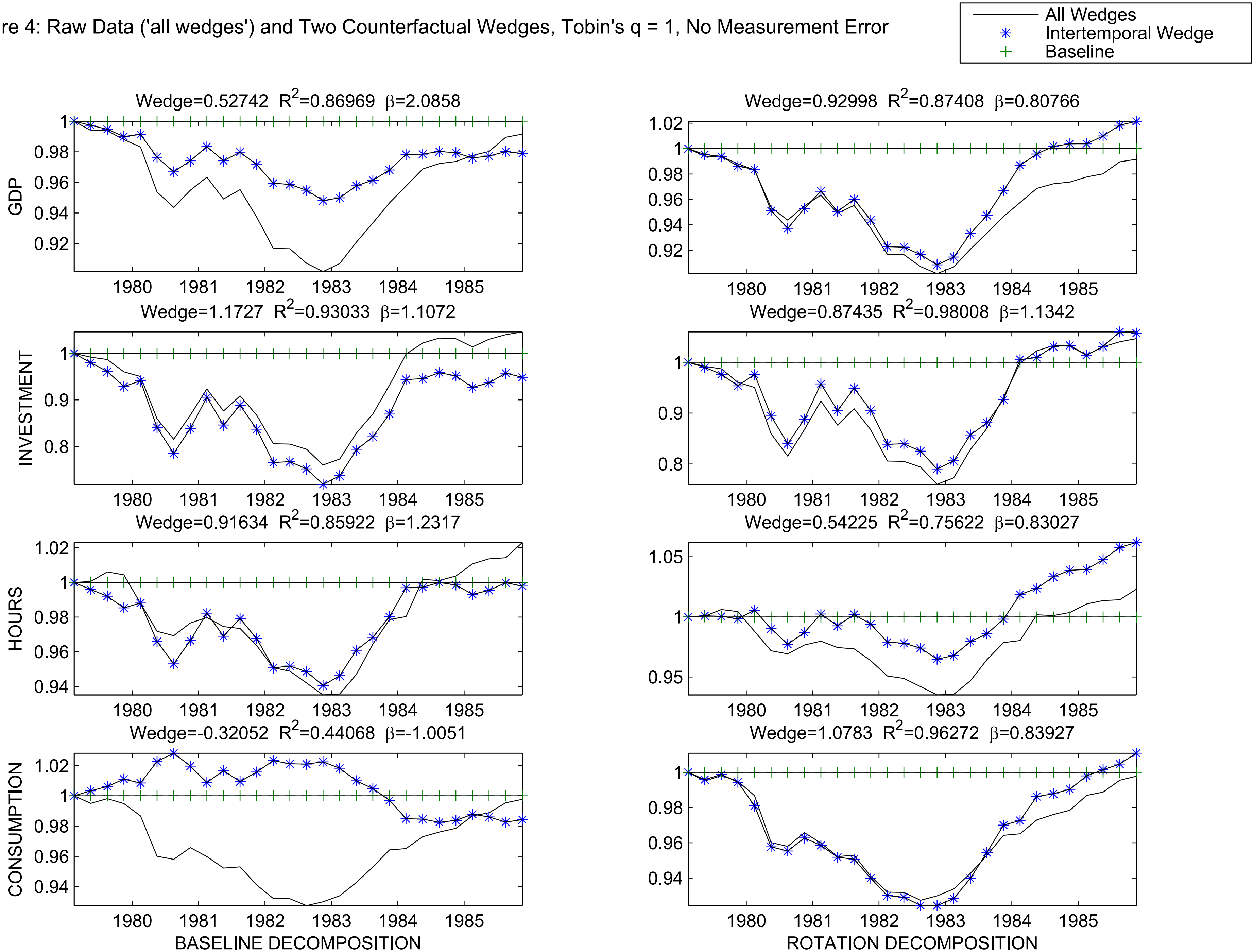


Figure 4: Raw Data ('all wedges') and Two Counterfactual Wedges, Tobin's q = 1, No Measurement Error



Note (1) wedge - fraction of fall in raw data at the trough for output accounted for by the intertemporal wedge; (2) R^2 - R-square in regression of raw data on wedge component throughout recession episode; (3) β - slope coefficient in preceding regression

Figure 5A: Percent Euler Errors, Output

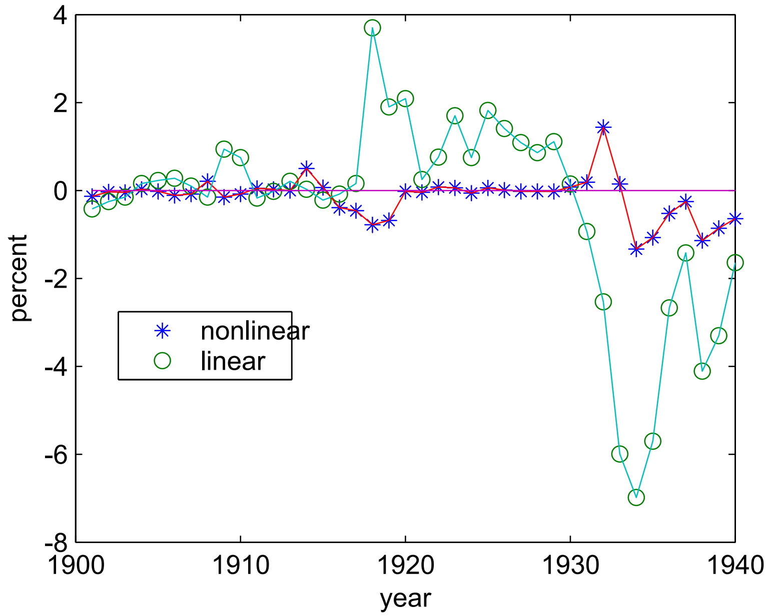


Figure 5B: Percent Euler Errors, Consumption

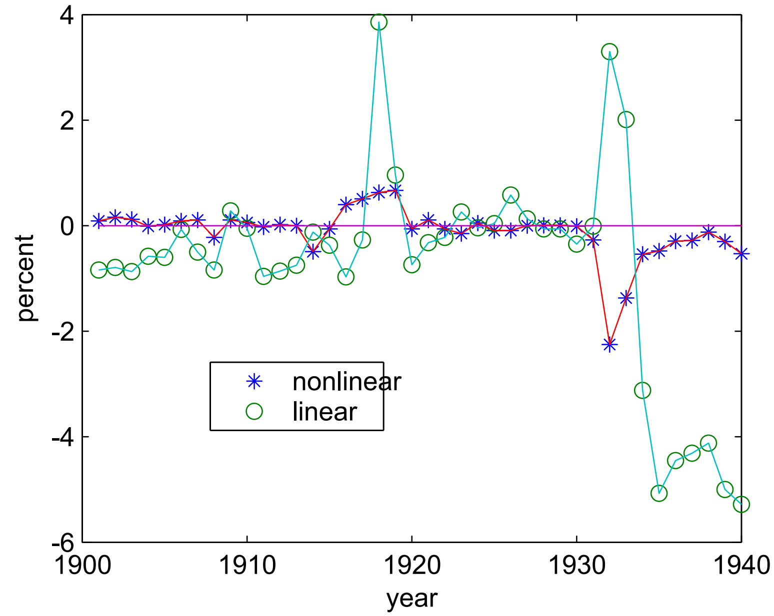


Figure 5C: Percent Euler Errors, Hours

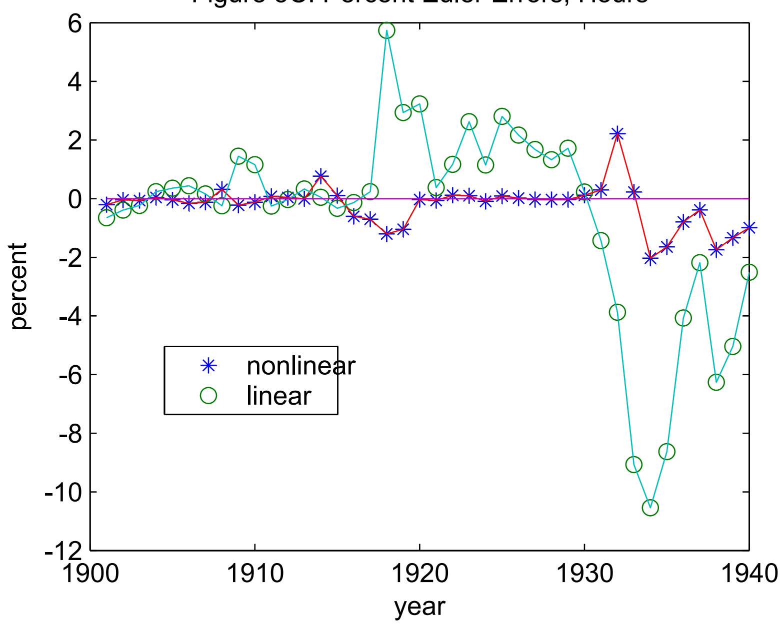


Figure 5D: Percent Euler Errors, Investment

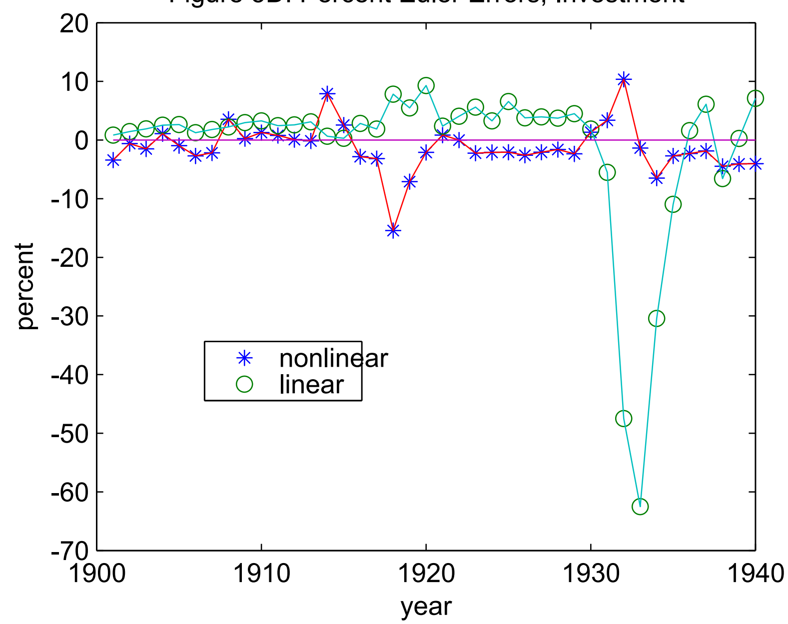
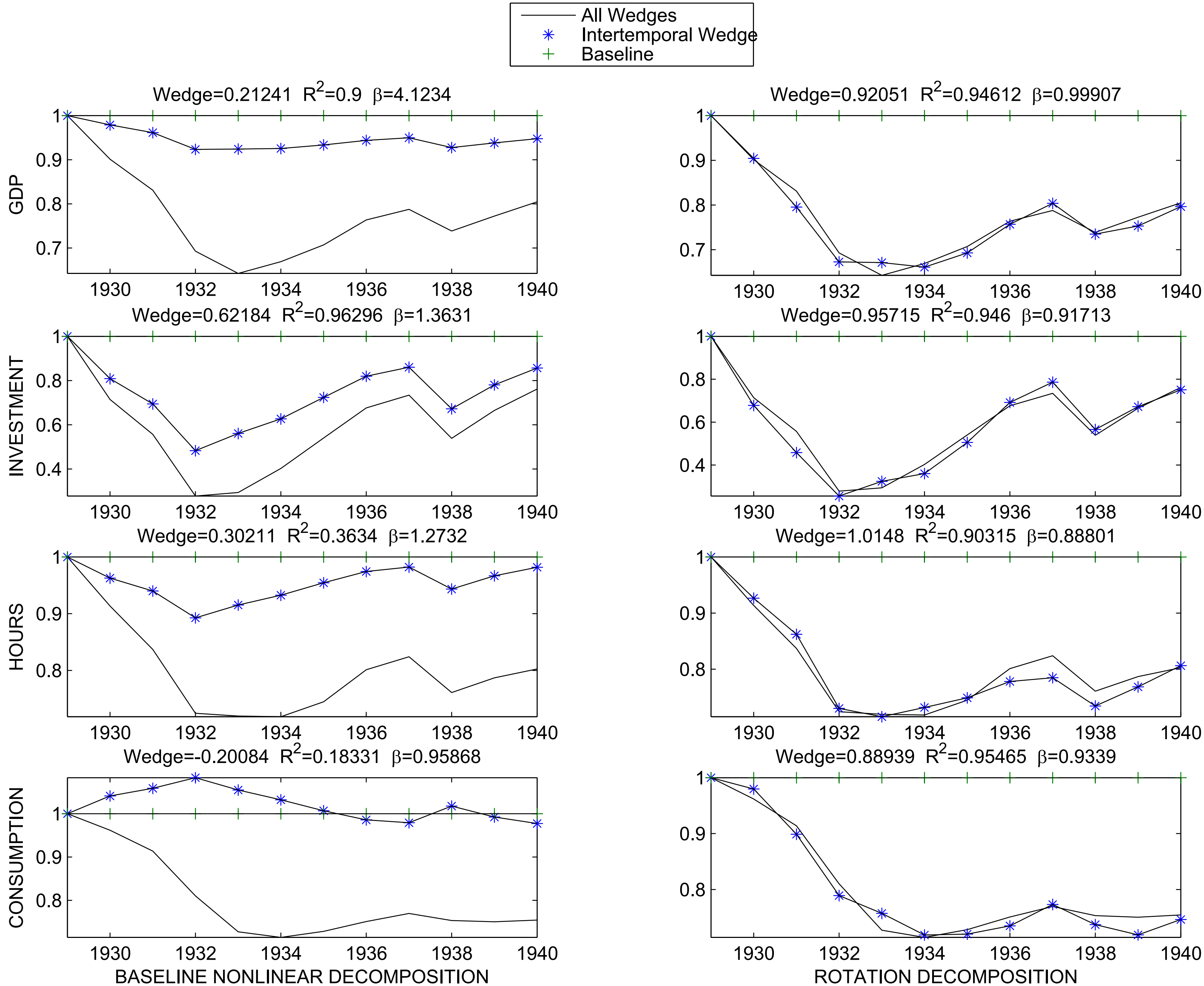
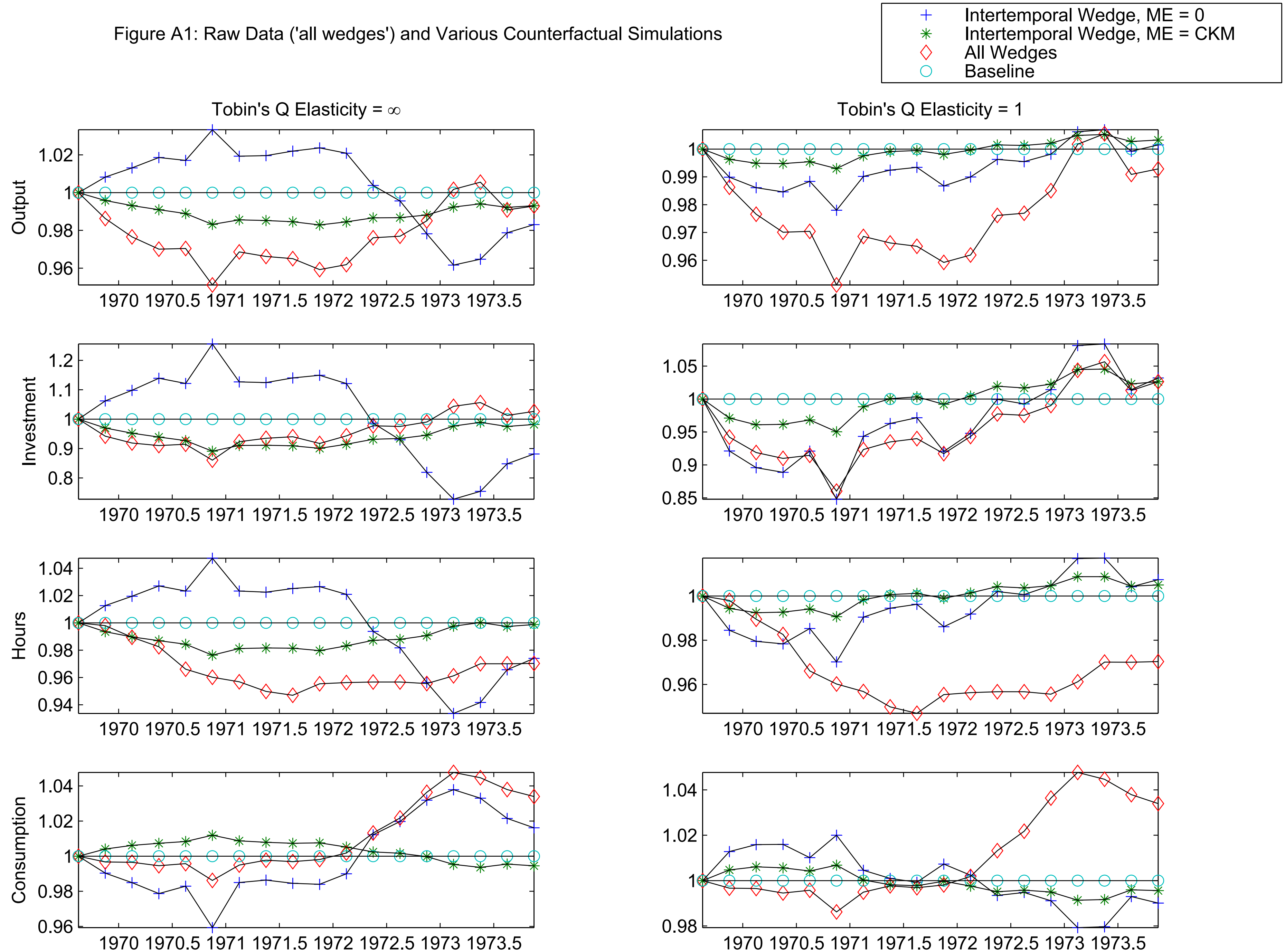


Figure 6: Wedges, US Great Depression Based on Second Order Approximation to Model, CKM Measurement Error, Tobin's q Elasticity = 1



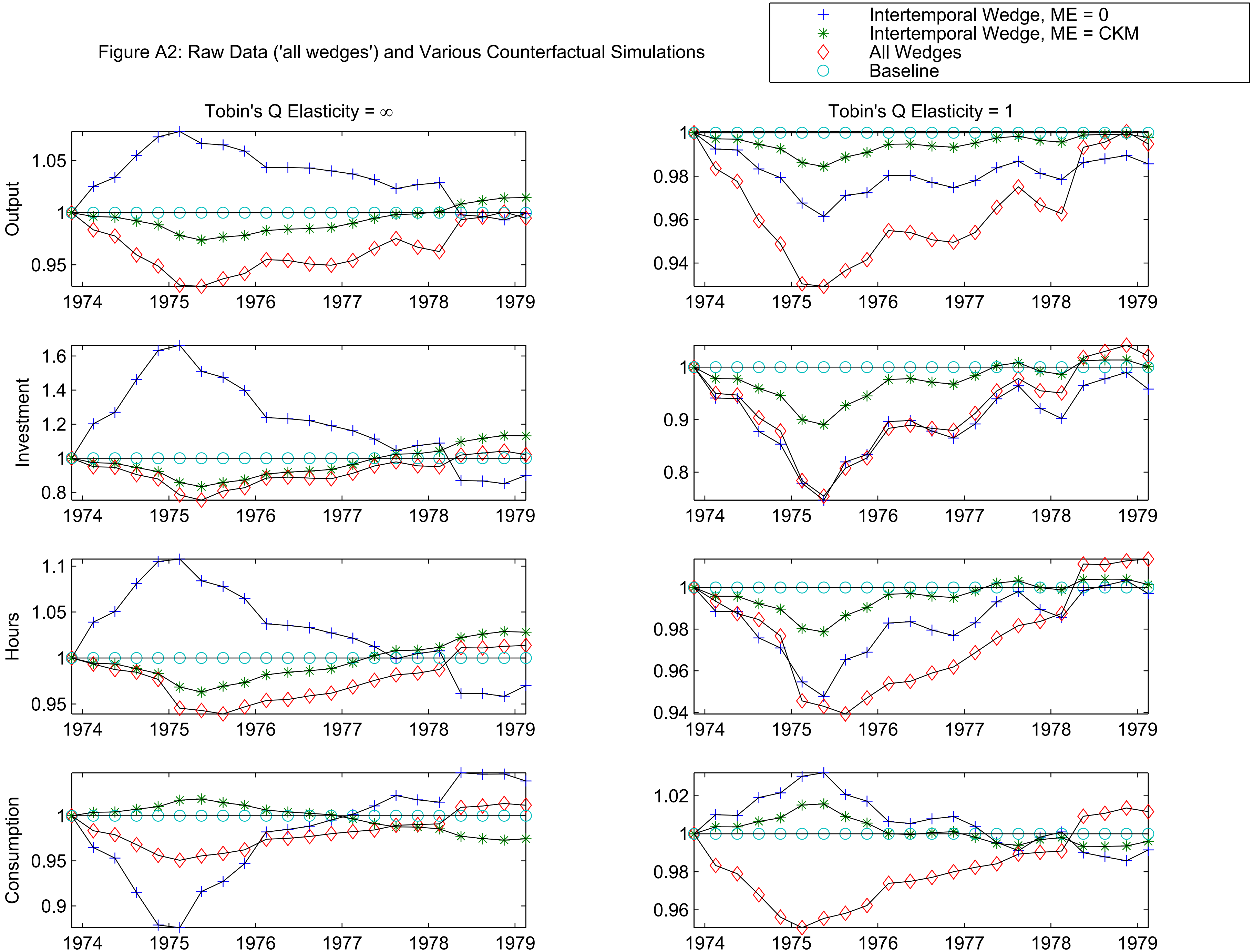
Note: (1) wedge - fraction of fall in raw data at the maximum for output accounted for by the intertemporal wedge; (2) R^2 - R-square in regression of raw data on wedge component throughout recession episode; (3) β - slope coefficient in preceding regression.

Figure A1: Raw Data ('all wedges') and Various Counterfactual Simulations



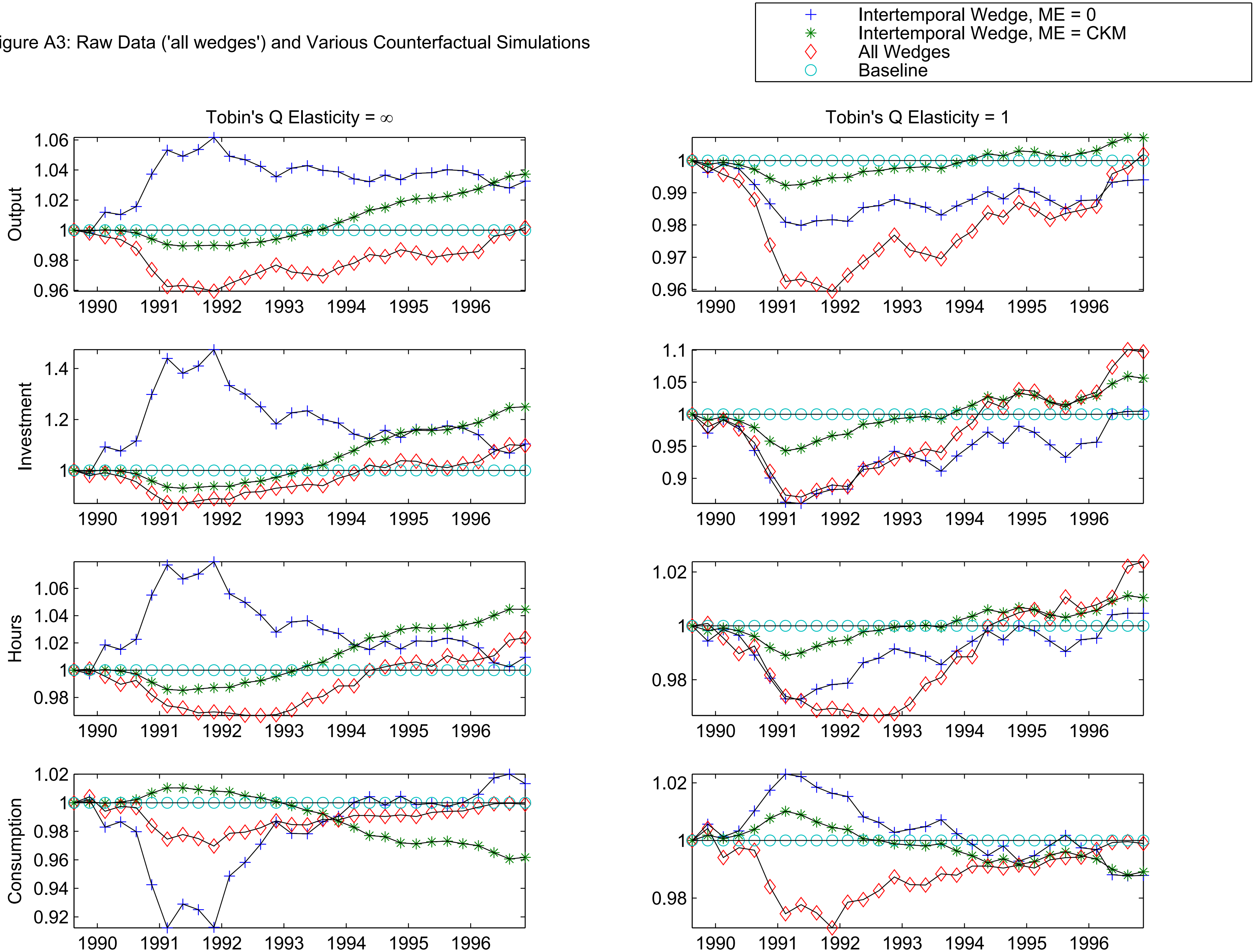
Notes: (1) ME = CKM - model estimation using CKM measurement error assumption, (2) ME = 0 - model estimation using no measurement error; (3) Tobin's Q elasticity = ∞ - investment adjustment costs set to zero, =1 means investment adjustment costs implies a unit Tobin's Q elasticity

Figure A2: Raw Data ('all wedges') and Various Counterfactual Simulations



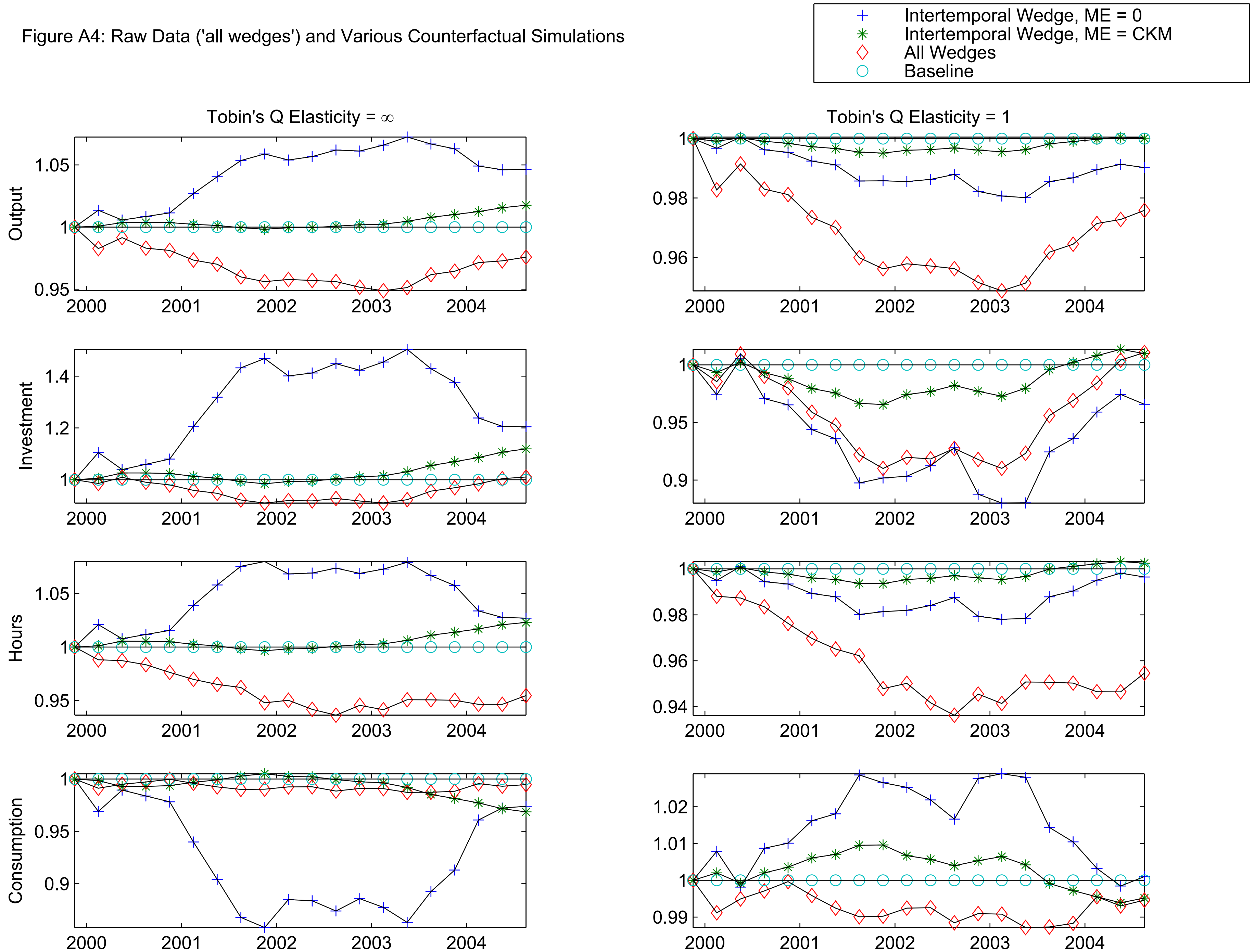
Notes: (1) ME = CKM - model estimation using CKM measurement error assumption, (2) ME = 0 - model estimation using no measurement error;
(3) Tobin's Q elasticity = ∞ - investment adjustment costs set to zero, =1 means investment adjustment costs implies a unit Tobin's Q elasticity

Figure A3: Raw Data ('all wedges') and Various Counterfactual Simulations



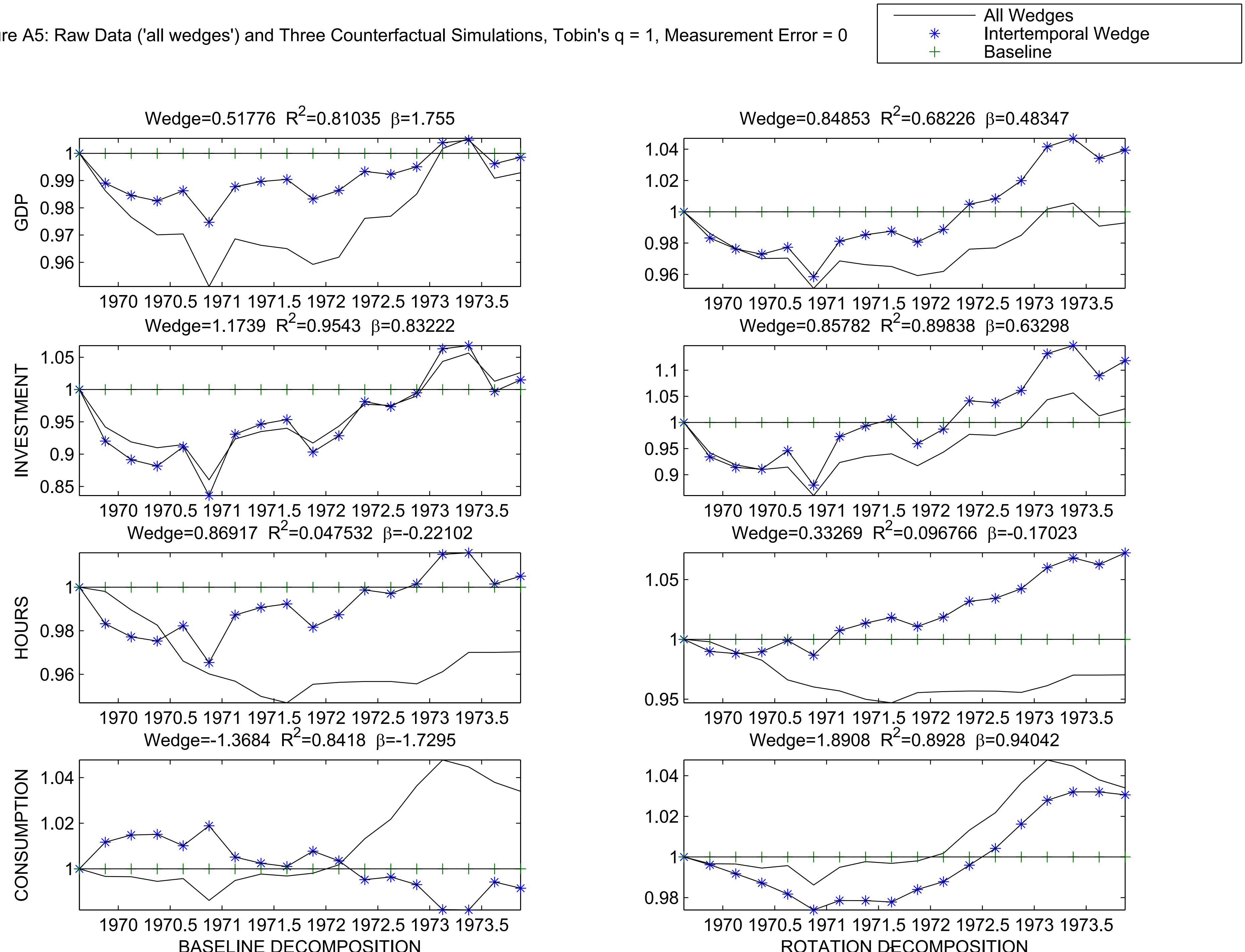
Notes: (1) ME = CKM - model estimation using CKM measurement error assumption, (2) ME = 0 - model estimation using no measurement error;
(3) Tobin's Q elasticity = ∞ - investment adjustment costs set to zero, =1 means investment adjustment costs implies a unit Tobin's Q elasticity

Figure A4: Raw Data ('all wedges') and Various Counterfactual Simulations



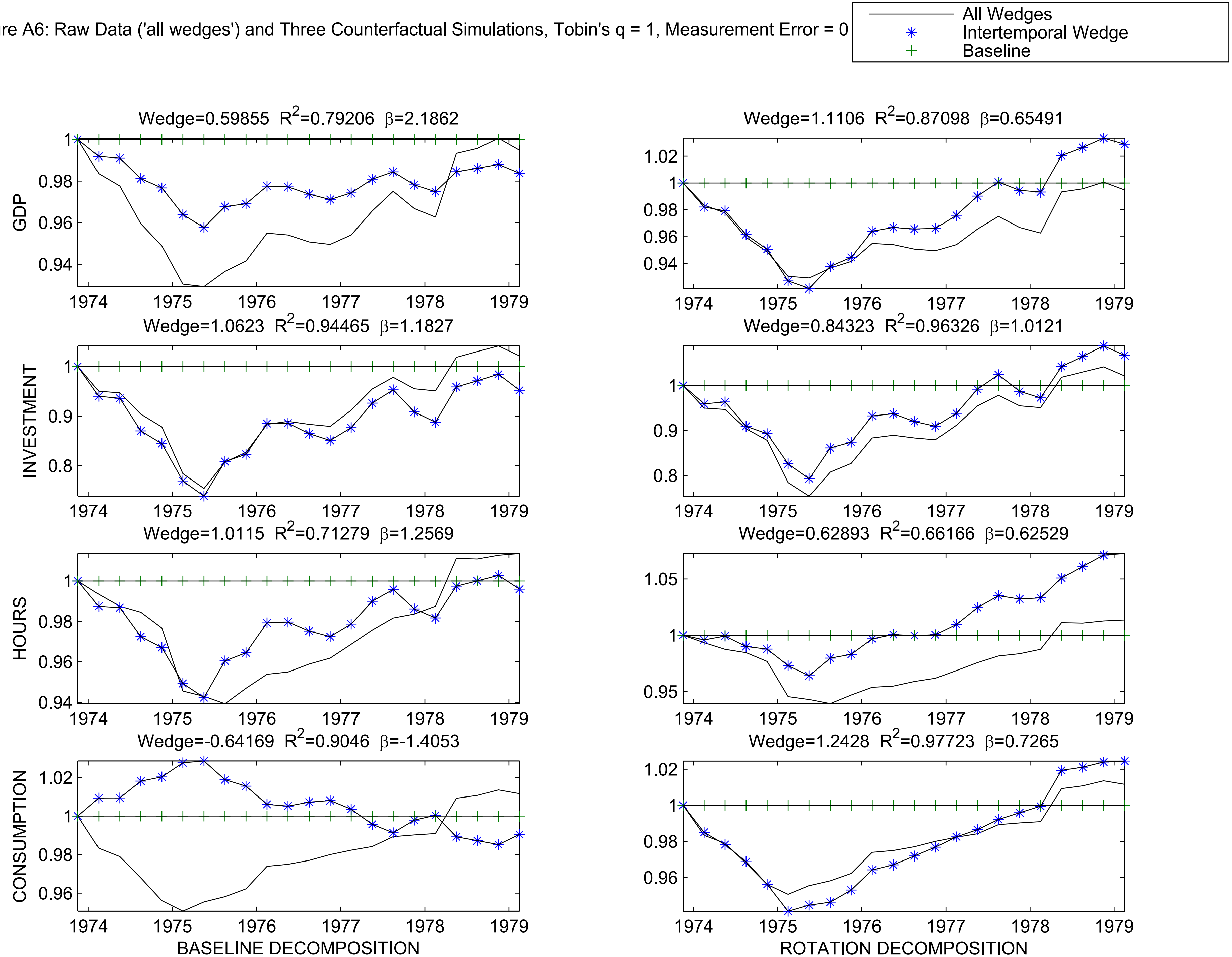
Notes: (1) ME = CKM - model estimation using CKM measurement error assumption, (2) ME = 0 - model estimation using no measurement error; (3) Tobin's Q elasticity = ∞ - investment adjustment costs set to zero, =1 means investment adjustment costs implies a unit Tobin's Q elasticity

Figure A5: Raw Data ('all wedges') and Three Counterfactual Simulations, Tobin's q = 1, Measurement Error = 0



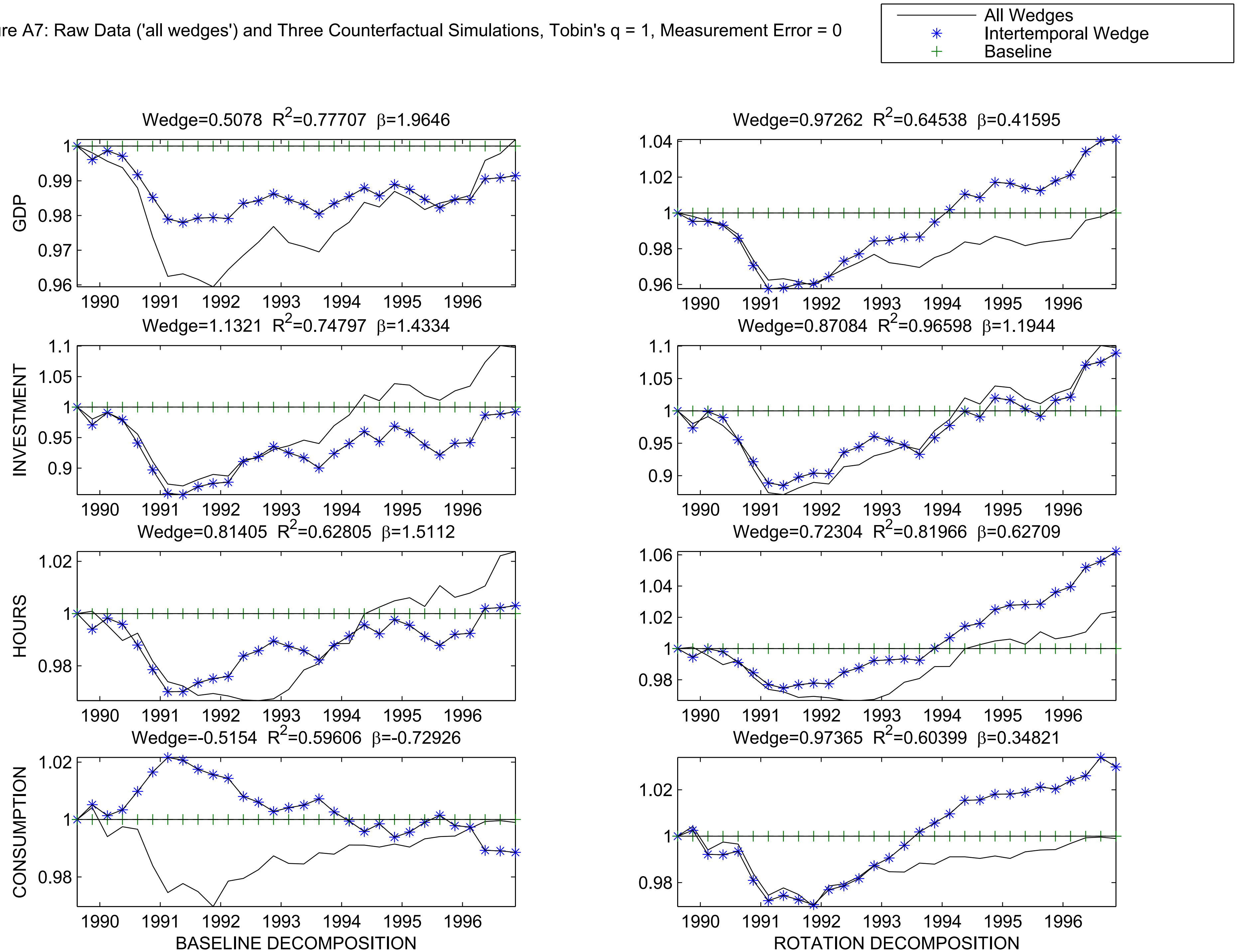
Note: (1) wedge - fraction of fall in raw data at the minimum for output accounted for by the intertemporal wedge; (2) R^2 - R-square in regression of raw data on wedge component throughout recession episode; (3) β - slope coefficient in preceding regression

Figure A6: Raw Data ('all wedges') and Three Counterfactual Simulations, Tobin's q = 1, Measurement Error = 0



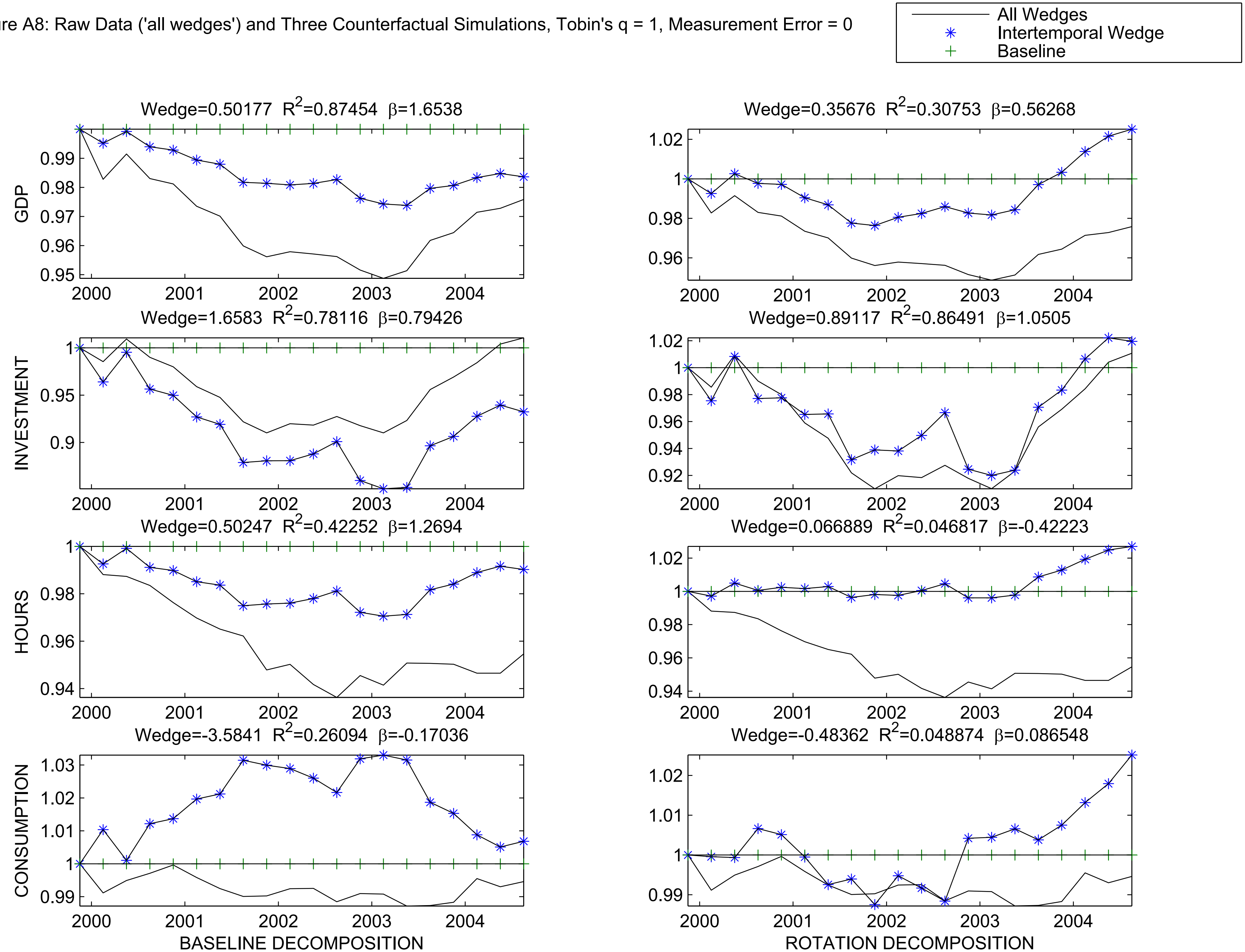
Note: (1) wedge - fraction of fall in raw data at the minimum for output accounted for by the intertemporal wedge; (2) R^2 - R-square in regression of raw data on wedge component throughout recession episode; (3) β - slope coefficient in preceding regression

Figure A7: Raw Data ('all wedges') and Three Counterfactual Simulations, Tobin's q = 1, Measurement Error = 0



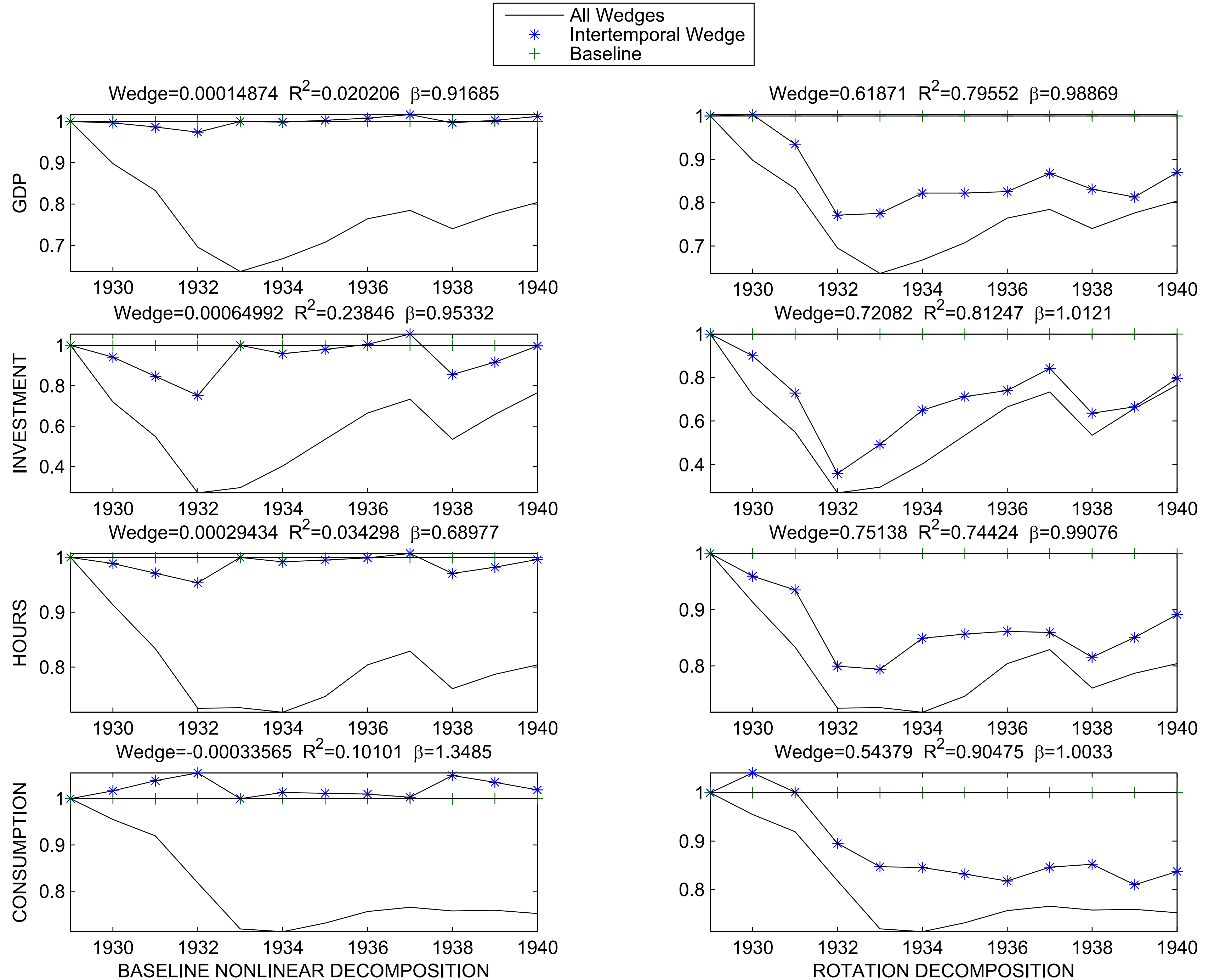
Note: (1) wedge - fraction of fall in raw data at the minimum for output accounted for by the intertemporal wedge; (2) R^2 - R-square in regression of raw data on wedge component throughout recession episode; (3) β - slope coefficient in preceding regression

Figure A8: Raw Data ('all wedges') and Three Counterfactual Simulations, Tobin's q = 1, Measurement Error = 0



Note: (1) wedge - fraction of fall in raw data at the minimum for output accounted for by the intertemporal wedge; (2) R^2 - R-square in regression of raw data on wedge component throughout recession episode; (3) β - slope coefficient in preceding regression

Figure A9: Wedges, US Great Depression Based on Second Order Approximation to Model, no Measurement Error, Tobin's q Elasticity = 1



Note: (1) wedge - fraction of fall in raw data at the maximum for output accounted for by the intertemporal wedge; (2) R^2 - R-square in regression of raw data on wedge component throughout recession episode; (3) β - slope coefficient in preceding regression.