that the best 8-ary constellation is either the 8-PSK, or 8-cross, or $(1, 7)$ depending on the desired spectral efficiency. As for the case of 16-ary constellations, the most attractive signal set is rectangular 16-QAM for all spectral efficiencies of practical interest. These results have been confirmed by computer simulations of BICM schemes combining turbo codes and several signal sets. The study carried out in this correspondence does not formally prove the optimality of these particular signal sets for BICM design. To the best of our knowledge, we have, however, considered in this work the 11 signal sets that present the best SER performance and/or the lowest $N_{\min}$, among all possible 8- and 16-ary constellations. Therefore, it is our belief that there is no constellation that significantly outperforms 8-PSK, 8-cross, $(1, 7)$, or 16-QAM when combined with an error-correcting code in a BICM system.

An important conclusion of this work is that signal sets having a simple structure, such as 8-PSK and 16-QAM, are very attractive for BICM. This result strongly suggests that some other higher order constellations such as cross 32-QAM, 64-QAM, cross 128-QAM, 256-QAM, etc., are also very much of interest for the design of power-efficient BICM schemes. On the other hand, signal sets displaying optimal error performance in the absence of coding are, generally, not of interest for BICM design.

ACKNOWLEDGMENT

The author wishes to thank the reviewers for their valuable and constructive comments that improved the presentation of this material. The author would also like to express his special gratitude to X. Li, E. Moore, A. Glavieux, and C. Berrou for their continuous encouragements.

REFERENCES

[1] G. Caire, G. Taricco, and E. Biglieri, “Bit-interleaved coded modulation,” *IEEE Trans. Inform. Theory*, vol. 44, pp. 927–946, May 1998.
[2] E. Zehavi, “8-PSK trellis codes for a Rayleigh fading channel,” *IEEE Trans. Commun.*, vol. 40, pp. 873–884, May 1992.
[3] C. Berrou, A. Glavieux, and P. Thitimajshima, “Near Shannon limit error correcting coding and decoding: Turbo codes,” in *Proc. IEEE int. Communications Conf. (ICC'93)*, Geneva, Switzerland, May 1993, pp. 1064–1070.
[4] S. Benedetto and G. Montorsi, “Unveiling turbo codes: Some results on parallel concatenated coding schemes,” *IEEE Trans. Inform. Theory*, vol. 42, pp. 409–428, Mar. 1996.
[5] R. G. Gallager, *Low-Density Parity-Check Codes*. Cambridge, MA: MIT Press, 1963.
[6] D. J. C. MacKay, “Good error-correcting codes based on very sparse matrices,” *IEEE Trans. Inform. Theory*, vol. 45, pp. 399–431, Mar. 1999.
[7] S. Le Goff, A. Glavieux, and C. Berrou, “Turbo codes and high spectral efficiency modulation,” in *Proc. IEEE Int. Communications Conf. (ICC'94)*, New Orleans, LA, May 1994, pp. 645–649.
[8] A. S. Barbulescu, W. Farell, P. Gray, and M. Rice, “Bandwidth-efficient turbo coding for high-speed mobile satellite communications,” in *Proc. Int. Symp. Turbo Codes and Related Topics*, Brest, France, Sept. 1997, pp. 119–126.
[9] T. M. Duman, “Turbo codes and turbo-coded modulation: Analysis and performance bounds,” Ph.D. dissertation, Northeastern Univ., Boston, MA, 1998.
[10] X. Li and J. A. Ritcey, “Bit-interleaved coded modulation with iterative decoding using soft feedback,” *Inst. Elec. Eng. Electron. Lett.*, vol. 34, pp. 942–943, May 1998.
[11] ——, “Bit-interleaved coded modulation with iterative decoding—Approaching turbo-TCM performance without code concatenation,” in *Proc. 32nd Conf. Information Sciences and Systems (CISS'98)*, Princeton, NJ, Mar. 1998.
[12] S. ten Brink, J. Speidel, and R.-H. Yan, “Iterative demapping for QPSK modulation,” *IEE Electron. Lett.*, vol. 34, pp. 1459–1460, July 1998.
[13] ——, “Iterative demapping and decoding for multilevel modulation,” in *Proc. IEEE Global Telecommunications Conf.*, Sydney, Australia, Nov. 1998, pp. 579–584.
[14] C. M. Thomas, M. Y. Weidner, and S. H. Durrani, “Digital amplitude-phase keying with M-ary alphabets,” *IEEE Trans. Commun.*, vol. COM-22, pp. 168–180, Feb. 1974.
[15] G. J. Foschini, R. D. Gitlin, and S. B. Weinstein, “Optimization of two-dimensional signal constellations in the presence of Gaussian noise,” *IEEE Trans. Commun.*, vol. COM-22, pp. 28–38, Jan. 1974.
[16] D. G. Forney, Jr., R. G. Gallager, G. R. Lang, F. M. Longstaff, and S. U. Qureshi, “Efficient modulation for band-limited channels,” *IEEE J. Select. Areas Commun.*, vol. SAC-2, pp. 632–647, Sept. 1984.
[17] X. Dong, N. C. Beaulieu, and P. H. Wittke, “Signaling constellations for fading channels,” *IEEE Trans. Commun.*, vol. 47, pp. 703–714, May 1999.
[18] R. D. Wesel, C. Komninakis, and X. Liu, “Toward optimality in constellation labeling,” in *Proc. Communication Theory Mini-Conf. at IEEE Global Telecommunications Conf.*, Phoenix, AZ, Nov. 1997, pp. 23–27.
[19] X. Liu and R. D. Wesel, “Profile optimal 8-QAM and 32-QAM constellations,” in *Proc. 36th Annu. Allerton Conf. Communication, Control, and Computing*, Monticello, IL, Sept. 1998, pp. 136–145.
[20] R. D. Wesel, X. Liu, J. M. Cioffi, and C. Komninakis, “Constellation labeling for linear encoders,” *IEEE Trans. Inform. Theory*, vol. 47, pp. 2417–2431, Sept. 2001.
[21] S. Le Goff, “Les turbo codes et leur application aux transmissions à forte efficacité spectrale,” Ph.D. dissertation (in French), Univ. Bretagne Occidentale, Brest, France, 1995.
[22] D. Divsalar and F. Pollara, “On the design of turbo codes,” *TDA Progr. Rep.*, pp. 42–123, Nov. 1995.
[23] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, “Parallel concatenated trellis-coded modulation,” in *Proc. IEEE Int. Communications Conf. (ICC'96)*, Dallas, TX, June 1996, pp. 974–978.
[24] G. Ungerboeck, “Channel coding with multilevel/phase signals,” *IEEE Trans. Inform. Theory*, vol. IT-28, pp. 56–67, Jan. 1982.
[25] P. Robertson and T. Wörz, “Bandwidth-efficient turbo trellis-coded modulation using punctured component codes,” *IEEE J. Select. Areas Commun.*, vol. 16, pp. 206–218, Feb. 1998.

Two Generalized Complex Orthogonal Space–Time Block Codes of Rates 7/11 and 3/5 for 5 and 6 Transmit Antennas

Weifeng Su, *Student Member, IEEE*, and
Xiang-Gen Xia, *Senior Member, IEEE*

***Abstract*—Space–time block codes from orthogonal designs have two advantages, namely, fast maximum-likelihood (ML) decoding and full diversity. Rate 1 real (pulse amplitude modulation—PAM) space–time codes (real orthogonal designs) for multiple transmit antennas have been constructed from the real Hurwitz–Radon families, which also provides the rate $1/2$ complex (quadrature amplitude modulation—QAM) space–time codes (complex orthogonal designs) for any number of transmit antennas.**

Manuscript received March 1, 2001; revised September 4, 2002. This work was supported in part by the Air Force Office of Scientific Research (AFOSR) under Grants F49620-00-1-0086 and F49620-02-1-0157 and by the National Science Foundation (NSF) under Grants MIP-9703377 and CCR-0097240. The material in this correspondence was presented at the IEEE International Symposium on Information Theory, Washington, DC, June 2001.

W. Su was with the Department of Electrical and Computer Engineering, University of Delaware, Newark, DE 19716 USA. He is now with the Institute for Systems Research, University of Maryland, College Park, MD 20742 USA (e-mail: weifeng@isr.umd.edu).

X.-G. Xia is with the Department of Electrical and Computer Engineering, University of Delaware, Newark, DE 19716 USA (e-mail: xxia@ee.udel.edu).

Communicated by R. Urbanke, Associate Editor for Coding Techniques.
Digital Object Identifier 10.1109/TIT.2002.806123

Rate $3/4$ complex orthogonal designs (space–time codes) for three and four transmit antennas have existed in the literature but no high rate $(> 1/2)$ complex orthogonal designs for other numbers of transmit antennas exist. In this correspondence, we present rate $7/11$ and rate $3/5$ generalized complex orthogonal designs for five and six transmit antennas, respectively.

***Index Terms*—Diversity, (generalized) complex orthogonal designs, space–time block codes.**

I. INTRODUCTION

Space–time coding for multiple transmit antenna systems in broad-band wireless communications has attracted considerable attention lately, see for example [1], [2]–[11]. To design "good" space–time block codes is a challenging problem. Since a space–time block code is a collection of some matrices, even for a small block size and a reasonable rate, the set of a space–time block code can be significantly large and therefore, its maximum-likelihood (ML) decoding may have a high complexity. On the other hand, the performance of a space–time block code depends on the diversity of the code. Therefore, a "good" space–time block code should possess two properties: i) the decoding at the receiver is reasonably fast; and ii) the diversity of the code is not small. Based on these two properties, space–time block codes based on orthogonal designs have been first proposed in Alamouti [3] for two transmit antennas and then generalized in Tarokh, Jafarkhani, and Calderbank [4] for $n \geq 2$ transmit antennas by connecting the space–time codes to orthogonal designs and the Hurwitz–Radon theory, see, for example, [12], [13]. A similar scheme was suggested in Ganesan and Stoica [6] from a maximum signal-to-noise ratio (SNR) approach.

Consider a space–time block code from complex orthogonal design G for n transmit antennas and block length p, i.e., G is a $p \times n$ generator. Consider a signal constellation $\mathcal{S}$, for example the binary phase shift keying (BPSK) $\{1, -1\}$ or the quaternary phase shift keying (QPSK) $\{1, -1, j, -j\}$. Let $x_1, x_2, \ldots, x_k$ be information symbols in $\mathcal{S}$. The entries of the complex orthogonal design of G are formed from complex linear combinations of $x_1, x_1^*, x_2, x_2^*, \ldots, x_k, x_k^*$ such that the columns of G are orthogonal to each other, where x^* is the complex conjugate of x. More detailed definition is reviewed later. See [4] for more about its encoding scheme and fast decoding algorithm. The rate of the code is $R = k/p$, which means that each codeword with block length p carries k information symbols. For a fixed n and rate R, it is desired to have the block length p as small as possible for decreasing the time delay in decoding. It was shown in [2], [4] that $R \leq 1$, i.e., $p \geq k$. Clearly, for a given k, the smallest possible block length p is k. When the signal constellation $\mathcal{S}$ has all real symbols, such as pulse amplitude modulation (PAM), rate $R = 1$, i.e., $p = k$, real orthogonal designs for any fixed transmit antenna number n have been given in, for example, [4] from the real Hurwitz–Radon families [12]. These real orthogonal designs also provide a method to construct complex orthogonal designs of rate $R = k/(2k) = 1/2$ for any fixed transmit antenna number n [4] where the block length $p = 2k$ and k depends on n (see details later). Examples of rate $3/4$ complex orthogonal designs for $n = 3$ and $n = 4$ transmit antennas have appeared in [4], [6]–[8]. It was proved in [4] that 4×4 complex orthogonal designs of rate 1 do not exist and a simpler proof was given in [6] by using the amicable design theory [12], [13]. This implies that the only square complex orthogonal design of rate 1 is the 2×2 complex orthogonal design proposed in [3]. To the best of our knowledge, there are no known high-rate $(R > 1/2)$ complex orthogonal designs for transmit antenna number $n \geq 5$.

The main contribution of this correspondence is to present rate $7/11$ and rate $3/5$ generalized complex orthogonal designs for $n = 5$ and $n = 6$ transmit antennas, respectively. We also present some constructions of rate $1/2$ complex orthogonal designs with smaller block length p than those presented in [4].

II. (GENERALIZED) COMPLEX ORTHOGONAL DESIGNS

This correspondence follows the terminologies of [4]. We first review the concept of the (generalized) complex orthogonal design and some known designs of rate greater than or equal to $1/2$[3], [4], [6]–[8]. We then present two generalized complex orthogonal designs of rate $7/11$ and rate $3/5$ for $n = 5$ and $n = 6$ transmit antennas, respectively. We also present some simpler complex orthogonal designs of rate $1/2$ for $n = 5, 6, 7, 8$ transmit antennas.

Definition 1: A generalized complex orthogonal design (GCOD) in variables $x_1, x_2, \ldots, x_k$ is a $p \times n$ matrix G such that:

- the entries of G are complex linear combinations of $x_1, x_2, \ldots, x_k$ and their complex conjugates $x_1^*, x_2^*, \ldots, x_k^*$;
- $G^{\mathcal{H}}G = D$, where $G^{\mathcal{H}}$ is the complex conjugate and transpose of G, and D is an $n \times n$ diagonal matrix with the (i, i)th diagonal element of the form
$$l_{i,1}|x_1|^2 + l_{i,2}|x_2|^2 + \cdots + l_{i,k}|x_k|^2$$
where all the coefficients $l_{i,1}, l_{i,2}, \ldots, l_{i,k}$ are strictly positive numbers.

The rate of G is defined as $R = k/p$. If
$$G^{\mathcal{H}}G = (|x_1|^2 + |x_2|^2 + \cdots + |x_k|^2)I_{n\times n}$$
then G is called a complex orthogonal design (COD).

Tarokh, Jafarkhani, and Calderbank [4] first mentioned that the rate of space–time block codes from generalized complex orthogonal designs cannot be greater than 1, i.e., $R = k/p \leq 1$. Later, it was proved in [9] that this rate must be less than 1 for more than two transmit antennas. For a fixed number of transmit antennas n and rate R, it is desired to have the block length p as small as possible.

The first space–time block code from complex orthogonal design was proposed in Alamouti [3] for two transmit antennas. It is the following 2×2 COD in variables x_1 and x_2

$$G_2 = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}. \tag{1}$$

Clearly, the rate of G_2 achieves the maximum rate 1. For space–time block codes from (generalized) complex orthogonal designs, rate 1 is achievable only for two transmit antennas.

For $n = 3$ and $n = 4$ transmit antennas, there are complex orthogonal designs of rate $R = 3/4$ [4], [6]–[8], for example,

$$G_3 = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ x_3^* & 0 & -x_1^* \\ 0 & x_3^* & -x_2^* \end{bmatrix} \tag{2}$$

for three transmit antennas, and

$$G_4 = \begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & x_3 \\ x_3^* & 0 & -x_1^* & x_2 \\ 0 & x_3^* & -x_2^* & -x_1 \end{bmatrix} \tag{3}$$

for four transmit antennas. In fact, G_3 is obtained by taking the first three columns of G_4. For $n \geq 5$, Tarokh, Jafarkhani, and Calderbank [4] gave a general construction for complex orthogonal design of rate $R = 1/2$. The block length p of this design is

$$p = 2\,\min\,(2^{4c+d})$$

where the minimization is taken over the set

$$\left\{c,\ d\colon 0 \leq d < 4,\ c \geq 0,\ \text{and}\ 8c + 2^d \geq n\right\}.$$

We observe that, when $5 \leq n \leq 8$, p is 16.

By now, the existing designs of rate greater than $1/2$ are only G_2, G_3, and G_4 with rates 1, $3/4$, and $3/4$, respectively. It has been proved in [4], [5] that if there exists a $p \times n$ GCOD in variables $x_1, x_2, \ldots, x_k$ such that $l_{i,1} = l_{i,2} = \cdots = l_{i,k}$ for each i, then there exists a COD in the same variables and of the same size. Notice that all of the existing designs are CODs. However, the constraint of CODs is not necessary to construct space–time block codes. GCODs can also provide the advantages of the fast ML decoding and the full diversity. In fact, the diagonal form of D guarantees the fast ML decoding, since the orthogonal columns of G can separate the transmitted symbols $x_1, x_2, \ldots, x_k$ from each other at the decoder. Also, the strictly positive coefficients $l_{i,1}, l_{i,2}, \ldots, l_{i,k}$ imply the full rank of G. This guarantees the full diversity advantage of coding. For more details about the coding scheme and the fast ML decoding, we refer the reader to [4].

A. Two GCODs of Rates $7/11$ and $3/5$ for $n = 5$ and $n = 6$ Transmit Antennas

We present here two generalized complex orthogonal designs of rate greater than $1/2$. The first one is an 11×5 matrix given by

$$G_5 = \begin{bmatrix} x_1 & x_2 & x_3 & 0 & x_4 \\ -x_2^* & x_1^* & 0 & x_3 & x_5 \\ x_3^* & 0 & -x_1^* & x_2 & x_6 \\ 0 & x_3^* & -x_2^* & -x_1 & x_7 \\ x_4^* & 0 & 0 & -x_7^* & -x_1^* \\ 0 & x_4^* & 0 & x_6^* & -x_2^* \\ 0 & 0 & x_4^* & x_5^* & -x_3^* \\ 0 & -x_5^* & x_6^* & 0 & x_1 \\ x_5^* & 0 & x_7^* & 0 & x_2 \\ -x_6^* & -x_7^* & 0 & 0 & x_3 \\ x_7 & -x_6 & -x_5 & x_4 & 0 \end{bmatrix} \tag{4}$$

for $n = 5$ transmit antennas. G_5 is constructed from G_4 as follows. At first, we keep G_4 as a 4×4 submatrix of G_5 and add symbols x_4, x_5, x_6, x_7 into the fifth column of G_5. Then, we arrange the entries of G_5 from the fifth row to the end such that all of the five columns are orthogonal to each other and the number of the total rows should be as small as possible. From the resulting matrix in (4), we can check that $G_5^{\mathcal{H}} G_5 = D$, where D is a 5×5 diagonal matrix with the (i, i)th diagonal element $D(i, i)$ of the form

$$D(1,1) = D(2,2) = D(3,3) = D(4,4) = \sum_{m=1}^{7} |x_m|^2$$

and

$$D(5,5) = 2\sum_{m=1}^{3} |x_m|^2 + \sum_{m=4}^{7} |x_m|^2.$$

Notice that the symbols x_1, x_2, x_3 and their complex conjugates x_1^*, x_2^*, x_3^* appear in the fifth column of G_5. Clearly, the rate of G_5 is $R = 7/11 = 0.6364$, and the block length $(p = 11)$ of G_5 is smaller than that $(p = 16)$ of rate $1/2$ complex orthogonal design given in [4] for $n = 5$ transmit antennas. A by-product of constructing G_5 is a 7×4 complex orthogonal design as follows:

$$\begin{bmatrix} x_1^* & 0 & 0 & -x_4^* \\ 0 & x_1^* & 0 & x_3^* \\ 0 & 0 & x_1^* & x_2^* \\ 0 & -x_2^* & x_3^* & 0 \\ x_2^* & 0 & x_4^* & 0 \\ -x_3^* & -x_4^* & 0 & 0 \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix} \tag{5}$$

where rate $R = 4/7 = 0.5714$, and the block length $p\,(=7)$ in this example is not a power of 2 as appeared in the previous examples of COD.

The second one is a 30×6 matrix given by

$$G_6 = \begin{bmatrix} x_1 & x_2 & x_3 & 0 & x_4 & x_8 \\ -x_2^* & x_1^* & 0 & x_3 & x_5 & x_9 \\ x_3^* & 0 & -x_1^* & x_2 & x_6 & x_{10} \\ 0 & x_3^* & -x_2^* & -x_1 & x_7 & x_{11} \\ x_4^* & 0 & 0 & -x_7^* & -x_1^* & x_{12} \\ 0 & x_4^* & 0 & x_6^* & -x_2^* & x_{13} \\ 0 & 0 & x_4^* & x_5^* & -x_3^* & x_{14} \\ 0 & x_5^* & -x_6^* & 0 & -x_1 & x_{15} \\ x_5^* & 0 & x_7^* & 0 & x_2 & x_{16} \\ x_6^* & x_7^* & 0 & 0 & -x_3 & x_{17} \\ x_7 & -x_6 & -x_5 & x_4 & 0 & x_{18} \\ x_8^* & 0 & 0 & -x_{11}^* & -x_{15}^* & -x_1^* \\ 0 & x_8^* & 0 & x_{10}^* & x_{16}^* & -x_2^* \\ 0 & 0 & x_8^* & x_9^* & -x_{17}^* & -x_3^* \\ 0 & 0 & 0 & x_{18}^* & x_8^* & -x_4^* \\ 0 & 0 & -x_{18}^* & 0 & x_9^* & -x_5^* \\ 0 & -x_{18}^* & 0 & 0 & x_{10}^* & -x_6^* \\ x_{18}^* & 0 & 0 & 0 & x_{11}^* & -x_7^* \\ 0 & -x_9^* & x_{10}^* & 0 & x_{12}^* & x_1 \\ x_9^* & 0 & x_{11}^* & 0 & x_{13}^* & x_2 \\ -x_{10}^* & -x_{11}^* & 0 & 0 & x_{14}^* & x_3 \\ -x_{12}^* & -x_{13}^* & -x_{14}^* & 0 & 0 & x_4 \\ -x_{16}^* & -x_{15}^* & 0 & -x_{14}^* & 0 & x_5 \\ -x_{17}^* & 0 & x_{15}^* & -x_{13}^* & 0 & x_6 \\ 0 & -x_{17}^* & -x_{16}^* & x_{12}^* & 0 & x_7 \\ 0 & x_{14} & -x_{13} & -x_{15} & x_{11} & 0 \\ x_{14} & 0 & -x_{12} & -x_{16} & x_{10} & 0 \\ -x_{13} & x_{12} & 0 & x_{17} & x_9 & 0 \\ x_{15} & -x_{16} & x_{17} & 0 & x_8 & 0 \\ -x_{11} & x_{10} & x_9 & -x_8 & x_{18} & 0 \end{bmatrix} \tag{6}$$

for $n = 6$ transmit antennas. Actually, G_6 is constructed form G_5 as follows. At first, we keep G_5 as an 11×5 submatrix of G_6 and add symbols $x_8, x_9, \ldots, x_{18}$ into the sixth column of G_6. Then, we arrange the entries of G_6 from the twelfth row to the end such that all of the six columns of G_6 are orthogonal to each other and the number of the total rows should be as small as possible. The resulting matrix in (6) is of size 30×6. By a tedious check, we have $G_6^{\mathcal{H}} G_6 = D$, where D is a 6×6 diagonal matrix with the (i, i)th diagonal element $D(i, i)$ of the form

$$D(1,1) = D(2,2) = D(3,3) = D(4,4) = \sum_{m=1}^{18} |x_m|^2$$

and

$$D(5,5) = \sum_{m=1}^{18} |x_m|^2 + \sum_{m=1}^{3} |x_m|^2 + \sum_{m=8}^{11} |x_m|^2$$

$$D(6,6) = 2\sum_{m=1}^{7} |x_m|^2 + \sum_{m=8}^{18} |x_m|^2.$$

Notice that the symbols $x_1, x_2, \ldots, x_7$ and their conjugates $x_1^*, x_2^*, \ldots, x_7^*$ appear in the sixth column of G_6. Clearly, the rate of G_6 is $R = 18/30 = 0.6$.

The same procedure may be used to construct generalized complex orthogonal designs for other numbers of transmit antennas. However, it is hard to obtain other designs with rate greater than $1/2$. For example, G_6 may be used to construct G_7 for seven transmit antennas as follow. We keep G_6 as a 30×6 submatrix of G_7 and add symbols $x_{19}, x_{20}, \ldots, x_{48}$ into the seventh column of G_7. However, it is hard to arrange the entries of G_7 from the thirty-first row to the end such that all of the seven columns are orthogonal to each other and the number of the total rows should be as small as possible. Notice that in this case, the symbols $x_1, x_2, \ldots, x_{18}$ and their complex conjugates $x_1^*, x_2^*, \ldots, x_{18}^*$ should appear in the seventh column of G_7. Therefore, the block length of G_7 will be at least $30 + 18 + 18 = 66$.

B. Simpler CODs of Rate $1/2$ for $n = 5, 6, 7, 8$ Transmit Antennas

For a $p \times n$ complex orthogonal design for n transmit antennas, the block length p is related to the time delay in decoding. Clearly, for a fixed transmit antenna number n and rate R, it is desired to have the block length p as small as possible.

In [4], the constructions of rate $1/2$ complex orthogonal designs for $n \geq 5$ take advantage of the real orthogonal designs of full rate 1 which are available for any $n \geq 2$. For n $(n = 5, 6, 7, 8)$ transmit antennas, the complex orthogonal design in [4] is given by taking the first n columns of the following 16×8 matrix:

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ -x_2 & x_1 & x_4 & -x_3 & x_6 & -x_5 & -x_8 & x_7 \\ -x_3 & -x_4 & x_1 & x_2 & x_7 & x_8 & -x_5 & -x_6 \\ -x_4 & x_3 & -x_2 & x_1 & x_8 & -x_7 & x_6 & -x_5 \\ -x_5 & -x_6 & -x_7 & -x_8 & x_1 & x_2 & x_3 & x_4 \\ -x_6 & x_5 & -x_8 & x_7 & -x_2 & x_1 & -x_4 & x_3 \\ -x_7 & x_8 & x_5 & -x_6 & -x_3 & x_4 & x_1 & -x_2 \\ -x_8 & -x_7 & x_6 & x_5 & -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* & x_5^* & x_6^* & x_7^* & x_8^* \\ -x_2^* & x_1^* & x_4^* & -x_3^* & x_6^* & -x_5^* & -x_8^* & x_7^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* & x_7^* & x_8^* & -x_5^* & -x_6^* \\ -x_4^* & x_3^* & -x_2^* & x_1^* & x_8^* & -x_7^* & x_6^* & -x_5^* \\ -x_5^* & -x_6^* & -x_7^* & -x_8^* & x_1^* & x_2^* & x_3^* & x_4^* \\ -x_6^* & x_5^* & -x_8^* & x_7^* & -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_7^* & x_8^* & x_5^* & -x_6^* & -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_8^* & -x_7^* & x_6^* & x_5^* & -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix} \tag{7}$$

where the block length $p = 16$.

For n $(n = 5, 6, 7, 8)$ transmit antennas, we have a simpler design of rate $1/2$ by taking the first n columns of the following 8×8 matrix:

$$\begin{bmatrix} x_1 & x_2 & x_3 & 0 & x_4 & 0 & 0 & 0 \\ -x_2^* & x_1^* & 0 & x_3 & 0 & x_4 & 0 & 0 \\ x_3^* & 0 & -x_1^* & x_2 & 0 & 0 & x_4 & 0 \\ 0 & x_3^* & -x_2^* & -x_1 & 0 & 0 & 0 & x_4 \\ x_4^* & 0 & 0 & 0 & -x_1^* & x_2 & -x_3 & 0 \\ 0 & x_4^* & 0 & 0 & -x_2^* & -x_1 & 0 & -x_3 \\ 0 & 0 & x_4^* & 0 & -x_3^* & 0 & x_1 & x_2 \\ 0 & 0 & 0 & x_4^* & 0 & -x_3^* & -x_2^* & x_1^* \end{bmatrix} \tag{8}$$

where the block length $p = 8$ which is smaller than that in (7).

III. Conclusion and Some Comments

Space–time block codes from (generalized) complex orthogonal designs have two advantages: fast ML decoding and full diversity. It has been proved in [9] that the rate of these space–time block codes must be less than 1 except for the case of wo transmit antennas. There is a systematic construction of complex orthogonal designs of rate $1/2$ for any number of transmit antennas based on the Hurwitz–Radon theory. The previously known designs of rate larger than $1/2$ and less than 1 were given only for three and four transmit antennas with rate $3/4$. In this correspondence, we presented two generalized complex orthogonal designs of rate $7/11$ and rate $3/5$ for five and six transmit antennas, respectively. Although the construction method may be used for other numbers of transmit antennas, it is hard to obtain other designs with rate larger than $1/2$.

As pointed out in [4], what we have known about orthogonal designs is only a tip of the iceberg. We hope that the two designs we presented will stimulate future work. Recently, it was proved in [10] that for more than two transmit antennas, the rate of CODs cannot be greater than $3/4$; and the rate of GCODs cannot be greater than $4/5$. A tutorial on space–time block codes from (generalized) complex orthogonal designs can be found in [11].

References

[1] J.-C. Guey, M. P. Fitz, M. R. Bell, and W.-Y. Kuo, "Signal design for transmitter diversity wireless communication systems over Rayleigh fading channels," in *Proc. IEEE Vehicular Technology Conf. (VTC'96)*, 1996, pp. 136–140.

[2] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space–time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–765, Mar. 1998.

[3] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, Aug. 1998.

[4] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space–time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1467, July 1999.

[5] ——, "Correction to 'Space–time block codes from orthogonal designs'," *IEEE Trans. Inform. Theory*, vol. 46, p. 314, Jan. 2000.

[6] G. Ganesan and P. Stoica, "Space–time block codes: A maximum SNR approach," *IEEE Trans. Inform. Theory*, vol. 47, pp. 1650–1656, May 2001.

[7] B. M. Hochwald, T. L. Marzetta, and C. B. Papadias, "A transmitter diversity scheme for wideband CDMA systems based on space–time spreading," *IEEE J. Select. Areas Commun.*, vol. 19, pp. 48–60, Jan. 2001.

[8] O. Tirkkonen and A. Hottinen, "Square-matrix embeddable space–time block codes for complex signal constellations," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1122–1126, Feb. 2002.

[9] X.-B. Liang and X.-G. Xia, "Nonexistence of rate one space–time block codes from generalized complex linear processing orthogonal designs for more than two transmit antennas," presented at the Internationa Symposium on Information Theory (ISIT'01), Washington, DC, June 2001.

[10] H. Wang and X.-G. Xia, "Upper bounds of rates of complex orthogonal space–time block codes," preprint, 2002.

[11] W. Su and X.-G. Xia, "On space–time block codes from complex orthogonal designs," Wireless Personal Commun., to be published.

[12] A. V. Geramita and J. Seberry, *Orthogonal Designs, Quadratic Forms and Hadamard Matrices (Lecture Notes in Pure and Applied Mathematics)*. New York /Basel: Marcel Dekker, 1979, vol. 43.

[13] D. B. Shapiro, *Compositions of Quadratic Forms*. New York: De Gruyter, 2000.