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Title: TWO-LANE TRAFFIC RULES FOR CELLULAR AUTOMATA: A SYSTEMATIC APPROACH

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Two-lane traffic rules for cellular automata: A systematic approach

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Microscopic modeling of multi-lane traffic is usually done by applying heuristic lane changing rules, and often with unsatisfying results. Recently, a cellular automaton model for two-lane traffic was able to overcome some of these problems and to produce a correct density inversion at densities somewhat below the maximum flow density. In this paper, we summarize different approaches to lane changing and their results, and propose a general scheme, according to which realistic lane changing rules can be developed. We test this scheme by applying it to several different lane changing rules, which, in spite of their differences, generate similar and realistic results. We thus conclude that, for producing realistic results, the logical structure of the lane changing rules, as proposed here, is at least as important as the microscopic details of the rules.

I. INTRODUCTION

Much progress has been made in understanding single lane traffic by using simple models [Nagel.92, Nagel.flow, Bando, Kerner.Konh, Krauss]. Although one could claim that these models also explain homogeneous multi-lane traffic, they definitely fail when traffic on different lanes behaves differently. If one wants to investigate lane specific dynamics, one has to address the question of how vehicles change from one lane to the other. Here we propose an elementary scheme to develop such rules and compare the simulation results of different realizations of this scheme with empirical data from the German highway.

The preferred approach in science is to start from first principles and then, using mathematics or simulation, to derive macroscopic relationships. In sciences which involve human beings this is hopeless: there are no first principles which govern human behavior. One alternative is to search heuristically for microscopically minimal "plausible" models which generate observed behavior on the macroscopic level. It is this approach that has often been used successfully when physics methods have

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been applied in the area of socio-economic systems. In this paper we want to go one step beyond that and look for systematic logical structures in the rule sets for lane changing.

Accordingly, we start out from real world data (Sec. 2), followed by a short review of traditional approaches to this problem in traffic science (Sec. 3). Sec. 4 outlines our approach. In the following three sections (Sec. 5 – 7), we describe simulation results with different rules. Sec. 8 looks closer into the mechanism at flow breakdown near maximum flow in the two-lane models. Sec. 9 is a discussion of our work, followed by a section showing how other multi-lane models for cellular automata fit into our scheme. The paper concludes with a short summary.

II. REAL WORLD MEASUREMENTS

FIG. 1. Traffic measurements in reality. Left column: as function of flow; right column: as function of density. Top: flow; middle: velocity; bottom: lane usage. The units for density are vehicles per kilometer per 2 lanes, and for flow they are vehicles per hour per 2 lanes. Each data point corresponds to a 1 minute average. Figure from Wiedemann, see [Wiedemann.mea] for further information.

As stated above, we are interested in *macroscopic* observations of traffic flow quantities related to lane changing behavior. A typical such measurement can look like Fig. 1. It contains measurements of density (in vehicles/km/2 lanes), flow (in veh/h/2 lanes), velocity (in km/h) and lane usage (in %), all averaged over one minute intervals. The left column shows velocity and lane usage as functions of flow; the right column shows flow, velocity, and lane usage as functions of density. For theoretical purposes, using flow as the control parameter has the disadvantage that for the same flow value one has two different regimes—at high density and at low density. For example in the lane usage plot, one cannot distinguish which data points belong to which regime. We will therefore concentrate on plots where density is the control parameter.

The top right plot shows the typical flow-density diagram. Flow first increases nearly linearly with density, until it reaches a maximum at $\rho \approx 40$ vehicles/km/2 lanes and $q \approx 3500$ vehicles/hour/2 lanes. From there, flow decreases with increasing density, but the scatter of the values is much larger than before. — The currently best explanation for this [Nagel.flow, Kerner:Konh, Krauss] is that, for low densities, traffic is roughly laminar and jams are short-lived. In consequence, the addition of vehicles does not change the average velocity much and flow is a linear function of density: $q = \rho v$. For high densities, traffic is an irregular composition of jam waves, and laminar outflow traffic between jams. Here, data points are arbitrary averages over these regimes, leading to a much larger variability in the measurements.[1]

The plot of the velocity vs. density confirms this: There is an abrupt drop in the average velocity at $\rho \approx 40$ veh/km/2 lanes. Yet, velocity is also not constant at lower densities, leading indeed to a curvature of the flow-vs.-density curve below the value $\rho \approx 40$ veh/km/2 lanes, which can be explained by the increasing influence of the slower vehicles in multi-lane traffic.

The lane usage shows a peculiarity which is particularly strong in Germany. As should be expected, at very low densities all traffic is on the right lane.[2] But with

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[1]Recent measurements indicate that there exists a third state called "synchronized" traffic near maximum flow where traffic is still fairly laminar but speeds between lanes are highly synchronized. Note that one needs multi-lane traffic for this characterization.

[2]For countries such as Great Britain or Australia, left and right have to be interchanged.

increasing density, eventually more than half of the traffic is on the left lane. Only at densities above the maximum flow point, this reverts to an equal distribution of densities between lanes.

Fig. 1 does not show the flows of the individual lanes. Ref. [Sparman.2lane] contains such plots. They show that the pointed peak of the overall flow is caused by a pointed peak in the flow of the left lane; flow on the right lane remains constant over a large density range.

All this suggests the interpretation that the flow breakdown mechanism on German autobahns is complicated, with flow breaking down on the left lane first and thus not allowing the right lane to reach its possible full capacity [Brilon.personal].

III. TRADITIONAL APPROACHES

Sparmann [Sparmann.2lane] discusses a lane changing implementation for the microscopic Wiedemann-model [Wiedemann]. Following Wiedemann's proposition, he distinguishes between the wish to change lanes and the decision to change lanes. For a lange change from right to left, these two parts are:

- **Wish** to change lanes if on any of the two lanes there is another vehicle ahead and obstructing.
- **Decision** to actually change lanes if there is enough space on the other lane.

Conversely, for changing from left to right:

- **Wish** to change lanes if on both lanes there is nobody ahead and obstructing.
- **Decision** to actually change lanes if there is enough space on the other lane.

According to the philosophy of the Wiedemann-approach, "obstructing" is defined in terms of so-called psycho-physiological thresholds, which depend mostly on speed difference and distance, and allow three outcomes: no obstruction, light obstruction, severe obstruction. Gipps [Gipps:mlane] reports a similar model.

The results are reported to be satisfying, yet unrealistic in at least one respect: The density inversion between right and left lane near maximum flow is not reproduced.

The Wiedemann-approach is a time-discrete formulation of a stochastic differential equation and therefore continuous in space. Some recent work in traffic has used a cellular automata approach, which is coarse-grained discrete both in time and space. Early lane changing rules in the context of cellular automata models for traffic flow are due to Cremer and co-workers [Cremer:Ludwig, Schuett]. Following Sparmann, they implemented lane changes in the following way: Lanes are changed to the left

- if a slower vehicle is less than l_l cells ahead,
- and if a gap of size Δx exists on the left lane;

lanes are changed to the right

- if, *on the right lane*, there is no slower vehicle less than l_r cells ahead,
- and there is a gap of size Δx on the right lane.

Again, they failed to reproduce the density inversion in the lane usage.

IV. OUR APPROACH

Which contribution can Statistical Physics make in such a situation? The strength of Statistical Physics is to explain how microscopic relationships *generate* macroscopic (i.e. aggregated) behavior. Thus, the contribution of Statistical Physics in traffic science (or in socio-economic systems in general) will be to investigate which microscopic rules contribute to certain aspects of macroscopic behavior and how.

As indicated in the introduction, this paper attempts to go one step beyond this. Since current psychological knowledge does not allow to define beyond doubt the set of microscopic rules involved in lane changing, we propose to construct these rules according to certain symmetries inherent in the problem. As we will point out, these symmetries simplify considerably the construction of consistent lane changing rules. Note that this has an interesting consequence: Models obtained from this approach may no longer be the most minimal ones, but because of the symmetry relation, they may have a more compact description. The argument thus is that, lacking sufficient microscopic knowledge, one should use the most compact model available which explains the macroscopic behavior.

Now, in spite of the absence of "first principles", it certainly still makes sense to have a "plausible" starting point. We thus state here what we will use as the elementary laws, and later, how we derive algorithmic rules from them. Similar to Ref. [Sparman:2lane], we propose that the basic ingredients are security, legal constraints, and travel time minimization. Security requires to leave enough space between all vehicles. The legal constraints depend on the country. Travel time minimization means that one chooses the optimal lane under these constraints.

Let us start with security. Security means that one leaves enough space in front of and behind oneself. As long as one stays on one lane, this is ensured by single-lane driving rules, as e.g. given by the rules in Refs. [Nagel.physcomp,Nagel.92.Schreck]. In the context of changing a lane this means that there must be enough space on the target lane. Technically, one can say that there must be a gap of size $gap_- + 1 + gap_+$. gap_+ is the gap on the target lane in front of the vehicle that wants to change the lane; gap_- is the gap on the target lane behind that vehicle.

Different choices for both parameters are possible. Throughout this paper we use $gap_+ = v$ and $gap_- = v_{max}$, where v is the speed of the vehicle which changes lanes.

Let us now go to legal constraints. For example in Germany, lane usage is regulated essentially by two laws: 1. The right lane has to be used by default, and 2. passing has to be on the left. In the United States, the second law is considerably relaxed. In this paper, we will use "Germany" and "United States" as placeholders for two somewhat extreme cases. We expect that the behavior of many other countries will be found somewhere in between.

Travel time optimization means that lane changes to the left are triggered by a slower vehicle in the same lane ahead and when the target lane is more attractive (because of optimization). Here we give two examples, first for changing to left:

(a) **German criterion.** In Germany passing is *not allowed* on the right. Hence, if there is a slow vehicle on the *left* lane, one has to change to the left, behind that slow vehicle. Thus one changes to the left if there is a slow car ahead on the same lane *or* on the left:

$$v_r \leq v \;.OR.\; v_l \leq v \;.[3]$$

[3] One should use "≤" instead of "<" here because of a technicality: Assume a situation where there are many cars at density one on the right lane, whereas on the left lane is free traffic. Then $v = 0$, and therefore it will never be smaller than any other velocity, and thus

v_r, v_l are taken within a certain distance one looks ahead, d, which is a free parameter. If there is no vehicle within this distance, the respective velocity is set to ∞.

(b) **American criterion.** By contrast, in America passing on the right is not explicitely forbidden. The left lane is only more attractive if the traffic there is faster than in one's own lane. Thus one changes from the right to the left if there is a slower car ahead in the same lane and if the next car in the left lane is faster than the car ahead:

$$v_r \leq v \ .AND. \ v_r \leq v_l \ .$$

The easiest implementation of the law to use the right lane by default is to make the criterion for changing back to the right lane the logical negation of the criterion to change to the left lane; i.e. whenever the reason to change to the left lane ceases to exist, one changes back.

- This means for Germany that a change back to the right lane is tried as soon as the velocities of the cars ahead in both lanes are sufficiently large:

$$v_r > v \ .AND. \ v_l > v \ .$$

- In America, the rule would mean that one tries to change back if there is a faster car than oneself (or no car at all) in the right lane, or if traffic in the right lane is running faster than on the left lane:

$$v_r > v \ .OR. \ v_r > v_l \ .$$

Thus, a reasonable way to classify lane changing rules is the following:

- (1) Reasons to change lane
  - check same lane [usually: slower vehicle ahead?]
  - check target lane [e.g.: target lane better? Germany: slower vehicle ahead on that lane?]
- (2) Security criterion: $gap_- \dots gap_+$ on target lane free?

One notes that the forward part of the security criterion and the target lane part of the reason to change lane look into the same space — often one will thus find only one or the other in a given rule set.

Note that there is an even more extreme case than our "American" rules: totally symmetric rules. Here, one changes lanes *only* when a slower vehicle is ahead; that is, vehicles stay on the left lane even when the right lane is completely empty. Symmetric rules describe *actual* American driving behavior fairly well, for example the fact that the rightmost lane is usually more empty than all other lanes [May]. American drivers often do not use the rightmost lane in order to avoid the repeated disturbances due to slow vehicles coming from on-ramps. In the words of totally symmetric rules: When these drivers encounter *one* slow vehicle from on an on-ramp, they switch to the middle lane and stay there until they run into a slower vehicle on the middle lane or until they want to get off the freeway. For that reason, TRANSIMS [TRANSIMS,TRANSIMS:flow:char] in its current microsimulation uses a totally symmetric

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these vehicles will remain in that dense queue forever. Latour has shown that this produces undesirable artifacts at high densities [Latour:thesis, Rickert:twolane].

lane-changing rule set. See Refs. [Nagatani:2lane,Rickert:diplom,Rickert:2lane] for symmetric lane changing rules.

Also note that our paper only treats uni-directional traffic, i.e. all vehicles are headed into the same direction. Refs. [Schuett,Simon:Gutowitz] are examples for the treatment of bi-directional traffic by cellular automata.

V. COMPUTER SIMULATIONS OF THE BASIC VELOCITY RULES

We now proceed to present computer simulations of the German rule-set to illustrate the above principles. Following Refs. [Latour.thesis, Rickert.thesis, Rickert.twolane], an update step of the whole system is divided into two major substeps: (i) lane changing, (ii) forward movement.

A. Lane changing

Lane changing here is implemented as a pure sideways movement. One should, though, better look at the overall result after the whole time step is completed, and then lane changing vehicles usually will have moved forwards, too. Still, the algorithm is underestimating the time vehicles usually need to change lanes: One CA iteration roughly corresponds to one second; lane changes in reality need about 3 sec [Sparman.2lane].

More specifically, the lane changing algorithm is an implementation of the following:

In *even* time steps, perform lane changes from right to left.[4] All vehicles on the right lane for which the ReasonToChangeLanes ($v_r \leq v$ $.OR.$ $v_l \leq v$) and the Security Criterion ($-v_{max} \ldots v$) are fulfilled are simultaneously moved to the left.

In *odd* time steps, perform lane changes from left to right. All vehicles on the left lane for which the ReasonToChangeLanges ($v_r > v$ $.AND.$ $v_l > v$) and the Security Criterion ($-v_{max} \ldots v$) are fulfilled are simultaneously moved to the right.

The number of sites one looks ahead for the ReasonToChangeLanes plays a critical role. Quite obviously, if one looks far ahead, one has a tendency to go to the left lane already far away from an obstructing vehicle, thus leading to a strong density inversion at low densities. Thus, this parameter can be used to adjust the density inversion. — The results described below were obtained with a lookahead of 16 sites, that is, if no vehicle was detected in that range on that lane, the corresponding velocity v_r or v_l was set to ∞.

B. Forward movement

The vehicle movement rules (ii) are taken as the single lane rules from Nagel and Schreckenberg [Nagel.92.Schreckenberg] which are by now fairly well understood [Nagel.95.flow, Sasvari, Eisenblaetter].

For completeness, we mention the single lane rules here. They are

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[4]We separate changes from left to right and changes from right to left in anticipation of three lane traffic. In three lane traffic, in a simultaneous update it is possible that a vehicle from the left lane and a vehicle from the right lane want to go to the same cell in the middle lane. From a conceptual viewpoint of simulation, this may be called a scheduling conflict. Such conflicts can be resolved by, e.g., different update schedulings (such as here) [Barrett.???].

- IF ($v < v_{max}$) THEN $v := v + 1$ (accelerate if you can)
- IF ($v > gap$) THEN $v := gap$ (slow down if you must)
- IF ($v \geq 1$) THEN WITH PROBABILITY p DO $v := v - 1$ (sometimes be not as fast as you can for no reason).

Throughout this paper, we use p equal to 0.25. All simulations were performed in a circle of length $L = 10\,000$. The maximum velocity is $v_{max} = 5$.

C. Results

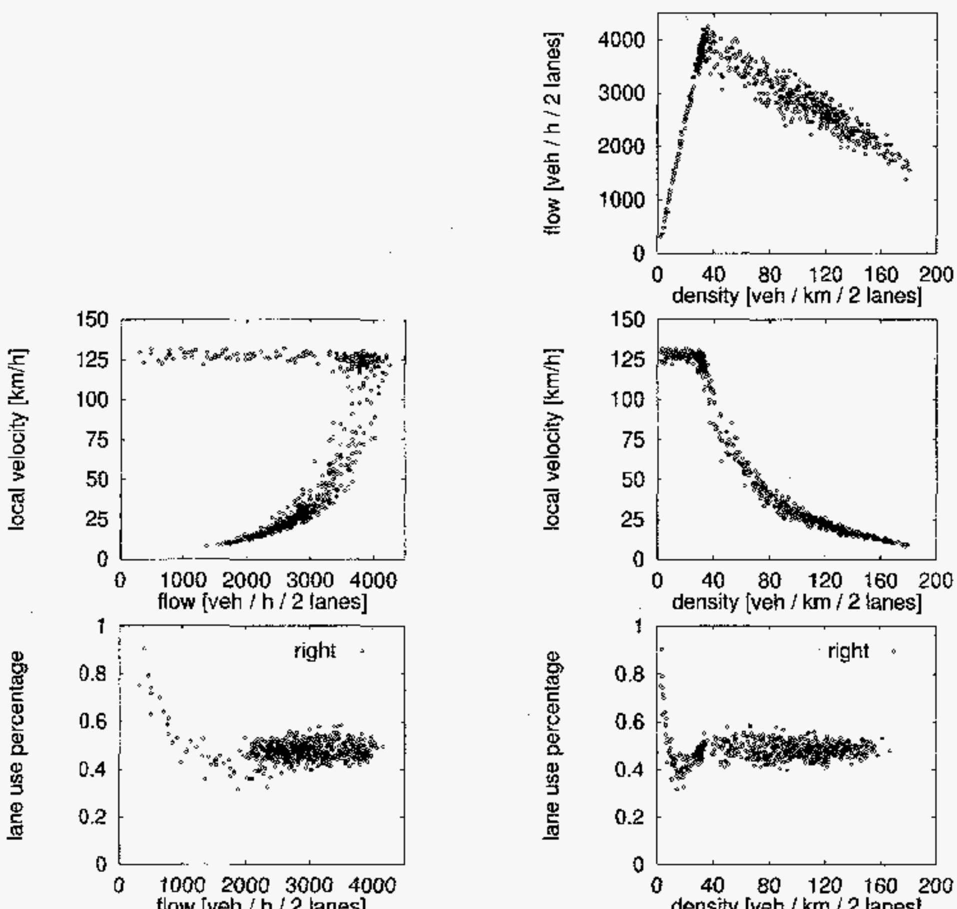


FIG. 2. Simulation results for basic version of the velocity-based lane changing rules. Same type of plots as in Fig. 1. Each data point is a one minute average, except for lane usage, where each data point is a three minute average.

As shown in Fig. 2, these rules generate reasonable relations between flow, density, and velocity. More importantly, they generate the density inversion below maximum flow which is a so important aspect of the dynamics on German freeways. Note that, maybe contrary to intuition, it is not necessary to have slow vehicles in the simulations in order to obtain the density inversion.

VI. COMPUTER SIMULATIONS OF GAP-RULES

For comparison, we also simulated a version of Wagner's "gap-rules" [Wagner:julich,Wagner:Nagel:Wolf], which is adapted to our classification schema above. The reason to change to the left then becomes

$$gap_r < v_{max} \ .OR. \ gap_l < v_{max} \ ,$$

i.e. one has a reason to change to the left when there is not enough space ahead either on the right or on the left lane.

As stated above, as reason to change to the right we take the negation, although we allow for some "slack" Δ:

$$gap_r \geq v_{max} + \Delta \ .AND. \ gap_l \geq v_{max} + \Delta \ ,$$

i.e. one changes from left to right if on both lanes there is enough space ahead.

The parameter Δ has been introduced in Ref. [Wagner:julich]. It denotes "slack" between the the two decisions, i.e. there are situations where the driver has no particular preference and then just stays on the lane where she already is. In these rules, Δ is used to adjust the degree of the lane inversion; note that the lookahead distance, d, played a similar rule in the basic velocity rules. We will use $\Delta = 9$, the same value as in Ref. [Wagner:Nagel:Wolf].

Note that this produces a conflicting rule set for the gap rules: The ReasonToChangeLeft says "go left if $gap_l < v + \Delta$", whereas the security criterion says "do *not* go left if $gap_l \leq v$". Using a $\Delta \leq 1$ would thus completely eliminate changes to the left due to this condition, i.e. only cars ahead on the right lane would trigger changes to the left. Quite in general, it can happen that a rule can fit into our logical scheme, but parts of the decision tree can never be reached so that parts of the rule can be omitted without changing anything.

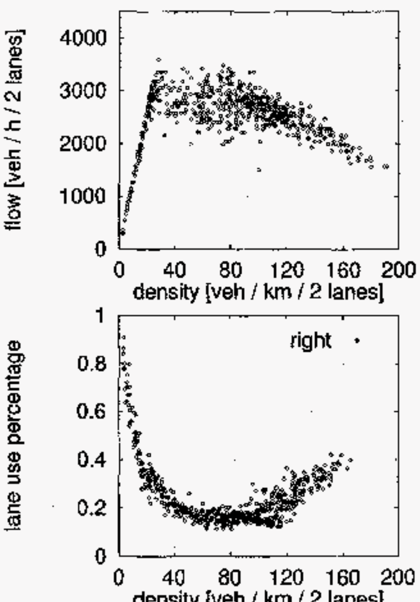


FIG. 3. Simulation results for gap-based lane changing rules. *Top:* Flow vs. density. *Bottom:* Lane usage vs. density.

Fig. 3 shows results of simulations with these rules. One immediately notes that these rules both qualitatively and quantitatively generate the correct density inversion at maximum flow, i.e. at $\rho \approx 38$ veh/km/2lanes; but from there on with further increasing density the density inversion increases further, contrary to reality. In order to compensate for this effect, Ref. [Wagner:Nagel:Wolf] introduced an additional symmetric rule which had its strongest effect at high densities.

VII. EXTENSIONS FOR REALITY

After having shown that both velocity-based and gap-based lane changing rules, based on the introduced logical scheme, can generate the density inversion effect, we now proceed to include more realism to bring the result closer to Wiedemann's data (Fig. 1).

A. Slack

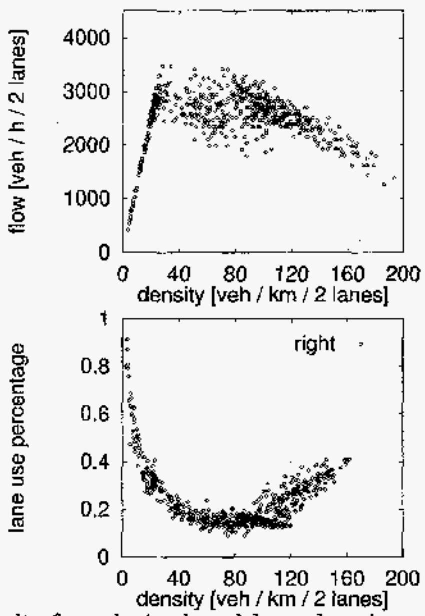


FIG. 4. Simulation results for velocity-based lane changing rules with slack (i.e. there is some "slack" between the ReasonToChangeLeft and the ReasonToChangeRight). *Top:* Flow vs. density. *Bottom:* Lane usage vs. density.

With the basic velocity-based rules, one can adjust the density inversion to the correct lane use percentage, but the maximum inversion is reached at too low densities (at approx. 16 veh/km/2lanes compared to approx. 28 veh/km/2lanes in reality). One possibility to improve this is to introduce some slack $\Delta = 3$ into the rules similar to the slack in the gap-based rules, i.e. vehicles change to the left according to the same rules as before, but the ReasonToChangeRight is not the exact inversion of this but relaxed. That is, the ReasonToChangeRight now reads

$$v_r > v + \Delta \ .AND.\ v_l > v + \Delta \ .$$

Since these rules tend to produce a stronger density inversion than before, we reduced the look ahead value to 7 to obtain realistic lane usage values. Results are shown in Fig. 4.

B. Slack plus symmetry at high densities/low velocities

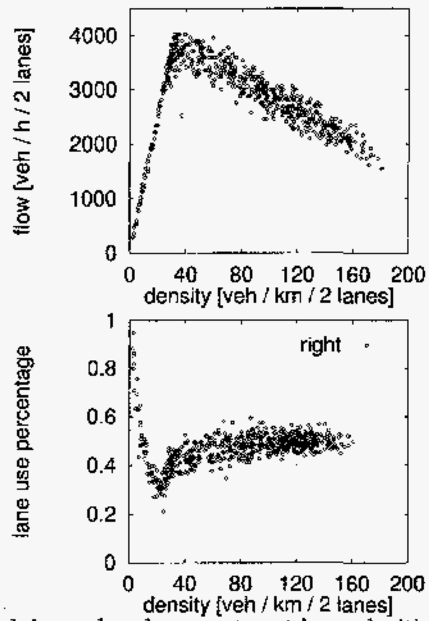
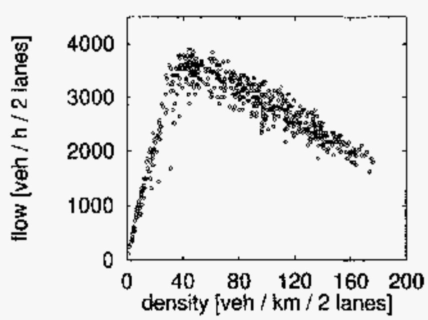


FIG. 5. Plots when slack is used and symmetry at low velocities included. *Top:* Flow vs. density. *Bottom:* Lane usage vs. density.

Including slack into the rules moved the density inversion to higher densities, but with the side effect that traffic never reverts to an equal lane usage, even at very high densities, similar to what we obtained with the gap-rules above. In order to improve this, we make the rule-set symmetric at zero speed. In technical terms, this means that a vehicle at speed zero only checks if the speed on the other lane is higher than on its own lane, and if so, attempts to change lanes (restricted by the security criterion). Other solutions are possible to achieve this (see, e.g., Ref. [Wagner:Nagel:Wolf]). Fig. 5 shows that our approach indeed works, i.e. the lane usage at high densities now goes indeed to approximately 50% for each lane.

C. Slow vehicles



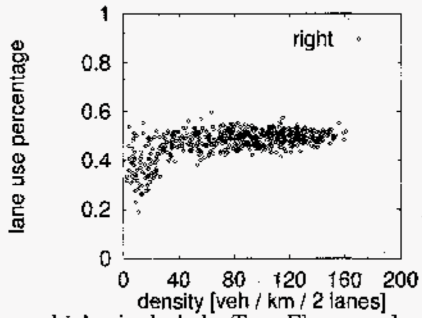
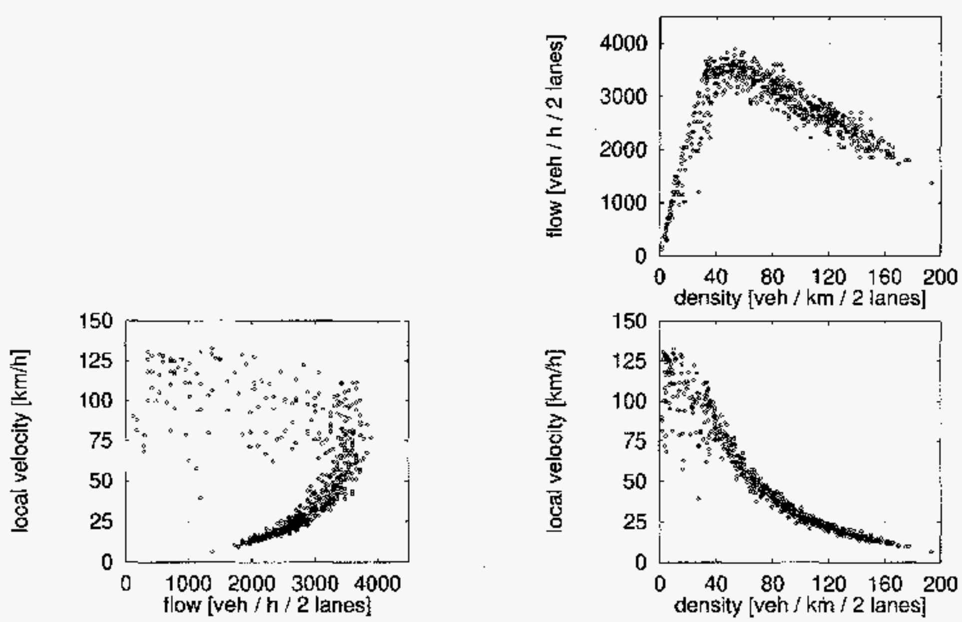


FIG. 6. Plots when slow vehicles included. *Top:* Flow vs. density. *Bottom:* Lane usage vs. density.

Wiedemann's data includes 10 percent trucks. We model the effect of trucks by giving 10 percent of the vehicles a lower maximum velocity [Rickert:masters,Latour:masters,Chowdhury:2lane]. Note that this only models the lower speed limit which is in effect for trucks in most European countries, but not the lower acceleration capabilities. The result for the flow-density curve and for the lane usage is shown in Fig. 6. The main difference to before is that the maximum flow is shifted towards higher densities, and there are more fluctuations in that region [Rickert:masters].

D. Combination of all extensions



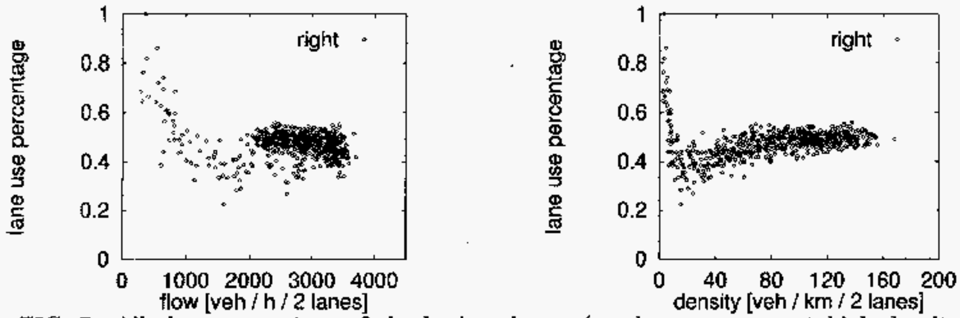


FIG. 7. All three extensions of the basic rule-set (trucks; symmetry at high density; slack) are included. Compare this figure to Fig. 1.

Last, we show simulation results where all the above improvements (trucks; symmetry at high densities; slack) are used simultaneously (Fig. 7). Indeed, the results are now close to reality (cf. Fig. 1).

VIII. THE FLOW BREAKDOWN MECHANISM NEAR MAXIMUM FLOW

One of the questions behind this research was to investigate if, in highly asymmetric two-lane systems, flow breakdown is indeed triggered by a single lane flow breakdown on the left lane. In order to address this question, we will, in the following, study space-time plots of the respective traffic dynamics as well as fundamental diagrams by lane. Since it turns out that traffic without slow vehicles is fundamentally different from traffic with slow vehicles, we will treat the two situations separately.

A. Maximum flow without slow vehicles

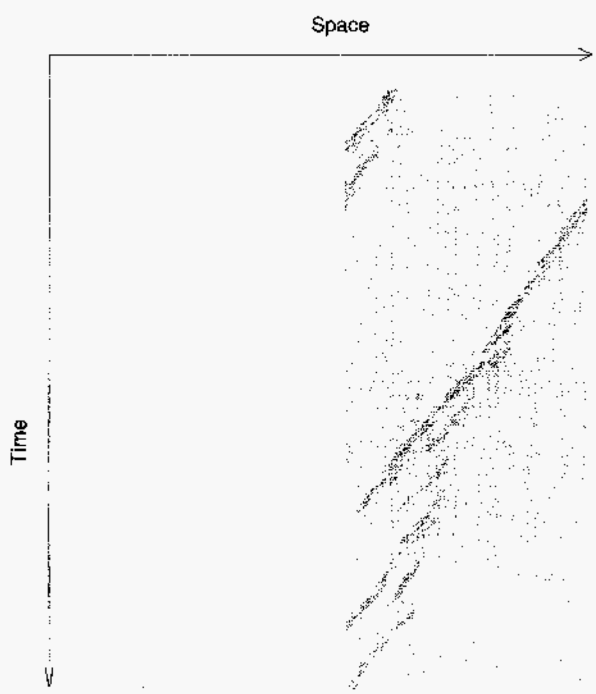


FIG. 8. Space-time plot of one-lane traffic without slow vehicles.

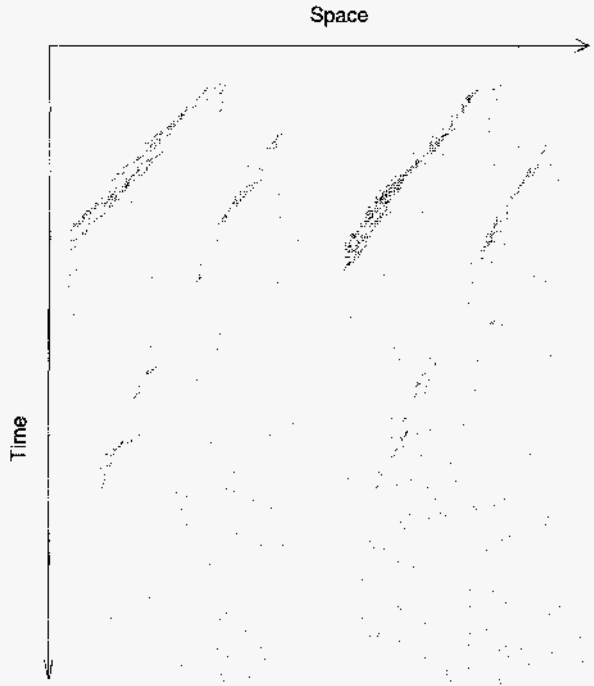
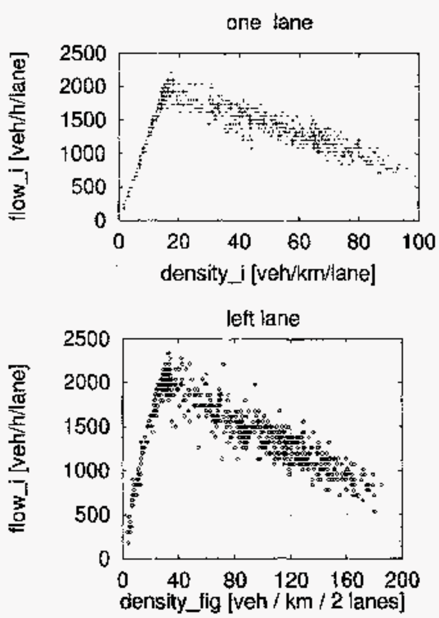


FIG. 9. Space-time plot of two-lane traffic with the "basic" lane changing rules without slow vehicles.



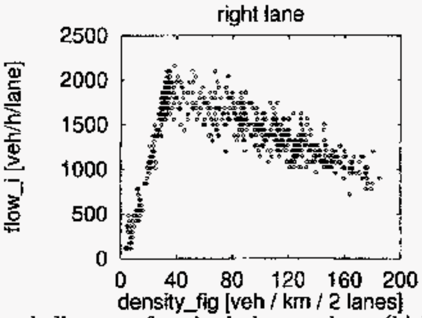


FIG. 10. *(a)* Fundamental diagram for single-lane rules. *(b)* Fundamental diagram for left lane of basic velocity two-lane rules, i.e. plotting flow on the left lane vs. density on both lanes for 1-minute averages. *(c)* Fundamental diagram for right lane of basic velocity two-lane rules.

Figs. 8 and 9 compare space-time plots from a one-lane situation with the two-lane situation using the "basic" velocity-based lane changing rules, in both cases approximately at maximum flow. Not much difference in the dynamics is detectable except that maybe the 2-lane plot is a bit more "noisy". This is confirmed by the single-lane fundamental diagrams for the systems (Fig. 10): The fundamental diagram for the left lane of basic velocity-based lane changing rules looks very similar to the corresponding 1-lane diagram, and also the right lane does not look much different. Also, the density inversion has reverted to 50:50 at maximum flow (Fig. 2).

Thus, the approach to maximum flow via increasing density is better described in the way that the left lane reaches maximum flow earlier than the right lane, and from then on all additional density is squeezed into the right lane. Only when the combined density of both lanes is above the maximum flow density, flow break-down happens. This argument gets confirmed by the observation that there are many measurement points near maximum flow in all fundamental diagrams, whereas at densities slightly higher than this significantly fewer data points exist. This should be compared to the situation which includes slower vehicles, which will be explained next.

B. Maximum flow with slow vehicles

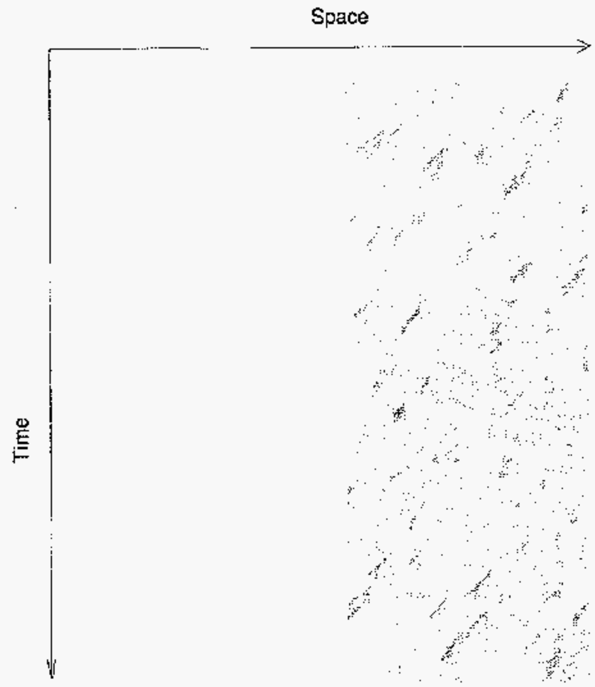


FIG. 11. Space-time plot of one-lane traffic near maximum flow including 10% slow vehicles.

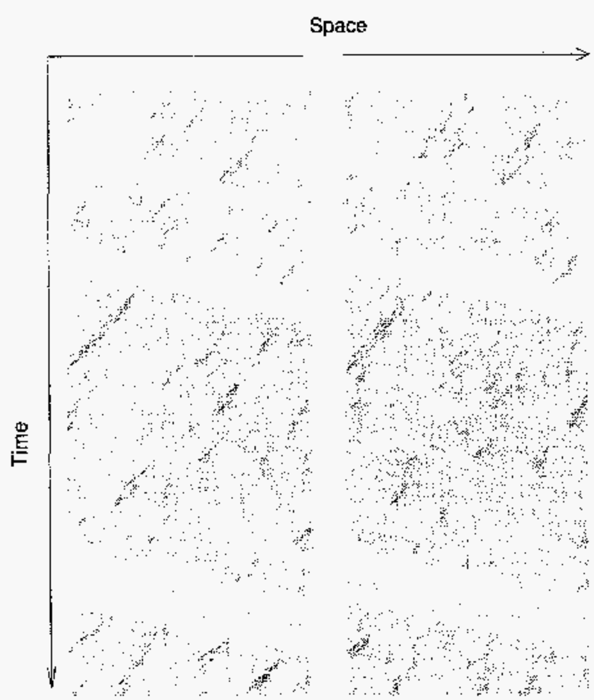


FIG. 12. Space-time plot of two-lane traffic near maximum flow including 10% slow vehicles using the "basic" velocity-based lane changing rules of this paper.

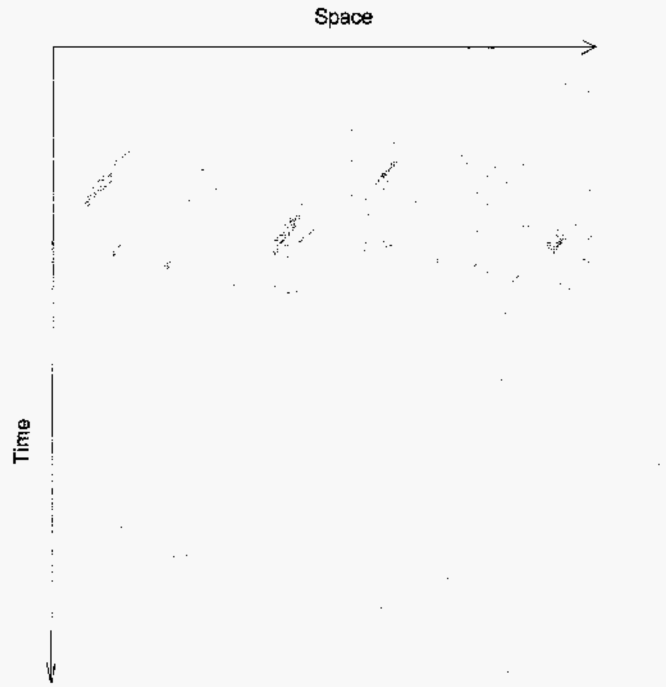


FIG. 13. Space-time plot of two-lane traffic at about half the density of maximum flow, including 10% slow vehicles, using the "basic" lane changing rules of this paper. Same as Fig. 12, except for the lower density.

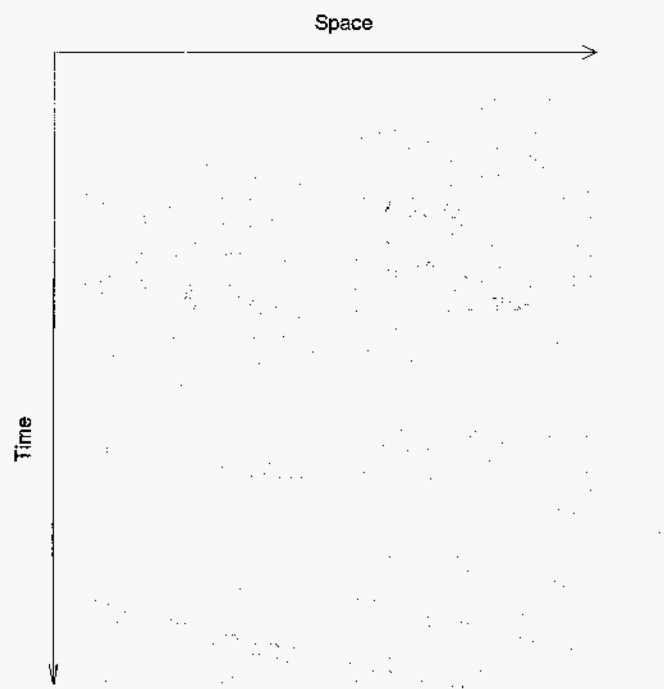


FIG. 14. Space-time plot of two-lane traffic at about half the density of maximum flow, including 10% slow vehicles, using the lane changing rules with slack and symmetrization.

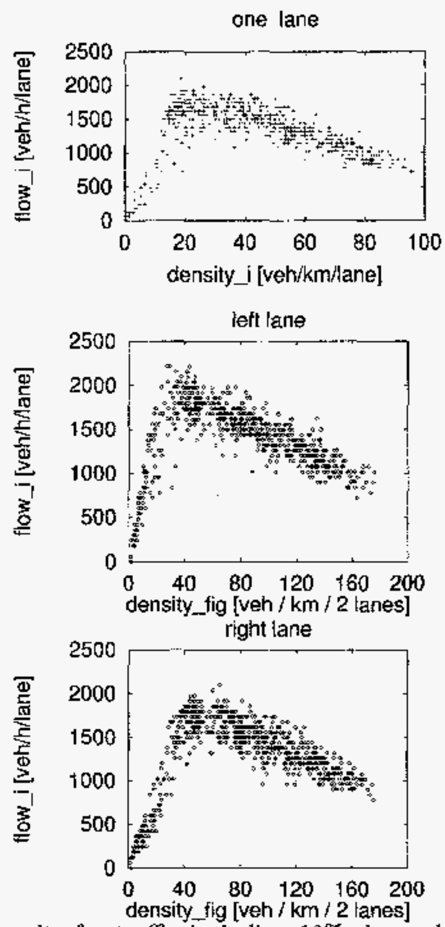


FIG. 15. Simulation results for traffic including 10% slow vehicles: *(a)* Fundamental diagram for single-lane rules. *(b)* Fundamental diagram for left lane of basic velocity-based two-lane rules, i.e. plotting flow on the left lane vs. density on the left lane for 1-minute averages. *(c)* Fundamental diagram for right lane of basic velocity two-lane rules.

The situation when slow vehicles are present is markedly different. The 2-lane situation with slow vehicles (Fig. 12) looks more like the 1-lane situation with slow vehicles (Fig. 11) than like the 2-lane situation without slow vehicles (Fig. 9). That means: *The presence of slow vehicles has a stronger influence on the dynamics than the difference between 1-lane and 2-lane traffic.* The dominating feature is that fast vehicles jam up behind slow vehicles and get involved in start–stop dynamics which gets worse with increasing distance from the leading slow vehicle. In the 2-lane situation, these "plugs" are caused by two slow vehicles side by side; a situation which is empirically known to happen regularly.

For the "basic" lane changing rules, the queues behind the "plugs" have similar length on both lanes, both near the density of maximum flow (Fig. 12) and at lower densities (Fig. 13). In contrast, when using the lane changing rules with slack and symmetrization, then in the same situation, there are more vehicles behind the truck on the left than there are behind the truck on the right (Fig. 14). Since from personal experience the latter seems to dominate, this indicates that the more complicated rule-set is the more realistic one here.

The lane-based fundamental diagrams (Fig. 15) confirm the observation that slow vehicles change the dynamics. The marked peak and the accumulation of data points near maximum flow are both gone; maximum flow is found over a wider density range than before. The flow on the left lane generally reaches higher values both than flow on the right lane, and than single-lane traffic flow.

Space-time plots (Figs. 12 and 13) show why this is the case. Traffic in this situation is composed of two regimes:

- "Plugs" of slow vehicles side by side, and faster vehicles queued up behind them.
- "Free flow" regions, where the slow vehicles stay on the right and the fast vehicles are mostly on the left.

At low density, there are mostly free flow regions and a couple of "plugs" with queues behind them. With increasing density, the share of the free flow regions *de*creases while the share of the queueing regions *in*creases. Eventually, the free flow regions get "eaten up" by the queueing regions, a 2-lane variant of the mechanism described in Ref. [Krug:Ferrari].

From visual inspection, it is clear that up to that density (approx. 40 veh/km/2 lanes) the left lane carries a higher flow since it only has fast cars in the free flow regions. Above this density, it is clear that now also the slow vehicles get slowed down by the end of the queue ahead of them. This argument for itself is not strong enough to guarantee maximum flow at this density; indeed, in Ref. [Krug:Ferrari] maximum flow occurs at a higher density. Yet, for our 2-lane traffic simulation maximum flow clearly occurs near this density, indicating that, analogous to the 1-lane situation [Nagel:flow], the parallel update in this paper is different from the random sequential update of Ref. [Krug:Ferrari].

IX. DISCUSSION

(i) In spite of widespread efforts, many earlier models were *not* able to reproduce the lane inversion. Why is that so? The reason is that the lane inversion is not a simple rate effect (more changes to left than to right), but a spatial correlation effect: "I stay on the left if there is somebody ahead on the left." Indeed, some of the earlier models [Schuett,Latour] do not contain this crucial rule. Sparmann [Sparmann] contains it but still does not reproduce the density inversion; so one would speculate that the weight for this rule was not high enough.

(ii) Real-world traffic seems to be more stable in the laminar regime than our simulated two-lane traffic. This can be seen in the "overshoot" (hysteresis, see Ref. [Treiterer:hysteresis]) of the low-density branch of the flow-density-plot which is more pronounced in reality than in the results of this paper. The single-lane model [Nagel:Schreckenberg] looked more realistic here. Yet, recent research shows that the hysteresis effect is actually related to the structure of the braking rules of the single-lane velocity rules [Krauss:etc:metastable, Krauss:Kreta]. More precisely: In models with more refined braking rules the laminar traffic does not break down that easily because small disturbances can be handled by small velocity adjustments.

In this context, it should be stressed that, as mentioned above, our plots actually show three minute averages for the lane usage plots whereas all other plots are generated from one minute averages. The reason for this is that one minute averages for lane usage had so much variance that the overall structure was not visible. Yet, in reality one minute averages are sufficient also for this quantity. This indicates that our models have, for a given two-lane density, a higher variation in the lane usage than reality has. — Also, the plots of velocity vs. flow indicate that the range of possible velocities for a given flow is wider in the simulations than in reality, again indicating that for a given regime, our model accepts a wider range of dynamic solutions than reality.

(iii) The fact that we needed space-time plots for resolving many of the dynamical questions indicates that the methodology of plotting short time averages for density, flow, and velocity, has shortcomings. The reason has been clearly pointed out in recent research [Kerner.Konh, Nagel.96.flow, Bando, Konh.complexity]: Traffic operates in distinctively different dynamic regimes, two of them being laminar traffic and jammed traffic. Averaging across time means that often this average will, say, contain some dynamics from the laminar regime and some dynamics from the jammed regime, thus leading to a data point at some intermediate density and flow, which though *dynamically* does not exist.

In transportation science, it seems that this problem is empirically known because people are using shorter and shorter time averages (1-min averages instead of 5-min averages used a couple of years ago or 15-min averages used ten or more years ago). It seems that one should try vehicle based quantities. Plotting $v/\Delta x$ as a function of $1/\Delta x$, where Δx is the front-bumper to front-bumper distance between two vehicles, is still a flow-density plot, but now individualized for vehicles. Instead of just plotting data point clouds, one would now have to plot the full distribution (i.e. displaying the number of "hits" for each flow-density value).

X. OTHER TWO-LANE MODELS

It is possible to review earlier lane changing models in the view of the scheme presented in this paper. In general, classifying some of the earlier rules into our scheme is sometimes difficult, but usually possible. For example, when one uses

$$gap_r < v_{max} \ .OR.\ gap_l < v_{max}$$

as a reason to change to the left, then the negation of that, i.e.

$$gap_r \geq v_{max} + \Delta \ .AND.\ gap_l \geq v_{max} + \Delta$$

would be the reason to change to the right (where Δ denotes "slack" in the negation). Let us also use a security criterion as follows:

$$gap_- = v_{back} + 1$$

(i.e. the distance to the car behind on the other lane should be larger than its velocity) and

$$gap_+ = \min[gap + 1, v_{max}]$$

(i.e. the distance to the car ahead on the target lane should be larger than either (i) the distance to the car ahead on the current lane, or (ii) the maximum velocity). With the exception of the addition of the second part of the ReasonToChangeToLeft, these are exactly the same rules as used in Ref. [Wagner.Nagel.Wolf].

Note, though, that this is not exactly easy to see. For example, the ReasonToChangeLeft "$gap_l > gap_r$" of Ref. [Wagner:Nagel:Wolf] is now in the security criterion. Also, for changes from left to right, the forward part of the security criterion could be left out, since $gap_r \geq v + \Delta$ from the ReasonToChangeLanes is a stronger criterion, at least for the values of Δ which have been used.

Indeed, many asymmetric lane changing rules investigated in the literature can be viewed through our characterization. Table I contains many asymmetric lane changing rules from the traffic cellular automaton literature. The underlined parts have been added to make the rules completely fit into our scheme, i.e. to make the reason to change to the right the logical negation (sometimes including "slack") of the reason to change to the left. It would be interesting to test if the neglected part of the rules would be used often or not if they were actually implemented.

XI. SUMMARY

This paper classifies the multitude of possible lane changing rules for freeway traffic. The first part of this follows Sparmann [Sparmann.2lane]: One can separate the rules into the "reason to change lanes" and a security critierion, which asks if there is enough space available on the target lane.

The second part of this is the observation that in countries with a default lane and a passing lane, the ReasonToChangeRight is just the logical negation of the ReasonToChangeLeft, with possibly some slack (inertia).

The security criterion seems to be universal for all reasonable lane changing rules: $gap_- \dots gap_+$ has to be empty on the target lane; the exact values of the parameters gap_- and gap_+ do not seem to matter too much as long as they are reasonably large. We used $gap_- = v_{max}$ and $gap_+ = v$.

For the ReasonToChangeLanes criterion we argue that its general structure for highly asymmetric traffic has to be "change to the left when either on your lane or on the left lane somebody is obstructing you", and "change back when this is no longer true". Since this usually leads to a generic density inversion at high densities, one has to add a symmetrizing rule for high density traffic. We simply used a symmetric ReasonToChangeLanes for vehicles with velocity zero.

Both velocity and gap based implementations of this give satisfying results.

Further, we showed that most asymmetric lane changing models in the physics literature fit into this scheme.

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APPENDIX A: TRANSFORMATION OF THE WAGNER'S RULES IN REF. [WAGNER:JULICH]

Finding a correspondance for the rules of Wagner in Ref. [Wagner.julich] is not straightforward. However, at closer inspection, the rules turn out to be inconsistent for certain choices of parameters. The forward part of the ReasonToChangeLanes is:

$$R \to L : gap_r < v_{max} \ .AND. \ gap_l > gap_r$$

$$L \to R : gap_r > v + \Delta' \ .AND. \ gap_l > v + \Delta'$$

Assume for example a case where $gap_r = 3$, $gap_l = 4$, $v = 0$, $v_{max} \geq 4$, and $\Delta = 0$. Then the vehicle does not want to be in either lane.

This problem gets resolved for $\Delta' \geq v_{max} - 1$; and indeed $\Delta' \geq 6$ was used.

Now, if one assumes $\Delta' \geq v_{max} - 1$, then one can simplify the rule-set. One can move the condition $gap_l > gap_r$ into the security criterion $gap_o \geq \min[gap + 1, v_{max}]$, and the remaining reasons to change lanes are:

$$R \to L : gap_r < v_{max} \ \underline{.OR. \ gap_l < v_{max}}$$

$$L \to R : gap_r \geq v_{max} + \Delta(v) \ .AND. \ gap_l \geq v_{max} + \Delta(v) \ ,$$

where, as in Table I, the underlined part is added to make the rule fit into the scheme. Note that in this interpretation, the slack now is $\Delta(v) = \Delta' + v$, i.e. a function of the velocity.

Schütt [Schuett]: $R \to L$: $v_r < v$ $(1, \dots, l_l = 9)$ $L \to R$: $v_r \geq v$ $(1, \dots, l_r = 15)$	$-4, \dots, 5 = v_{max}$
Rickert asym1 Master thesis [Rickert:masters]: $R \to L$: $gap_r < \min[v+1, v_{max}]$ $.OR.$ $gap_l \leq 2 \min[v+1, v_{max}]$ $L \to R$: $gap_r \geq \min[v+1, v_{max}]$ $.AND.$ $gap_l > 2 \min[v+1, v_{max}]$	$-v_{max}, \dots, \min[v+1, v_{max}]$
Rickert asym2 Master thesis [Rickert:masters]: $R \to L$: $gap_r < \min[v+1, v_{max}]$ $.OR.$ $gap_l \leq 2v_{max}$ $.OR.$ $v < v_{max} - 1$ $L \to R$: $gap_r \geq \min[v+1, v_{max}]$ $.AND.$ $gap_l > 2v_{max}$ $.AND.$ $v \geq v_{max} - 1$	$-v_{max}, \dots, \min[v+1, v_{max}]$
Latour 1 Master thesis [Latour:masters]: $R \to L$: $gap_r < v$ $L \to R$: $gap_r \geq v$	0
Latour 2 Master thesis [Latour:masters]: $R \to L$: $gap_r < f(v)$ $.AND.$ $gap_l > gap_r$, $f(v) = v, v+1, v+2, v_{max}$ $L \to R$: $gap_r \geq f(v)$ $.OR.$ $gap_l \leq gap_r$	a) 0 b) $-2, \dots, 0$
Rickert et al Physica A [Rickert:etc:twolane]: $R \to L$: $gap_r < v+1$ $L \to R$: $gap_r \geq v+1$	$-(v_{max}+1), \dots, v+1$
Wagner Jülich original [Wagner:julich]: $R \to L$: $gap_r < v_{max}$ $.AND.$ $gap_l > gap_r$ $L \to R$: $gap_r \geq v + \Delta'$ $.AND.$ $gap_l \geq v + \Delta'$	$-(v_{back}+1), \dots, 0$
Wagner Jülich transformed:[‡] $R \to L$: $gap_r < v_{max}$ $.OR.$ $gap_l < v_{max}$ $L \to R$: $gap_r \geq v_{max} + \Delta(v)$ $.AND.$ $gap_l \geq v_{max} + \Delta(v)$; $\Delta(v) = \Delta' - v_{max} + v$	$-(v_{back}+1), \dots, \min[gap+1, v_{max}]$
Wagner et al Physica A original [Wagner:etc:twolane]: $R \to L$: $gap_r < v_{max}$ $.AND.$ $gap_l \geq gap_r$ $L \to R$: $gap_r > v_{max} + \Delta'$ $.AND.$ $gap_l > v_{max} + \Delta'$	$-v_{max}, \dots, 0$
Wagner et al Physica A transformed: $R \to L$: $gap_r < v_{max}$ $.OR.$ $gap_l < v_{max}$ $L \to R$: $gap_r \geq v_{max} + \Delta$ $.AND.$ $gap_l \geq v_{max} + \Delta$; $\Delta = \Delta' + 1$	$-v_{max}, \dots, \min[gap, v_{max}]$
Chowdhury et al Physica A [Chowdhury:etc:twolane]: $R \to L$: $gap_r < v$ $.OR.$ $v_d > v_{d,ahead}$ (distance v_{fast}) $L \to R$: $gap_r > v$ $.AND.$ $v_d \leq v_{d,ahead}$ (distance v_{fast})	$-v_{max}, \dots, gap+1$
This paper (velocity): $R \to L$: $v_r \leq v$ $.OR.$ $v_l \leq v$ $L \to R$: $v_r > v + \Delta$ $.AND.$ $v_l > v + \Delta$	$-v_{max}, \dots, v$
This paper (gap): $R \to L$: $gap_r < v_{max}$ $.OR.$ $gap_l < v_{max}$ $L \to R$: $gap_r \geq v_{max} + \Delta$ $.AND.$ $gap_l \geq v_{max} + \Delta$	$-v_{max}, \dots, v$

TABLE I. Lane changing rules in the literature. The left column gives the "reasons to change lane" for the indicated lane change right to left ($R \to L$) or left to right ($L \to R$). The right colomn gives the security criterion, i.e. the sites on the target lane that need to be empty. Remarks: ‡ See appendix of this paper.